Nuclear spin-orbit splitting from an intermediate Δ excitation

Kohichi Ohta,* Tokuo Terasawa, and Mitsuru Tohyama
Institute of Physics, University of Tokyo, Komaba, Meguro-ku, Tokyo 153, Japan

(Received 2 July 1980)

The strength of the single particle spin-orbit potential is calculated from the two pion exchange box diagrams involving an intermediate Δ(1232) resonance excitation by taking account of the exclusion principle for the intermediate nucleon states. The effect of the ρ meson is also considered. The predicted strength is found to account for a substantial part of the empirical spin-orbit splittings.

[NUCLEAR STRUCTURE Calculated spin-orbit splittings; effect of an intermediate Δ excitation.]

The problem of finding the origin of the spin-orbit coupling in the nuclear shell model is an old one, and various processes have been examined in many investigations.[1-6] Recently it has been shown[4] that the spin-orbit splittings in ^{16}O and ^{40}Ca can reasonably be understood through the combined effect of the effective nucleon-nucleon (*NN*) spin-orbit interaction in first order and the effective *NN* tensor interaction in second order. Quantitatively, however, it does not seem to be conclusive as yet. Furthermore, there remain large discrepancies between theory and experiment for splittings in nuclei in which the $j=l+\frac{1}{2}$ subshell is filled but the $j=l-\frac{1}{2}$ subshell is empty.[5,6] Therefore, it is worthwhile to look for any other sources of the nuclear spin-orbit splittings in the light of present knowledge about the *NN* interaction.

Recent work has indeed brought about a deeper understanding of the *NN* force in a field theoretic way. Among others, two-pion exchange processes containing one nucleon and one Δ(1232) isobar in intermediate states have been found[7] to generate the attractive force at intermediate distances which was accounted for by a fictitious scalar meson in one-boson exchange models. Within the framework of the coupled-channel formalism,[7] the Δ excitation is described by a local transition potential which is dominantly tensor. Therefore we notice at once that the transition potential will also give a spin-orbit coupling in second-order perturbation theory in a way quite similar to the ordinary tensor force.[1-4] In the latter case the main contribution comes from the exclusion principle effect in intermediate states. The purpose of this paper is to show that a considerable part of the observed splittings may be obtained from the two-pion exchange via a virtual *N*Δ excitation if we take into account the exclusion principle for intermediate nucleon states.

Consider the Feynman diagrams in Fig. 1. They involve the integration

$$(i/2\pi)\int_{-\infty}^{\infty} dQ_{10}(m-Q_{10}-i\epsilon)^{-1}(\omega_k^{\,2}-k_0^{\,2}-i\epsilon)^{-1} \times(\omega_q^{\,2}-q_0^{\,2}-i\epsilon)^{-1}(m^*-Q_{20}-i\epsilon)^{-1}\,, \quad (1)$$

where $\omega_k=(k^2+\mu^2)^{1/2}$, m, m^*, and μ are the masses of the nucleon, Δ, and pion, respectively, and we have used the static forms for the nucleon and Δ propagators. Energy conservation at each vertex implies that $k_0=q_0=m-Q_{10}$ and that $Q_{20}=m+k_0$ for the direct box diagram (a) and $Q_{20}=m-k_0$ for the crossed box diagram (b). We have taken the static limit for all the external nucleons. Carrying out the Q_{10} integration, we get the result which is equal to the sum of the twelve time-ordered diagrams. Among them only four diagrams having an intermediate state without pions are included in the coupled-channel calculation. In the present case the remaining diagrams will be found to be equally important.

A large nucleus may be approximated by an ideal Fermi gas. In this model, the nucleon propagator in Eq. (1) is replaced by

$$\frac{1-n(Q_1)}{m-Q_{10}-i\epsilon}+\frac{n(Q_1)}{m-Q_{10}+i\epsilon} =(m-Q_{10}-i\epsilon)^{-1}-2\pi i n(Q_1)\delta(m-Q_{10}), \quad (2)$$

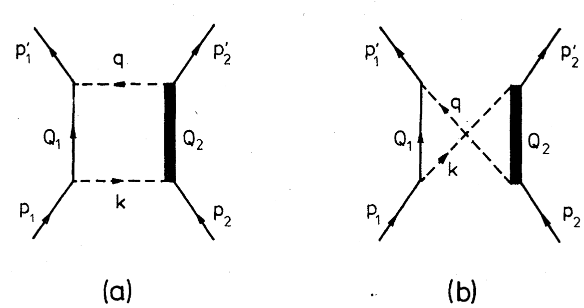

FIG. 1. Direct (a) and crossed (b) two-pion exchange diagrams involving *N*Δ intermediate states. The Δ resonance is indicated by a heavy solid line.

