Systematics of particle-nucleus reactions. II. Nuclear recoil in nonfission reactions induced by 1-GeV to 400-GeV protons

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Experimental results from nonfission reactions, induced by protons with energies of 1 GeV or more, were analyzed by simple nuclear-reaction models. A set of parameters, described in the preceding paper, was obtained from this analysis for the purpose of studying the systematics of nuclear recoil in these reactions. Spallation reactions were assumed to occur in two steps: an initial fast process in which momentum is transferred to the struck nucleus in the forward direction, followed by an isotropic random-walk process. In another possible type of reaction–fragmentation–the observed nucleus is formed in a single fast event, corresponding to the first step of a spallation reaction. Since a model for determining the parameters of a fragmentation reaction from the experimental data was lacking, all of the results were analyzed by the two-step model. The parameters were divided into three groups. The first two sets of parameters correspond to the two steps described above, and the third is based on an overall energy inventory of each reaction. The general properties of the reactions studied are indicated by the systematics of these parameters. Criteria are presented for distinguishing between spallation, fragmentation, and fission. More detailed information about possible mechanisms for specific reactions is given.

[NUCLEAR REACTIONS Proton induced; recoil systematics at $E_p = 1$–400 GeV.]

A systematic review of measurements of particle-nucleus reactions is proposed in the preceding paper, based on parameters from simple nuclear-reaction models.[1] That report is an introduction to a series of reports on this subject. The present report, the second in the series, begins the examination of the literature with the recoil systematics of nonfission reactions induced by protons with energies of 1 GeV or more. I have limited this initial study in this way in order to minimize the number of variables that have to be examined. Later reports in this series will take up excitation functions, fission reactions, reactions induced by other particles, etc.

The parameters, which are the basis of the systematics, are divided into three groups. The first two sets of parameters correspond to the two steps in the Serber model[2] and the third set is based on overall energy inventory of each reaction. The preceding report defines these parameters and describes the overall analysis.[1]

I. FIRST STEP OF THE REACTION

The relevant parameters for the first step of the reaction, when the energy of the incident proton is in the GeV region, are E^*, $E^*/\Delta A$, Δm, and $\Delta A/\Delta m$. These parameters are obtained from the collective-tube (CT) model,[1,3] which assumes that the incident particle interacts with the nucleons in its path as if they were a single object of mass Δm. The excitation energy E^* of the residual nucleus from the first step of the reaction is given by

$$pv_p = E^*(1 + \Delta mc^2/E)\,, \qquad (1)$$

where p is the forward momentum of this nucleus and v_p and E are the velocity and total energy of the incident proton. In this expression E^* includes the kinetic energy of the residual nucleus and the separation energy of the ejected nucleons.

The determination of E^* and Δm from Eq. (1) requires the measurement of p for two or more values of E in the GeV region. Relatively few measurements of this type have been made, see Table I.

The kinetic energies E_p of the incident protons are given in the first column of the table. The target nucleus and the final nucleus are listed in the next two columns. The values of E^* are given in the fourth column. The values of $\Delta A = A$ (mass number of the target nucleus) $- A_{REC}$ (mass number of final nucleus), and $E^*/\Delta A$ are listed in the next two columns. The values of Δm in mass-number units and $\Delta A/\Delta m$ follow. The numbers in the last column give the references for each reaction in the table.

Measurements have been reported for the reaction $p + {}^{27}Al \rightarrow {}^{7}Be$ but were not of sufficient quality to include in Table I.[4] A quantity related to E^*, namely E_0^*, which is derived from the single-collision model,[1] is given in Ref. 4 for this reaction and for the first four reactions in Table I.

An examination of Table I reveals a number of features of nonfission reactions induced by high-

TABLE I. Values of E^*, Δm, and related parameters.

E_p (GeV)	Target nucleus	Recoil nucleus	E^* (MeV)	ΔA	$\frac{E^*}{\Delta A}$ $\left(\frac{\text{MeV}}{\text{nucleon}}\right)$	Δm	$\frac{\Delta A}{\Delta m}$	Ref.
>1	^{27}Al	^{24}Na	54	3	18.0	0.9 ± 0.2	3	3,4
>1		^{22}Na	83	5	16.6	0.7 ± 0.2	(7)	4
≥1		^{18}F	156	9	17.3	-0.1 ± 0.2		4
≥1		^{11}C	227	16	14.2	-0.3 ± 0.1		4
3,28	^{63,65}Cu	^{28}Mg	294	36	8.2	1.7 ± 0.3	21	5,6
3,28		^{24}Na	316	40	7.9	1.3 ± 0.3	31	5,6
≥1	^{181}Ta	^{149}Tb	214	32	6.7	2.4 ± 0.3	13	7
3,11.5	^{197}Au	^{171}Lu	179	26	6.9	1.4 ± 0.3	19	8
3,11.5		^{167}Tm	163	30	5.4	2.9 ± 0.4	11	8
>1		^{149}Tb	269	48	5.6	3.1 ± 0.4	15	3,7,9,10
11.5–300		^{139}Ce–^{149}Gd	224	53	4.2	3.8 ± 0.7	14	8
11.5–300		^{28}Mg	458	169	2.7	10.4 ± 1.5	16	8
11.5–300		^{24}Na	513	173	3.0	9.3 ± 1.5	19	8
11.5–300		^{22}Na	545	175	3.1	12.7 ± 1.5	14	8
						average (^{197}Au) =	15 ± 3	
3,6.2	^{209}Bi	^{149}Tb	316	60	5.3	2.9 ± 0.5	21	7

energy protons.

(1) The excitation energy E^* varies directly with ΔA. This result is to be expected. The energy needed to remove nucleons in a nonfission reaction is a function of the number of emitted nucleons.

(2) There is an overall decrease in the values of $E^*/\Delta A$ with ΔA. This observation indicates that the binding energy per nucleon of the emitted particles and hence their average mass increases with the total number of emitted nucleons.

(3) In general, the values of Δm increase with ΔA (except for the first four reactions). Thus, the "hole" that is formed in the first step of the reaction, according to the CT model, increases in size with the total number of emitted nucleons. These results are consistent with the CT model.

(4) The reactions of $p+{}^{197}$Au which produce ^{22}Na, ^{24}Na, and ^{28}Mg are probably fragmentation reactions, at least in part, according to the observations in the next section on the second step of the reaction. The Serber and related models are probably not valid for describing fragmentation, since the formation of a fragment is expected to occur in a single fast event. Nevertheless, these reactions are seen to fit the overall pattern shown in Table I. Especially noteworthy is the observation that all values of $\Delta A/\Delta m$ for a gold target are essentially the same, 15 ± 3. From this we can conclude that approximately 14 additional nucleons are emitted for each target nucleon ejected in the primary interaction in these reactions on gold.

