

Pion coupling to the $A = 6$, $A = 12$, and $A = 14$ nuclei

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The values of the coupling constants $f_{\pi^+ 6\text{Li}^6\text{Be}}^2 = 0.07$, $f_{\pi^+ 12\text{C}^{12}\text{N}}^2 = 0.004$, and $f_{\pi^+ 14\text{N}^{14}\text{O}}^2 = 0.00014$ have been extracted from the polelogical treatment of the ${}^6\text{Li}(p,n){}^6\text{Be}$, ${}^{12}\text{C}(p,n){}^{12}\text{N}$, and ${}^{14}\text{N}(p,n){}^{14}\text{O}$ reactions. A critical discussion of our present knowledge of the pion couplings to these nuclei and of existing methods of their determination is given.

[NUCLEAR REACTIONS Initial nucleus-pion-final nucleus coupling constants. PCAC hypothesis. Analyticity. ${}^6\text{Li}(p,n){}^6\text{Be}$, ${}^{12}\text{C}(p,n){}^{12}\text{N}$, ${}^{14}\text{N}(p,n){}^{14}\text{O}$ at 0.144 GeV.]

I. INTRODUCTION

In the elementary particle approach to nuclei it is convenient to introduce the initial-nucleus-pion-final nucleus vertex function $f_{\pi ab}^2(q^2)$, the pion-nucleus coupling constant being the corresponding vertex function for a pion on its mass shell, i.e., $f_{\pi ab}^2(-m_\pi^2)$. The coupling constants $f_{\pi ab}^2 \equiv f_{\pi ab}^2(-m_\pi^2)$ play an important role in the description of pion-nucleus and nucleus-nucleus scattering processes. One of the basic questions here is the question of whether the coupling constants $f_{\pi ab}^2$ are smaller or larger than their elementary analogue $f_{\pi NN}^2 = 0.081$. The first possibility would indicate the presence of expected "shadowing effects" in nuclei.¹ In fact, different analyses of scattering data involving even such light nuclei as ${}^3\text{He}$, ${}^3\text{H}$ show²⁻⁴ a rather significant reduction of the coupling constant $f_{\pi^+ 3\text{He}^3\text{He}}^2$. It would be interesting to explore how this reduction changes with the size of the nuclear system. The study of the pion coupling to the $A = 6$, $A = 12$, and $A = 14$ nuclei would help clarify this question.

The paper is organized as follows. In Sec. II we summarize our knowledge about the pion coupling to the $A = 6$, $A = 12$, and $A = 14$ nuclei obtained by applying the partially conserved axial-current (PCAC) hypothesis to the evaluation of the matrix elements of the axial weak current between the relevant nuclear states. In Sec. III the information about these couplings obtained on the basis of studying the π^+ photoproduction near threshold on ${}^6\text{Li}$ and ${}^{12}\text{C}$ by means of low-energy theorems is discussed. In Sec. IV we present our own polelogical treatment of the $p + {}^6\text{Li} \rightarrow n + {}^6\text{Be}$, $p + {}^{12}\text{C} \rightarrow n + {}^{12}\text{N}$, and $p + {}^{14}\text{N} \rightarrow n + {}^{14}\text{O}$ reactions with the extraction of the coupling constants $f_{\pi^+ 6\text{Li}^6\text{Be}}^2$, $f_{\pi^+ 12\text{C}^{12}\text{N}}^2$, and $f_{\pi^+ 14\text{N}^{14}\text{O}}^2$. This is followed by a general discussion and conclusions in Sec. V.

II. PCAC HYPOTHESIS

The PCAC hypothesis leads (see, e.g., Ref. 5) to the following relations:

$$f_{\pi ab}^2(0) = a_\pi^{-2} F_{A;ab}(0), \quad (1)$$

$$f_{\pi ab}^2 = a_\pi^{-2} F_{A;ab}(-m_\pi^2) [1 + \epsilon_{ab}(-m_\pi^2)]^2 = f_{\pi ab}^2(0) \frac{F_{A;ab}^2(-m_\pi^2)}{F_{A;ab}^2(0)} [1 + \epsilon_{ab}(-m_\pi^2)], \quad (2)$$

where $a_\pi = 0.94 \pm 0.01$ is the pion decay constant defined by and determined from the observed value of the $\pi^+ \rightarrow \mu^+ + \nu_\mu$ decay rate, $F_{A;ab}(q^2)$ is the axial nuclear form factor, and $\epsilon_{ab}(-m_\pi^2)$ is fixed by the observed rate of $\mu^+ + b \rightarrow \nu_\mu + a$. We will discuss here neither the basis nor the derivation of these relations but will proceed straightforwardly to their applications to specific processes.

