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Nuclear excitation function and particle emission from complex nuclei following muon capture

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A model for the calculation of the nuclear excitation function following muon capture in complex nuclei and of the ensuing emission is presented. The capturing nucleus is treated as consisting of quasifree nucleons moving in a momentum-dependent potential and having an effective nucleon-momentum distribution. The emission of protons, deuterons, α particles, and neutrons is calculated by considering both pre-equilibrium and compound-nucleus emission. The model accounts well for the observed charged particles emission rates from capturing nuclei over the wide range $23 \leq A \leq 209$. The proton emission is shown to become mainly of pre-equilibrium nature for the heavier nuclei. The results for neutron emission are also in satisfactory agreement with experimentally measured multiplicities and spectra.

NUCLEAR REACTIONS Calculated probabilities of (μ^-, pxn), (μ^-, α) $E_\mu = 0$ reactions for various targets with $23 \leq A \leq 209$; inclusive and exclusive channels; pre-equilibrium emission; compound-nucleus emission; neutron multiplicities; nuclear excitation distribution following μ capture.

I. INTRODUCTION

In muon capture from a $1s$ atomic orbit by a nucleus (A, Z) through the weak interaction process

$$\mu^- + (A, Z) \to \nu_\mu + X ,$$

about 100 MeV is made available to the reaction products. While the bulk of energy is carried away by the neutrino, approximately 20 MeV on the average is retained by the nuclear product X. In most cases, an excited nucleus is formed which deexcites by emission of neutrons and (or) γ's, and to a much lesser extent, by emitting charged particles, i.e., protons, deuterons, tritons, and α's.

The neutron emission process has been studied extensively, both experimentally and theoretically; on the other hand, only a limited amount of work was done on the infrequent processes of charged particles emission and the mechanism of their emission is less understood. The state of the field was reviewed lately by several authors.[1]

In a recent letter,[2] we proposed a model which successfully accounts for the new experimental findings of Wyttenbach *et al.*[3] on charged particles emission rates following muon capture in a wide range of nuclei, as well as for the older data on p and α spectra and rates from emulsion experiments.[4-7] In the present article, we give details of the work sketched in the letter[2] and we expand it by considering the proton emission from an additional large number of nuclei, which were used in several recent activation experiments.[3,8,9] Predictions on the emission of deuterons and α particles from various light and intermediate nuclei are also presented. Furthermore, we extend our model[2] to the calculation of neutron emission following muon capture. The agreement with experiment of the new calculations on charged particles emission rates from the additional large number of nuclei is of the same good quality as in the sample presented in the letter.[2] The results on neutron emission are generally in better agreement with experiments, especially for the high multiplicities, than in previous calculations.[1] These results thus lend additional support for the model on particle emission following μ capture in complex nuclei, which we suggested in Ref. 2. One should mention, however, that in its present form this model cannot account for the extreme high-energy tail of the charged particles spectrum, as reported in recent experiments.[17]

Before going into the details of the present work, we review summarily the background situation. As we mentioned, the neutron emission has been dealt with more extensively and the emission multiplicities and energy spectra have been measured for a wide range of nuclei in various experiments. The energy spectrum[10] has a low-energy part characteristic of evaporation ($E_n \leq 6$ MeV) and a high-energy tail, amounting in intermediate and heavy nuclei to ~10%, which originates from a more direct ejection process. On top of these, there is some evidence[1] for spectral lines, indicative of giant resonance excitations.

The gross features of neutron emission can be understood[11] by combining the emission from the compound nucleus formed after muon capture with the direct emission which is evaluted by an optical model. The nuclear excitation energy is calculated by using a simplified Hamiltonian with the weak matrix element represented by an effective constant, whereby the proton and neutron-hole nuclear-momentum distributions were assumed to have a Gaussian form. With effective nucleon masses, as obtainable from Brueckner theory of $\simeq 0.7$ M, the calculated[1,11] neutron multiplicities are in qualitative agreement with experiments,[10,12] although some remaining discrepancies are to be noted: (a) the calculated average neutron emission is 20–30% lower than the experimental one; (b) there is no satisfactory agreement for the zero-neutron-emission transition; and (c) the observed high multiplicities ($n \geq 3$) are larger than the calculated ones. Improvements on the above picture have been made by Schroder,[13] who has treated more rigorously the once scattered neutrons, and by Hadermann and Junker.[14] The latter authors generate the nucleon-momentum distribution from a Woods-Saxon potential and modify phenomenologically its high-momentum tail to account for short-range correlations. Although their results are closer to experiment, some discrepancies still remain, especially concerning points (b) and (c) above. It should be mentioned that a better fit can be achieved by leaving the effective nucleon mass M^* as a free parameter. However, the required value of $M^* \simeq 0.4\, M$ (see MacDonald *et al.*, Ref. 12) is incompatible with nuclear matter calculations and the evidence from other independent experiments.

The charged particles emission following muon capture had received less attention until recently, when the availability of meson-factory μ beams rendered its detailed investigation more feasible. The early emulsion experiments of Morinaga and Fry[4] had established that muon capture in the heavy elements of the emulsion (Ag, Br) is accompanied in 2.2% of the events by one proton emission and in 0.4–0.5 % by α emission. These findings were confirmed in subsequent emulsion experiments.[5-7] From these and later experiments[3,8,9,15] one concludes that the charged particles emission ranges from $\simeq$7–15 % in light nuclei to 0.3–3 % in heavy and intermediate ones. In the early emulsion experiments[4,5] the spectra of the emitted protons and α's have been recorded and in a later experiment[16] with semiconductor detectors the high-energy part of the charged particles spectrum ($E > 15$ MeV) has been measured for μ capture in Si, S, Ca, and Cu. In some of these experiments[6,7,16] protons and other charged particles with sizable energies of several tens of MeV have been detected. Very recently, two new counter experiments[17] have confirmed the occurrence of very energetic charged particles from μ capture in additional medium and heavy nuclei.

The early attempt of Ishii[18] to calculate the charged particles emission by using a finite temperature Fermi gas for the capturing nucleus and statistical emission from a compound nucleus failed to account, by a factor of ten, for the observed proton emission. The model[11] which we described above for the neutron emission is also found, as will be demonstrated in the present work, to be inadequate for the charged particles emission by factors varying from 3 to 10 from the intermediate to the heavier nuclei. Alternative models have considered the effect of correlations on the emission process and have postulated[19] direct proton emission from capturing pseudo-deuterons on the nuclear surface or throughout the nuclear volume.[20] The latter calculation failed to reproduce the observed rate by orders of magnitude, while the former entailed mainly a qualitative picture. Bernabéu *et al.*[21] have reconsidered muon capture by correlated nucleon pairs and have estimated the high-energy tail of proton emission by relating the processes of muon and pion capture. Kozlowski and Zglinski[22] have considered the muon capture as proceeding by excitation of giant resonances in the daughter nucleus and have used these states to compute the excitation energy distribution from which pre-equilibrium emission of particles occurs. Reasonably good agreement is achieved for several neutron spectra and rates and likewise for several channels of charged particle emission, although for intermediate and heavy nuclei the experimental rates of the p and pxn emission are unaccounted for by factors between 2 and 5. The calculated (μ, ν) rates are also off by similar factors in this model. The situation on the charged particles emission from very light nuclei, where transitions to specific configurations can in principle be calculated and measured, has been discussed by Batusov and Eramzhyan[1] and we shall not refer to these nuclei in this paper. We remark only that the agreement between theory and experiment is generally quite unsatisfactory.

An analysis of the above situation leads us to consider[2] a model for particle emission following muon capture in which two improvements are incorporated: (a) a new description for the nucleon-momentum distribution, and (b) in the emission process both pre-equilibrium and statistical emission are included.

The Gaussian form previously used for the nucleon-momentum distribution, although containing more high-momentum components than Fermi-gas or shell-model wave functions, is believed now to be still inappropriate for describing the high-energy tail of the nuclear-momentum distribution, as apparent in various experiments.

The discrepancies enumerated above concerning neutron and charged particles emission appear to all indicate that the nuclear excitation function resulting from muon capture has a higher average energy and extends to higher energies than the one obtained[1,11] with Gaussian nucleon-momentum distribution. The use of a more "realistic" distribution should remedy these deficiencies and we henceforth consider the nucleon-momentum distribution $n(p)$ recently suggested by Amado[23]

$$n(p)=N/\cosh^2\gamma p\,, \tag{1}$$

where N is a normalization constant and γ a momentum scale.

