Comparison between NN potential derived from dispersion relations and a model field theory

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We compare the solution of a model field theory which relates off-shell amplitudes for elastic NN and πN scattering to a similar relation between on-shell amplitudes derived from a dispersive approach. The first is a dynamical realization of s-channel unitarity, whereas the second starts with and exploits $t(u)$ channel unitarity. Provided that T_{NN} satisfies a Mandelstam representation, equivalent approximations ought to produce equivalent results and we perform the comparison for the actually calculated T_{NN} and the subsequently extracted potentials V_{NN}. Interpreting V_{NN} we caution against the simultaneous retention of contributions with pions (crossed and uncrossed) linking intermediate nucleons as well as nucleons and physical isobars.

[NUCLEAR REACTIONS NN scattering, comparison between elastic amplitude and potential derived from model field theory and dispersive approach.]

INTRODUCTION

In a study of the $NN\pi$ system which is based on a model field theory, we recently derived integral equations for amplitudes coupling the πd, NN, and $N\Delta$ (generally $N[\pi N]$) channels.[1,2] Eliminating, for instance, all but the elastic NN amplitude T_{NN}, there emerges a generally nonlocal, energy-dependent NN potential V_{NN} with contributions added to the one-boson exchange (OBE) potentials. Comparisons have been made in Ref. 1 (henceforth cited as I) with related coupled-channel models for V_{NN}.[3-5] In the present note we extend the comparison to dispersion theoretical calculations, which do not have such an apparent relation.

In the dispersive approach to the elastic NN amplitude T_{NN} one starts from a Mandelstam representation for T_{NN} and implements fundamental principles like unitarity, crossing, and analyticity.[6-8] For that reason it is also generally believed that the potential V_{NN} ("the Paris potential"), parts of which are extracted from the thus calculated T_{NN}, is the most reliable NN potential available today.

Also the model field theory discussed in I is from the outset inferior to the above mentioned dispersion approach, because the spelled-out basic requirements are at most in part fulfilled in the derivation of T_{NN}. Yet we shall reach the unexpected conclusion that not T_{NN}, but corresponding parts of the extracted NN potentials V_{NN} are essentially the same. Our demonstration will rest on the fact that the solutions of the model field theory appear to satisfy s-channel unitarity, whereas in the dispersion approach one has chosen to exploit the equivalent $t(u)$-channel unitarity. We then argue that, contrary to the construction of the on-shell amplitude T_{NN}, one loses some aspects of a fully relativistic theory in any practical extraction of V_{NN}. Also, though not the case for T_{NN}, the construction of V_{NN} requires some dynamical information. These observations appear to be vital points of contact between the two approaches and help to explain the similarity between parts of the extracted V_{NN}.

The comparison of T_{NN} and the extracted potentials V_{NN} will be our main concern. However, in its course we shall meet different notions of isobars and in particular an isobar appearing together with a nucleon in intermediate states. We shall show that some double counting may occur when one retains in V_{NN} uncrossed and crossed pion contributions, which connect two nucleon as well as nucleon-isobar pairs.

I. THE DISPERSIVE APPROACH

With the sole aim of juxtaposing the dispersive and the field theoretical approaches, we briefly outline the former. No essential feature of our reasoning depends on spin and isospin; these will be disregarded for simplicity.

In order to, for instance, emphasize different and decreasing ranges contributing to the amplitude T_{NN} (stu), one starts with a dispersion relation in the squared momentum transfer t. Thus ($\mu, M, \ldots$ are masses of $\pi, N, \ldots$)

$$T_{NN}(stu)=\frac{1}{\pi}\int_{4\mu^2}^{\infty}\frac{T_t(s,t')}{t'-t}dt' + (u\text{-channel contributions}). \qquad (1.1)$$

Next one evaluates the spectral function T_t, Eq. (1.1), in the "$n_t \leq 2$" approximation, defined to comprise single π, 2π, and the ω part of 3π exchanges. Denoting masses and coupling strengths by μ_i, g_i^2, t-channel unitarity in that approximation implies

$$T_t(st) \equiv \mathrm{Im}T(s,t+i\epsilon) = (2i)^{-1}[T(s,t+i\epsilon) - T(s,t-i\epsilon)]$$
$$\approx \sum_{i=\pi,\omega} g_i^2[\delta(t-\mu_i^2)+\delta(u-\mu_i^2)] + T_t^{2\pi}(st), \quad (1.2)$$
$$T_t^{2\pi} = \sum_\alpha \langle N\bar{N}|T^\dagger|2\pi,\alpha\rangle\langle 2\pi,\alpha|T|N\bar{N}\rangle,$$

where α is any 2π state conserving four-momenta, having the same quantum numbers as the $N\bar{N}$ system. One then concentrates on the $N\bar{N}\to\pi\pi$ annihilation amplitude. The latter is directly related to elastic $\pi\pi$, and by crossing to elastic πN scattering. The continuation to the physical region is then provided by a fixed-t dispersion relation:

$$t_{\pi N}(st) = \frac{1}{\pi}\int \frac{\sigma_{\pi N}(s't)}{s'-s}ds'$$
$$= \frac{g^2}{s-M^2} + \frac{g^2}{u-M^2} + \frac{1}{\pi}\int_{(M+\mu)^2}^\infty \frac{\sigma_{\pi N}'(s't)}{s'-s}ds'. \quad (1.3)$$