In Ref. 6 the values $f_{\pi^+ 6\text{Li}^6\text{Be}}^2(0) = 0.059$, $f_{\pi^+ 12\text{C}^{12}\text{N}}^2(0) = 0.011$, and $f_{\pi^+ 14\text{N}^{14}\text{O}}^2(0) = 0.000002$ have been worked out on the basis of Eq. (1) using information on $F_{A;6\text{Li}^6\text{Be}}(0)$, $F_{A;12\text{C}^{12}\text{N}}(0)$, and $F_{A;14\text{N}^{14}\text{O}}(0)$ from the beta decays ${}^6\text{Be} \rightarrow {}^6\text{Li} + e^- + \bar{\nu}_e$, ${}^{12}\text{N} \rightarrow {}^{12}\text{C} + e^- + \bar{\nu}_e$, and ${}^{14}\text{O} \rightarrow {}^{14}\text{N} + e^- + \nu_e$. No attempt was made to determine the physical coupling constants by means of Eq. (2).

In Ref. 5 using the observed rates of ${}^{12}\text{B} \rightarrow {}^{12}\text{C} + e^- + \bar{\nu}_e$ and ${}^{14}\text{C} \rightarrow {}^{14}\text{N} + e^- + \bar{\nu}_e$ the values $f_{\pi^+ 12\text{C}^{12}\text{C}}^2(0) = 0.024$ and $f_{\pi^+ 14\text{C}^{14}\text{N}}^2(0) = 0.0000008$ have been deduced. (Our values of f^2 are by definition 4π times smaller than those used in Ref. 5.) Moreover, using the impulse approximation

$$\frac{F_{A;ab}(q^2)}{F_{A;ab}(0)} \approx \frac{F_{M;ab}(q^2)}{F_{M;ab}(0)}, \quad (3)$$

where $F_{M;ab}(q^2)$ is the weak-magnetism nuclear form factor, and the conserved-vector-current (CVC) hypothesis

$$\frac{F_{M;ab}(q^2)}{F_{M;ab}(0)} = \frac{\mu(q^2)}{\mu(0)}, \quad (4)$$

where $\mu(q^2)$ is the transition magnetic moment determined from the observed cross section for the inelastic electron scattering $e^- + b \rightarrow e^- + b^*$ and the observed rate for $b^* \rightarrow b + \gamma$ decay, the values for the coupling constants $f_{\pi^{12B}12C} = 0.149^{+0.104}_{-0.076}$, and $f_{\pi^{14C}14N} \approx 0.009$ have been worked out on the basis of Eq. (2).

III. PION PHOTOPRODUCTION AT THRESHOLD AND LOW-ENERGY THEOREMS

This approach allows one to determine the pion-nucleus coupling constants from the measured $\gamma + a \rightarrow b + \pi^+$ cross section near threshold exploiting low-energy theorems. These theorems are based to a considerable extent on the PCAC hypothesis. Strictly speaking, they are only applicable to off-shell amplitudes involving soft (zero four-momentum) pion. However, some procedures have been proposed and used^{7,8} to relate these soft pion results to the real world.

Let us state, for example, Low's theorem: the two lowest terms—of order K^{-1} and of order zero in K —of the series expansion, in powers of the photon four-momentum K , of the scattering amplitude depend only on the $f_{\pi ab}$ coupling constant and on the electromagnetic constants (charges and anomalous moments) of the participating nuclei a and b .