(5) The values of Δm for the reactions of $p+{}^{27}$Al which produce ^{11}C and ^{18}F are negative. The increase in the values of E_0^* with the energy of the incident proton for these reactions (instead of the decrease or constancy for the other reactions in Table I) is a related effect.[4] This result indicates that the CT model is not valid in very light nuclei, when ΔA is relatively large. The value of Δm will be assumed to be one nucleon mass for all of the reactions with an aluminum target.

Many nuclei between ^{28}Mg and ^{139}Ce were detected in the gold work.[8] These were not included in Table I in order to exclude fission reactions, which will be considered later in this series of papers.[11]

The values in Table I provide us with the beginning of a systematics of nonfission reactions based on the Serber and other models. More experiments are needed to fill in the gaps in the table for other values of A and ΔA. The systematics of these reactions, based on parameters from the second step of the reaction, will be considered next.

II. SECOND STEP OF THE REACTION

The relevant parameters for the second step of the reaction are $\langle T\rangle$, ϵ_S, and ϵ_F.[1] The average recoil energy imparted to the observed nucleus in this step is given by $\langle T\rangle$. The other two parameters are the energies of the recoil partners to this nucleus for spallation [Eq. (2)] or for breakup into two fragments [Eq. (3)]:

$$\epsilon_S=\frac{\langle T\rangle}{\Delta A/A}, \qquad (2)$$

$$\epsilon_F = \frac{\langle T \rangle}{\Delta A / A_{REC}} . \quad (3)$$

Previous studies have shown that the values of $\langle T \rangle$ and related quantities increase with ΔA.[8,9,12-17] I will reexamine this relationship for nonfission reactions from the perspective of Eqs. (2) and (3). I have chosen those cases from the literature for determining these quantities that satisfy the followlowing criteria:

(1) The energy of the incident protons was 1 GeV or more. Exceptions were made in cases where a single value of $\langle T \rangle$ was given for E_p values extending from below to greater than 1 GeV, or where data of specific interest (Table V) were available only below 1 GeV.

(2) The target nucleus is specified. This is the case for elements having only one stable isotope. In those cases where two isotopes predominate, the mass was taken to be the weighted average.

(3) The observed recoil nucleus is specified. In those cases where the results for different nuclei of the same element were combined, the mass number was taken to be the average of the two mass numbers.

(4) The target nucleus is ^{27}Al or a heavier nucleus. (The Serber model may not be valid for very light target nuclei.)

(5) The observed nucleus is probably not a fission product.

(6) The value of $\langle T \rangle$ is either given or can be readily determined from the experimental results.

Recoil-range experiments will be taken up first. Results from counter experiments will then be considered.

Recoil-range experiments

The values of $\langle T \rangle$ have been determined in thick-target experiments with ^{27}Al at proton energies up to 400 GeV (Ref. 4) and with ^{181}Ta at 19 GeV (Ref. 13) and can be obtained from measurements with copper at 3 GeV (Ref. 5) and 28 GeV,[6] with silver at 3 GeV,[5] and with ^{197}Au at 1 to 300 GeV.[8] The values for the recoil nuclei ^{24}Na and ^{28}Mg in the copper and silver experiments were calculated with the aid of subsequently published range-energy tables.[18,19]

The ranges of nuclei recoiling from thin foils have been measured at 90° to the beam, or at

TABLE II. Values of $\langle T \rangle$ from recoil-range experiments.

Index number in Figs. 2 and 3	E_p (GeV)	Target nucleus	Recoil nucleus	$\frac{\Delta A}{A}$	$\langle T \rangle$ (MeV)	Ref.
0	19	^{181}Ta	^{173}Lu–^{145}Eu	0.044–0.199	0.6–3.9	13
1	11.5	^{197}Au	^{196}Au	0.005	0.10	8
2	11.5	^{197}Au	^{194}Au	0.015	0.25	8
3	11.5	^{197}Au	^{185}Os	0.061	0.73	8
4	11.5	^{197}Au	^{183}Os	0.071	0.88	8
5	11.5	^{197}Au	^{182}Os	0.076	0.96	8
6	28	^{63,65}Cu	^{58}Co	0.089	1.3	6
7	0.18–400	^{27}Al	^{24}Na	0.111	1.85	4
8	0.45–6.2	^{181}Ta	^{149}Tb	0.177	2.5	7
9	28	^{63,65}Cu	^{52}Mn	0.183	3.0	6
10	3–300	^{27}Al	^{22}Na	0.185	2.9	4
11	2.9	^{107,109}Ag	^{83}Sr	0.231	3.2	21
12	28	^{63,65}Cu	^{48}V	0.246	3.6	6
13	28	^{63,65}Cu	^{44}Sc	0.309	4.6	6
14	2.9	^{35,37}Cl	^{24}Na	0.324	4.3	22
15	1–11.5	^{27}Al	^{18}F	0.333	5.9	4
16	2.9	^{107,109}Ag	^{61,64}Cu	0.421	5.9	21
17	2.9	^{51}V	^{24}Na	0.529	6.1	22
18	3.0,28	^{63,65}Cu	^{28}Mg	0.560	10.7	5,6
19	1–11.5	^{27}Al	^{11}C	0.593	8.0	4
20	2.9	^{107,109}Ag	^{43,44}Sc	0.597	9.4	21
21	2.9,3.0,28	^{63,65}Cu	^{24}Na	0.623	9.8	5,6,22
22	2.2	^{27}Al	^{8}Li	0.704	18.6	20
23	1–11.5	^{27}Al	^{7}Be	0.741	16.7	4
24	3.0	^{107,109}Ag	^{28}Mg	0.741	23.2	5
25	2.9,3.0	^{107,109}Ag	^{24}Na	0.778	20.8	5,21
26	2.0	^{63,65}Cu	^{8}Li	0.874	23.6	23
27	2.0,2.2	^{107,109}Ag	^{8}Li	0.926	28.9	20,23
28	2.0,2.2	^{197}Au	^{8}Li	0.959	34.0	20,23
29	2.2	^{238}U	^{8}Li	0.966	40.5	20

angles that were symmetrical around 90°.[7,20-23] An example of this type of experiment is the use of emulsions to detect the recoil-range distribution of ^{8}Li formed in various reactions at 2.0 GeV (Ref. 23) and at 2.2 GeV.[20] The value of $\langle T \rangle$ was obtained from the resulting energy distributions. In some of the cases, the distributions for several angles, averaging 90°, were combined for better statistics. The forward motion from the first step of the reaction has a small effect on measurements at 90°, which was disregarded.