Concerning the emission process, it became evident during recent years that in nuclear reactions at excitation energies of 15–80 MeV there is sizable emission of particles during the equilibration process.[24,25] As the excitation spectrum of the nucleus following muon capture covers this same energy range (the average excitation energy being close to 20 MeV), it is imperative to consider both pre-equilibrium and equilibrium emission. This should be especially relevant for charged particles emission and for the high-energy tail of the neutron spectrum and the neutron high multiplicities emission. Henceforth we shall employ the exciton model[26] to follow the equilibration process during which particle emission may also occur, and our explicit calculations of p and n emission are performed with the hybrid model formulation of Blann and Mignerey.[27]

The plan of the paper is as follows: In Sec. II we describe the calculation of the nuclear excitation function resulting from muon capture. Section III contains a description of the formalism employed in calculating the emission processes. In Secs. IV and V we present our results for charged particles and neutron emission, respectively. Section VI is devoted to a discussion of our model and of the results we obtain for various nuclei, as well as to an analysis of the sensitivity to the parameters used in the calculations.

II. THE NUCLEAR EXCITATION FUNCTION

We assume that the elementary capture process

$$\mu^- + p \to n + \nu_\mu \tag{2}$$

proceeds on nuclear protons which can be treated as quasifree, independently moving nucleons in a momentum-dependent nuclear potential. The nuclear wave function is expressed as an antisymmetric product of individual nucleon wave functions, the influence of other nucleons being taken into account[11] through the effective mass approximation[28] and the use of an effective nucleon-momentum distribution.

We use a simplified weak-interaction Hamiltonian

$$H_W = G\tau_l \sum_{i=1}^{A} \tau_i \delta(\vec{r}_i - \vec{r}_\mu)\,, \tag{3}$$

where τ_l changes a muon into a neutrino and τ_i changes a proton into a neutron, and vanish otherwise. After summing over spins and using the value of the muon wave function at origin one arrives[11] at the rate

$$\Lambda = G^2 N |\psi_\mu(0)|^2 \int d^3k\, d^3p\, d^3q\, g(p)[1-h(q)] \times \delta(\vec{p}-\vec{k}-\vec{q})\delta(E_0-k-E)\,, \tag{4}$$

where N is a constant and we use units with $\hbar = c = 1$. $\vec{k}$, $\vec{p}$, and $\vec{q}$ are the three-momenta of the neutrino, capturing proton, and born neutron, respectively. $g(p)$ and $1-h(q)$ are the proton and neutron-hole normalized momentum distributions, the latter accounting for the Pauli exclusion principle. E is the excitation energy of the $(A, Z-1)$ nucleus, and the energy available to the process is

$$E_0 = M(A,Z) + m_\mu - M(A,Z-1) - B_\mu\,, \tag{5}$$

where B_μ is the binding energy of the muon in the K orbit. In order to relate E to the nucleon variables one assumes that the excitation energy of the $(A, Z-1)$ nucleus is given by the difference in kinetic energies of the created neutron and the capturing proton

$$E = (2M^*)^{-1}(q^2 - p^2) = E_0 - k\,. \tag{6}$$

For the effective nucleon mass we do not use its detailed momentum dependence,[28] as we find that an average constant value suffices for our purpose. In all our calculations we take $M^* = 0.68\,M$ for both the capturing and the formed nucleon, which value is consistent with nuclear calculations.[28] Integrating over d^3q and the angle between $\vec{k}$ and $\vec{p}$, and changing variables to $dE = -dk$, one obtains

$$\Lambda = K\int_0^{E_0} (E_0 - E)dE \int_{p_0}^{\infty} p g(p)[1-h(q_0)]dp\,, \tag{7}$$

where

$$p_0 = \tfrac{1}{2}\left|k - 2M^* k^{-1}(E_0 - k)\right|, \quad q_0 = (p^2 + 2M^*E)^{1/2}. \tag{8}$$

Assuming an explicit form for $g(p)$, $h(q)$ one can reexpress (7) in terms of the nuclear excitation function $I(E)$, defined by

$$\Lambda = K\int_0^{E_0} I(E)dE\,. \tag{9}$$

The single-particle momentum distribution $g(p)$

[or $h(q)$] is the probability density for finding a particle of momentum p in the nucleus, i.e.,

$$g(p)=\int |\psi(\vec{p},\vec{p}_2,\vec{p}_3,\ldots,\vec{p}_A)|^2 \times \delta\left(\vec{p}+\sum_{i=2}^{A}(\vec{p}_i)\right)\prod_{i=2}^{A}d^3p_i\,, \tag{10}$$

where ψ is the ground state nuclear wave function. This distribution is an important quantity in many nuclear processes at low, intermediate, and high energies; however, little is known about its accurate form, especially in the large momenta region.

Most of the previous calculations of the nuclear excitation following μ capture which used the above single-particle capture model had assumed[1] Gaussian distributions for $g(p)$ and $h(q)$, sometimes with certain corrections to allow for nuclear correlations. As we have stated in the Introduction, the excitation functions so calculated do not have a high-energy component of appropriate strength to account correctly for high neutron multiplicities and charged particles emission.

Recently, the nucleon-momentum distribution in the nucleus has been the subject of several analyses.[23,29] Amado and Woloshyn[23] have investigated $g(p)$ in general as well as with the aid of an ideal one-dimensional model of N bosons interacting via δ-function forces.[30] Generally, they find that the momentum distribution function behaves asymptotically like p^{-4-2m}, where m depends on the form of the two-body potential and can be zero, as for example, with the δ-function forces. They also show that in the one-dimensional model there is an intermediate (though quite limited) region in which $g(p)$ decreases exponentially. In particular, an exponential decrease obtains in the Hartree approximation of the one-dimensional problem where the exact solution found for $g(p)$ is just our Eq. (1), $g(p)$ behaving asymptotically like $\exp(-2\gamma p)$. These results were derived in an ideal model and the inclusion of Fermi statistics and realistic nucleon-nucleon forces is expected[23] to lead to a more rapid asymptotic decrease than the power law, thus indicating the possible validity of (1) as an appropriate description of the nucleon-momentum distribution in nuclei. In Ref. 29 additional support for these conclusions can be found.

Amado and Woloshyn have used[31] the distribution (1) to explain the energetic (0.1–0.4 GeV) backward protons observed[32] in $A(p,p')X$ reactions with 0.6–0.8 GeV protons incident on various targets. Assuming a single scattering mechanism, they first determined γ from quasielastic electron scattering and then showed that with the same value of this parameter a good fit to Frankel's data[32] is obtained. Further analyses[33] of these experiments have strengthened the view that the nucleon-momentum distribution decreases exponentially in p rather than like a Gaussian. It should be mentioned, however, that alternative descriptions for the results of Ref. 32 have also been put forward,[34] and that in any case an unambiguous interpretation of these experimental results in terms of a nucleon-momentum distribution of the target nuclei is generally a difficult proposition.[31]

In our problem, a wide range of momenta is involved, though it is largely the high-energy tail of $g(p)$ which previously was poorly accounted for. We conjecture now, for simplicity, that (1) can be used for the full nucleon-momentum distribution in nuclei and we calculate with it the nuclear excitation function $I(E)$ of Eq. (9). In Fig. 1 we give the nucleon-momentum distribution (1) with $\gamma=0.8$ fm (which is the value chosen for our calculations) and for comparison a widely used Gaussian momentum distribution $f(p)=\exp(-p^2/\alpha^2)$ with $\alpha^2/2M=20$ MeV. In Fig. 2 we present the nuclear excitation function $I(E)$ for muon capture in ^{79}Br, calculated with Gaussian and Amado distributions (with $\gamma=0.8$ fm and 0.5 fm).

The average excitation energy $\langle E\rangle$ of the nucleus after muon capture is found to be approximately 18 MeV with the excitation function derived from (1) and $\gamma=0.8$ fm, while a Gaussian nucleon-momentum distribution gives an average excitation energy of 4–5 MeV less. Some typical results for the average excitation energy from $A=28$ to the heaviest nuclei are given in Table I, where a comparison is made with a Gaussian distribution and

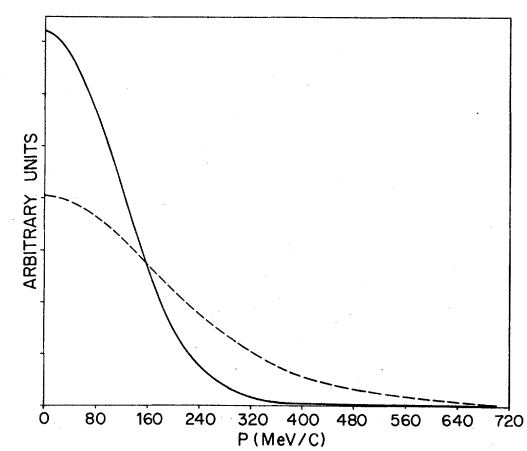

FIG. 1. Comparison of the Amado nucleon-momentum distribution $f(p)=N\cosh^{-2}\gamma p$ for $\gamma=0.8$ fm which is used in the present model (dashed curve) with a Gaussian nucleon-momentum distribution $f(p)=N'\exp(-p^2/\alpha^2)$ for $\alpha^2/2M=20$ MeV (solid curve).