Here

$$\sigma'(st) \equiv \sigma(st) - \pi g^2[\delta(s-M^2)+\delta(u-M^2)] \quad (1.4)$$

is the absorptive part of $t_{\pi N}$ from which the direct and crossed N-poles contributions have been subtracted. Substituting (1.3) back into (1.2) one obtains for $T_t^{2\pi}$, the two-pion part of the spectral function $T_t(st)$,

$$T_t^{2\pi}(st) = \frac{1}{\pi}\int_{4M^2}^\infty \frac{ds'}{s'-s}\iint ds_1 ds_2 K(s_1s_2,s't)\sigma(s_1t)\sigma(s_2t)$$
$$+\tau^{\pi\pi}(st), \quad (1.5)$$

with K some kinematical factor.[6,7] The second term in (1.5) is due to $\pi\pi$ scattering. Substituting into (1.1) the $n_t \le 2$ approximation (1.2) for the spectral function T_t, one finds by means of Eqs. (1.2)–(1.5) a relation between, on the one hand T_{NN}, and on the other hand elastic $\pi N(\pi\pi)$ amplitudes supplemented by one-boson exchange (OBE) parameters. It is expected to hold for peripheral, on-shell NN partial wave amplitudes. In order to establish similar relations for low partial wave NN phases, higher mass exchanges beyond correlated, single-range (ω) contributions are needed, but their construction is extremely complicated (see Ref. 9). One further observes that no explicit dynamics is required to establish the relation just discussed.

Before starting the discussion on an NN potential, it is worthwhile to recall that the effective interaction in nuclei (and nuclear matter) is often $\langle p_1p_2|T_{NN}(s)|p_1'p_2'\rangle$ with some of the particles off their mass, or at least off their energy shell [respectively, $E_{\vec{p}_i} \ne (\vec{p}_i^{\,2}+M^2)^{1/2}$, $(E_{\vec{p}_1}+E_{\vec{p}_2})^2 \ne s \ne (E_{\vec{p}_1'}+E_{\vec{p}_2'})^2$].

Dispersion or S-matrix approaches deal only with on-shell amplitudes and a determination of off-shell extensions necessarily requires external information, usually of a dynamical nature. Two problems arise when one follows the standard procedure and introduces an effective NN interaction to drive an equation for NN scattering. First, one has, in principle, to tackle a fully relativistic formulation, i.e., a Bethe-Salpeter equation, but its complications render practical only three-dimensional reductions of the Blankenbecler-Sugar–Logunov-Tavkhelidze (BSLT)[10] type. In these some aspects of relativity got lost. Moverover, a solution of such an equation

$$\langle\vec{p}'|T_{NN}(s)|\vec{p}\rangle = \langle\vec{p}'|V_{NN}(s)|\vec{p}\rangle$$
$$+\int\frac{d\vec{p}}{8\pi^3E_{\vec{p}''}}\langle\vec{p}|V_{NN}(s)|\vec{p}''\rangle G(s,p'')$$
$$\times\langle\vec{p}''|T_{NN}(s)|\vec{p}\rangle \quad (1.6)$$

[$s^{1/2}$ the total energy and $\vec{p}' = (\vec{p}'-\vec{p}_2')/2 \ne \vec{p} = (\vec{p}_1-\vec{p}_2)/2$, initial and final relative momenta] is only possible if T_{NN} is given off shell. These are just the amplitudes which necessitate the introduction of V_{NN}. To break a vicious circle one relates in the developments of Refs. 6–8 portions of T and V having inverse ranges μ^{-1}, μ_ω^{-1} and $(2\mu)^{-1}$, $(3\mu)^{-1}\ldots$, which are, respectively, pole positions and branch points of $T_{NN}(t)$, Eq. (1.1). One thus writes

$$T = T_\pi + T_\omega + T_{2\pi} + T_{3\pi} + \cdots,$$
$$V = T_\pi + T_\omega + V_{2\pi} + V_{3\pi} + \cdots \quad (1.7a)$$

and determines the components of V by substitution in (1.6). Grouping terms with the same inverse range one finds ($\mu_\omega \sim 5.6\ \mu$)

$$V_{2\pi} = T_{2\pi} - T_\pi G T_\pi,$$
$$V_{3\pi} = T_{3\pi} - V_{2\pi}GT_\pi - T_\pi GV_{2\pi} - T_\pi GT_\pi GT_\pi, \quad (1.7b)$$
$$\vdots$$

T_{NN} on shell determines single-mass exchange parts T_π, T_ω, Eq. (1.2), but a calculation of the off-shell elements of T_π, $V_{2\pi}\ldots$ in (1.7b) requires additional input. At this point one involves dynamical information as well as parametrizations (for shorter range parts). These are then tested against on-shell NN data and imply, of course, off-shell behavior for T_{NN}.