The values of $\langle T \rangle$ are listed in Table II.[24] The entries for ^{181}Ta are limited to values of $\Delta A < 40$. The overall results for 19 reactions with these values of ΔA are given in the table with the index number zero. The entries for ^{197}Au are limited to $\Delta A \leq 15$ and to the recoil nucleus ^{8}Li. The latter nucleus is the only one included for ^{238}U. These cases were chosen to minimize the possible contribution of fission reactions.

The values under $\Delta A/A$ are the number of nucleons emitted, divided by the mass number of the target nucleus. The values of $\langle T \rangle$ are arranged in order of increasing $\Delta A/A$ in Table II and are shown in Fig. 1. With few exceptions, the values of $\langle T \rangle$ increase with $\Delta A/A$. These exceptions are partly due to experimental error, e.g., errors in the range-energy data needed to obtain recoil energies from range measurements. Discrepancies of 20% or more in the Northcliffe-Schilling ranges and stopping powers, compared to directly measured values, have been observed in some cases.[18,19] In view of the variety of target and recoiling nuclei and stopping materials, it is not surprising that some values of $\langle T \rangle$ are out of line with the general trend.

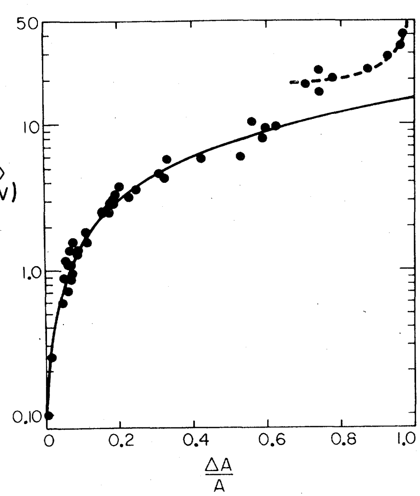

FIG. 1. $\langle T \rangle$ from Table II as a function of $\Delta A/A$. The solid line is given by $\langle T \rangle = 15\Delta A/A$ MeV. The dashed line is the least-squares fit to the last eight points; see Fig. 2.

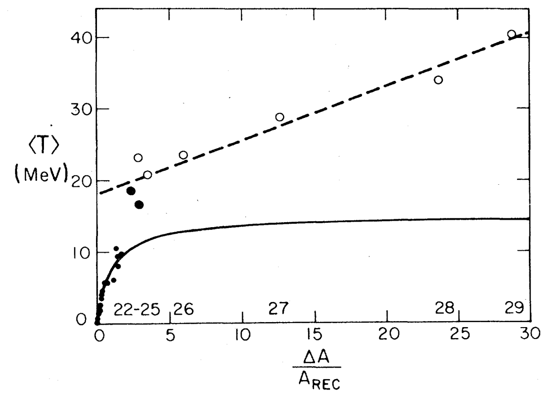

FIG. 2. Values of $\langle T \rangle$ as a function of $\Delta A/A_{\rm REC}$. The small points are for reactions indentified by index numbers 1 to 21 in Table II. The large points are identified by index numbers 22 to 29. The large solid points are results with an aluminum target. The solid line is given by $\langle T \rangle = 15\Delta A/A$ MeV. The dashed line is the least-squares fit to the large points.

The apparent one-to-one relationship between $\langle T \rangle$ and $\Delta A/A$ arises from two effects which lead to Eqs. (2) and (3):

(1) The recoiling nucleus gets a momentum "kick" and increase in kinetic energy with each particle that is emitted by the target nucleus. As a result, the value of $\langle T \rangle$ increases as ΔA increases.

(2) From momentum conservation, the recoil energy is inversely proportional to the mass number of the recoil nucleus $A_{\rm REC}$.

These two effects lead to the functional relationship between $\langle T \rangle$ and $\Delta A/A_{\rm REC}$, seen in Fig. 2. As a result, $\langle T \rangle$ is also a function of $\Delta A/A$, since $\Delta A/A_{\rm REC} = (\Delta A/A)/(1 - \Delta A/A)$. These observations provide a way to systematize a large variety of nuclear reactions induced by high-energy particles. The significance of this relationship can be seen in Fig. 3, where ϵ_S, given by Eq. (2), is shown, and in Fig. 4, where values of ϵ_F, given by Eq. (3), are plotted.

The numbers in Figs. 2 and 3 identify reactions from Table II. The results for 19 recoiling nuclei from the interaction of 19-GeV protons with ^{181}Ta are combined in Figs. 3 and 4 (point with flags), but are plotted individually in Fig. 1. These points are not shown in Fig. 2.

The reactions in Table II can be classified as spallation or fragmentation, since fission reactions are not included. The systematic variation of $\langle T \rangle$, ϵ_S, and ϵ_F with the parameter $\Delta A/A$ for a variety of target and recoil nuclei and proton energies indicates that the reaction mechanism in these cases depends primarily on this parameter. The values of $\langle T \rangle$ and ϵ_S for reactions included in

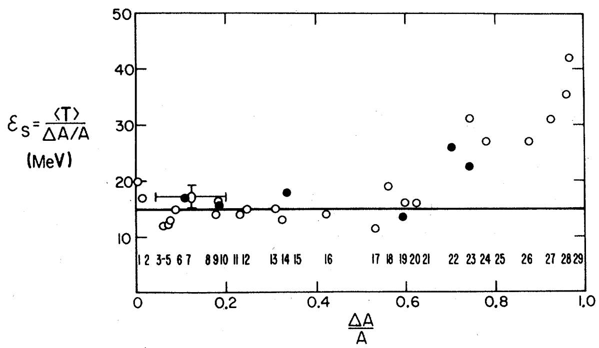

FIG. 3. Values of ϵ_S. The numbers identify the reactions; see Table II. The results with an aluminum target are given by the solid points ● and with other targets by open points ○. The solid line is for $\epsilon_S = 15$ MeV.

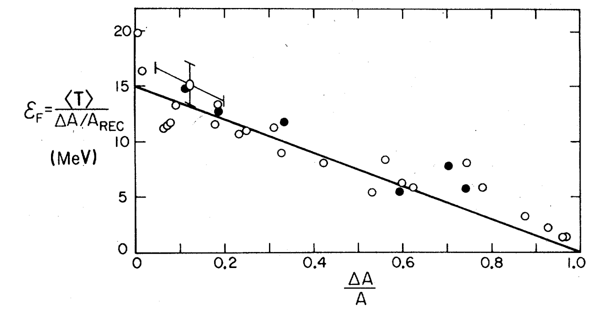

FIG. 4. Values of ϵ_F. The results with an aluminum target are given by the solid points ● and with other targets by open points ○. The solid line is for $\epsilon_S = 15$ MeV or, from Eqs. (2) and (3), $\epsilon_F = 15 A_{REC}/A$ MeV.