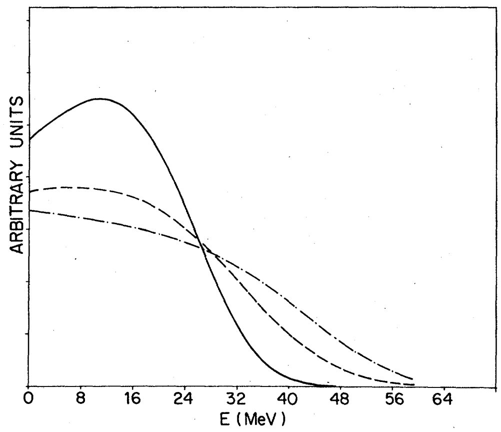

FIG. 2. The nuclear excitation function from muon capture in ^{79}Br calculated with different nucleon-momentum distributions: Amado distribution [Eq. (1)] with $\gamma = 0.5$ fm (dash-dotted curve), Amado distribution with $\gamma = 0.8$ fm (dashed curve), Gaussian distribution (solid curve).

with results of Christillin *et al.*[35] An examination of Table I shows that the $\langle E \rangle$ we calculated is essentially constant around 17–18 MeV over the periodic table, the small scatter having to do mainly with individual nuclear structure effects. Christillin *et al.*[35] have determined the average excitation energies required to reproduce the experimental muon capture rates in their formula. Their values exhibited in Table I agree quite well with ours. (The values of Ref. 35 for $\langle E \rangle$ refer to natural elements, while ours are for specific isotopes; the corrections are usually only a few tenths of an MeV.) Thus, our results give support to their contention of an essentially constant $\langle E \rangle$ over most of the periodic table, implying a decreasing average neutrino energy $\langle k \rangle$ where $\langle k \rangle = m_\mu - B_\mu + M(A,Z) - M(A,Z-1) - \langle E \rangle$. This fact has implications[1,35] for the theoretical approaches used to calculate total muon capture rates. Conversely, our calculated $\langle E \rangle$ could, in principle, be used to check the accuracy of the formula for capture rates of Ref. 35. However, it should be kept in mind that the model we use is not refined enough to include all the specific properties of individual nuclei in the derivation of the excitation energy. Our calculated $\langle E \rangle$ agree generally also with those determined by Evseev and Mamedov,[36] who obtained values of $\langle E \rangle \simeq 17$–$19$ MeV from the neutron spectra emitted in μ capture, for several intermediate and heavy nuclei. Finally, Baertschi *et al.*[37] estimate the average excitation energy of the compound nucleus formed in μ capture in U^{238} to be around 18 MeV, from the peak-to-valley ratio in the mass distribution of the fission products. Thus, one can conclude that the average excitation energies calculated from our excitation function are well corroborated by various other sources.

III. THE EMISSION PROCESS

The nuclear excitation function after μ capture (Fig. 2) spans a range of energies extending beyond 60 MeV, with an average close to 20 MeV (Table I). During the last ten years it became evident[24,25] that in nuclear reactions at similar energies, the extreme situations of direct reactions and equilibrium emission are unable to account fully for the observed features of the spectra of emitted particles. There is strong evidence from (α,p), (p,p'), (p,n), and (n,p) reactions that a sizable amount of particle emission occurs during the equilibration process. The recent review of Blann[25] contains many convincing examples and an extensive analysis of the pre-equilibrium emission. We are thus naturally led to argue that an attempt to account for the emission rates and spectra following muon capture should include the possibility of pre-equilibrium emission, in addition to the emission from the compound stage. Since in this respect the situation encountered in muon capture should not be different from the one in which a comparable amount of energy is injected into the nucleus by the use of other means, we expect that models accounting for nuclear

TABLE I. Average excitation energy $\langle E \rangle$ of the nucleus following μ capture.

Capturing nucleus	^{28}Si	^{39}K	^{55}Mn	^{56}Fe	^{59}Co	^{60}Ni	^{79}Br	^{107}Ag	^{127}I	^{133}Cs	^{181}Ta	^{197}Au	^{208}Pb	^{209}Bi
$\langle E \rangle$ in MeV														
Present model [Eq. (1) with $\gamma = 0.8$ fm]	17.9	18.3	18.0	17.8	18.2	17.9	18.3	18.0	17.6	17.7	17.0	16.8	16.0	16.7
Gaussian momentum distribution (with $\alpha^2/2M = 20$ MeV)	13.9	14.3	14.1	13.9	14.2	14.0	13.5	13.9	13.7	13.4	13.2	13.1	12.4	13.0
Ref. 35			16.9	15.8	17.4			20	18.8			17.6	15.5	17.9

reactions should also be appropriate for our calculation. We shall therefore use the same formalism which proved successful in nuclear reactions, without changes in parameters. In the following, we describe the formulations we employed for the pre-equilibrium and statistical emission.

Griffin has proposed[26] the exciton model in order to deal with the equilibration process, which is achieved thereby by a series of two-body interactions. Each intermediate stage is characterized by the number of excitons (particles plus holes) and at each stage there is a fraction of unbound particles, whose probability for being emitted can be calculated. The model has been further developed by Blann, Gadioli, Miller, and co-workers.[38-40] In our calculation we employ the hybrid model approach[25] to pre-equilibrium emission. In this approach one assumes that transitions among excited states occur always towards the more complicated states ($n \rightarrow n+2$), the excited particle populations during equilibration are calculated by use of partial state densities, and intranuclear transitions rates of the excited particles are determined by use of nucleon mean free paths in nuclear matter. The pre-equilibrium emission probability of particle x with energy ϵ after μ capture is then given by

$$P_x(\epsilon)d\epsilon = \int_0^{E_0} I(E)dE \sum_{\substack{n=n_0 \\ (\Delta n=2)}}^{\bar{n}} {}_nf_x\left[\frac{\rho_{n-1}(U,\epsilon)}{\rho_n(E)}g\right] \times \left[\frac{\lambda_c(\epsilon)}{\lambda_c(\epsilon)+\lambda_{n+2}(\epsilon)}\right]D_n d\epsilon \equiv \int_0^{E_0} I(E)dE \sum_n {}_nP_x(\epsilon, E)d\epsilon\,, \tag{11}$$

in which the decay probability as derived by Blann *et al.*[27,38] is weighted with the nuclear excitation spectrum $I(E)$ [Eq. (9)]. ${}_nf_x$ is the number of nucleons of type x in an n-exciton state and the expression in the first bracket gives the fraction of the n-exciton state population having one particle in an unbound level with energy ϵ, which if emitted leaves the daughter nucleus with excitation energy U. $\lambda_c(\epsilon)$ is the transition rate into continuum and $\lambda_{n+2}(\epsilon)$ is the transition rate from the n to the $n+2$ exciton state. The second bracket thus gives the fraction of particles which are emitted with energy ϵ rather than undergoing an internal transition. D_n is the depletion factor, expressing the fraction of the population surviving emission from states $m<n$:

$$D_n = \prod_{n'=n_0}^{n}\left[1-\sum_{x=p,n}\int_{(n'-2)} P_x(\epsilon)d\epsilon\right]. \tag{12}$$

Using for the calculation of internal transitions the average mean free path of excited nucleons in nuclear matter as given by Kikuchi and Kawai,[41] and the Ericson expression for the intermediate state densities (the exciton number m equals the number of holes h plus the number of particles in state m)

$$\rho_m(E) = \frac{g(gE)^{m-1}}{(m-h)!\,h!\,(m-1)!}\,, \tag{13}$$

where $g_x = (A/28)$ (MeV^{-1}) (x = protons or neutrons), Blann and Mignerey[27] arrive at the following numerical expression for the pre-equilibrium decay probability of a particle $x=n,p$, with energy ϵ:

$${}_nP_x(\epsilon, E) = \frac{(n-1)\,{}_nf_x}{E}\left(\frac{U}{E}\right)^{n-2} D_n\left\{\frac{\sigma_x\epsilon/g_x}{\sigma_x\epsilon/g_x + k^{-1}[1890(\epsilon+B_x)-8(\epsilon+B_x)^2]}\right\}. \tag{14}$$

Here σ_x is the inverse reaction cross section in mb and ϵ and B_x (the binding energy of particle x) are in MeV. With $k=1$ one reproduces the mean-free-path values of Ref. 41 of 3–4 fm in our energy range, while larger values of k would imply larger mean-free-path values which could reflect, for instance, the inclusion of surface effects. A very similar expression is obtained[38] if, instead of using the values of Kikuchi and Kawai[41] λ_{n+2} is calculated from the nuclear optical potential.