We return to $T_t^{2\pi}$, Eq. (1.5) and to its dependence on πN amplitudes. In actual calculations[6-8] these have been taken from measured πN phases. However, in attempts to illustrate the derived $V_{2\pi}$, one sometimes emphasizes selected singular components like the N poles in (1.3) and further the crossing-symmetric δ approximation (1.4) for σ' (Ref. 11):

$$\sigma'(st)\sim\sigma'_{3/2,3/2}(s,u)=g_{\Delta_0}{}^2[\delta(s-m_{\Delta_0}{}^2)+\delta(u-m_{\Delta_0}{}^2)]\,,$$

$$t_{\pi N}\approx g_N{}^2\left(\frac{1}{s-M^2}+\frac{1}{u-M^2}\right)+g_{\Delta_0}{}^2\left(\frac{1}{s-m_{\Delta_0}{}^2}+\frac{1}{u-m_{\Delta_0}{}^2}\right). \tag{1.8}$$

That assumption implies s- and u-channel zero-width resonances ("Δ_0") as part of $t_{\pi N}$. Neither m_{Δ_0}, the mass of the bare Δ_0, nor its strength $g_{\Delta_0}{}^2$ in (1.8) relate directly to the observed $\frac{3}{2}\frac{3}{2}$ phase shift $\delta_\Delta(s)$ to which the separated u-channel N-pole in Eqs. (1.3) and (1.8) contributes significantly. Once made, the approximation (1.8) leads to a V_{NN} with the following components (Fig. 1) (see also Ref. 12):

(a) OBE (π,ω) exchange parts;

(b) uncrossed 2π contribution properly corrected for $V_\pi^{(2)}$, the second order contribution due to $V_{\rm OPE}$;

(c) the crossed 2π contribution;

(d) the analog of (b) with a $N\Delta_0$ intermediate state; and,

(e) the same with Δ_0;

(f) the pion crossed analogs of (d) and (e); and

(g) $\pi\pi$ scattering contributions to (c), (d), (e), and (f).

Approximate calculations of $T_{3\pi}$ are discussed in Ref. 9.

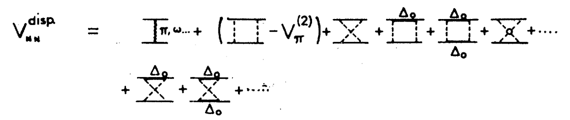


FIG. 1. Representation of V_{NN} resulting from dispersion approach in $n_t\le 2$ approximation (but including ω exchange). "Δ_0" stands for a bare 33 resonance. The open circle denotes the full $t_{\pi\pi}$.

II. A FIELD THEORETICAL MODEL

We briefly recall the essentials of a model field theory used for a description of the $NN\pi$ system.[1,2] The starting point is a Hamiltonian[13]

$$H=H_0+v'_{NN}+w'_{\pi N}+U_{NN\pi}+H_{\rm CT}\,, \tag{2.1}$$

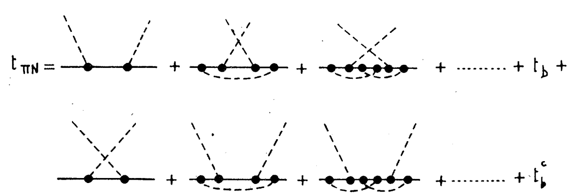
where H_0 describes free pions and nucleons, the latter with their observed mass M. The $NN\pi$ vertex U is of the standard γ^5 type, but for the discussion it suffices to write in terms of nucleon and pion creation (annihilation) operators

$$U_{NN\pi}=\sum_{\vec k\vec k'\vec q}g(\vec k',\vec k\vec q)a^\dagger_{\vec k'}a_{\vec k}b_{\vec q}+\text{H.c.} \tag{2.2}$$

One further finds in (2.1) v'_{NN}, $w'_{\pi N}$, which are parts of the full NN and πN interaction not generated by $U_{NN\pi}$ (heavy-boson exchange potentials, etc.). The same holds for the screened $NN\pi$ vertex in (2.2). Finally, $H_{\rm CT}$ in (2.1) is a counterterm.