Table I, but not in Table II, plus recent results,[24] are listed in Table III. Most of the reactions in Table III do not fit the criteria for inclusion in Table II, but still show the same functional dependence on $\Delta A/A$.

Examination of Fig. 3 and Table III indicates the presence of two regions:

(1) In the region $\Delta A/A < 0.67$, the value of $\epsilon_S \simeq 17$ MeV is essentially constant. This value for ϵ_S is the average of the value for reactions with index numbers 1–21 in Table II and Fig. 3 (15 MeV), the value for reactions on tantalum with index number 0 (17 MeV), and the value for reactions in Table III (18 MeV). The observation that this value of ϵ_S is essentially the same for all reactions with these values of $\Delta A/A$ implies that the mechanism is the same in these reactions at proton energies of 1 GeV or more. This observation is also evidence for the random-walk process, on which Eq. (2) is based, in the second step of the reaction. Reactions with these properties are spallation reactions. The nature of the recoil angular distribution for deep spallation is further evidence for this designation.[1]

Thus, $\Delta A/A < 0.67$ and $\epsilon_S \simeq 17$ MeV, or $\langle T \rangle \simeq 17\ \Delta A/A$ MeV [Eq. (2)], are criteria for designating a reaction as spallation. The solid line in Figs. 1 to 4 was obtained by setting $\epsilon_S = 15$ MeV (the average value for reactions 1–21 in Table II and Fig. 3).

These observations confirm the validity of the method of analysis for spallation reactions and the Serber model on which it is based. The observation that the product nuclei from ^{27}Al with $\Delta A/A < 0.67$ fall in line with the general trend indicates that this model is valid for target nuclei with mass numbers of 27 or more.

TABLE III. Values of $\langle T \rangle$ not included in Table II.

E_p (GeV)	Target nucleus	Recoil nucleus	$\frac{\Delta A}{A}$	$\langle T \rangle$ (MeV)		ϵ_S (MeV)	Ref.
0.6,10.5	^{89}Y	^{84}Rb	0.056	1.2		21	17
0.6,10.5	^{89}Y	^{83}Rb	0.067	1.4		21	17
1–11.5	^{197}Au	^{171}Lu	0.132	2.4		18	8
1–11.5	^{197}Au	^{167}Tm	0.152	2.6		17	8
0.6–21	^{107,109}Ag	^{84}Rb	0.222	3.7		17	17
0.6–21	^{107,109}Ag	^{83}Rb	0.231	4.4		19	17
1–300	^{197}Au	^{149}Gd	0.244	4.0		16	8
1–300	^{197}Au	^{149}Tb	0.244	3.8		16	7,9,10
1–300	^{197}Au	^{146}Gd	0.259	4.3		17	8
1–300	^{197}Au	^{145}Eu	0.264	4.6		17	8
11.5,300	^{197}Au	^{143}Pm	0.274	4.7		17	8
3,6.2	^{209}Bi	^{149}Tb	0.287	4.9		17	7
11.5–300	^{197}Au	^{139}Ce	0.294	4.8		16	8
				average	=	18 ± 2	
11.5–300	^{197}Au	^{28}Mg	0.858	46.5		54	8
11.5–300	^{197}Au	^{24}Na	0.878	39.3		45	8
11.5–300	^{197}Au	^{22}Na	0.889	45.8		52	8

(2) In the region $\Delta A/A > 0.67$, the value of ϵ_S increases appreciably; see Fig. 3. This implies that a new mechanism is becoming increasingly important as $\Delta A/A$ increases. This increase in the value of ϵ_S must be due to the emission of larger nuclei, or fragments. Further evidence for this conclusion is given in Sec. III on the probability of various reaction modes.

Both spallation and fragmentation occur in this region of $\Delta A/A$. However, the Serber model is probably not valid for describing fragmentation, since the formation of a fragment is expected to occur before the deexcitation process usually associated with the second step of the reaction.

This conclusion is confirmed by the results shown in Figs. 1, 2, and 4. There appears to be a change in curvature at $\Delta A/A \simeq 0.67$ in Fig. 1. This effect is clearly noticeable in Fig. 2, where $\langle T \rangle$ is plotted against $\Delta A/A_{\mathrm{REC}}$. Below $\Delta A/A_{\mathrm{REC}} = 2$, corresponding to $\Delta A/A = 0.67$, the values of $\langle T \rangle$ increase rapidly from 0.1 to 10 MeV. Above $\Delta A/A_{\mathrm{REC}} = 2$ the values of $\langle T \rangle$ increase slowly with $\Delta A/A_{\mathrm{REC}}$. An abrupt change in the dependence of $\langle T \rangle$ on this parameter is apparent. The values of ϵ_F as a function of $\Delta A/A$ in Fig. 4 show this effect in another way.

Agreement between $\langle T \rangle$ and the solid line in Figs. 1 and 2 is evidence for a spallation process. Values of $\langle T \rangle$, which are above this line, indicate the presence of a fragmentation process or fission. The largest values of $\langle T \rangle$ are found in fission (~50–100 MeV),[25-30] corresponding to the region of Figs. 1 and 2 above the solid and dashed lines.

The observation made in previous studies and noted above, that $\langle T \rangle$ and related quantities increase with ΔA, was made for cases where $\Delta A/A \ll 1$. This observation breaks down if cases where $\Delta A/A \to 1$ are included. Thus, $\Delta A/A$ in Eq. (2), which was derived for a random-walk process,[1,15] is a more relevant parameter than ΔA for comparing spallation with other types of reaction.

Alpha-particle emission

In general, the mass of a particle emitted in a high-energy nuclear reaction is expected to be intermediate between the extremes represented by ϵ_S and ϵ_F. The emission of only nucleons, where $\epsilon \simeq \epsilon_S$, is shown to be unlikely in reactions studied on tantalum, gold, and bismuth.[7,9] This point is also discussed in Sec. III below. At the other extreme, breakup into two nuclei, where $\epsilon = \epsilon_F$, represents only one of the possible reaction modes.

