Systematics of particle-nucleus reactions. I. Parameters

Lester Winsberg
Department of Physics, University of Illinois at Chicago Circle, Chicago, Illinois 60680
and W. K. Kellogg Radiation Laboratory, California Institute of Technology, Pasadena, California 91125
(Received 5 June 1979)

A set of parameters for studying the systematics of nuclear recoil, induced by high-energy protons, is proposed. A simple version V_0 of the two-velocity model is the basis of this analysis. The velocity of the observed recoil nucleus is assumed to be the vector sum of an initial velocity v, which is constant and in the forward direction from the first step of the reaction, and a velocity V, which has a distribution of values and is isotropic from the second step. The evaluation of recoil experiments yields values of v and p/p_{CN} (the fractional momentum transfer) for the first step of the reaction; $\langle T\rangle = 1/2M_{\mathrm{REC}}\langle V^2\rangle$, ϵ_S and ϵ_F for the second step; and $T_{\max}/\langle T\rangle$ and $P_{\max}$ for the overall reaction. The mass of the observed recoil nucleus is M_{REC}; ϵ_S and ϵ_F are the average kinetic energy of the emitted particles in a random-walk process or in two-fragment breakup, respectively; $T_{\max}$ is the maximum value of T for a given reaction mode, and $P_{\max}$ is the upper limit to the probability of this reaction mode. The combination of the single-collision model and the V_0 approximation results in the additional parameter $E_0^* = p\,E_{\mathrm{CN}}/p_{\mathrm{CN}}$ for the first step of the reaction, where E_{CN} is the excitation energy plus a small kinetic energy in a compound-nucleus reaction. An analysis based on the collision-tube model, in which one or more nucleons may be emitted in the first step, gives the excitation energy E^*, $E^*/\Delta A$, the mass Δm of the effective target, and $\Delta A/\Delta m$. The value of E^* includes the kinetic energy of the excited nucleus and the separation energy of the nucleons emitted in the first step. The total number of nucleons emitted in the reaction is ΔA.

[NUCLEAR REACTIONS Proton induced; introduction to recoil systematics.]

In this series of papers I will examine the systematics of nuclear reactions induced by protons and other particles with energies of ~100 MeV or more. The overall objective will be to determine the general characteristics of these reactions, as directly as possible, from measurements in the literature. The results of these measurements will be analyzed with simple nuclear-reaction models to obtain several reaction parameters. A systematic comparison of these parameters for a variety of nuclear reactions will give us a general picture of the mechanism of these reactions. In this paper I will introduce the overall procedure to be followed. Because of the extensive literature on this subject, the next report in the series will be limited to the systematics of recoil measurements for nonfission reactions, induced by protons with energies between 1 GeV and 400 GeV.[1] Later reports will cover the systematics of excitation functions and fission reactions.

Inelastic nuclear reactions in this energy range are of three general types—spallation, fragmentation, and fission.[2] In spallation, one or more light particles (nucleons, alpha particles, etc.) are emitted by the target nucleus to form the observed recoil nucleus. Fragmentation is characterized by the emission of one or a few pieces of nuclear matter, called fragments and consisting of many nucleons. In fission, the target nucleus divides into two fairly large nuclei. In general, the struck nucleus emits the lightest particles in spallation and the heaviest in fission (in addition to some light particles), with fragments having an intermediate mass.

Spallation reactions can be divided into two types: simple reactions—(p,p'), (p,n), (p,pn), $(p,2p)$, etc.—and deep spallation in which many particles are emitted. Strictly speaking, (p,p') and (p,n) reactions are not spallation, since the mass number remains unchanged. However, they will be included in the spallation category along with the other simple reactions.

A convenient starting point in the analysis is the model proposed by Serber to describe nuclear reactions induced by high-energy particles.[3] This model, designated S, assumes a sequence of two events.

(1) The incident particle initiates a cascade of nucleon-nucleon interactions. In this process the target nucleus may lose one or more nucleons (or none) and is left with the excitation energy E^*.

(2) This excited nucleus loses mass and excitation energy to form the final recoil nucleus.