where the occupation number $n(Q_1)$ is one for $Q_1 \leq p_F$ and zero for $Q_1 > p_F$ ($p_F = 1.36\ \mathrm{fm}^{-1}$ is the Fermi momentum of normal nuclear matter). The term proportional to $n(Q_1)$ in the propagator represents the exclusion effect which entails a modification of the NN interaction in the nucleus. When inserted into Eq. (1), this term yields $n(Q_1)/\omega_k{}^2\omega_q{}^2\Delta m$ both for the direct and crossed diagrams. Here Δm is the $N-\Delta$ mass difference. Thus the direct and crossed diagrams contribute the same amount to the effective interaction in the nuclear medium, at least for static nucleons.

For two free nucleons, the amplitude for Fig. 1 is a function of the momentum transfer $\vec{\Delta}(=\vec{p}_1 - \vec{p}_1' = \vec{p}_2' - \vec{p}_2 = \vec{k} - \vec{q})$ alone in static theory. In the nucleus, however, the exclusion principle requires the correction term $V(\vec{\Delta})$ which has additional dependence on $\vec{p}_1$ through $n(Q_1) = n(|\vec{p}_1 - \vec{k}|)$. In coordinate space, $V(\vec{\Delta})$ is transformed into $V(\vec{r} = \vec{r}_1 - \vec{r}_2)$ and $\vec{p}_1$ becomes an operator acting on ket wave functions. Collecting all the necessary factors, we find

$$V(\vec{r}) = \alpha^2 \frac{\mu^2}{\Delta m} \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{r} + i\vec{q}\cdot\vec{r}} n(|\vec{p}_1 - \vec{k}|) k^{-2} w(k) q^{-2} w(q) \tau_{1i} \tau_{1j} \vec{\sigma}_1 \cdot \vec{q}\, \vec{\sigma}_1 \cdot \vec{k} \times (T_{2i}^\dagger T_{2j} \vec{S}_2^\dagger \cdot \vec{q}\, \vec{S}_2 \cdot \vec{k} + T_{2j}^\dagger T_{2i} \vec{S}_2^\dagger \cdot \vec{k}\, \vec{S}_2 \cdot \vec{q}), \quad (3)$$

where $\vec{\tau}_1$ and $\vec{\sigma}_1$ are the isospin and spin operators of particle 1, $\vec{T}_2$ and $\vec{S}_2$ are the transition operators transforming particle 2 from N to Δ, and $\alpha = ff^*/4\pi$ (f and f^* are the πNN and $\pi N\Delta$ coupling constants). In Eq. (3), we have used the notation

$$k^{-2} w(k) = 4\pi \mu^{-3} \omega_k{}^{-2} F^2(k^2) \quad (4)$$

for the regularized static pion propagator. The form factors $F^2(k^2)$ come from each end of the propagator. The function $w(k)$ is related to the radial factor $u_T(r)$ of the tensor part of the transition potential through

$$w(k) = 4\pi \int_0^\infty dr\, r^2 j_2(kr) u_T(r) \quad (5)$$

with j_2 being the spherical Bessel function of order 2.

The spin-dependent term of $V(\vec{r})$ is cast in the form

$$V(\vec{r}) = \tfrac{8}{3} i\alpha^2 \frac{\mu^2}{\Delta m} \int \frac{d^3k}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} e^{-i\vec{k}\cdot\vec{r} + i\vec{q}\cdot\vec{r}}\, n(|\vec{p}_1 - \vec{k}|)\, \vec{q}\cdot\vec{k}\, \vec{\sigma}_1 \cdot \vec{q} \times \vec{k}\, k^{-2} w(k)\, q^{-2} w(q) . \quad (6)$$

As we are considering the effective interaction of particle 1 with all others in the spin-saturated closed shells, we have dropped the $\vec{\sigma}_2$ term which will vanish after the spin average. The isospin-dependent term is also neglected, since it does not constitute the major contribution to the spin-orbit splittings.