Because of their large binding energy, alpha particles are expected to comprise a significant fraction of the emitted particles.[31] The average energy ϵ_α of an alpha particle can be determined from Eq. (6) of the preceding paper,[1] if the energies of the other emitted particles and the order of particle emission are known. I have made the following assumptions in order to determine ϵ_α for the reactions in Table II with index numbers 3 to 6 and 8 to 29:

(1) If an even number of protons is emitted, the reaction is $(p, pN_\alpha \alpha N_n n)$, where N_α and N_n are the number of alpha particles and neutrons, respectively. In one case (index number 22), $(p, p4\alpha{}^3\mathrm{He})$ is the assumed reaction, with the energy of ^{3}He taken to be 10 MeV.

(2) If an odd number of protons is emitted, the reaction is $(p, 2pN_\alpha \alpha N_n n)$. The number of unbound nucleons emitted is $N_n + 1$.

(3) The alpha particles are emitted first, followed by nucleons.

(4) The average energy of each type of particle is constant. The energy of nucleons is assumed to be 5 MeV.

The values of ϵ_α are plotted in Fig. 5. The energy of ^{3}He from the reaction $^{27}\mathrm{Al} \to {}^{24}\mathrm{Na}$ (index number 7) is included. These results confirm the previous conclusion that a change in mechanism occurs at $\Delta A/A = 0.67$. Below this value, $\epsilon_\alpha \simeq 17$ MeV. Above this value, ϵ_α rises rapidly with increasing values of $\Delta A/A$.

Counter and emulsion experiments, $A_{\mathrm{REC}} < 5$

Values of $\langle T \rangle$ as a function of $\Delta A/A$ can be obtained from counter experiments if A_{REC} is known.[32-44] In several such experiments the recoil-energy spectra at various lab angles were analyzed to obtain the T spectra in the frame of

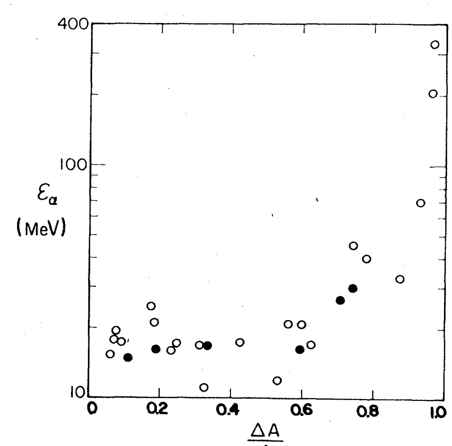

FIG. 5. Values of ϵ_α. The results with an aluminum target are given by the solid points ● and with other targets by open points O.

TABLE IV. Values of $\langle T \rangle$ from counter experiments.

		$A_{REC} > 5$			
E_p (GeV)	Target nucleus	Recoil nucleus	$\frac{\Delta A}{A}$	$\langle T \rangle$ (MeV)	Ref.
4.9	^{27}Al	^{9,10}Be	0.648	11	43
4.9		^{7}Be	0.741	15	43
1	^{107,109}Ag	^{12}B	0.889	28	35
1–5.5		^{11}B	0.898	30	33–35
1		^{11}Be	0.898	21	35
1		^{10}B	0.907	30	35
1–5.5		^{10}Be	0.907	27	33–35
1–5.5		^{9}Be	0.917	27	33–35
1,5.5		^{9}Li	0.917	27	33,35
1,5.5		^{8}Li	0.926	27	33,35
1–5.5		^{7}Be	0.935	31	33–35
1–5.5		^{7}Li	0.935	27	33–35
1–5.5		^{6}Li	0.944	27	33–35
1,5.5		^{6}He	0.944	22	33,35

reference of the nucleus formed in the first step of the reaction. In other cases, $\langle T \rangle$ was calculated from the spectrum obtained at a lab angle of 90° or at several angles averaging 90°. The resulting values for nonfission reactions, induced by protons with energies of 1 GeV or more, are given in Table IV for reactions with $A_{REC} > 5$. Since data in this energy range of incident protons are incomplete for $A_{REC} < 5$, results from experiments below 1 GeV are included in Table V.[45]

The recoil energies of neutrons and of hydrogen and helium nuclei, including ^{4}He from the reaction of 1–3-GeV protons with AgBr in emulsions,[31] are given in Table V. Except for neutrons and protons, these values are in good agreement with the value $\epsilon_S = (17 \pm 3)$ MeV obtained in reactions with $\Delta A/A < 0.67$. This agreement is evidence for the following conclusions about these reactions:

(1) The nuclei with $A_{REC} < 5$ are the randomly emitted recoil partners to the final nuclei in reactions with $\Delta A/A < 0.67$. Other light nuclei, e.g., ^{6}He, Li nuclei, etc., may also be emitted at the same time to give an overall value of $\epsilon_S \simeq 17$ MeV. The emission of the latter, see Table IV, along with neutrons and protons could give this result.

(2) The conclusion that these are spallation reactions is confirmed.

(3) The agreement between $\langle T \rangle$ for the lightest nuclei and ϵ_S for $\Delta A/A < 0.67$ is evidence that the analysis based on the simple models described here is self-consistent and that no major errors have been made in the analysis.

Recoil nuclei with $A_{REC} > 5$ and $\Delta A/A > 0.67$

In the preceding discussion we concluded that both spallation and fragmentation occur for recoil nuclei with $\Delta A/A > 0.67$. We now consider nuclei in this category with the additional property that $A_{REC} > 5$. The results for this group of nuclei are included in Tables II to IV and in the figures. The values for ^{7}Be and ^{9,10}Be from aluminum, given in Table IV, fall in line with the overall variation of $\langle T \rangle$ with $\Delta A/A$. The remaining values in this table appear to be correlated in a different way. All are for a silver target with $\langle T \rangle \simeq 27$ MeV.

Several observations can be made about this group of nuclei:

(1) This region is very small for reactions in aluminum; see Table V of Ref. 4. It is confined to isotopes of Li and Be.

(2) In the case of a silver target this region extends from the recoil nucleus ^{28}Mg (Table II) into the Li-B group of nuclei (Table IV). The constancy

TABLE V. Values of $\langle T \rangle$ from counter and emulsion experiments in MeV.

			$A_{REC} < 5$					
E_p (GeV)	Target nucleus	n	^{1}H	^{2}H	^{3}H	^{3}He	^{4}He	Ref.
4.9	^{27}Al		9	12	14		12	43
0.66	^{58}Ni					15	12	36,42
0.66	^{64}Ni					16	13	36,42
0.19	natural Ni	2.8						45
1–3	AgBr						17	31
0.19	^{107,109}Ag	2.3						45
1.0	^{107,109}Ag					28 [a]	16	35
5.5	^{107,109}Ag		11	16	20	21	19	33
0.66	^{112}Sn					17	15	36,42
0.66	^{124}Sn					17	16	36,42
0.19	^{197}Au	2.3						45
	Average	2.5	10	14	17	17	15	

[a]Not included in average value.