I will try to determine the range of target and product nuclei where this model is valid.

An alternate reaction mode is the process of fragmentation. However, a model has not been developed for determining the parameters of a fragmentation reaction from the experimental data. The Serber model will, therefore, be the basis of the analysis. The presence of fragmentation in a reaction should result in deviations from the predictions of this model.

Extensive cascade-evaporation calculations, based on the S model, have been compared with many cross-section measurements and with the kinetic properties of the reaction products in order to determine the reaction mechanism.[2,4] This comparison has been partially successful in confirming the nature of these reactions.

My approach is different. I will examine the systematics of a few parameters, which can be obtained directly from recoil measurements, to determine how they depend on the energy of the incident proton, on the target nucleus, and on the observed recoil nucleus. From this comparison will come an experimental test for distinguishing between spallation, fragmentation, and fission.

Recoil-range experiments are of two general types:

(1) Thin-target experiments.[2,5,6] In this type of experiment the thickness of the target is much smaller than the recoil range to be measured. The product nuclei recoil at various angles into some material, usually aluminum or an organic plastic film. If the catcher consists of many thin films, the recoil-range distribution can be measured at each angle.

(2) Thick-target experiments.[2,5-10] A foil assembly containing a target foil, sandwiched between two or more catcher foils, is aligned perpendicular to the incident beam. The target foil must be thicker than the largest recoil range to be measured. The observed nuclei must not be formed directly in the catcher foils, since the beam passes through the entire foil assembly. The fraction of nuclei, which recoil into the forward and backward foils, are denoted by F and B, respectively. The remainder come to rest within the target foil. In one variation of this experiment, the foil assembly is aligned parallel, or nearly parallel, to the incident beam, and the fraction P of nuclei, which recoil into either catcher foil, is measured.

Another source of information are counter and emulsion experiments for measuring the angular and energy distribution of the recoiling nuclei.[11-26] The type of information obtained from these experiments complements the recoil-range results.[1]

Many review articles have been written on nuclear reactions induced by high-energy protons.[2,5,27-33] The results of recoil experiments are often analyzed by the two-velocity model, denoted V (also called the two-step velocity vector model).[6-10] The two steps in this model are similar to the two events described in the S model.

(1) In the first step, the particle interacts with the target nucleus (mass number $=A$) to form an excited nucleus with the velocity v, momentum p, and excitation energy E^*.

(2) In the second step, the excited nucleus loses mass and excitation energy to form the final recoiling nucleus (mass number $=A_{REC}$). This nucleus acquires the additional velocity V, which in general will have a distribution of values and directions.

In most reactions, v and p are expected to have a component perpendicular to the direction of the beam, as well as parallel to it. Also, V may have a nonisotropic angular distribution. In many thick-target experiments only F and B are measured, which does not provide enough information to determine the perpendicular component of v or p or the nature of the angular distribution of V. For these cases, the following additional assumptions can be made:

(1) The quantities v and p in the first step are constant[34] and in the forward direction.

(2) The velocity in the second step is isotropic.

The distribution in V was shown to be Maxwellian (or nearly so) for several cases.[10,35] The corresponding energy T will have a minimum value T_0, which may be greater than zero because of the Coulomb barrier. I will denote the two-velocity model with these additional assumptions and with a distribution in V as the V_0 approximation; see Fig. 1. An analysis of thick-target experiments, based on these assumptions, was recently reported.[10]

In the V_0 approximation, the angular distribution of the recoil products is the result of a forward motion from the first step and an isotropically distributed motion from the second step. These two effects result in an angular distribution in the laboratory that is a maximum at 0° and decreases monotonically to a minimum value at 180°. This type of angular distribution has been observed for proton-induced deep spallation.[12,15,36-38] However, a peak has been observed between 30° and 90° in

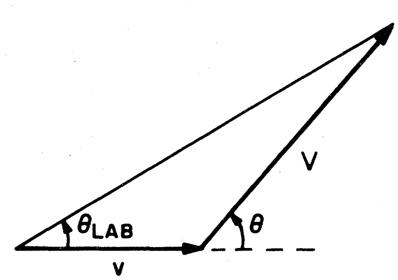

FIG. 1. The V_0 approximation of the two-velocity model.

some, but not all, (p,n), (p,pn), and other simple reactions[38-45] and in some fission or fragmentation reactions with heavy target nuclei.[14,22,46-49] In these cases, the V_0 approximation is, strictly speaking, not valid. Nevertheless, parameters, based on this approximation, may be useful in studying the systematics of these reactions.