Consider first the scattering of a π from an isolated N in an approximation which does not restrict the total number of pions in the s channel. Using reduction techniques and assuming $\nu_s\le 1$, with ν_s the number of pions in any given intermediate (s-channel) state, we showed [I (3.9); see also Ref. 13] that an effective πN interaction can be constructed which drives the two-body BSLT equation.[10] That $v_{\pi N}^{\rm eff}$ is, apart from a background contribution $w'_{\pi N}$, just the direct and crossed N pole (Chew-Low terms for finite M).

Since the number ν_s is not a crossing-invariant concept, it is desirable to lift the restriction on it and to define $v_{\pi N}^{\rm eff}$ as the sum of all πN irreducible diagrams which can be constructed by means of (2.1). Terms for the corresponding $t_{\pi N}$ are



(2.3)

with t_b, t_b^c components due to the background interaction $w'_{\pi N}$.

The $\frac{3}{2}\frac{3}{2}$ projection of the graphs in the second line above, including the crossed N pole, describe in the relevant energy region the Δ. A satisfactory, though not crossing-invariant representation of the off-shell t matrix there is

$$(t_{\pi N})_{3/2,3/2}\sim g_\Delta G_\Delta g_\Delta$$
$$G_\Delta(s)=|G_\Delta(s)|e^{i\delta_\Delta(s)}\,, \tag{2.4}$$

where the Δ propagator $G_\Delta(s)$ carries the observed phase.

From the discussion above it is evident that the diagrams in the first line of (2.3) contain the crossed Δ and that its description requires inclusion of the direct N pole. We shall later return to these observations.

Consider now NN scattering. We neglect for a moment the πd channel and suggest as an approximation $\nu_s\le 1$, i.e., no more than a single pion is permitted in a given intermediate state, but their total number remains unrestricted. Notice that such an intermediate state might contain an N and a Δ. Any number of intermediate pions may be present in the representation (2.3) for the Δ.

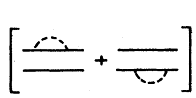
For the imaginary parts of amplitudes with $T_N^{\rm lab} \leq 700$ MeV, $\nu_s \leq 1$ suffices (negligible 2π production) but we have no argument other than simplicity to offer for the assumption in general.

We proceed as in I and Ref. 14, and without proof we state the following result (see Ref. 2 for an alternative derivation)

$$T_{NN} = v^{\rm OBE}(1 + \tfrac{1}{2}G_{NN}T_{NN}) + g_N^{*\rm as}G_0 t_{\pi N}G_0 U_{(\pi N)N,NN}\,,$$
$$U_{(\pi N)N,NN} = g_N^{\rm as}(1 + \tfrac{1}{2}G_{NN}T_{NN}) + t_{\pi N}G_0 U_{(\pi N)N,NN}\,. \tag{2.5}$$

The superscript "as" indicates antisymmetrization of expression involving two nucleons in the same state.[1,14] T_{NN} and $t_{\pi N}$ above are now the amplitudes for generally off-energy shell, elastic NN, and πN scattering, and G_{NN} is the NN propagator with self-energy insertions, in line with the approximation $\nu_s \leq 1$. As shown by Avishai and Mizutani,[2] G_{NN} satisfies an integral equation, schematically written as

$$G_{NN} = G_{NN,0} + G_{NN,0}\left[\;\cdots\;\right] G_{NN}\,, \tag{2.6}$$



where $G_{NN,0}$ is the propagator of two free nucleons. ($G_{NN} \to G_{NN,0}$ was erroneously used in I.) Finally $U_{(\pi N)N,NN}$ is the transition operator for $NN \to N(\pi N)$, where the πN pair interacts last.[14,15] When multiplying $U_{(\pi N)N,NN}$ with $g_\Delta G_0$ [g_Δ being the $\Delta N\pi$ form factor as in (2.4)] and integrating over the πN relative momentum, one is led to the standard $NN \to N\Delta$ production amplitude. (One thus checks that for a separable approximation (2.4) for $P_{3/2,3/2}[t_{\pi N}]$, (P is a projection operator) one recovers I, Eq. (3.22) which are equations coupling amplitudes for the NN, $N\Delta$ (and πd) channels.)

We resume a more detailed analysis of (2.5), not immediately invoking (2.4), a single, separable approximation for the dominant Δ component of $t_{\pi N}$. We first formally eliminate $U_{(\pi N)N,NN}$ in (2.5), which results in

$$T_{NN} = [v^{\rm OBE} + g_N^{*\rm as}G_0 t_{\pi N}G_0(1 - t_{\pi N}G_0)^{-1}g_N^{\rm as}] \times (1 + \tfrac{1}{2}G_{NN}T_{NN})\,. \tag{2.7}$$

Notice first that (2.7) is an equation for a generally off-shell NN amplitude. When free from intermediate NN states the first factor in brackets in (2.7) would by definition by V_{NN} for the model defined by (2.1) used in the spelled-out approximations. However, the direct N pole in $P_{(1/2)^+1/2}[t_{\pi N}]$, Eq. (2.3), introduces into those brackets an NN-reducible part. As a consequence the recognition of V_{NN} in (2.7) is no longer immediate, but that equation still defines a generally nonlocal, energy dependent V_{NN} without any further assumption. In order to perform the comparison with the dispersion method, we discuss some of the lowest order terms of V_{NN} which result from T_{NN}.