In the center of mass system the differential cross section at threshold is^{3,7}

$$\begin{aligned} \lim_{|q| \rightarrow 0} \frac{|\vec{K}|}{|\vec{q}|} \frac{d\sigma}{d\Omega} &\equiv \frac{\alpha_N^i}{4\pi} \\ &= \frac{1}{137m_\pi^2} f_{\pi ab}^2 \frac{(1 + \frac{1}{2} m_\pi/m_a)^2}{(1 + m_\pi/m_a)^3} |M|^2, \end{aligned} \quad (5)$$

where \vec{K} and \vec{q} are the photon and the pion momentum and M is the reduced matrix element

$$M(\gamma + a \rightarrow b + \pi^+) = -\sqrt{2} \left(1 - \frac{1}{2} \frac{m_\pi}{m_a} + \frac{\mu_a + \mu_b}{4} \cdot \frac{m_\pi^2}{m_a^2} \right). \quad (6)$$

Here μ_a and μ_b are the total magnetic moments of a and b , respectively.

In Refs. 7 and 8 the values $f_{\pi^{6He}6Li} = 0.020$ and $f_{\pi^{12B}12C} = 0.006$ have been deduced by analyzing the measured $\gamma + {}^6\text{Li} \rightarrow {}^6\text{He} + \pi^+$ and $\gamma + {}^{12}\text{C} \rightarrow {}^{12}\text{B} + \pi^+$ cross sections near threshold by means of Eqs. (5) and (6).

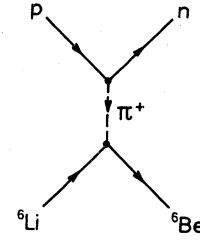


FIG. 1. The t channel π^+ pole in the reaction $p + {}^6\text{Li} \rightarrow n + {}^6\text{Be}$. The pole is at $\cos\theta = 1.049$.

IV. POLOLOGICAL TREATMENT OF THE $p + {}^6\text{Li} \rightarrow n + {}^6\text{Be}$, $p + {}^{12}\text{C} \rightarrow n + {}^{12}\text{N}$, AND $p + {}^{14}\text{N} \rightarrow n + {}^{14}\text{O}$ REACTIONS

In principle, the most model-independent way of determining the pion-nuclei coupling constants would be by the exploitation of the analytic properties of the scattering amplitude either in the form of forward dispersion relations or as an analytic continuation of $d\sigma/d\Omega$ in the $\cos\theta$ plane. A detailed account of the use of analyticity in nuclear physics can be found in Ref. 9.

Recently¹⁰ the differential cross sections of the three reactions, $p + {}^6\text{Li} \rightarrow n + {}^6\text{Be}$, $p + {}^{12}\text{C} \rightarrow n + {}^{12}\text{N}$, and $p + {}^{14}\text{N} \rightarrow n + {}^{14}\text{O}$ have been measured at 144 MeV. These data invite one to make an attempt to extract the coupling constants $f_{\pi^{6Li}6Be}$, $f_{\pi^{12C}12N}$, and $f_{\pi^{14N}14O}$ by continuing analytically $d\sigma/d\Omega$ to the π^+ pole. The method is well known. For example, it has been used¹¹ to determine the coupling constant $f_{\pi^{3He}3He}$ from data on the $p + {}^3\text{H} \rightarrow n + {}^3\text{He}$ differential cross section.

The nearest singularities of $d\sigma/d\Omega$ in the $\cos\theta$ plane for the three reactions are shown in Figs. 1–9. The relevant formulas for finding the positions of these singularities can be found, e.g., in Ref. 12.

In order to carry out the extrapolation to the pion pole in an optimal way we first symmetrize the positions of the nearest singularities by means of the bilinear transformation

$$w(x) = \frac{x(x_{t\text{-anom}} - x_{u\text{-pole}})}{x(x_{t\text{-anom}} + x_{u\text{-pole}}) - 2x_{t\text{-anom}}x_{u\text{-pole}}}, \quad (7)$$

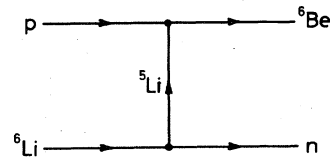


FIG. 2. The u channel ${}^5\text{Li}$ pole is in the reaction $p + {}^6\text{Li} \rightarrow n + {}^6\text{Be}$. The pole is at $\cos\theta = -3.00$.

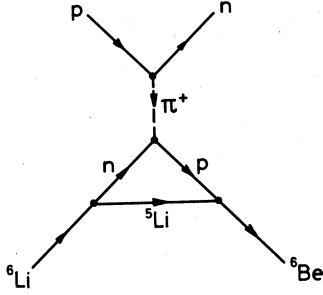


FIG. 3. The graph corresponding to the lowest t channel anomalous cut in the reaction $p + {}^6\text{Li} \rightarrow n + {}^6\text{Be}$. The cut starts at $\cos\theta = 1.127$.

where $x \equiv \cos\theta$.