Fission reactions are not included in the initial study of nuclear-reaction systematics.[1] The other known cases, where a peak has been observed in the angular distribution, are excluded in that study by restricting the proton energies to 1 GeV or more.

The V model has been evaluated for angular and range measurements of ^{149}Tb from the interaction of 2.2-GeV protons with gold.[37] The authors of that study concluded that the V model is consistent with the data, if a positive correlation between V and the forward component of v is included. These results can be compared with other range measurements made on this reaction with tantalum, gold, and bismuth foils.[50] In the latter study, the forward component of v was found to be constant, independent of the value of V. Although this conclusion, which is consistent with the V_0 approximation, is different from that of the 2.2-GeV study, the final values of parameters related to v and V agree within experimental uncertainty; see Table I.

I have concluded, on the basis of this comparison and the shape of the angular distribution, as noted above, that the V_0 approximation is valid for most spallation reactions induced by high-energy protons. As is pointed out in Ref. 1, the values of parameters obtained with this approximation provide a clue as to the nature of the reaction. The values of parameters obtained for fragmentation reactions, where the V_0 approximation is not expected to be valid, are indeed found to be out of line with those for spallation reactions.

In the analysis of recoil experiments, the range of the recoiling nucleus is usually taken to have the form

$$R = kT^{N/2}. \quad (1)$$

The constants k and N can be derived from range-energy data.[51,52] The analysis of the experimental data, based on the V_0 approximation gives the values of v, p, $\langle V\rangle$, $\langle T\rangle$, and related quantities.[10] (Average values are indicated by $\langle\ \rangle$.)

The S and V models are closely related, since each step in the V model corresponds to the same step in the S model. Despite this overall similarity, the two models differ significantly in several ways.

(1) The S model requires the input of a large amount of data on particle-particle collisions, nuclear-energy levels, etc., much of which is not known and must be estimated.[4] The large number of variables in the S model makes it difficult to extract unambiguous information from a comparison of calculations based on it with measured cross sections and kinetic properties of the reaction products.

(2) The analysis based on the V model provides a way of determining reaction parameters (v, $\langle T\rangle$, and related quantities) directly from the experimental results.

(3) The V_0 approximation assumes that all of the forward momentum transfer from the incident particle occurs in the first step, and all of the isotropic processes occur in the second step. In the S model no such separation occurs, since there can be a perpendicular component in the first step and a nonisotropic distribution of recoils in the second step.

The paper that follows will first consider the recoil results of proton-induced nonfission reactions[1] and then correlate them with cross-section measurements.[53] A study of fission systematics and of systematics of reactions induced by other particles will also be given.

TABLE I. $p + {}^{197}\mathrm{Au} \rightarrow {}^{149}\mathrm{Tb}$.

E_p GeV	$\langle v\rangle$ $\left(\frac{\text{MeV}}{\text{nucleon}}\right)^{1/2}$	$\left\langle\frac{v}{V}\right\rangle$	$\langle T\rangle$ MeV	Ref.
0.7	0.120	0.65	3.9	50
1.0	0.106	0.59	3.8	50
1.7	0.108	0.57	4.2	50
2.2	0.0923	0.510	3.88	37
3.0	0.085	0.48	3.7	50
4.5	0.067	0.38	3.6	50
6.2	0.067	0.37	4.0	50

RECOIL PARAMETERS

The results of recoil experiments can be expressed in various ways. In order to make a systematic survey of the results of these experiments, parameters with one or both of the following properties are particularly useful:

(1) The parameter indicates the nature of the nuclear reaction directly.