There is no problem in principle to evaluate and to interpret for a general $t_{\pi N}$, Eq. (2.7), and the resulting V_{NN}. However, as in the previous section, an interpretation is facilitated if $t_{\pi N}$ is approximated by its dominant parts. For instance, in the resonance region one certainly ought to retain the $\frac{3}{2}^+\frac{3}{2}$ partial wave, which is well represented by (2.4), although that form violates crossing symmetry. In addition, we also keep out of the full $t_{\pi N}$ the direct N pole. Their sum leads to the following long and medium range components of V_{NN} (see Ref. 16 for a relativistic calculation of some parts of $V_{2\pi}$ without construction of a closed system of complex amplitudes):

(a′) OBE exchange parts (including ρ exchange);
(b′) the uncrossed $n\pi$ exchange parts corrected for iterated lower order parts, e.g., for $n_t = 3$

$$V^{(3)} = V_{3\pi}^{\rm dir} - V_\pi G_{NN}V_\pi G_{NN}V_\pi - V_\pi G_{NN}V_{2\pi} - V_{2\pi}^{\rm dir}G_{NN}V_\pi\,; \tag{2.8}$$

(c′) crossed 2π, 3π... contributions from which the $\frac{3}{2}^+\frac{3}{2}$ projections have been removed; and
(d′) contributions due to single and multiple intermediate $N\Delta$ states where the Δ represents the *physical* resonance, properly described by the observed $\frac{3}{2}^+\frac{3}{2}$ phase shift.

The theory further accommodates contributions due to intermediate πd states which for simplicity have been disregarded.

Components (c′) and (d′) require spectial attention. Since the physical Δ draws on the $\frac{3}{2}^+\frac{3}{2}$ projection of the crossed N pole, the latter ought to be removed from contributions containing two (generally n) crossed pions.

It is further in the nature of the chosen approximation $t_{\pi N} \approx t_N^{\rm pole} + t_\Delta$, Eq. (2.4), that no crossed, physical Δ occurs. We recall the remark made after Eq. (2.4) and assert that when wishing to do so, part of the nondirect N pole will be included in the crossed Δ and $V^{2\pi}$ should be correspondingly corrected.

Finally, as done in (1.8), one may as an alternative approximation split $t_{\pi N}$ into a N and zero-width Δ_0 contributions. Instead of contribution (c′) above one now has (Fig. 3)

(c″) 2, 3... crossed π contributions;
(d″) contributions to crossed and uncrossed 1, 2..., $N\Delta_0$, and $\Delta_0\Delta_0$ intermediate states.

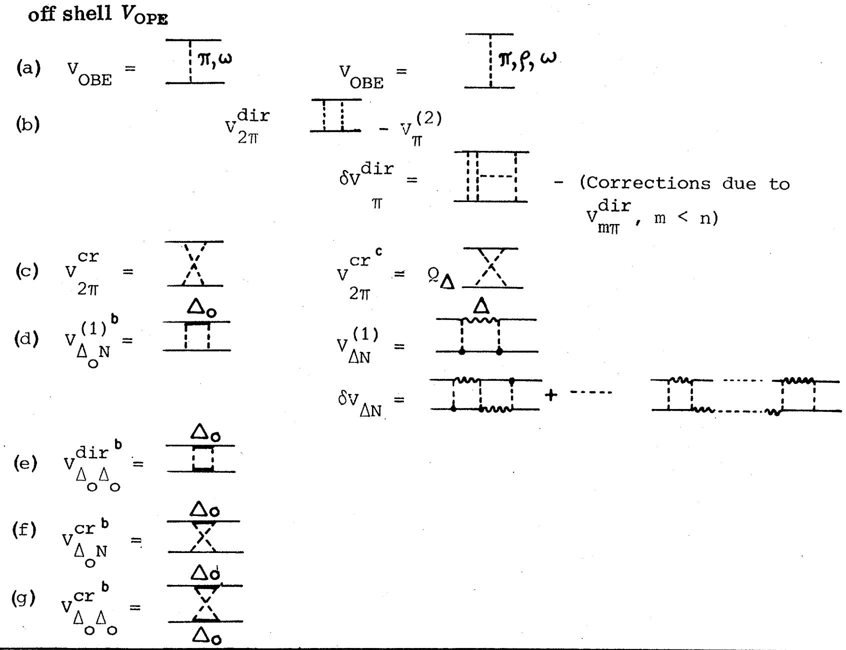
Notice that all contributions are calculated with a screened $NN\pi$ vertex (filled circles) which is not necessarily a virtue and often a necessity. Apart from improving convergence it should help in a parametrization of $t_{\pi N}$. We further remark that nucleon lines in NN intermediate states contain π insertions permitted by $\nu_s \leq 1$.[2] Table I summarizes our findings.