In the muon capture process, the initial exciton number is $n_0=2$, with one proton hole and one excited neutron from the elementary reaction $\mu^- + p \rightarrow n + \nu_\mu$. From this stage only neutron emission can occur, the first exciton state from which a proton can be emitted being $n=4$.

For the fraction of the reaction cross section which does not emit during equilibration, we use the well-known statistical model of Weisskopf and Ewing,[42] according to which the probability of emission of particle x with energy ϵ_x from a compound nucleus of excitation energy E is

$$P_x(\epsilon_x) = [(2s_x+1)/\pi^2]m_x\epsilon_x\sigma_x(\epsilon_x)[\rho_x(U)/\rho_c(E)]\,, \tag{15}$$

where s_x and m_x are the spin and mass of the emitted particle, $\rho_x(U)$ is the level density function of the residual nucleus of excitation energy U, and $\rho_c(E)$ the level density function of the compound nucleus. The probability of emitting particle x after μ capture is given by

$$P_x = \int_0^{E_0} I(E)dE \frac{\int P_x(\epsilon_x, E)d\epsilon_x}{\sum_i \int P_i(\epsilon_i, E)d\epsilon_i} \tag{16}$$

and the summation was performed over $i = n, p, d, \alpha$, thus neglecting the small amount of ^{3}He, ^{3}H emission. Since we are interested in the gross features of the process for a wide range of nuclei, we found it sufficient for our purpose to use for σ_x in (14) and (16) the parametrization of Dostrovsky *et al.*,[43] which is known to be adequate for a wide range of nuclei in the energy regime explored here. Specifically, we used

$$\sigma_n(\epsilon) = \sigma_g \alpha(1 + \beta/\epsilon), \tag{17a}$$

$$\sigma_g = \pi R^2, \quad R_n = 1.5A^{1/3} \text{ fm}, \tag{17b}$$

$$\alpha = 0.76 + 2.2A^{-1/3}, \quad \beta = (2.12A^{-2/3} - 0.05)/\alpha, \tag{17c}$$

and for the charged particles

$$\sigma_c(\epsilon) = \sigma_g(1 + K_c)(1 - V_{\text{eff}}/\epsilon), \tag{18a}$$

$$R_c = 1.7A^{1/3} \text{ fm}, \quad c = p, d, \alpha, \tag{18b}$$

$$V_{\text{eff}} = k_c V_c, \quad V_c = z_c Z e^2/(R_c + \rho_c), \tag{18c}$$

where $\rho_c = 0$ for protons and $\rho_c = 1.2$ fm for α's and deuterons. Values of the parameters K_c and k_c are given in Ref. 43 and for the missing nuclei we interpolated.

For the level density function of the residual nucleus we use the following expression, which was found[44] to be an adequate representation in a large number of nuclear reactions at energies of a few tens of MeV:

$$\rho(U) \alpha \frac{a^{1/2} \exp[2(aU)^{1/2}]}{A^{5/2}(U+t)^2}, \tag{19}$$

where a is proportional to the single-particle level density and we used $a = A/10$ for nuclei with $A \leq 90$, and $a = A/15$ for $A > 90$. These expressions are known to represent quite well the fitted values[36, 43-45] of a in nuclear reactions of appropriate energies and ranges of masses. The thermodynamic temperature t relates to the excitation energy by

$$U = at^2 - t. \tag{20}$$

The excitation energy of the residual nucleus was taken as

$$U = E - B_x - \epsilon_x - \delta_x, \tag{21}$$

and for the pairing energy δ_x we took $\delta = 2.8$ MeV for even and $\delta = 1.4$ MeV for even-odd nuclei.[46]

IV. CHARGED PARTICLES EMISSION

Our main goal in this work was to achieve a satisfactory explanation for the previously unaccounted charged particles emission rates following muon capture. We start therefore the presentation of our results with the charged channels, Sec. V being devoted to the application of our model to neutron emission.

Calculations were performed for proton, deuteron, and α emission. For protons, we calculated both the precompound and the statistical emission by using formulas (11), (14), and (15), (16), respectively. From these expressions, the probability of emitting a proton as the primary particle in each of the processes is calculated. The probability of emitting a proton as the second particle is also obtainable, by determing the energy excitation function of the residual nucleus which is left after the first (n or p) particle emission. This turns out, however, to be only a small correction in our energy range because of the Coulomb barrier, and it was neglected. The experimental result of Batusov *et al.*[7] on the rate of two-charged particles emission from the heavy emulsion nuclei ($\lesssim 0.2\%$) confirms this approach, at least for this particular channel.

We have calculated single as well as inclusive proton emission and we use for these modes the notation (μ, p) and $(\mu, \tilde{p})$, respectively, where

$$(\mu, \tilde{p}) \equiv (\mu, p) + (\mu, pn) + (\mu, p2n) + \cdots + (\mu, d) + (\mu, dn) + \cdots.$$

The deuteron emission is included in the results we give for the inclusive proton channel, since most of the experimental results available are from activation experiments which do not distinguish between (μ, pn) and (μ, d), etc. The calculation of the (μ, p) channel is achieved by limiting in (11) and (16) to an excitation energy of the residual nucleus $U < B_m$, where B_m is its smallest separation energy. The protons emitted through the (μ, p) channel are therefore those having the highest energies.

The calculation of the statistical emission is done only for the nuclei reaching the equilibrium stage without prior emission, which we find to happen in 70–90% of the cases, depending on A and on the particulars of the nucleus. Thus, the small amount of equilibrium emission from nuclei which emitted during equilibration is neglected. The magnitude of the correction can be traced by referring to Chevarier *et al.*[47] who find by using a similar formalism that the statistical emission from nuclei which emitted during equilibration represents a 20% addition to the evaporation process in (α, p) reactions at approximately 60 MeV excitation energy. As we deal with an excitation spectrum of 20 MeV average energy, the inclusion of this emission will probably increase our results by less than 10% in the medium nuclei and will have no practical effect on our results for the heavier nuclei, where the pre-equilibrium emission dominates the proton channels. Hence, the total emission rate for single-proton and proton-inclusive reactions is given in our formalism by

$$(\mu,p)_T=(\mu,p)_{\rm PE}+q(\mu,p)_{\rm ST}\,, \tag{22a}$$

$$(\mu,\tilde{p})_T=(\mu,\tilde{p})_{\rm PE}+q(\mu,\tilde{p})_{\rm ST}\,, \tag{22b}$$

where PE and ST stand for pre-equilibrium and statistical emission, respectively, and T stands for total emission. q gives the probability that the $(A, Z-1)$ excited nucleus formed in the capture process does not emit during the equilibration process, i.e.,

$$q=1-(\mu,\tilde{p})_{\rm PE}-(\mu,\tilde{n})_{\rm PE}\,, \tag{23}$$

where $(\mu,\tilde{n})_{\rm PE}$ is the rate for inclusive precompound neutron emission, with the neutron as the first emitted particle. In Table II we present our results for proton emission in the single (μ,p)

TABLE II. Calculated probabilities per muon capture for the reaction ${}^{A}_{Z}X(\mu,p)\,{}^{A-1}_{Z-2}X$ and for inclusive proton emission $(\mu,\tilde{p})$. The experimental data are from Ref. 3, except when otherwise referenced. For $(\mu,\tilde{p})$ the experimental figures are lower limits, determined from the actually measured channels. The figures in parentheses are estimates for the total inclusive rate derived from the measured exclusive channels by the use of the approximate regularity noted in Ref. (3): $(\mu,p):(\mu,pn):(\mu,p2n):(\mu,p3n)=1:6:4:4$. PE stands for pre-equilibrium.