(2) The parameter depends on the fewest number of variables.

First step of the reaction: the parameters p/p_{CN}, E_0^*, E^*, $E^*/\Delta A$, Δm, and $\Delta A/\Delta m$

The parameter p/p_{CN} has the first property. It is the fractional momentum transfer, or the ratio of p to the value it would have for a compound-nucleus reaction. In such a reaction this parameter is equal to unity. In most reactions induced by protons with energies of 1 GeV or more, $p/p_{\mathrm{CN}} \ll 1.0$.[10] This parameter is most useful for incident energies below 1 GeV.

Another quantity of interest is the excitation energy of the recoiling nucleus from the first step of the reaction. Several reaction models have been proposed for determining this energy.

1. The single-collision (SC) *model.* This model was proposed by Turkevich for reactions induced by nucleons (quoted in Ref. 8). In this model, the incident nucleon collides with a nucleon inside the nucleus and escapes. The target nucleon remains inside the nucleus with its kinetic energy becoming E_0^* = excitation energy plus kinetic energy (usually small) of the recoiling nucleus. A discussion of this model is given by Kaufman, Steinberg, and Weisfield.[54]

From the model we get

$$E_0^* = \frac{p}{p_{\mathrm{CN}}} E_{\mathrm{CN}} . \tag{2}$$

Here, E_{CN} is the excitation energy plus kinetic energy of the recoiling nucleus for a compound-nucleus reaction.

Intranuclear cascade calculations indicate a linear relationship between the excitation energy and the fractional momentum transfer, given by[54,55]

$$E_1^* = 0.8 E_0^* . \tag{3}$$

The factor 0.8 is, in a sense, a correction to account for the prompt emission of other particles, in addition to the incident nucleon.

The values of E_0^* for the interaction of protons with ^{27}Al to form ^{7}Be, ^{11}C, ^{18}F, ^{22}Na, and ^{24}Na are given in the preceding paper.[56] This parameter appears to be correlated with the excitation function for these reactions. The most complete measurements were made for ^{24}Na. In this case the values of E_0^* and the excitation function increase together with E_p, the energy of the incident proton, near the threshold of the reaction. At higher values of E_p, E_0^* remains essentially constant.

2. The collective-tube (CT) *model.* The SC model is probably not valid for deep-spallation reactions. The values of E_0^* may vary with E_p, as a result. The reactions of $p + {}^{27}$Al which produce ^{11}C and ^{18}F and of $p + {}^{181}$Ta, ^{197}Au, and ^{209}Bi which produce ^{149}Tb are examples of reactions in which this effect is present.[1,56]

Cumming has recently applied the collective-tube (CT) model to the analysis of nuclear reactions induced by protons of energies greater than several GeV.[57] In this model the incident particle interacts with the nucleons in its path as if they were a single object. As a result, the effective target may have a mass Δm of one or more nucleons. Cumming derived the following expression, relating the product of p and v_p (the velocity of the incident proton) to the excitation energy,

$$p v_p = E^*(1 + \Delta m c^2/E) . \tag{4}$$

In this expression, E^* includes the excitation energy of the target nucleus with a "hole" left by the ejected nucleons, its kinetic energy, and the separation energy of the ejected nucleons. The total energy of the incident proton is given by $E = E_p + m_p c^2$. Equation (4) reduces to Eq. (2) for $\Delta m = m_n$, the nucleon mass.

Cumming plotted $p v_p$ vs E^{-1} to obtain E^* and Δm. For the reaction of protons with aluminum[56] to produce ^{24}Na, he obtained $E^* = (54 \pm 2)$ MeV and $\Delta m = (0.9 \pm 0.2) m_n$. For ^{149}Tb from gold,[50,58] $E^* = (269 \pm 12)$ MeV and $\Delta m = (3.1 \pm 0.4) m_n$. The first value of Δm is consistent with the SC model; the second is not.

The values of E^* (and E_0^*) vary with $\Delta A = A - A_{\mathrm{REC}}$, the total number of nucleons emitted in the reaction. The parameter $E^*/\Delta A$, which gives the excitation energy per emitted nucleon, varies relatively little with ΔA.[1] It thus possesses the two properties given above for a useful parameter.