The cut w plane is then transformed into the unit circle by means of the mapping

$$u(w) = \frac{(1+w)^{1/2} - (1-w)^{1/2}}{(1+w)^{1/2} + (1-w)^{1/2}}, \quad (8)$$

which maps the cuts onto the circle. Finally, the unit circle is mapped into itself by means of the bilinear transformation

$$z(u) = \frac{u + \lambda}{1 + \lambda u}, \quad (9)$$

where λ is chosen so as to symmetrize the region containing the fitted data around the origin in the z plane where expansion in powers of z converges most rapidly, $\lambda(p + {}^6\text{Li} \rightarrow n + {}^6\text{Be}) = -0.448$, $\lambda(p + {}^{12}\text{C} \rightarrow n + {}^{12}\text{N}) = -0.495$, and $\lambda(p + {}^{14}\text{N} \rightarrow n + {}^{14}\text{O}) = -0.510$.

Inside the unit circle in the z plane the differential cross section has a second order pole at $z = z_r$. We eliminate it by multiplying $d\sigma/d\Omega$ by the factor $(z - z_r)^2$ and use the expansion

$$(z - z_r)^2 \frac{d\sigma(z)}{d\Omega} = f_{\pi ab}^2 \cdot C \cdot \left(\frac{dz}{dx} \Big|_{x=x_r} \right)^2 + \sum_{n=1}^N a_n (z^n - z_r^n) \quad (10)$$

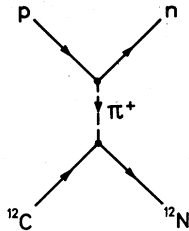


FIG. 4. The t channel π^+ pole in the reaction $p + {}^{12}\text{C} \rightarrow n + {}^{12}\text{N}$. The pole is at $\cos\theta = 1.046$.

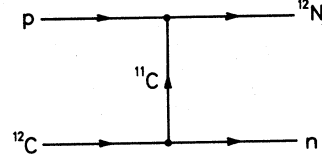


FIG. 5. The u channel ${}^{11}\text{C}$ pole in the reaction $p + {}^{12}\text{C} \rightarrow n + {}^{12}\text{N}$. The pole is at $\cos\theta = -6.12$.

to represent the experimental data. It converges inside the whole circle. Here C is a factor depending on kinematics. Its explicit form is the following:

$$C = f_{\pi mb}^2 \cdot \frac{m_a^2 m_b^2}{m_\pi^4} \cdot \frac{20(\hbar c)^2}{S} \cdot \frac{K'}{K} \cdot \frac{q^2}{2J_a + 1} \times \sum_{i=1}^4 |f_i(x_r)|^2, \quad (11)$$

where $f_{\pi mb}^2 = 0.162$, $\hbar c = 0.1973$ GeV fm,

$$S = 2m_a(T + m_b) + m_b^2 + m_a^2, \quad (12)$$

$T = 0.144$ GeV,

$$K^2 = \frac{[S - (m_a + m_b)^2][S - (m_a - m_b)^2]}{4S}, \quad (13)$$

$$K'^2 = \frac{[S - (m_b + m_n)^2][S - (m_b - m_n)^2]}{4S}, \quad (14)$$

$$q = \begin{cases} 1/K' & (0^+ \rightarrow 1^+ \text{ transition}), \\ 1/K & (1^+ \rightarrow 0^+ \text{ transition}), \end{cases} \quad (15)$$

J_a is the spin of the target nucleus, and

$$|f_1(x_r)|^2 = \beta_+ \frac{E_1^2}{M_1^2} (1 + x_r) (x_r - \alpha)^2, \quad (16)$$

$$|f_2(x_r)|^2 = \beta_+ \frac{E_1^2}{M_1^2} |1 - x_r| (x_r - \alpha)^2, \quad (17)$$

$$|f_3(x_r)|^2 = \beta_- |1 - x_r| (1 + x_r)^2, \quad (18)$$

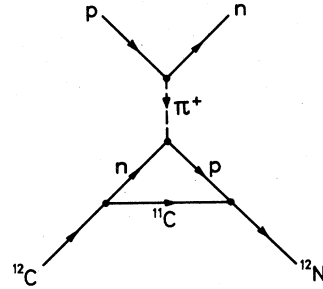


FIG. 6. The graph corresponding to the lowest t channel anomalous cut in the reaction $p + {}^{12}\text{C} \rightarrow n + {}^{12}\text{N}$. The cut starts at $\cos\theta = 1.121$.