In a Thomas-Fermi type approximation for the single-particle density matrix, the one-body spin-orbit force is given by[8]

$$V_{ls}(\vec{r}_1) = \int d^3r_2\, \rho(\vec{r}_2) V(\vec{r}_1 - \vec{r}_2) . \quad (7)$$

Following the standard procedure,[3] we expand the nuclear density $\rho(\vec{r}_2)$ around $\vec{r}_1$ and integrate over $\vec{r}_2$ to obtain

$$V_{ls}(\vec{r}_1) = \tfrac{1}{2} \kappa \vec{\sigma}_1 \cdot \vec{r}_1 \times \vec{p}_1 \frac{1}{r_1} \frac{d\rho(r_1)}{dr_1} . \quad (8)$$

The strength of the spin-orbit force is given by

$$\kappa = \frac{16\alpha^2}{3\Delta m} \frac{\mu^2}{p_1^2} \int_{|\vec{p}_1 - \vec{k}| \leq p_F} \frac{d^3k}{(2\pi)^3} \vec{p}_1 \cdot \vec{k}\, k^{-2} w^2(k) . \quad (9)$$

As κ is a function of $p_1{}^2$, it becomes a complicated operator in the nuclear configuration space. For a nucleon just above the last closed shell, however, one may replace p_1 by the Fermi momentum p_F. Our final expression is

$$\kappa = \frac{2\alpha^2}{3\pi^2 p_F} \frac{\mu^2}{\Delta m} \int_0^{2p_F} dk\, k \left[1 - \left(\frac{k}{2p_F}\right)^2\right] w^2(k) . \quad (10)$$

For the numerical computation of Eq. (10), we need the pion form factor $F(k^2)$. We use the monopole form factor $F(k^2) = (\Lambda^2 - \mu^2)/(\Lambda^2 + k^2)$ with $\Lambda^2 = 72\,\mu^2$ (Ref. 10). The $F(k^2)$ is normalized to unity at $k^2 = -\mu^2$ and the corresponding coupling

TABLE I. Strength of the spin-orbit force calculated from the two-pion exchange processes.

	V_{ls} (MeV) Static	V_{ls} (MeV) Nonstatic
$f^{*2}/4\pi = 0.36$	13.2	11.2
$f^*(0)^2/4\pi = 0.246$	8.5	7.2

constants are given by $f^2/4\pi = 0.08$ and $f^{*2}/4\pi = 0.36$. In the first column of Table I we give the resulting value for $V_{ls} \equiv \kappa\rho_0/r_0^2$ where $\rho_0 = 0.17$ fm^{-3} is the nuclear density at the center and $r_0 = 1.1$ fm.

The above value for f^* is calculated from the Δ width in lowest order perturbation theory and is known to be too large compared with the quark model prediction, $f^{*2}/4\pi = 0.23$. We also calculate V_{ls} using the form $\Lambda^2/(\Lambda^2+k^2)$ for $F(k^2)$ which is normalized at the soft-pion limit $k^2=0$. The coupling constants at the unphysical point $k^2=0$ are estimated from the Goldberger-Treiman relation $f(0)=0.942f$ (Ref. 11) and from the dispersion relation[12]

$$\frac{f^*(0)^2}{4\pi\mu^2} = \left(\frac{2m^*}{m+m^*}\right)^2 \frac{3}{\pi}\int_m^\infty dW q^{-3}\sin^2\delta_{33}\,, \qquad (11)$$

where the integration is over the πN center-of-mass energy W, $q=(W^2-m^2)/2W$, and δ_{33} is the phase shift for the nucleon and zero-mass pion in the 3-3 channel. Direct numerical evaluation of the integral, using the experimental phase shift[13] and the method of Gerstein and Lee[12] for going off the pion mass shell, gives $f^*(0)^2/4\pi = 0.246$.[14] The result for V_{ls} is given in the second row of Table I.[15]

We have assumed all the nucleons and Δ to be static, but this approximation can easily be avoided. The factor $1/\Delta m$ in Eq. (9), when put into the $\vec{k}$ integration, is rewritten as the sum of the direct box diagram contribution

$$\frac{3}{8\pi p_F^3}\int_{p_2\le p_F} d^3p_2(\Delta m+\epsilon_{Q_1}+\epsilon_{Q_2}-\epsilon_{p_1}-\epsilon_{p_2})^{-1} \qquad (12)$$

and the crossed box diagram contribution

$$\frac{3}{8\pi p_F^3}\int_{p_2\le p_F} d^3p_2(\Delta m+\epsilon_{p_1}+\epsilon_{Q_2}-\epsilon_{Q_1}-\epsilon_{p_2})^{-1} \qquad (13)$$

with ϵ's for the kinetic energies of N and Δ. The results of direct computations are shown in the second column of Table I.