III. CORRESPONDENCE AND DIFFERENCES

In actual evaluations of the approaches one introduces approximations. Thus in the dispersion approach, $n_t \leq 2$ implies limitation to single and double pion t-channel exchanges (actually including ω representing 3π exchange), whereas $\nu_s \leq 1$ limits s-channel states to NN, $NN\pi$, and $N\Delta$. In the thus calculated T_{NN} one then recognizes some but not all intermediate states corresponding to $n_t = 2$ exchanges retained in the dispersion approach. Comparing these compatible approximations for T_{NN}, one sees (cf. Figs. 1–3 as well as Table I) that similar parts for V_{NN} result. This is unexpected since the two underlying descriptions seem to have little in common. The fully relativistic dispersive approach to T_{NN} exploits t-channel unitarity, starts from $T(N\bar{N} \to \pi\pi)$ and ultimately relates to the *on*-shell ampli-

TABLE I. Comparison of tools for the derivation of V_{NN} and of results, from dispersive and field theoretic approaches.

1. Tool	Mandelstam representation, exploitation of $t(u)$ channel unitarity leading to on-shell $T_{NN}(st)$	Field theoretical model Hamiltonian H leading to integral equations for coupled off-shell amplitudes, satisfying (off-shell) s-channel unitarity for $T_{NN}(s)$
2. Major approximation	$n \leq 2$, i.e., no more than $\pi\pi$ in s channel	$\nu_s \leq 1$, any number of π in s channel but no more than a single one in any intermediate state
3. Input	(i) v_{HBE} (ii) $\mathcal{L}_{NN\pi}$ (iii) $t_{N\pi}$ on shell (iv) $t_{\pi\pi}$	off shell (constructed from H) ρ
4. Construction V_{NN}	Using analyticity arguments Born terms $T^{2\pi}$ can be recognized. $V^{2\pi}$ requires dynamical model for half off shell V_{OPE}	Directly from elimination of all but NN channels in coupled integral equations
5. Content V_{NN}[a]		



[a]Common contributions featuring the $NN\pi$ vertex and NN propagator are calculated with, respectively, undressed (dispersion) and dressed (field theory) vertices and N lines.

[b]All contributions include $\pi\pi$ rescattering.

[c]Q_Δ projects out the $\frac{3}{2}\frac{3}{2}$ crossed π contribution on an N line present in $V_{N\Delta}^{(1)}$.

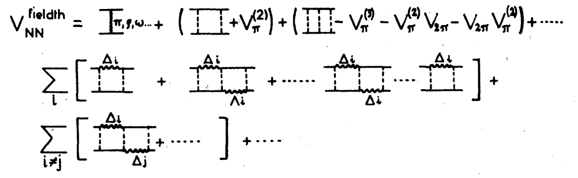


FIG. 2. Representation of V_{NN} resulting from model field theory in $\nu_s \leq 1$ approximation (but including ω exchange and unlimited ν_s terms for proper description of $t_{\pi N}$). Δ signifies (off-shell) $\frac{3}{2}\frac{3}{2}N$ amplitude (dominant part except for direct N pole).

tudes for elastic NN and πN ($\pi\pi$) scattering. The second approach is based on a dynamical model with definitely poorer requirements (actually without antinucleons vital for the exploitation of t-channel unitarity) and relates *off*-energy shell amplitudes for elastic NN and πN scattering [Eq. (2.5)]. Yet the extracted NN potential V_{NN} is largely the same.

The link between the two models is readily exposed, once it is recalled that the coupled integral equations (2.5) [and similar ones embracing πd and more $N(N\pi)$ channels[1,2]] satisfy two- and three-body s-channel unitarity. This is evident from the choice of a complete set of $N\pi$ and $NN\pi$ states used[14] as well as from the explicit proof given by Avishai and Mizutani[2] who followed the reasoning used in Ref. 17.