Capturing nucleus	(μ,p) Present calculation % of PE emission	(μ,p) Present calculation 10^3 times total emission	(μ,p) 10^3 times experiment	$(\mu,\tilde{p})$ Present calculation % of PE emission	$(\mu,\tilde{p})$ Present calculation 10^3 times total emission	$(\mu,\tilde{p})$ 10^3 times experiment
${}^{27}_{13}$Al	2.2	9.7	(4.7)	7.1	40	>28 ± 4 (70)
${}^{28}_{14}$Si	1.9	32	53 ± 10[a]	6.9	144[b]	150 ± 30[b]
${}^{31}_{15}$P	2.4	6.7	(6.3)	7.8	35	>61 ± 6 (91)
${}^{39}_{19}$K	1.8	19	32 ± 6[a]	9.2	67	
${}^{41}_{19}$K	4.0	5.1	(4.7)	10	30	>28 ± 4 (70)
${}^{51}_{23}$V	5.0	3.7	2.9 ± 0.4	12	25	>20 ± 1.8 (32)
${}^{55}_{25}$Mn	7.4	2.4	2.8 ± 0.4	17	16	>26 ± 2.5 (35)
${}^{59}_{27}$Co	6.4	3.3	1.9 ± 0.2	14	21	>37 ± 3.4 (50)
${}^{60}_{28}$Ni	5.2	8.9	21.4 ± 2.3[c]	16	49	40 ± 5[c]
${}^{63}_{29}$Cu	6.0	4.0	2.9 ± 0.6	13	25	>17 ± 3 (36)
${}^{65}_{29}$Cu	13	1.2	(2.3)	23	11	>35 ± 4.5 (36)
${}^{75}_{23}$As	11	1.5	1.4 ± 0.2	18	14	>14 ± 1.3 (19)
${}^{79}_{35}$Br	11	2.7		14	22	[22][d]
${}^{94}_{40}$Zr	31	0.48	(0.75)	35	9.2	(11)
${}^{107}_{47}$Ag	13	2.3		19	18	[11][d]
${}^{115}_{49}$In	27	0.63	(0.77)	30	7.2	>11 ± 1 (12)
${}^{121}_{51}$Sb	39	0.33	(0.49)	40	4.2	>7 ± 0.6
${}^{133}_{55}$Cs	27	0.75	0.48 ± 0.07	29	8.7	>4.9 ± 0.5 (6.7)
${}^{165}_{67}$Ho	58	0.26	0.30 ± 0.04	47	4.1	>3.4 ± 0.3 (4.6)
${}^{181}_{73}$Ta	73	0.15	0.26 ± 0.04	57	2.8	>0.7 ± 0.1 (3.0)
${}^{208}_{82}$Pb	~100	0.14	0.13 ± 0.02	~100	1.1	>3.0 ± 0.8 (4.1)
${}^{209}_{83}$Bi	~100	0.04	0.08 ± 0.01	88	1.4	(1.2)

[a] Reference 8.

[b] Reference 15. The experimental and theoretical figures for $(\mu,\tilde{p})$ refer in this case to total charged particles emission.

[c] Reference 9.

[d] Interpolation values given by Ref. 3.

and inclusive $(\mu,\tilde{p})$ channels, respectively. We take $k=1$ in Eq. (14), which corresponds to the mean-free-path values of Kikuchi and Kawai[41] and was used by Blann and collaborators in all their calculations.[25,27] The experimental results quoted are from Ref. 3, except when otherwise stated. In this experiment partial rates for (μ,pxn) have been measured with $x=0,1,2,3$. However, for most nuclei investigated, only some of these channels are actually measurable by the activation method. Hence, the experimental figures for $(\mu,\tilde{p})$ are mostly lower limits, determined from a summation over the measured channels. In addition, we give in parenthesis "calculated" experimental figures, which were determined with the aid of the actually measured partial rates by means of the following approximate regularity for a given target noted by the authors of Ref. 3:

$$(\mu,p):(\mu,pn):(\mu,p2n):(\mu,p3n)=1:6:4:4\,. \qquad (24)$$

The experimental numbers for Ag and Br in the square brackets are estimates of Ref. 3 by interpolation.

Turning now to α-particle emission, our results for the (μ,α) channel are presented in Table III and comparison is made again with the experiment of Ref. 3. These calculations include statistical α emission only, after taking into account the pre-equilibrium emission of protons and neutrons from the $(A,Z-1)$ excited nucleus [i.e., the q factor of Eq. (23)]. Thus, in our model

$$(\mu,\alpha)_T=q(\mu,\alpha)_{\mathrm{S\,T}}\,. \qquad (25)$$

It is known[44] that in nuclear reactions at similar energies the compound nucleus emission is the dominating process for α emission from nuclei with $A<80$. As the existing data[3] on α-particle emission following μ capture indeed refer to such nuclei, and moreover, a satisfactory treatment for compound-particle pre-equilibrium emission is still lacking,[25] we chose to consider statistical emission solely, in dealing with α's and d's. The remarkable agreement of our calculation with experiment as demonstrated in Table III confirms the assumption that for the regime considered here, this is indeed the dominant mechanism.

TABLE III. 10^3 times the calculated probabilities per muon captured for the reaction ${}^{A}_{Z}X(\mu,\alpha){}^{A-4}_{Z-3}X$, compared to 10^3 times the experimental data of Ref. 3.

Capturing nucleus	Present calculation	Experiment
^{23}Na	10	11 ±1.5
^{27}Al	7.3	7.6 ±1.1
^{28}Si	17	
^{31}P	10	13 ±2
^{39}K	20	
^{51}V	1.6	1.5 ±0.2
^{55}Mn	2.3	1.6 ±0.2
^{56}Fe	3.8	4.6 ±0.7
^{59}Co	1.4	
^{65}Cu	0.36	0.7 ±0.2
^{75}As	0.40	>0.28±0.04
^{79}Br	0.48	

TABLE IV. 10^3 times the calculated probabilities per muon captured for the reaction ${}^{A}_{Z}X(\mu,d){}^{A-2}_{Z-2}X$ and for inclusive deuteron $(\mu,\tilde{d})$ and alpha-particle $(\mu,\tilde{\alpha})$ emission.

Capturing nucleus	(μ,d)	$(\mu,\tilde{d})$	$(\mu,\tilde{\alpha})$
^{23}Na	4.6	9.0	29
^{27}Al	6.0	12	20
^{28}Si	8.2	21	34
^{31}P	3.9	8.2	26
^{39}K	3.6	14	50
^{51}V	1.2	6.4	5.8
^{55}Mn	0.34	2.2	11
^{56}Fe	1.3	4.3	14
^{59}Co	0.32	2.2	7.4
^{65}Cu	0.12	1.1	2.9
^{75}As	0.11	1.2	3.5
^{79}Br	0.18	1.7	3.8

We remark that in Table II the inclusive figure we give for ^{28}Si is, in fact, for the total charged emission, i.e., $(\mu,\tilde{p})+(\mu,\tilde{d})+(\mu,\tilde{\alpha})$, since the experimental value quoted refers to this quantity. The 14.4% charged particles emission we calculated consists of 8.8% $(\mu,\tilde{p})$, 2.2% $(\mu,\tilde{d})$, and 3.4% $(\mu,\tilde{\alpha})$.

Of special interest is the emission from the heavy nuclei of nuclear emulsion, which the earlier experiments[4-6] recorded. Our results are

$$(\mu,\tilde{p})_{\mathrm{Ag,Br}}=1.9\%,\quad (\mu,\tilde{\alpha})_{\mathrm{Ag,Br}}=0.38\%\,, \qquad (26)$$

while the experimental figures are 2.2 ± 0.4 % and 0.5 ± 0.1 %, respectively. The calculated spectrum of protons is in very good agreement with the experimental one and it was presented, together with the α spectrum, in our previous report.[2] The slight discrepancy for α's is probably due to the fact that for these heavier nuclei, there is also a small amount of pre-equilibrium emission.

Finally, we have calculated the inclusive α and d as well as single-d emission from some 10 nuclei with $A<80$ with the assumption embodied by Eq. (25). These results are given in Table IV. Their measurement should provide a good test for our model.

V. NEUTRON EMISSION

The model we proposed[2] was shown in the previous sections to account satisfactorily for the

TABLE V. Calculated neutron multiplicities compared to the experimental data of MacDonald *et al.* (Ref. 12).

Multiplicity F_i	Ag Theory $\theta=1.47$ MeV	$\theta=1.0$ MeV	Experiment	I Theory $\theta=1.28$ MeV	$\theta=1.0$ MeV	Experiment
F_0	0.430	0.420	0.360 ± 0.021	0.425	0.418	0.396 ± 0.021
F_1	0.388	0.379	0.456 ± 0.023	0.382	0.376	0.474 ± 0.023
F_2	0.141	0.150	0.144 ± 0.017	0.146	0.152	0.087 ± 0.015
F_3	0.036	0.045	0.031 ± 0.009	0.041	0.046	0.035 ± 0.009
F_4	0.004	0.006	0.007 ± 0.005	0.005	0.007	0.007 ± 0.005
F_5	0.0003	0.0005	0.002 ± 0.004	0.0004	0.0006	0.0002 ± 0.0004
$\langle n\rangle$	1.46	1.54	1.615 ± 0.060	1.50	1.56	1.436 ± 0.056

	Au Theory $\theta=1.37$ MeV	$\theta=1.0$ MeV	Experiment	Pb Theory $\theta=1.20$ MeV	$\theta=1.0$ MeV	Experiment
F_0	0.416	0.406	0.370 ± 0.015	0.393	0.388	0.324 ± 0.022
F_1	0.379	0.370	0.425 ± 0.016	0.382	0.376	0.483 ± 0.025
F_2	0.154	0.162	0.156 ± 0.012	0.166	0.171	0.137 ± 0.018
F_3	0.044	0.052	0.032 ± 0.006	0.050	0.054	0.045 ± 0.010
F_4	0.006	0.009	0.014 ± 0.004	0.008	0.010	0.011 ± 0.006
F_5	0.0005	0.0009	0.003 ± 0.003	0.0007	0.0009	
$\langle n\rangle$	1.56	1.63	1.662 ± 0.044	1.65	1.70	1.709 ± 0.066

charged particles emission. A stringent test would then be its applicability to the main de-excitation process, the neutron emission, since a valid model for the particle emission following μ capture should indeed be able to account for the major features of both types of emission.