TABLE VI. Values of $\langle T\rangle$ for nuclei with $\Delta A/A > 0.67$ and $A_{REC} > 5$.

Target nucleus	Recoil nuclei	$\langle T\rangle$ (MeV)	$3.1(Z/A^{1/3})$ (MeV)
^{27}Al	^{7}Be, ^{8}Li	16 ± 3	13
^{63,65}Cu	^{8}Li	24 ± 3	22
^{107,109}Ag	^{6}Li-^{12}B	28 ± 4	31
^{197}Au	^{8}Li-^{28}Mg	41 ± 5	42
^{238}U	^{8}Li	41 ± 5	46

of values of $\langle T\rangle$ for nuclei from ^{6}Li to ^{12}B (except for ^{11}Be) is an unexpected result.

(3) This region contains ^{8}Li from various nuclei (Table II) and ^{22}Na, ^{24}Na, and ^{28}Mg from gold (Table III).

The second observation, concerning the recoil products of reactions on silver, indicates that $\Delta A/A$ is not the appropriate parameter for this group of nuclei. The following empirical relationship appears to correlate the recoil properties of nuclei with $A_{REC} > 5$ and $\Delta A/A > 0.67$:

$$\langle T\rangle = 3.1 Z/A^{1/3}, \qquad (4)$$

where Z = atomic number of the target nucleus. This equation was suggested by the expression for the Coulomb repulsion of two charged objects. Although its theoretical basis is dubious, Eq. (4) provides a way to systematize recoil energies if fragmentation occurs; see Table VI.

Katcoff, Baker, and Porile measured and analyzed the energy spectra, angular distributions, and cross sections of ^{8}Li, formed from copper, silver, and gold targets with 2-GeV incident protons.[23] They concluded, "A significant fraction of the ^{8}Li fragments is emitted as a result of a fast fragmentation rather than a slower evaporation process. Alternatively, the emission of light fragments may take place by a mechanism intermediate between these two extremes." Their conclusions apply equally to all of the reactions, with $A_{REC} > 5$ and $\Delta A/A > 0.67$, studied here.

III. PROBABILITY OF VARIOUS REACTION MODES

Each reaction we have studied is characterized by the target nucleus and by the final recoil nucleus and its kinetic energy. We have been able to determine the average kinetic energy of the recoil partners to the measured nucleus for a random-walk process, as would occur in spallation, and for two-fragment breakup, as in fragmentation or fission. In any given reaction a combination of various reaction modes is possible. In this section I will examine the constraints imposed on several reaction modes by the available energy in each case.

TABLE VII. Values of $T_{max}/\langle T\rangle$ and P_{max}.

Target nucleus	Recoil nucleus	Nucleon emission[a] $T_{max}/\langle T\rangle$	Nucleon emission[a] P_{max}	^{4}He and ^{3}He emission: Assumed reaction	^{4}He and ^{3}He emission: $T_{max}/\langle T\rangle$	^{4}He and ^{3}He emission: P_{max}	Two-fragment breakup[b] $T_{max}/\langle T\rangle$
^{27}Al	^{24}Na	0.44	0.22	$(p,p)^{3}$He	1.82	0.88	1.82
	^{22}Na	0.39	0.18	$(p,p)\alpha n$	2.7	0.97	
	^{18}F	0.38	0.18	$(p,p)2\alpha n$	2.9	0.98	7.2
	^{11}C	0.27	0.10	$(p,n)4\alpha$	3.8	1.00	14.3
	^{8}Li	0.12	0.02	$(p,p)4\alpha^{3}$He	1.75	0.86	8.7
	^{7}Be	0.17	0.04	$(p,n)5\alpha$	2.6	0.97	11.8
^{63,65}Cu	^{28}Mg	–	0	$(p,pn)8\alpha^{3}$H	1.19	0.69	13.7
	^{24}Na	–	0	$(p,p)9\alpha 4n$	1.14	0.66	18.7
^{181}Ta	^{149}Tb	–	0	$(p,pn)4\alpha 15n$	0.58	0.33	9.9
^{197}Au	^{171}Lu	<0.01	0	$(p,p)4\alpha 10n$	0.73	0.43	9.5
	^{167}Tm	–	0	$(p,p2n)5\alpha 8n$	0.65	0.38	9.3
	^{149}Tb	–	0	$(p,p2n)7\alpha 18n$	0.51	0.27	15.8
	^{139}Ce-^{149}Gd[c]	–	0	$(p,2p2n)9\alpha 14n$	0.43	0.21	15.1
	^{28}Mg	–	0	$(p,4p6n)32\alpha 32n$	0.01	0	8.0
	^{24}Na	–	0	$(p,3p6n)33\alpha 33n$	0.02	0	10.9
	^{22}Na	–	0	$(p,3p10n)33\alpha 31n$	0.02	0	9.4
^{209}Bi	^{149}Tb	–	0	$(p,p4n)9\alpha 20n$	0.44	0.22	17.6

[a] $E^* + Q < 0$ for all cases indentified by the symbol – in the third column.

[b] $P_{max} = 1.00$ for all reactions listed in this column, except for the reaction ^{27}Al → ^{24}Na + ^{3}He, where $P_{max} = 0.88$. No value was determined for ^{27}Al → ^{22}Na because of ^{5}He instability.

[c] ^{144}Nd was taken to represent the average nucleus for these nuclei.

I will assume that the values of Δm and E^* be determined from an analysis of the recoil measurements, based on the two-velocity and collision-tube models. This procedure may not be valid for fragmentation reactions, as mentioned above. A model for determining the parameters of a fragmentation reaction from the experimental data is needed. Lacking such a model, I will conclude the analysis of nonfission reactions with the calculation of the upper limit P_{max} to the probability of a few reaction modes, based on the models used here. In two-fragment breakup these models lead to $P_{max}=1.00$ (except for $^{27}Al \rightarrow {}^{24}Na$, see Table VII). This result is, of course, consistent with any model.