Another parameter of interest is the total number of nucleons emitted divided by the number of target nucleons in the initial interaction. This ratio $\Delta A/\Delta m$, with Δm in mass-number units, provides a comparison between the first step and the overall reaction.

Second step of the reactions: the parameters $\langle T \rangle$, ϵ_S, and ϵ_F

The average recoil energy $\langle T \rangle$, imparted to the observed nucleus in the second step of the reaction, is found to be constant with E_p for spallation reactions[50,56,58-60] and is thus a convenient parameter for studying reaction systematics. The determination of ϵ, the average energy of a particle (nucleon, alpha particle, etc.) emitted in this step of the reaction, from the value of $\langle T \rangle$ is given by Cumming and Bachmann for a random-walk process.[61] Their derivation is repeated here in a slightly different fashion.

The average recoil velocity squared, corresponding to $\langle T \rangle$, is

$$\langle V^2 \rangle = \sum_{i=1}^{n} V_i^{\,2} , \tag{5}$$

where n is the number of particles emitted and V_i is the velocity imparted to the residual nucleus by the ith particle.

With momentum conservation, we get

$$\langle T\rangle = A_{\mathrm{REC}} \sum_{i=1}^{n} \frac{a_i \epsilon_i}{A_i^2}, \tag{6}$$

where a_i and ϵ_i are the mass number and average kinetic energy of the ith particle and A_i is the mass number of the resulting nucleus. If the particles have the same value for a, given by

$$a = \Delta A/n \tag{7}$$

and for ϵ, Eq. (6) gives

$$\epsilon = \frac{\langle T\rangle}{a A_{\mathrm{REC}} \sum (A - i a)^{-2}}. \tag{8}$$

The excited nucleus from the first step is assumed to be the target nucleus.

In the limit as $a/A \to 0$, ϵ becomes

$$\epsilon_S = \frac{\langle T\rangle}{\Delta A/A}. \tag{9}$$

Emission of small particles in this way is typical of spallation reactions. The average energy $\epsilon_{\mathrm{NUCLEON}}$ for the emission of nucleons only is obtained from Eq. (8) by setting $a=1$. The value of ϵ_S is larger than this value by a small amount, which depends on the value of A.

If the excited nucleus breaks up into two nuclei of mass numbers A_{REC} and ΔA, ϵ becomes

$$\epsilon_F = \frac{\langle T\rangle}{\Delta A/A_{\mathrm{REC}}}. \tag{10}$$

This type of breakup is typical of a fission or fragmentation process.

For the general case, the total kinetic energy of the emitted particles is $\sum n_j \epsilon_j$, where n_j is the number of particles of a given type and ϵ_j is their average kinetic energy. Each set of n_j particles has the same mass number a_j and atomic number z_j. Thus,

$$\epsilon = \frac{1}{n} \sum n_j \epsilon_j. \tag{11}$$

In most cases of interest, $j_{\max} = 2$ or 3. If only one kind of particle is emitted, the total kinetic energy is $n\epsilon$ with ϵ given by Eq. (8). If two or more kinds of particles are emitted, one value of ϵ_j can be determined from Eq. (6), if the other values are known or can be estimated.

The value of $\sum n_j \epsilon_j$ is needed in an energy inventory of the overall reaction. Two more parameters will be obtained later from this inventory.

The parameters ϵ_S and ϵ_F represent the two extremes of particle emission in a nuclear reaction. They are related by the expression

$$\epsilon_F = \frac{A_{\mathrm{REC}}}{A} \epsilon_S. \tag{12}$$

For small values of $\Delta A/A$, $\epsilon_F \simeq \epsilon_S$.

If a/A is small, the value of ϵ will depend only weakly on the average mass of the emitted particles. Thus, if ten neutrons and ten protons are emitted and if $\epsilon_S = 15$ MeV, the total kinetic energy of the emitted particles will be ~300 MeV for unbound nucleons and ~75 MeV for alpha particles. In either case, the average kinetic energy of a particle is ~15 MeV. This result agrees with the fact that alpha particles are more effective than unbound nucleons in imparting recoil energy to a residual nucleus.