Approximately 1.5 neutrons are emitted per μ capture in intermediate and heavy nuclei, and their energy spectrum is composed of an evaporation part and a high-energy one, the latter extending from $\simeq5$ MeV to several tens of MeV. These fast neutrons represent about 7–10% of the neutron intensity in heavy nuclei.[10] We proceed to calculate with our model the probabilities for emitting 0, 1, 2,... neutrons (the so-called "multiplicities") as well as the average emission per μ capture, and compare them with the data of MacDonald *et al.*[12]

The high-energy part is given in our model primarily by the pre-equilibrium neutron emission from the $n=2$ exciton state, which is the main source of "direct neutrons" in this approach. Thus, we take

$$(\mu,n)_{\text{direct}}=(\mu,n)_{\text{PE}\,(n=2)}\,. \tag{27}$$

The calculated probability of emitting one direct neutron by pre-equilibrium emission for several selected heavy nuclei is then obtained from (14) and (27) to be

$$^{107}\text{Ag}:5.6\%,\quad ^{127}\text{I}:5.3\%,\quad ^{197}\text{Au}:4.3\%,\quad ^{208}\text{Pb}:5.2\%. \tag{28}$$

The emission from the compound nucleus is estimated by assuming an evaporation spectrum of the form $n(\epsilon)\sim\epsilon\exp(-\epsilon/\theta)$, where θ is the nuclear temperature of the residual nucleus, with θ taken to be constant for successive emissions. This simple form allows then the derivation of a closed expression[11,12] for the probability P_i of emitting at least i neutrons from the compound nucleus having excitation energy E,

$$P_i(E)=1-\exp\left(-\frac{E-B_i}{\theta}\right)\sum_{n=0}^{2i-3}\frac{1}{n!}\left(\frac{E-B_i}{\theta}\right)^n \tag{29}$$

for $i\geqslant2$ and

TABLE VI. Calculated probabilities $W_{\mu,\nu}$ per muon captured for the reaction $^A_ZX(\mu)^A_{Z-1}X$ compared to the experimental data of Ref. 48.

Nucleus	^{27}Al	^{51}V	^{127}I	^{181}Ta	^{197}Au	Pb
$W_{\mu,\nu}$						
Present calculation	0.11	0.12	0.16	0.15	0.16	0.14
Experiment (Ref. 48)	0.10 ± 0.01	0.10 ± 0.01				0.09 ± 0.015

$$P_0 = 1, \quad P_1 = \begin{cases} 1 & \text{for } E \geq B_1, \\ 0 & \text{for } E < B_1, \end{cases} \tag{30}$$

where B_i is the binding energy of i neutrons. The probability M_i of emitting i neutrons from the compound nucleus formed in μ capture is then

$$M_i = \int_{B_i}^{E_0} P_i(E) I(E) dE - \int_{B_{i+1}}^{E_0} P_{i+1}(E) I(E) dE \,. \tag{31}$$

Before a comparison can be made with the data, the neutron multiplicities M_i have to be converted to the observed ones F_i, which include the observational efficiency of $\eta = 0.545$ in the experiment of MacDonald *et al.*[12] The conversion formula is[12]

$$F_i = \eta^i \sum_{k=i} M_k (1-\eta)^{k-i} \frac{k!}{i!\,(k-i)!} \,. \tag{32}$$

For the compound-nucleus-emission calculation one has to choose an appropriate temperature. There is evidence[10, 36] that the effective temperatures determined from the neutron evaporation spectrum following μ capture are substantially higher (by factors of 1.5–2.5) than those determined from (n, n') reactions at an energy similar to the average energy of the μ capture excitation spectrum (except for the Pb region where the values from both types of processes agree). The authors of Ref. 36 have determined temperatures of 1.2–1.5 MeV for heavy nuclei ($125 < A < 210$) in μ capture, while in Ref. 10 the temperatures determined from μ capture in Tl, Pb, and Bi are between 1.05 and 1.25 MeV. We perform our calculations for each nucleus at two temperatures (the higher one being from the compilation of Ref. 36) thus putting into evidence its effect on the multiplicity values and on the average neutron emission. The results for multiplicities F_i and average neutron emission $\langle n \rangle$ from four heavy nuclei calculated with the inclusion of both evaporation and direct neutrons [Eqs. (27)–(32)] are compared in Table V with the experiment of MacDonald *et al.*[12] We remark that our calculation does not include the pre-equilibrium neutron emission from the higher exciton states ($n \geq 4$), which we estimate to be at most a 10% correction to the figures of Table V.

A quantity of special interest is the probability $W_{\mu,\nu}$ that muon capture is unaccompanied by nuclear particle emission, i.e., transitions to ground state or excited states with $E < B_i$ in the

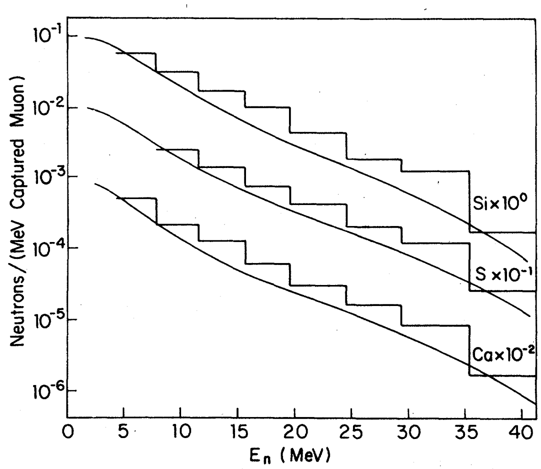

FIG. 3. Calculated neutron energy spectrum from muon capture in Si, S, and Ca, versus the experimental results of Ref. 49.

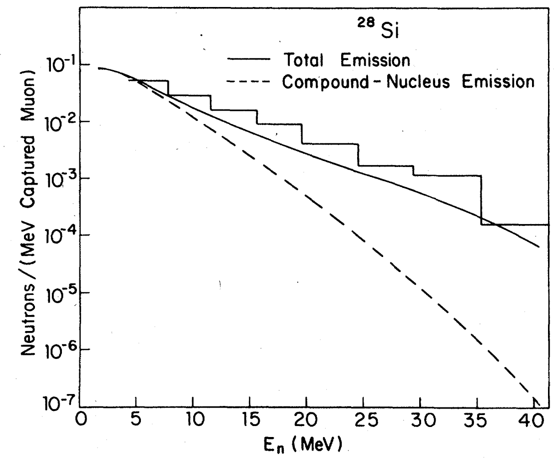

FIG. 4. Calculated neutron energy spectrum emitted from the compound nucleus formed after muon capture in ^{28}Si and the total neutron emission calculated from pre-equilibrium and compound-nucleus emission. The experimental histogram is from Ref. 49.

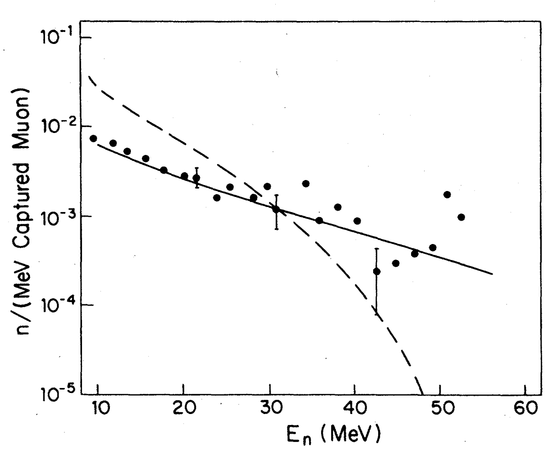

FIG. 5. The neutron energy spectrum from muon capture in Pb calculated with the present model (dashed curve), compared with the theoretical curve of Ref. 51 (solid curve) and with the experimental results of Ref. 50.

$(A, Z-1)$ nucleus. There appears to be some disagreement between the experiments[12,48] on the strength of the (μ, ν) channel, which is reported to range between 0 and 10% in intermediate and heavy nuclei. The figures calculated with our model (Table VI) are generally in good agreement with the experimental findings of Bunatyan *et al.*[48] who measure these transitions directly in activation experiments.