The evaluation of P_{max} is based on Eq. (15) in the preceding report[1] for the emission of two kinds of particles in the second step of the reaction

$$T_{max}+n_1\epsilon_{1\,max}+n_2\epsilon_{2\,max}=E^*+Q\,, \tag{5}$$

where n_1 and n_2 are the number and $\epsilon_{1_{max}}$ and $\epsilon_{2_{max}}$ the maximum kinetic energy of each type of emitted particle and T_{max} = maximum value of T. The Q value in Eq. (5) was calculated for the overall reaction.[46,47]

I will assume that

$$\epsilon_{2\,max}/\epsilon_{1\,max}=\epsilon_2/\epsilon_1=r_{21}\,, \tag{6}$$

with the ratio r_{21} based on $\epsilon=\langle T\rangle$ in Table V. From Eq. (5),

$$T_{max}+(n_1+r_{21}n_2)\epsilon_{1\,max}=E^*+Q\,. \tag{7}$$

I will take r_{21} to be unity for ^2H, ^3H, ^3He, and ^4He, since the values of $\langle T\rangle$ for these particles agree within experimental error. For neutrons and alpha particles this ratio is $r_{n\alpha}=\frac{1}{6}$; for protons and alpha particles, $r_{p\alpha}=\frac{2}{3}$; and for neutrons and protons, $r_{np}=\frac{1}{4}$.

The initial interaction between the incident proton and the target nucleus is assumed to occur in the following way. The incident proton strikes Δm nucleons in its path in the target nucleus and escapes. One of the struck nucleons stays behind and the remaining $\Delta m-1$ nucleons escape. The mass number of the resulting excited nucleus is $A+1-\Delta m$. Another possibility is for a (p,n) reaction to occur in the first collision and for the neutron to escape. These reactions can be written as $[p,p(\Delta m-1)$ nucleons$]$ or $[p,n(\Delta m-1)$ nucleons$]$, respectively, or for either case as $(p, \Delta m$ nucleons$)$. This picture is consistent with the single-collision model, where $\Delta m=1$.

I will assume for the second step that particles with the largest mass and charge will be emitted first and those with the smallest values will be emitted last. A second expression relating T_{max} and $\epsilon_{1_{max}}$ is given by Eq. (6) of the preceding report.[1]

$$T_{max}=A_{REC}\left[a_1\sum_{i=1}^{n_1}(A+1-\Delta m-ia_1)^{-2}+r_{21}a_2\sum_{i=1}^{n_2}(A+1-\Delta m-n_1a_1-ia_2)^{-2}\right]\times\epsilon_{1\,max}\,. \tag{8}$$

We can determine T_{max} from Eqs. (7) and (8).

Values of $T_{max}/\langle T\rangle$ for reactions listed in Table I and in Ref. 4 are given in Table VII. The values of $\langle T\rangle$ were taken from Tables II and III. The results for reactions that produce only nucleons and the recoil nucleus are given in the third column. I assume that proton emission precedes neutron emission in the second step in these reactions.

Protons are found to have the largest formation cross section of the charged particles and alpha particles the next largest value.[37] As a result, the emission of alpha particles is expected to be a major cause for nuclear recoil in the reactions studied here. Some representative alpha-particle (and ^3He) emitting reactions are listed in the fifth column of Table VII. The initial interaction, based on the value of Δm from Table I, is enclosed in parentheses and is followed by the particles emitted in the order indicated in the second step. Thus, the initial reaction in copper to form ^{28}Mg is a (p,pn) reaction, followed in the second step by the emission of eight alpha particles and finally ^3H.

The values of $T_{max}/\langle T\rangle$ for two-fragment breakup of the intermediate excited nucleus are given in the last column of Table VII. For example, in the formation of ^{28}Mg from copper, where $A\simeq 64$, the (p,pn) product splits into ^{28}Mg and ^{35}Cl nuclei.

An indication of the probability of various reaction modes can be obtained from the values of $T_{max}/\langle T\rangle$, if the recoil-energy spectrum is known. An analysis of differential range measurements for several typical spallation reactions gives the distribution[1,48]

$$P(T)dT=4x\,e^{-2x}dx\,, \tag{9}$$

where $x=T/\langle T\rangle$. The maximum value that $P(T)$ can have for this type of spectrum is given by

$$P_{max}=\int_0^{T_{max}/\langle T\rangle}4x\,e^{-2x}dx\,. \tag{10}$$

Values of P_{max} are given in the fourth and seventh columns of Table VII. In two-fragment breakup $P_{max}=1.00$ for all of the reaction listed, except the first. The actual probability of a given reaction mode will in general be less than the value in Table VII, because of competing reactions.

The results in Table VII can be summarized as follows:

1. *Nucleon emission.* The emission of nucleons only to form the observed recoil nucleus is a minor factor at best in any of the reactions in the table. In the case of an aluminum target, this process comprises at most ~20% of the cross section for producing ^{18}F, ^{22}Na, and ^{24}Na and much less for ^{8}Li and ^{7}Be. With the possible exception of ^{197}Au → ^{171}Lu, insufficient energy is available for any of the other reactions in the table to occur by this process.

2. *The emission of ^{4}He and ^{3}He.* The emission of helium nuclei can account for most of the events that result in the formation of the observed recoil nuclei from aluminum. Less than 70% of the reactions that result in the formation of ^{24}Na and ^{28}Mg from copper can be explained by this process. The value of P_{max} is even smaller for heavy nuclei formed from gold and bismuth and is essentially zero for the formation of light nuclei (^{22}Na, ^{24}Na, and ^{28}Mg) from gold.

3. *Two-fragment breakup.* The value of P_{max} for the formation of ^{24}Na from aluminum is 0.88. No value was determined for the formation of ^{22}Na from aluminum by this process because of ^{5}He instability. In all of the other reactions listed in the table $P_{max} = 1.00$.

In general, intermediate values of $T_{max}/\langle T\rangle$ and P_{max} will be obtained for reaction modes in which an intermediate number of particles are emitted. Thus, $T_{max}/\langle T\rangle = 0.84$ and $P_{max} = 0.50$ for the reaction ^{27}Al$(p,p)^2$H^{1}H^{24}Na. These values lie between values for nucleon emission and those for the reaction ^{27}Al$(p,p)^3$He^{24}Na. Similarly, reactions with the emission of particles heavier than ^{4}He will generally have larger values of P_{max} than is the case for alpha-particle emission.

The results in Table VII indicate why the values of $\langle T\rangle$ for ^{22}Na, ^{24}Na, and ^{28}Mg from gold are much larger than the other values in Table II. Since $P_{max} = 0$ for these reactions for alpha-particle emission, the ejection of heavier nuclei must have occurred. A possible production mode for ^{28}Mg from gold is the reaction ^{197}Au$(p, 4p6n)$ 8^{20}O^{28}Mg, where $T_{max} = 47$ MeV and $P_{max} = 0.60$. The corresponding maximum kinetic energy for ^{20}O is 37 MeV in this process. When 4 ^{40}S nuclei are emitted, instead of 8 ^{20}O, $P_{max} = 0.99$. With the emission of heavier nuclei to form the same observed recoil nucleus, $P_{max} \simeq 1.00$. Similar results are obtained for gold producing sodium nuclei.