We next consider the effect of the ρ meson. When one or both of the two pions in Fig. 1 is replaced by a ρ meson, its contribution to V_{ls} is simply given by replacing $w(k)$ in Eq. (10) by $w(k)-(\mu_\rho\alpha_\rho/\mu\alpha)w_\rho(k)$, where $\alpha_\rho = f_\rho f_\rho^*/4\pi$, μ_ρ is the ρ-meson mass, and $w_\rho(k) = 4\pi\mu_\rho^{-3}k^2F_\rho^2(k^2)/(k^2+\mu_\rho^2)$. We employ the form factor $F_\rho(k^2)=\Lambda_\rho^2/(\Lambda_\rho^2+k^2)$ with $\Lambda_\rho = 1450$ MeV (Ref. 16) and the coupling constant $f_\rho = g_\rho(1+\kappa_\rho)\mu_\rho/2m$ with $g_\rho^2/4\pi = 0.52$ and $\kappa_\rho = 3.7$ from the vector-dominance model, while the $\rho N\Delta$ coupling constant $f_\rho^* = (f_\rho/f)f^*$ from the quark model.[17] In column (I) of Table II, we show

TABLE II. Predictions of V_{ls} including the ρ-meson contributions for (I) $g_\rho^2/4\pi = 0.52$, $\kappa_\rho = 3.7$, $f_\rho^{*2}/4\pi = 5.7$, (II) $g_\rho^2/4\pi = 0.55$, $\kappa_\rho = 6.6$, $f_\rho^{*2}/4\pi = 15.6$, and (III) $f_\rho^2/4\pi = 1.3$, $f_\rho^{*2}/4\pi = 3.74$.

	V_{ls} (MeV) I	II	III
$f^{*2}/4\pi = 0.36$	9.3	4.4	10.5
$f^*(0)^2/4\pi = 0.246$	6.0	2.9	6.8

the predictions for V_{ls} including the ρ-meson effect. There is considerable cancellation between the π and ρ contributions.

Unfortunately, the vector coupling constant g_ρ and the ratio κ_ρ of the tensor to vector coupling have uncertainties. A dispersion theoretic analysis by Höhler and Pietarinen[18] gets $g_\rho^2/4\pi = 0.55$ and $\kappa_\rho = 6.6$. With these values two thirds of the contribution are canceled by the ρ contribution, as shown in column (II) of Table II. On the other hand Kisslinger[19] deduced $f_\rho^{*2}/4\pi = 3.74$ from a Regge pole analysis which gives accurate fits to $\pi N \to \pi\Delta$. With this small value, the ρ contribution is much diminished, as seen from column (III) of Table II.

Recently Grein and Kroll[20] have suggested that the small value of κ_ρ found in many investigations of NN scattering is related to the neglect of the 3π discontinuity of the $N\bar{N}$ cut. When they allow a normalization of the P-wave $N\bar{N}\to\pi\pi$ amplitude to fit the discrepancy function which represents all the information available from experiment on the discontinuities, they find $\kappa_\rho = 3.5$–4.0 instead of 6.0–6.1. This means that the choice of the ρ-meson coupling constants depends on the model used for constructing the NN interaction. The reaction we have considered, $N\bar{N}\to\pi\rho$ by N and Δ exchange, constitutes a model for the 3π cut. It is not evident *a priori* that the ρ-meson coupling constants to be used in this reaction are the same as in the 2π channel, since they may reflect the neglect of 4π exchange. We cannot therefore determine which values of κ_ρ should be adopted for the $N\bar{N}\to\pi\rho$ process unless a consistent fit to NN scattering is made.

Seeing that the effect of the ρ meson on V_{ls} is sensitive to the coupling constants, one may think that short-range dynamical correlations should have a great influence on V_{ls} as ρ dominates at short distances. Actually, the result is insensitive to the short-range correlations since the integration in Eq. (10) is limited to $k \le 2p_F$ and therefore $w(k)$ at large k [i.e., $u_T(r)$ at short distances] does not contribute to V_{ls}. To see this more explicitly, we multiply $u_T(r)$ in Eq. (5) by the correlation function $1-e^{-r^2/b^2}$ with

$b = 0.4$ fm and compute V_{ls} to find that this effect is 5% at most for case (II). The reason for this is that since the r^{-3} singularity of $u_T(r)$ is moderated to r by the inclusion of the regulators $F(k^2)$ and $F_\rho(k^2)$, the integrand in Eq. (5) behaves like $\sim r^4 u_T(r) \sim r^5$ for small r so that the short range part is effectively cut off without the correlation function and thus V_{ls} is little influenced by it.