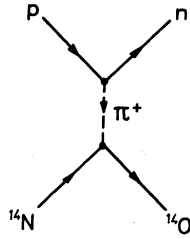


FIG. 7. The t channel π^+ pole in the reaction $p + {}^{14}\text{N} \rightarrow n + {}^{14}\text{O}$. The pole is at $\cos\theta = 1.040$.

$$|f_A(x_\pi)|^2 = \beta_\pm (1 + x_\pi)(1 - x_\pi)^2, \quad (19)$$

$$\beta_\pm = \frac{[K(E' + m_n) \pm K'(E + m_p)]^2}{(E + m_p)(E' + m_n)}, \quad (20)$$

$$\alpha = \begin{cases} \frac{K'E_0}{KE_1} & (0^+ \rightarrow 1^+ \text{ transition}), \\ \frac{KE_0}{K'E_1} & (1^+ \rightarrow 0^+ \text{ transition}), \end{cases} \quad (21)$$

$$E = (K^2 + m_p^2)^{1/2}, \quad (22)$$

$$E' = (K'^2 + m_n^2)^{1/2}, \quad (23)$$

E_0 is the c.m. energy of the spin-zero nucleus, E_1 is the c.m. energy of the spin-one nucleus, and M_1 is the mass of the spin-one nucleus.

Equation (11) has been deduced on the basis of the decomposition of the general form of the Born amplitudes corresponding to the Feynman diagrams shown in Figs. 1, 4, and 7 into helicity amplitudes as given in Ref. 6.

The results of the fits are shown in the Table I. According to the χ^2/ndf criterion we conclude that $f_{\pi^+ {}^6\text{Li} {}^6\text{Be}}^2 = 0.07$, $f_{\pi^+ {}^{12}\text{C} {}^{12}\text{N}}^2 = 0.004$, and $f_{\pi^+ {}^{14}\text{N} {}^{14}\text{O}}^2 = 0.00014$. The errors given in Table I reflect only the statistical and the 7% normalization errors on $d\sigma/d\Omega$ once N is fixed.

V. DISCUSSION AND CONCLUSIONS

Our value $f_{\pi^+ {}^6\text{Li} {}^6\text{Be}}^2 = 0.07$ is more than three times larger than the value $f_{\pi^+ {}^6\text{He} {}^6\text{Li}}^2 = 0.02$ obtained in Ref. 7 by studying the photoproduction

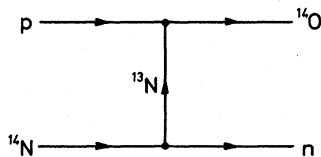


FIG. 8. The u channel ${}^{13}\text{N}$ pole in the reaction $p + {}^{14}\text{O} \rightarrow n + {}^{14}\text{N}$. The pole is at $\cos\theta = -6.95$.

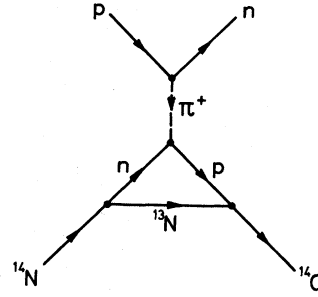


FIG. 9. The graph corresponding to the lowest t channel anomalous cut in the reaction $p + {}^{14}\text{N} \rightarrow n + {}^{14}\text{O}$. The cut starts at $\cos\theta = 1.122$.

of π^+ on ${}^6\text{Li}$ near threshold. We cannot explain this difference.