It is obviously of interest to know whether unitarity alone implies the dynamical equations (2.5). For a strict three-particle (and thus nonrelativistic) theory with separable pair potentials this is indeed the case, but there are already numerous ambiguities if the dynamical equations describing a relativistic three-body potential model are of the covariant Blankenbecler-Sugar-Logunov-Tavhkelidze type[10,7] (see Ref. 18 for a discussion). The field theoretical model defined by Eq. (2.1) is decidedly more complex and allows also two-particle states. For that reason one cannot apply the reasoning of Ref. 17, where the form of dynamical equations is derived from unitarity. It does not

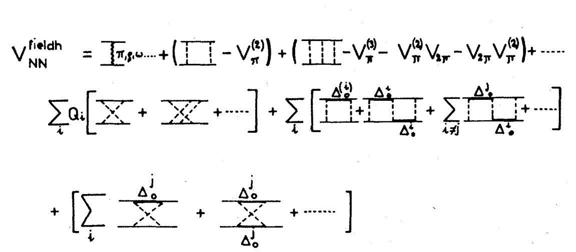


FIG. 3. Same as Fig. 2 but with $t_{\pi N}$ approximated by crossed and uncrossed N and "Δ_0" (bare isobar) poles, Eq. (1.8).

come as a surprise that starting from a dynamical equation derived in a restricted space, unitarity can be derived under the same restrictions. However, implication in the reverse sense has not been proved.

Putting aside the question of uniqueness, our model furnishes at least one realization of off-shell s-channel unitarity in the $\nu_s \leq 1$ approximation, i.e., T_{NN} satisfies in a schematic notation

$$\mathrm{Im}T_{NN,NN}(s) = \rho_{NN}|T_{NN,NN}(s)|^2 + \rho_{\pi d}|T_{NN,\pi d}(s)|^2 + \rho_{NN\pi}|T_{NN,NN\pi}(s)|^2, \quad (3.1)$$

where ρ_α stands for appropriate phase space factors.

Instead of two completely different approaches one deals in principle with a different exploitation of unitarity, crossing, and analyticity of T_{NN}. A Mandelstam representation for T_{NN} then guarantees the same content whether the emphasis is on the s or on the $t(u)$ channel. Differences in T_{NN} should therefore, in principle, only be due to inequivalent approximations in actual treatments of the s and t channels. We repeat that these approximations do not only involve n_t, ν_s, but, as is the case with the solution of (2.1), an approximate treatment of crossing and relativistic effects as well. Roughly speaking this is reflected in the use of BSLT rather than Bethe-Salpeter equations. Thus far for T_{NN}.

After the discussion in Sec. II on the construction of V_{NN} we can be brief as to the correspondence in V_{NN}. Our model uses from the onset coupled on-mass, off-energy shell equations of the BSLT type, while in the dispersion approach there is no need to use an approximation comparable with the BSLT on-mass, off-energy shell reduction. However, the explicit introduction of such an equation is unavoidable if one desires to extract an effective V_{NN} from an on-shell T_{NN} as provided by the dispersion approach of Refs. 6–8. It is in that step where to our taste some of the clear initial advantages of the dispersion approaches appear to escape and where an extraction of V_{NN} is no more immediate. We recalled above that (except for single mass exchange contributions) one needs dynamical information in order to calculate the required off-shell amplitudes. No such information is needed for T_{NN}.

As opposed to that road stands off-energy shell (s-channel) unitarity and its realization in equations coupling all relevant amplitudes. Their off-shell behavior is determined by the dynamics underlying the chosen model. In particular, Eqs. (1.7) and (2.7) define V_{NN} without the need of further information. The remarks above only relate to the extraction of V_{NN} and do not bear on the quality of that potential, which of course also lacks short-range parts.

We now discuss some differences in V_{NN}. These are inessential indeed and are due to different approximations. For instance the dispersive approach in the $n_t \leq 2$ approximation allows $\pi\pi$ rescattering. A corresponding description in the field theoretical model requires an additional $\pi\pi$ potential in (3.1) so constructed as to reproduce in the $J=I=0$ and $J=I=1$ channels characteristics of the σ and ρ (these should then be removed from $V^{\rm HBE}$). Conversely, that latter model with unlimited total number of pions but with $\nu_S \leq 1$, (i.e., no more than a single π in any intermediate state) accommodates multicrossed-pion contributions and further parts with any number of crossed and uncrossed "$N\Delta_0$ states." These in turn are part of the $n_t \geq 3$ terms in the dispersive development.

From the discussion above it should be clear that provided the Δ_0 contribution to $t_{\pi N}$ is carefully defined, the two models do not diverge in the interpretation of $N\Delta_0$ or $\Delta_0\Delta_0$ contributions to V_{NN}. At this point we wish to caution against too liberal use of sums of effective Lagrangians

$$\mathcal{L} = \mathcal{L}_{NN\pi} + \mathcal{L}^{\rm eff}_{\Delta N\pi} + \mathcal{L}^{\rm eff}_{NN\rho} + \mathcal{L}^{\rm eff}_{\Delta N\rho} + \cdots$$

with $\mathcal{L}^{\rm eff}$ treated with Feynman rules also beyond the permitted second order term. If, for instance, the Δ can (in part) be built from $\mathcal{L}_{NN\pi}$, a calculation of V_{NN} from $\mathcal{L}_{NN\pi} + \mathcal{L}^{\rm eff}_{\Delta N\pi}$ (Refs. 19–21) runs, as demonstrated the danger of over counting. Even at this elementary level of discussion one is apparently confronted with questions of elementarity of a particle and their role played in unitarity.