We can rule out some processes which cannot occur for reasons of energy conservation, i.e., those with $E^* + Q < 0$. Others are, in effect, eliminated because $P_{max} \simeq 0$. Both of these factors may account for the large values of $\langle T\rangle$ for reactions with $\Delta A/A > 0.67$ in Tables II to IV and probably play a key role in determining whether a reaction proceeds via spallation or fragmentation.

In relatively simple reactions, where only a few reaction modes are possible, a more detailed picture may emerge. If we make the assumption that the probability of a mechanism is proportional to P_{max}, we get the cross sections listed in Table VIII for the reaction modes resulting in the formation of ^{24}Na from aluminum. These values can be compared with the cross sections for the interaction of 0.6-GeV protons with aluminum to form hydrogen and helium nuclei; see Table IX.[37] The emission of ^{2}H and ^{3}He are not rare events and can account for the reaction modes listed in Table VIII.

No mention has been made of mechanisms in which new particles, e.g., pions, are formed. In the collision-tube model these processes occur in the initial interaction after the incident proton and the ejected nucleons had left the struck nucleus and would thus not affect the subsequent course of the reaction.

IV. SUMMARY

The recoil properties of nonfission reactions, induced by protons with energies of 1 GeV or more, were analyzed by simple nuclear-reaction models to determine the systematics of these reactions. The parameters, which were selected for this purpose, were E^*, $E^*/\Delta A$, Δm, and $\Delta m/\Delta A$ for the first step of the reaction; $\langle T\rangle$, ϵ_S, ϵ_F, and ϵ_α for the second step; and $T_{max}/\langle T\rangle$ and P_{max} for the overall reaction. Some of the reactions studied may proceed by a fragmentation process, which probably occurs in one fast step. Since a model for determining the parameters of a fragmentation reaction was lacking, all of the experiments were analyzed by the two-vector model. Systematic deviations of the resulting parameters were attributed to the presence of fragmentation.

First step of the reaction. The analysis of the experimental measurements by means of the two-

TABLE VIII. Postulated cross sections for reaction mechanisms for $p + {}^{27}$Al → ^{24}Na.

	σ (mb)
$(p,p)2pn$ } $(p,n)3p$ }	1
$(p,p)^2$Hp	3
$(p,p)^3$He	4
Total [a]	8

[a]Reference 4.

TABLE IX. Cross sections for the production of light nuclei from the interaction of 0.6-GeV protons with aluminum.[a]

	σ (mb)
^{1}H	670
^{2}H	169
^{3}H	44
^{3}He	80
^{4}He	295

[a]Taken from Ref. 37.

velocity and collision-tube models results in the following observations:

(1) The excitation energy E^* increases with ΔA.

(2) The values of $E^*/\Delta A$ decreases with ΔA. This indicates that the binding energy per nucleon and thus the average mass of the emitted particles increase with ΔA.

(3) There is an overall increase in Δm with ΔA.

(4) For a gold target $\Delta A/\Delta m = 15 \pm 3$ when ^{22}Na, ^{24}Na, ^{28}Mg, and ^{139}Ce to ^{171}Lu are formed.

(5) In some reactions with aluminum Δm is negative, indicating that the collision-tube model is not valid in light nuclei when ΔA is large.

Except for the last observation these results are consistent with the two-velocity and collision-tube models. However, the results which follow indicate that the interaction of protons with gold to produce ^{22}Na, ^{24}Na, and ^{28}Mg is probably a fragmentation process, at least in part.

Second step of the reaction. The values of $\langle T\rangle$, ϵ_S, and ϵ_F for a variety of nonfission reactions were shown to be a function of $\Delta A/A$ and $\Delta A/A_{REC}$. The following criteria distinguish spallation from fragmentation and both from fission:

(1) Spallation is indicated if $\Delta A/A < 0.67$ and if $\langle T\rangle \simeq 17\ \Delta A/A$ MeV.

The values of ϵ_α and the results of counter experiments confirm this conclusion. The average kinetic energy of ^{2}H, ^{3}H, ^{3}He, and ^{4}He was found to agree with $\epsilon_S \simeq 17$ MeV for reactions with $\Delta A/A < 0.67$. This result is evidence for a random-walk process caused by the emission of these light particles.

The energy of neutrons and protons is appreciably smaller than 17 MeV. Thus, the emission of nucleons only cannot account for the observed values of $\langle T\rangle$ in deep-spallation reactions. This conclusion is confirmed by the small value of P_{max} for this reaction mode. A combination of nucleon emission and the emission of heavier nuclei could account for the observed recoil energies.

(2) A combination of fragmentation and spallation is indicated, if $\Delta A/A > 0.67$, if $A_{REC} > 5$, and if $\langle T\rangle = 20$ to 50 MeV.

Counter experiments show that $\langle T\rangle \simeq 27$ MeV for nuclei of mass numbers from 6 to 12 ($\Delta A/A > 0.8$) from proton bombardment of silver. This suggests that the value of $\langle T\rangle$ in these reactions depends only on the target nucleus. The empirical expression

$$\langle T\rangle = 3.1 Z/A^{1/3}\ \text{MeV} \quad (4)$$

correlates the values of $\langle T\rangle$ for several target nuclei. The largest values of $\langle T\rangle$ are found in fission (~50 to 100 MeV).

The probability of various reaction modes. The maximum available energy for nuclear and particle recoil is $E^* + Q$. Values of $T_{max}/\langle T\rangle$, based on this quantity, were calculated for several reaction modes. The value of P_{max} for the emission of nucleons only is ~0.2 when ^{18}F, ^{22}Na, and ^{24}Na are formed from aluminum, and drops close to zero when ^{7}Be and ^{8}Li are formed. For the other reactions studied, $P_{max} = 0$ for this reaction mode.

The values of P_{max} for the emission of ^{3}He and ^{4}He from aluminum are ~0.9. These values decrease overall with both ΔA and A. In the reactions $^{197}\text{Au} \rightarrow {}^{22}$Na, ^{24}Na, and ^{28}Mg, where $\Delta A \simeq 170$, $P_{max} = 0$ for alpha-particle emission. The recoil partners in these reactions must be heavier than alpha particles. For example, $P_{max} = 0.60$ for the reaction ^{197}Au$(p, 4p6n)\ 8^{20}$O^{28}Mg. When nuclei heavier than ^{20}O are emitted, $P_{max} \rightarrow 1.00$. For all but one of the reactions studied, $P_{max} = 1.00$ for two-fragment breakup.

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