Energy inventory: the parameters $T_{\max}/\langle T\rangle$ and $P_{\max}$

The experiments described here do not provide enough information for making a detailed energy inventory for a nuclear reaction. Instead, I will make an approximate energy balance, assuming that the excited nucleus from the first step is the target nucleus. This assumption is dropped in the following paper, where cases with $\Delta m > m_n$ are considered.[1] The neglect of a perpendicular component of v is not expected to affect the results of the analysis appreciably for most spallation reactions.

The energy of the incident proton, based on these assumptions, is distributed as follows:

$$E_p = E_p' + \tfrac{1}{2} M v^2 + \langle T\rangle + \sum n_j \epsilon_j - Q + \Delta E, \tag{13}$$

where E_p' is the energy of scattered proton, M is the mass of the excited nucleus formed in the first step of the reaction, Q is the mass energy of reactants minus mass energy of products, excluding masses of particles, e.g., pions, formed in the reaction, and ΔE is the "excess energy" (energy of x rays, gamma rays, radioactive decay, etc.).

The excitation energy (in this case $E^* = E_0^*$) is given by the last four terms in Eq. (13). The total recoil energy is given by

$$\langle T\rangle + \sum n_j \epsilon_j = E^* + Q - \Delta E. \tag{14}$$

The terms $\sum n_j \epsilon_j$ and Q can be calculated for various combinations of emitted particles.[62] Reactions with negative values of $E^* + Q$ cannot occur because of energy conservation. We get the maximum recoil energy by setting $\Delta E = 0$ in Eq. (14):

$$T_{\max} + \left(\sum n_j \epsilon_j\right)_{\max} = E^* + Q. \tag{15}$$

If the distribution in T and ϵ_j is known, an upper limit to the probability of each reaction mode can be determined. This calculation can be illustrated for the case represented by Eq. (8). With Eq. (15) we get

$$T_{\max} = \frac{E^* + Q}{1 + n/[a A_{\mathrm{REC}} \sum (A - i a)^{-2}]}. \tag{16}$$

In the limit as $a/A \to 0$, we can use Eq. (9) to get the less accurate expression

$$T_{max} = \frac{E^* + Q}{1 + nA/\Delta A} . \tag{17}$$

The upper limit to the probability of a given reaction mode is given by

$$P_{max} = \int_0^{T_{max}} P(T) dT , \tag{18}$$

where $P(T)dT$ is the distribution in T. The probability of a given reaction mode will be less than this value, because of competing mechanisms and because ΔE is greater than zero in general.

The expression $P(T)$ was found to be of the form given by Eq. (19) for several types of spallation reaction.[10]

$$P(T) = \frac{4T \exp(-2T/\langle T \rangle)}{\langle T \rangle^2} . \tag{19}$$

The value of P_{max} depends only on the parameter $T_{max}/\langle T \rangle$. Both of these quantites are useful parameters for describing the overall reaction.

The systematic survey that follows of nuclear reactions induced by high-energy particles is based on the parameters presented here.[1,53]

ACKNOWLEDGMENTS

I wish to thank those who have worked in this field over the years. I am grateful to John Alexander and Ellis Steinberg for a critical reading of this and the following manuscript. Much of this work was performed while on sabbatical leave at Cal Tech, where I enjoyed the warm and generous hospitality of Professor Willy Fowler. This work was supported in part by the National Science Foundation (PHY76-83685) at Cal Tech and in part by the Division of Nuclear Physics of the U. S. Department of Energy at the University of Illinois.