In a final remark we wish to focus on some approximations used. One expects that with the same content in principle, one ought to be able to work out the theories to precisely the same approximations. We already mentioned approximate $n_t = 3$ extensions within the dispersive approach.[9] We are further aware of some successful solutions of a simpler field theory in the $\nu_s \leq 2$ approximation, with also the total number of pions ≤ 2.[22] We have till now not succeeded to realize s-channel unitarity in the form of coupled integral equations like (2.5) which include $NN\pi\pi$. One should look forward to further progress.

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[1]A. W. Thomas and A. S. Rinat, Phys. Rev. C 20, 216 (1979), referred to as I.

[2]Y. Avishai and T. Mizutani, Nucl. Phys. A326, 352 (1979) and unpublished.

[3]H. J. Weber, J. M. Eisenberg, and M. D. Shuster, Nucl. Phys. A278, 491 (1977).

[4]S. Jena and L. S. Kisslinger, Ann. Phys. (N.Y.) 85, 251 (1979).

[5]A. M. Green, *Mesons in Nuclei*, edited by M. Rho and D. H. Wilkinson (North-Holland, Amsterdam, 1979), p. 229.

[6]D. Amati, E. Leader, and B. Vitale, Nuovo Cimento 17, 68 (1960); *ibid.* 18, 409 (1960).

[7]W. N. Cottingham and R. Vinh Mau, Phys. Rev. 130, 735 (1963); W. N. Cottingham, M. Lacombe, B. Loiseau, J. M. Richard, and R. Vinh Mau, Phys. Rev. D 8, 800 (1973); M. Lacombe, B. Loiseau, J. M. Richard, R. Vinh Mau, P. Pires, and R. de Tourreil, *ibid.* 12, 1495 (1975).

[8]M. Chemtob, J. W. Durso, and D. O. Riska, Nucl. Phys. B38, 141 (1972).

[9]K. Tsu, Ph.D. thesis, Paris, 1978 (unpublished); J. Heitzmann, Ph.D. thesis, Paris, 1978 (unpublished).

[10]A. A. Logunov and A. N. Tavkhelidze, Nuovo Cimento 29, 380 (1963); R. Blankenbecler and R. Sugar, Phys. Rev. 142, 1051 (1966).

[11]M. Cini and S. Fubini, Ann. Phys. (N.Y.) 3, 352 (1960).

[12]R. Vinh Mau, J. M. Richard, B. Loiseau, M. Lacombe, and W. N. Cottingham, Phys. Lett. 44B, 1 (1972); R. Vinh Mau, see Ref. 5, p. 152.

[13]T. Mizutani, Ph.D. thesis, University of Rochester, 1975 (unpublished); T. Mizutani and D. S. Koltun, Ann. Phys. (N.Y.) 109, 1 (1978).

[14]A. S. Rinat, Nucl. Phys. A287, 399 (1977).

[15]J. L. Ballot and F. Becker, Phys. Rev. 164, 1285 (1967).

[16]M. Hossein Partovi and E. L. Lomon, Phys. Rev. D 2, 1999 (1970).

[17]D. Z. Freedman, C. Lovelace, and J. M. Namysłowski, Nuovo Cimento 43A, 258 (1966); R. Aaron, R. D. Amado, and J. M. Young, Phys. Rev. 174, 2022 (1968).

[18]A. S. Rinat and A. W. Thomas, Nucl. Phys. A282, 365 (1977); A. S. Rinat, in *Common Problems in Low- and Medium-Energy Nuclear Physics*, Proceedings of the NATO Advanced Study Institute, Alberta, 1978, edited by B. Castel, B. Goulard, and F. C. Khanna (Plenum, New York, 1979).

[19]J. W. Durso, M. Saarela, G. E. Brown, and J. D. Jackson, Nucl. Phys. A278, 445 (1977).

[20]G. E. Brown and W. Weise, Phys. Rep. 22C, 279 (1975).

[21]K. Hohlinde, R. Machleidt, M. R. Anastasio, A. Faessler, and K. Mütter, Phys. Rev. C 18, 871 (1978).

[22]M. Stingl and A. T. Stelbovic, J. Phys. G 4, 1371 (1978); 4, 1389 (1978); Nucl. Phys. A229, 391 (1978).