Our value $f_{\pi^+ {}^{12}\text{C} {}^{12}\text{N}}^2 = 0.004$ is in fair agreement with the value $f_{\pi^+ {}^{12}\text{B} {}^{12}\text{C}}^2 = 0.006$ obtained in Ref. 7 by studying the photoproduction of π^+ on ${}^{12}\text{C}$ near threshold. However, both these numbers are in violent disagreement with the value $f_{\pi^+ {}^{12}\text{B} {}^{12}\text{C}}^2 = 0.149_{-0.076}^{+0.104}$ obtained in Ref. 5 on the basis of the PCAC hypothesis. One possible explanation for this serious discrepancy could be that the parametrization

$$\frac{F_{M; {}^{12}\text{B} {}^{12}\text{C}}(q^2)}{F_{M; {}^{12}\text{B} {}^{12}\text{C}}(0)} = \frac{1}{(1 + q^2/2.7m_\pi^2)^2}, \quad (24)$$

which was used in Ref. 5 both for the description of experimental data in the "physical region" and for the extrapolation to the unphysical point $q^2 = -m_\pi^2$ [see Eqs. (2) and (3)] should not be used for the latter purpose. In fact, in Ref. 4 it was shown that one gets a large reduction in the coupling constant $f_{\pi^+ {}^3\text{He} {}^3\text{He}}^2$ by choosing another prescription for extrapolation instead of Eq. (24). We also take here the opportunity to note the discrepancy in the values $f_{\pi^+ {}^{12}\text{B} {}^{12}\text{C}}^2(0) = 0.024$ and $f_{\pi^+ {}^{12}\text{C} {}^{12}\text{N}}^2(0) = 0.011$ used by the authors of Refs. 5 and 6, respectively.

Finally, our values $f_{\pi^+ {}^{14}\text{N} {}^{14}\text{O}}^2 = 0.00014$ is in qualitative agreement with the value $f_{\pi^+ {}^{14}\text{C} {}^{14}\text{N}}^2 = 0.009$ deduced in Ref. 5 on the basis of the PCAC hypothesis. Here we call attention to the discrepancy in the values $f_{\pi^+ {}^{14}\text{C} {}^{14}\text{N}}^2(0) = 8 \times 10^{-8}$ and $f_{\pi^+ {}^{14}\text{N} {}^{14}\text{O}}^2(0) = 2 \times 10^{-6}$ used by the authors of Refs. 5 and 6, respectively.

Our analysis shows that the pion coupling constants to the $A = 6$, $A = 12$, and $A = 14$ nuclei are smaller than the elementary coupling constants $f_{\pi NN}^2$. However, one cannot interpret this as presence of significant shadowing effects in these nuclei. The point is that we have determined the coupling of the pion to the ground states of the

TABLE I. The results of the fits of Eq. (10) to the experimental data on $d\sigma/d\Omega$.

N	$p + {}^6\text{Li} \rightarrow n + {}^6\text{Be}$		$p + {}^{12}\text{C} \rightarrow n + {}^{12}\text{N}$		$p + {}^{14}\text{N} \rightarrow n + {}^{14}\text{O}$	
	χ^2/ndf	$f_{\pi} {}^6\text{Li} {}^6\text{Be}^2$	χ^2/ndf	$f_{\pi} {}^{12}\text{C} {}^{12}\text{N}^2$	χ^2/ndf	$f_{\pi} {}^{14}\text{N} {}^{14}\text{O}^2$
0					2.7/3	0.00014 ± 0.00001
1	53/10	0.031 ± 0.001	7.4/9	0.0023 ± 0.0001	0.2/2	0.00005 ± 0.00006
2	3.3/9	0.070 ± 0.006	2.7/8	0.004 ± 0.001		
3	2.7/8	0.088 ± 0.024	2.0/7	0.008 ± 0.005		

nuclei and not the effective coupling constants. The latter are determined by means of the forward dispersion relations (see Ref. 9) and are directly comparable to the elementary coupling constant. Only in light nuclei such as $A = 3$, where there are few excited states the pion-nucleus coupling constant, e.g., $f_{\pi} {}^3\text{He} {}^3\text{He}^2$, obtained from the analysis of the charge exchange $p + {}^3\text{H} \rightarrow n + {}^3\text{He}$, can be directly compared to $f_{\pi NN}^2$. On the other hand, our results could be analyzed in terms of a quenching of the pion coupling constant in nuclear matter by means of some solvable models¹³ (this work is in progress¹⁴) or indirectly con-

nected to the critical opalescence of the nuclear pion field.¹⁵

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