[1] L. Winsberg, Phys. Rev. C 22, 2123 (1980), following paper.
[2] J. Hudis, in *Nuclear Chemistry*, edited by L. Yaffe, (Academic, New York, 1968), Vol. I, p. 169; J. M. Alexander, *ibid.*, p. 273.
[3] R. Serber, Phys. Rev. 72, 1114 (1947).
[4] H. W. Bertini, A. H. Culkowski, O. W. Hermann, N. B. Gove, and M. P. Guthrie, Phys. Rev. C 17, 1382 (1978) refer to earlier work.
[5] B. G. Harvey, Annu. Rev. Nucl. Sci. 10, 235 (1960).
[6] L. Winsberg and J. M. Alexander, in Ref. 2, p. 340.
[7] N. Sugarman, M. Campos, and K. Wielgoz, Phys. Rev. 101, 388 (1956).
[8] N. T. Porile and N. Sugarman, Phys. Rev. 107, 1410 (1957).
[9] N. Sugarman, H. Munzel, J. A. Panontin, K. Wielgoz, M. V. Ramaniah, G. Lange, and E. Lopez-Manchero, Phys. Rev. 143, 952 (1966).
[10] L. Winsberg, Nucl. Instrum. Methods 150, 465 (1978).
[11] S. Katcoff, Phys. Rev. 114, 905 (1959).
[12] E. W. Baker, S. Katcoff, and C. P. Baker, Phys. Rev. 117, 1352 (1960).
[13] S. Katcoff, E. W. Baker, and N. T. Porile, Phys. Rev. 140, B1549 (1965).
[14] A. M. Poskanzer, G. W. Butler, and E. K. Hyde, Phys. Rev. C 3, 882 (1971).
[15] E. K. Hyde, G. W. Butler, and A. M. Poskanzer, Phys. Rev. C 4, 1759 (1971). References to the emulsion literature are given here.
[16] R. G. Korteling, C. R. Toren, and E. K. Hyde, Phys. Rev. C 7, 1611 (1973).
[17] E. N. Vol'nin, A. A. Vorob'ev, and D. M. Seliverstov, Zh. Eksp. Teor. Fiz. Pis'ma Red. 19, 691 (1974) [Sov. Phys.—JETP Lett. 19, 357 (1974)].
[18] V. I. Bogatin, V. K. Bondarev, V. F. Litvin, O. V. Lozhkin, N. A. Perfilov, Yu. P. Yakovlev, and V. P. Bochin, Yad. Fiz. 19, 32 (1974) [Sov. J. Nucl. Phys. 19, 16 (1974)].
[19] J. P. Alard, A. Baldit, R. Brun, J. P. Costilhes, J. Dhermain, J. Fargeix, L. Fraysse, J. Pellet, G. Roche, J. C. Tamain, A. Cordaillat, and A. Pasinetti, Nuovo Cimento 30A, 320 (1975).
[20] E. N. Volnin, G. M. Amalsky, D. M. Seleverstov, N. N. Smirnov, A. A. Vorobyov, and Yu. P. Yakovlev, Phys. Lett. 55B, 409 (1975).
[21] G. M. Raisbeck, P. Boerstling, R. Klapisch, and T. D. Thomas, Phys. Rev. C 12, 527 (1975).
[22] L. P. Remsberg and D. G. Perry, Phys. Rev. Lett. 35, 361 (1975).
[23] G. W. Butler, D. G. Perry, A. M. Poskanzer, J. B. Natowitz, F. Plasil, and L. P. Remsberg, in *High Energy Physics and Nuclear Structure—1975*, Proceedings of the Sixth International Conference, Santa Fe and Los Alamos, edited by D. E. Nagle *et al.* (AIP, New York, 1975).
[24] V. I. Bogatin, V. F. Litvin, O. V. Lozhkin, N. A. Perfilov, and Yu. P. Yakovlev, Nucl. Phys. A260, 446 (1976).
[25] G. D. Westfall, R. G. Sextro, A. M. Poskanzer, A. M. Zebelman, G. W. Butler, and E. K. Hyde, Phys. Rev. C 17, 1368 (1978).
[26] R. Green and R. G. Korteling, Phys. Rev. C 18, 311 (1978).
[27] J. M. Miller and J. Hudis, Annu. Rev. Nucl. Sci. 9, 159 (1959).
[28] N. A. Perfilov, O. V. Lozhkin, and V. P. Shamov, Usp. Fiz. Nauk 60, 3 (1960) [Sov. Phys. Usp. 3, 1 (1960)].
[29] J. B. Cumming, Annu. Rev. Nucl. Sci. 13, 261 (1963).
[30] J. R. Grover and A. A. Caretto, Annu. Rev. Nucl. Sci. 14, 51 (1964).
[31] E. K. Hyde, *The Nuclear Properties of the Heavy Elements* (Prentice-Hall, Englewood Cliffs, 1964), Vol. III.
[32] J. Saudinos and C. Wilkin, Annu. Rev. Nucl. Sci. 24, 341 (1974).
[33] L. B. Church, *International Review of Science*, *Ser. 2*,