Allowing for ambiguities, the combined effect of the Δ excitation and the exclusion principle amounts to $V_{ls} = 3-10$ MeV, which should be compared to the observed spin-orbit coupling strength ≈ 17 MeV.[9] Consequently it may be concluded that the present effect must be included in a quantitative calculation of the spin-orbit splittings. It would be quite interesting to examine the relative importance of the three contributions from the effective NN spin-orbit and tensor forces and the Δ effect we have considered. It is also desired that all these effects on V_{ls} should be calculated consistently with the construction of field theoretic nuclear forces.

The authors wish to thank Professor M. Ichimura, Professor L. S. Kisslinger, and Professor N. Onishi for interesting discussions.

---

*Present address: Center for Theoretical Physics, Laboratory for Nuclear Science and Department of Physics, Massachusetts Institute of Technology, Cambridge, Massachusetts 02139.

[1]T. Terasawa, Prog. Theor. Phys. 23, 87 (1960), and references cited therein; A. Arima and T. Terasawa, *ibid.* 23, 115 (1960).

[2]P. Goldhammer, Rev. Mod. Phys. 35, 40 (1963).

[3]T. T. Kuo and G. E. Brown, Nucl. Phys. 85, 40 (1966).

[4]R. R. Scheerbaum, Phys. Lett. 61B, 151 (1976).

[5]C. W. Wong, Nucl. Phys. A108, 481 (1968).

[6]R. R. Scheerbaum, Phys. Lett. 63B, 381 (1976); A. L. Goodman and J. Borysowicz, Nucl. Phys. A295, 333 (1978).

[7]H. Sugawara and F. von Hippel, Phys. Rev. 172, 1764 (1968); D. O. Riska and G. E. Brown, Nucl. Phys. A153, 8 (1970); for a recent review, see A. M. Green, Rep. Prog. Phys. 39, 1109 (1976), and in *Mesons in Nuclei*, edited by M. Rho and D. Wilkinson (North-Holland, Amsterdam, 1979), Vol. 1, p. 227.

[8]The average of the effective interaction arising from the Δ excitation on particle 1 does not provide a source of the spin-orbit coupling of particle 1.

[9]A. Bohr and B. R. Mottelson, *Nuclear Structure*, (Benjamin, New York, 1975), Vol. 1, p. 238.

[10]J. W. Durso, A. D. Jackson, and B. J. Verwest, Nucl. Phys. A282, 404 (1977).

[11]H. F. Jones and M. D. Scadron, Phys. Rev. D 11, 174 (1975).

[12]I. S. Gerstein and B. W. Lee, Phys. Rev. 152, 1418 (1966); R. Gatto and G. Veneziano, Phys. Lett. 20, 439 (1966); H. J. Schnitzer, Phys. Rev. 158, 1471 (1967).

[13]A. A. Carter *et al.*, Nucl. Phys. B26, 445 (1971).

[14]For the real pion, we have $f^{*2}/4\pi = f^*(0)^2/4\pi \times (0.942)^{-2} = 0.278$ which is close to 0.264 predicted by the fixed-t dispersion relation, G. Höhler, H. P. Jakob, and R. Strauss, Nucl. Phys. B39, 237 (1972).

[15]J.-I. Fujita and H. Miyazawa, Prog. Theor. Phys. 17, 366 (1957). They calculated the spin-orbit coupling strength from the three-body force due to an intermediate Δ excitation. The mechanism is in effect similar to ours but they predicted much smaller splittings.

[16]W. Jaus, Nucl. Phys. A314, 287 (1979).

[17]P. Haapakoski, Phys. Lett. 48B, 307 (1974).

[18]G. Höhler and E. Pietarinen, Nucl. Phys. B95, 210 (1975). See also G. Gustafson, H. Nielsen, and G. C. Oades, cited by M. M. Nagels *et al.*, Nucl. Phys. B147, 189 (1979).

[19]L. S. Kisslinger, in *Theoretical Methods in Medium-Energy and Heavy Ion Physics*, edited by K. W. McVoy and W. A. Friedman (Plenum, New York, 1978), p. 533; B. D. Keister and L. S. Kisslinger, Nucl. Phys. A326, 445 (1979).

[20]W. Grein and P. Kroll, Nucl. Phys. A338, 332 (1980).