Finally, we give in Fig. 3 the neutron energy spectrum calculated with our model for capture in Ca, S, and Si, compared to the experimental one of Sundelin and Edelstein.[49] In Fig. 4 we display the separate contribution of the statistical emission to the spectrum in ^{28}Si. As expected, the emission becomes essentially of pre-equilibrium nature for $E_n \geq 15$ MeV. In Fig. 5 we compare our calculated spectrum for capture in Pb with the data of Krieger[50] and with the spectrum derived by Singer, Mukhopadhyay, and Amado[51] by relating it to an inclusive strong nuclear process. The theoretical curves are normalized in this case to experiment at $E_n = 30$ MeV.

VI. DISCUSSION

We have presented a model for particle emission following muon capture, based on the following main assumptions: (a) The nuclear excitation function can be calculated by considering the nucleons as quasifree particles moving in a momentum-dependent potential,[28] with a "realistic" nucleon-momentum distribution in the nucleus as given by Eq. (1). (b) The deexcitation of the nucleus proceeds via both pre-equilibrium and compound-nucleus emission. The agreement obtained for the rates of single and inclusive proton emission (Table II), α emission (Table III), and neutron emission (Table V) as well as spectra (Figs. 3–5 and Ref. 2) is most remarkable indeed and shows that the emission of various kinds of particles following muon capture is accountable by this approach. This picture should be viewed in light of the fact that we have treated a very large number of nuclei spanning the range $23 \leq A \leq 209$ and using *the same parameters* for all nuclei in the equations leading to the excitation function and governing the emission process [Eqs. (1), (14), and (16)]. No attempt was made of parameter fitting in order to take into account specific properties of individual nuclei, beyond the use of the general parametrization of Eqs. (17) and (18). Thus, one would not expect more than rough overall agreement, provided the approach is basically correct, and in fact the results we obtained are in this sense beyond expectation. With the hoped for improvement in data availability, one should consider more detailed theoretical treatments of these processes for specific nuclei.

Our results on proton emission (Table II) show that both pre-equilibrium and compound-nuclear emission are essential in providing an explanation for the rates and spectra. The compound-nucleus emission is important in light nuclei and it is slowly taken over by pre-equilibrium emission as A increases. The latter accounts for a few percent of the proton emission in light nuclei, increasing to several tens of percent for $100 < A < 180$ and dominating completely the process in very heavy nuclei. This trend is similar to what one encounters in nuclear reactions at similar energies.[24,25] Also, it is the pre-equilibrium emission which dominates the higher-energy part of the spectrum [see Fig. 1(a) of Ref. 2 and Figs. 3–5 of this paper for the neutron spectrum]. The results of Table III on α-particle emission are in impressive agreement with the available measurements in eight nuclei with $23 \leq A < 80$. This shows that the compound-nucleus emission, which is the only one considered for the (μ, α) channel, is indeed the major process in this region. Most probably, the pre-equilibrium emission will play a role for heavier nuclei or at higher energies.[52] It would be valuable to have experiments to check our predictions of Table IV, which were calculated under the same assumption, as well as experimental energy spectra. Finally, the results on neutron multiplicities are an improvement over previous calculations[1]; however, although the overall picture is good, there still remains some disagreement for multiplicities zero and one.

We turn now to a discussion of the sensitivity of the model to its ingredients, which will be followed by a series of remarks on specific points.

The nuclear excitation function is shaped in our model by the form assumed for the nucleon-momentum distribution. We are not concerned here with the question whether this quantity is a true momentum distribution of the nuclear ground state (see Refs. 23, 31, and 32 for detailed discussions on this point). It suffices for our purpose to say that Eq. (1) is the effective momentum distribution, containing also the effect of correlations among nucleons, which is to be used in a treatment of quasifree nucleons in the nucleus. The particular functional form of Eq. (1) (or some equivalent form) is essential in obtaining a nuclear excitation function with a sizable high-energy tail, which can produce the observed rates of proton emission. We have performed similar calculations using a Gaussian momentum distribution and we find then the calculated rates for proton emission to be generally lower than in Table II by approximately one order of magnitude. The results for α and neutron emission are also poorer. A similar conclusion on the unsuitability of the Gaussian distribution was reached in Ref. 22.

The parameter γ of Eq. (1) was determined[31] from low-energy electron scattering to be ~1 fm. We have presented our calculations with $\gamma = 0.8$ fm, which gives calculated rates higher by ~20% on the average than with $\gamma = 1$ fm. Changing this parameter to $\gamma = 0.5$ fm (see Fig. 2 for the resulting nuclear excitation function), one achieves on the average a doubling of the precompound emission in $(\mu, \tilde{p})$. This will mainly affect the rates in heavy nuclei, usually in the right direction. However, we feel that one should hold to approximately the same value of γ which was used in treating various other processes[31,32] and was found recently to be appropriate also in a model of heavy-ion interactions (see Hatch and Koonin, Ref. 29). A detailed investigation would be needed to find the possible variation of γ when going from light to the heaviest nuclei. In any case, it is gratifying that the good overall agreement we presented is obtained with essentially the same γ as used by other authors, thus strengthening the credibility of Eq. (1) as a suitable effective nucleon-momentum distribution. In the calculation of the pre-equilibrium emission we have employed the hybrid approach of Blann and Mignerey,[27] using Eq. (14) with $k = 1$, as they did in accounting successfully for various nuclear reactions. A case has been made[40] for the use of higher values such as $k = 3$–4, implying very large mean free nuclear paths. Such change would decrease λ_{n+2} and consequently increase the pre-equilibrium emission and reduce the probability of compound-nucleus formation. We have performed calculations with various values of k and we find that in going from $k = 1$ to $k = 2$, the precompound emission is increased by factors ranging from 1.5–2 for the various nuclei. Although there is probably some increase in k in going towards heavy nuclei, there is no solid evidence on the form of such variation and therefore we chose to restrict ourselves in the present work to the same value $k = 1$ for all nuclei as used in nuclear reactions calculations.[25,27,38]

In the formulas for emission [Eqs. (14) and (15)] one is faced with dealing with the appropriate expressions for the inverse cross sections σ_x and the level density function of the residual nucleus $\rho(U)$. We used for σ_x the parametrization of Dostrovsky *et al.*[43] [Eqs. (17) and (18)], which is widely employed and one is fairly confident that it does not cause major error. A better approach would be to use the experimental cross sections; however, we felt that the computational effort is not required at the level of our approach. For $\rho(U)$ we have taken Eq. (19) which is again found to be a good representation in a wide variety of nuclear reactions.[44] There is some doubt regarding the appropriate value for a, especially as in our process $I(E)$ covers a wide range of excitation energies. We use $a = A/10$ for $A \leq 90$ and $a = A/15$ for the heavier nuclei, as deduced from a scan of the literature. A more detailed treatment of this point is warranted, including shell effects.

Although the overall picture achieved is a positive one, it appears that our nuclear excitation function still lacks in strength in the high-energy tail. This may be concluded from several observations, such as the recent counter experiments[16,17] in which the spectra of high-energy protons following muon capture have been measured. These experiments are not in complete mutual agreement, the results of the Los Alamos group[17] for the amount of protons with energies higher than 40 MeV being larger by a factor of ~3 than those of the Dubna group.[17] Still, the common message from these experiments is of sizable emission of very energetic protons. Calculating the number of protons emerging with energies above 15 MeV in our model N_f and comparing it with the data of Budyashov *et al.*[16] one finds the following: $(N_f^{\text{th}})_{\text{Si}} = 3.2 \times 10^{-3}$ vs $(N_f^{\text{exp}})_{\text{Si}} = 8.8 \pm 0.6 \times 10^{-3}$, $(N_f^{\text{th}})_{\text{Ca}} = 1.9 \times 10^{-3}$ vs $(N_f^{\text{exp}})_{\text{Ca}} = (13.0 + 1.1) \times 10^{-3}$, $(N_f^{\text{th}})_{\text{Cu}} = 0.6 \times 10^{-3}$ vs $(N_f^{\text{exp}})_{\text{Cu}} = (6.0 \pm 0.7) \times 10^{-3}$. [Remember, however, that for Cu our total rate for $(\mu, \tilde{p})$ is also below the experimental one, cf., Table II.] We have calculated energy spectra and we find that for energies above 20–25 MeV they fall significantly below the experimental points of Ref. 16.

This does not affect the good agreement we have on rates, as the high-energy part constitutes only a small fraction of the charged particle emission, but it is also worth stressing that our calculated rates appear to give the correct values up to the heaviest nuclei, where the pre-equilibrium emission which is responsible for the higher-energy part of the spectrum, is totally dominant.