edited by A. G. Maddock (Buttersworth, London, 1975), Vol. 8, p. 197.
[34]Actually, v may have a distribution in value, except for small second-order effects if v and V are uncorrelated; see Ref. 10.
[35]In earlier work, V was taken to be constant. The effect of a distribution in V can be neglected if $v \ll \langle V \rangle$.
[36]A. M. Poskanzer, J. B. Cumming, and R. Wolfgang, Phys. Rev. 129, 374 (1963).
[37]V. P. Crespo, J. B. Cumming, and J. M. Alexander, Phys. Rev. C 2, 1777 (1970).
[38]M. A. Molecke and A. A. Caretto, Phys. Rev. C 15, 719 (1977).
[39]J. A. Panontin, L. L. Schwartz, A. F. Stehney, E. P. Steinberg, and L. Winsberg, Phys. Rev. 145, 754 (1966).
[40]J. A. Panontin, L. L. Schwartz, A. F. Stehney, E. P. Steinberg, and L. Winsberg, Phys. Rev. 169, 851 (1968).
[41]L. P. Remsberg, Phys. Rev. 174, 1338 (1968).
[42]L. P. Remsberg, Phys. Rev. 188, 1703 (1969).
[43]Y. W. Yu, A. A. Caretto, and L. B. Church, Nucl. Phys. A152, 295 (1970).
[44]E. P. Steinberg and L. Winsberg, Phys. Rev. C 7, 1410 (1973), also quoted in Ref. 33.
[45]S. D. Baker *et al.*, Phys. Lett. 52B, 57 (1974).
[46]J. B. Cumming, R. J. Cross, J. Hudis, and A. M. Poskanzer, Phys. Rev. 134, B167 (1964).
[47]V. P. Crespo, J. B. Cumming, and A. M. Poskanzer, Phys. Rev. 174, 1455 (1968).
[48]D. R. Fortney and N. T. Porile, Phys. Rev. C 14, 1652 (1976).
[49]D. R. Fortney and N. T. Porile, Phys. Lett. 76B, 553 (1978).
[50]L. Winsberg, Phys. Rev. 135, B1105 (1964).
[51]L. C. Northcliffe and R. F. Schilling, Nucl. Data Tables A7, 233 (1970).
[52]L. Winsberg, At. Data Nucl. Data Tables 20, 389 (1977).
[53]L. Winsberg (unpublished).
[54]S. B. Kaufman, E. P. Steinberg, and M. W. Weisfield, Phys. Rev. C 18, 1349 (1978).
[55]N. T. Porile, Phys. Rev. 120, 572 (1960).
[56]L. Winsberg, E. P. Steinberg, D. Henderson, and A. Chrapkowski, Phys. Rev. C 22, 2108 (1980), preceding paper.
[57]J. B. Cumming, Phys. Rev. Lett. 44, 17 (1980).
[58]L. Winsberg, M. W. Weisfield, and D. Henderson, Phys. Rev. C 13, 279 (1976).
[59]S. Singh and J. M. Alexander, Phys. Rev. 128, 711 (1962).
[60]P. L. Reeder, Phys. Rev. 178, 1795 (1969).
[61]J. B. Cumming and K. Bachmann, Phys. Rev. C 6, 1362 (1972).
[62]A. H. Wapstra and K. Bos, At. Data Nucl. Data Tables 19, 177 (1977).