Our spectra for the high-energy part of the proton emission, as given in Fig. 6, if fitted approximately by a single exponential (which is not, however, a very good representation of the function) have slopes of $E_0 \simeq 3.7$–4.2 MeV. The slopes reported by Krane *et al.*[17] are twice as large, while in the earlier experiment on Si a slope of $E_0 \simeq 4.6$ MeV was reported[15] for the charged particles emissions. Thus, the results of Ref. 17, if verified in future experiments, pose a new and interesting challenge. An additional puzzle is the

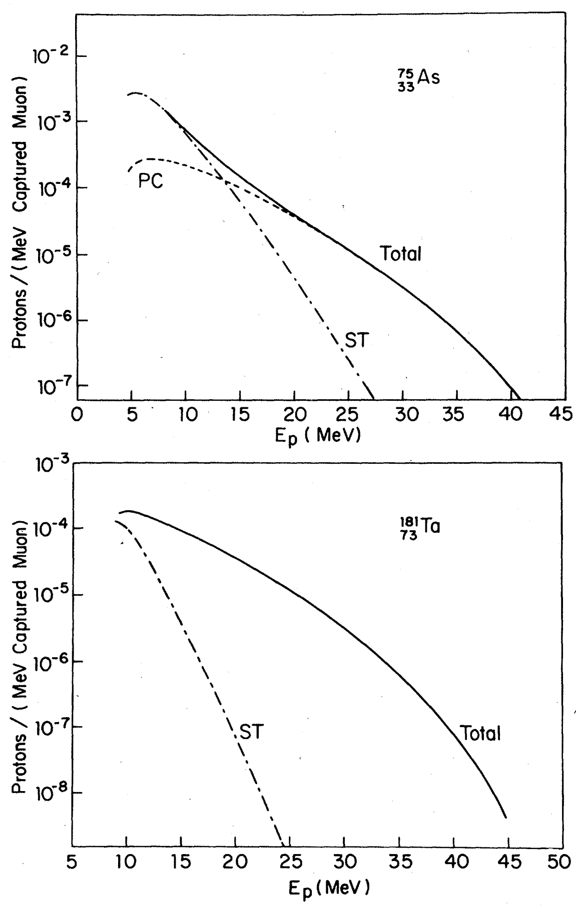

FIG. 6. Calculated proton energy spectra following μ capture in ^{75}As and ^{181}Ta. The pre-equilibrium (PC) and compound-nucleus (ST) contributions are indicated.

fact that the proton spectra of Krane *et al.*[17] have the same slope as the high-energy neutron spectra. This is again difficult to understand in our present calculation (the same holding for Ref. 22), as the direct neutrons are coming primarily from the $n = 2$ exciton, while the protons from the $n = 4$ ones, thus leading to different energy spectra with the proton spectrum falling faster with energy. The high-energy spectra observed[16,17] might then be a different manifestation of the nuclear correlations, as embodied, for example, in the pseudodeuteron models.[19,21]

The inspection of our results for neutron emission seems also to demand a somewhat stronger high-energy tail in the excitation function, although here the evidence is less clear. The energy spectra results might possibly require it (Figs. 3–5); however, the very good results for high neutron multiplicities do not. On the other hand, it emerges from Table V that one would like to have a certain shift in the lower-energy part of the excitation function, so as to increase the probability of emitting one neutron compared to the no-neutron emission. In any case, the existing disagreement between experiments[10,49,50] on the slope of the fast neutron energy spectrum prevents more definite conclusions at present.

We add now several short remarks:

(1) In the first report of this work[2] we have used for the heavy nuclei ($A > 100$) values of k larger than one in the precompound emission formula (14); however, we took for all nuclei a constant value of $a = A/10$ in the level density expression (19). This accounts for the somewhat different values for Sb, Cs, Ho, Ta, and Pb which we present in Table II, versus those previously reported.[2] For the same reason, the theoretical values given for inclusive emission from the heavy nuclei of emulsions in Eq. (26) supersede those of Ref. 2.

(2) Table II shows very good agreement with the results of Wyttenbach *et al.*[3] whenever direct measurements are involved, as it is the case for the (μ, p) channel. It is more difficult to assess the situation for the inclusive $(\mu, \tilde{p})$ channel, where only experimental lower limits are usually available. The figures in parentheses, which were calculated from some specific measured channel by the empirical rule[3] $(\mu, p) : (\mu, pn) : (\mu, p2n) : (\mu, p3n) = 1{:}6{:}4{:}4$ should be taken only as a rough indicator, since from some measured channels one knows that deviations from it are sometimes quite large. For instance, in Si^{28} where our calculation agrees well with experiment (in this case the total charged particle emission was measured) one has $(\mu, p) : (\mu, \tilde{p}) \simeq 1{:}3$ instead of 1:14 as given by the rule.

(3) Wyttenbach *et al.*[3] have plotted their results for the reaction probabilities of various channels against the Coulomb barrier between the outgoing particle and the residual nucleus. One finds that the probability for every reaction decreases exponentially with increasing barrier. Some points, however, deviate from the straight line; we find that in most of these cases our calculation corroborates the deviation, which is usually caused by the specifics of the particular nucleus making themselves felt in the calculation through the binding energies B_i. We give the following examples: ^{63}Cu deviates upward, having $^{63}\mathrm{Cu}(\mu, p)_{\mathrm{exp}} = (2.9 \pm 0.6) \times 10^{-3}$ and we calculate $^{63}\mathrm{Cu}(\mu, p)_{\mathrm{th}} = 4 \times 10^{-3}$. ^{209}Bi has a strong below the line deviation, with $^{209}\mathrm{Bi}(\mu, p)_{\mathrm{exp}} = (8 \pm 1) \times 10^{-5}$. The calculation gives $^{209}\mathrm{Bi}(\mu, p)_{\mathrm{th}} = 4 \times 10^{-5}$.

(4) The activation results of Vil'gel'mova *et al.*,[8] $^{28}\mathrm{Si}(\mu, p)_{\mathrm{exp}} = (5.3 \pm 1.0) \times 10^{-2}$, $^{39}\mathrm{K}(\mu, p) = (3.2 \pm 0.6) \times 10^{-2}$, are mentioned by the authors of Ref. 3 to be higher by an order of magnitude from what they expected on the basis of the above

mentioned curve. We find that our calculations support the findings of Vil'gel'mova *et al.*[8] within a factor of less than 2, as our model gives $^{28}Si(\mu,p)_{th} = 3.2\times 10^{-2}$, $^{39}K(\mu,p)_{th} = 1.9\times 10^{-2}$.

(5) In another activation experiment Heusser and Kirsten[9] find $^{60}Ni(\mu,p)_{exp} = (21.4\pm 2.3)\times 10^{-3}$. This result also deviates upward from the systematics mentioned by a factor of approximately 10. We find $^{60}Ni(\mu,p)_{th} = 8.9\times 10^{-3}$, which shows that indeed a large deviation was to be expected for μ capture in this even-even nucleus leading to an excited odd-odd one (see Table II). Heusser and Kirsten[9] also measure $^{58}Ni(\mu,\alpha)_{exp} = (28\pm 4)\times 10^{-3}$, which is considerably larger than in neighboring nuclei. Our model gives $^{58}Ni(\mu,\alpha)_{th} = 10.5\times 10^{-3}$ which is also substantially higher than for μ capture in the odd-even neighboring nuclei, though again, not quite as high as experimentally reported. The ^{56}Fe nucleus has also a relatively high rate for (μ,α), which is well accounted theoretically. Thus, one evidently sees an odd-even effect in this region.

(6) As we mentioned in the Introduction, previous calculations for charged particles emission were unable to reproduce even the experimental order of magnitude of the reaction, an exception being the recent model of Ref. 22. A few words of comparison are in order. Their approach is very different from ours in as much the calculation of the nuclear excitation function is concerned, as this is derived[22] by considering transitions to nuclear collective states. Nevertheless, the excitation function obtained in their model is quite similar to the one calculated in the present model (in their paper, comparison is made only with the excitation function of the old quasifree particle model.)[11] For the emission process, they also consider both pre-equilibrium and equilibrium emission. The excitation function calculated in Ref. 22 has somewhat more high-energy strength than ours, thus proton energy spectra with a richer high-energy tail are obtained. On the other hand, some of their results for (μ,α) deviate by factors of up to 2.5 from experiment in contrast to what is presented in our Table III, and even worse disagreement occasionally appears in the (μ, pxn) channels (see Table III of Ref. 22).

(7) In Table V the effect of changes in nuclear temperature on the neutron emission multiplicities is put into evidence. The higher values of θ reported by Evseev and Mamedov[36] do not improve the agreement, usually to the contrary. However, one cannot draw conclusions on this before additional calculations are confronted with more experimental data.

In concluding, we remark that improvements on the approximations made in this work, such as the inclusion of some variation of γ and k, consideration of proton emission as second particle, the neutron "direct" contribution from higher exciton states, the inclusion of compound-nucleus emission from nuclei which emitted also during equilibration, are all expected to contribute a small share in the direction of improving the agreement with experiment. Detailed comparison is hampered at present by the lack of sufficient experimental data, especially in the domain of energy spectra of all types of particles emitted. An effort in this direction is very timely.

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