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# Dynamical theory for the $P$-wave pion-nucleon interaction

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We present a dynamical theory of low energy $P$-wave $\pi N$ interaction. The theory is an extension of the Chew-Low theory, with three additional features, namely, (1) inclusion of the full nucleon recoil effect, (2) inclusion of the $Z$ graphs, and (3) inclusion of $P11$ channel inelasticity. The second feature is new in a $P$-wave theory based on the Low expansion. We are able to fit the $P33$ and the $P11$ phase shifts in the energy region $0 < T_\pi < 250$ MeV quite well. In particular, we show that it is possible to explain the change of sign of the $P11$ phase shift, $T_\pi \sim 150$ MeV, in terms of the strong inelasticity present in the channel. For the $P13$ and $P31$ channels, where the phase shifts are the smallest, we do reasonably well at threshold, but not so well as the energy increases. We have also examined the structure of the off-mass-shell amplitudes in the $P11$ and $P33$ channels. Only the $P33$ channel amplitude is found to be factorable. A major result of our work is that the $P33$ form factor is very hard. If one used a monopole form, the form factor mass should be $10\, m_\pi$ or larger.

[NUCLEAR REACTIONS Pion-nucleon interaction, $Z$-graph term, sigma exchange term, $P33$ resonance, form factor of $P33$ amplitude, $P11$ inelasticity, nonstatic kinematics.]

## I. INTRODUCTION

In the preceding paper, we have discussed the results of using nonstatic nucleon kinematics in treating the pion-nucleon interaction. The main result of that paper is that in the nonstatic theory the $P33$ resonance cannot be produced with the nucleon pole terms only. Other mechanisms must be included.

In this paper we present an extension of the Chew-Low theory[1] using nonstatic nucleon kinematics and two additional terms of the Low expansion over and above the usual ones. The latter are the nucleon poles and the elastic and sometimes inelastic rescattering terms. The new terms which we include are the so-called $Z$ graphs and a scalar-isoscalar seagull term. The $Z$ graphs describe the virtual process of a pion converting into a nucleon-antinucleon pair and then the antinucleon annihilating the initial nucleon to produce the final pion. In the Low expansion these graphs appear as the disconnected parts of the term involving two-nucleon–one-antinucleon intermediate states. In an earlier work on the pion nucleon $S$-wave interaction[2,3] one of us (M.K.B.) and Cammarata were able to explain satisfactorily the isovector part of the interaction as coming almost entirely from the $Z$ graphs. In the course of that work it was found that $|g_\pi(4M^2)| \equiv \bar{g}_\pi = 11.7$. Of course, it is well known that the $Z$ graph contributes $-(\bar{g}_\pi{}^2/4\pi M)$ to the isoscalar scattering length, $M$ being the nucleon mass. This is about two orders of magnitude larger than the experimental value. So, this enormous repulsion must be very nearly canceled out by introducing the exchange of a scalar-isoscalar boson field between the pion and the nucleon. This is called *pair suppression*. We will follow the popular practice of calling the field the $\sigma$ field. In the present theory, it is not necessary that it be an independent field. The $\sigma$ field may be a local function of the pion fields.[4] In the work of Ref. 2, explicit references to the isoscalar part of the $Z$ graph and its near cancellation by the $\sigma$ exchange were avoided by eliminating the combination with a soft pion limit.[5] Because of their roles in the $S$ wave, we continue to carry both mechanisms in the $P$ wave. As we shall see later, both mechanisms provide attraction in the $P$ wave, but the $\sigma$ exchange is less important than the $Z$ graph.

As soon as the $Z$ graphs are included, the $P33$ resonance is easily generated. This is shown in Fig. 1. The nonstatic kinematics produce two important features in the equation for the $P33$ amplitude. First, the crossed contributions from the other three $P$-wave channels are vastly reduced. Second, the rescattering contribution integrals, both the right-hand and left-hand side, converge rapidly. We thus have great confidence in our low energy $P33$ amplitude. Besides, we obtain a very accurate solution of our nonlinear equation, which enables us to determine the $P33$ off-shell amplitude form factor. We find that the mass parameter of a monopole form factor is at least larger than $10\, m_\pi$.

The $P13$ and $P31$ amplitudes are very small at low energy. We fit both quite well near threshold (Fig. 1), and then our theoretical values differ

 

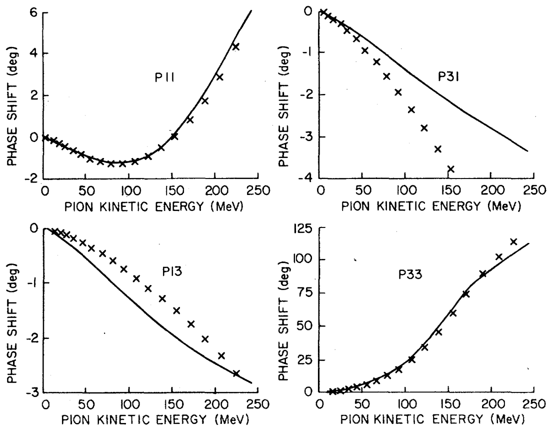


FIG. 1. Comparison of theoretical and experimental phase shifts. The solid lines are results. The crosses are values from Ref. 6.

from the experimental values[6] as energy increases. Since these are small in the region we are studying, minor errors in the high energy contributions to the rescattering integrals can have big influence on the solutions. This type of effect is the least at the threshold where we do have good fits. Thus, we consider our $P13$ and $P31$ solutions to be satisfactory in the present context.

The $P11$ phase shifts are known to be difficult to explain. Without the inelastic contributions one cannot reproduce the experimentally observed change of sign of the phase shift. Also, the $P11$ scattering volume is predicted to be much too negative in comparison with the experimental value, while for all other $P$ waves we have good scattering volume fits. These values are listed in Table I. Thus, the elastic theory cannot describe the $P11$ amplitude correctly. This special situation for the $P11$ channel is not surprising, since it is known experimentally that inelasticity is more important in the $P11$ channel than in the other. Therefore, we include the $P11$ inelastic channel contribution to the rescattering terms in our theory as an additional dynamical input. We do this by an ansatz which permits us to use the values of the $P11$ channel inelastic cross section obtained from the phase shift analysis. With this inelastic contribution, we are able to get a very good fit for the $P11$ phase shifts.

## II. DEVELOPMENT OF THE ELASTIC EQUATION

We start the derivation with the Low expansion[8] of the half-off-mass-shell amplitude

$$F_{\beta\alpha}(p_i,p_f,k) \equiv {}_{\rm out}\langle \pi_\beta(k), N(p_f)|j_\alpha(0)|N(p_i)\rangle .$$

$k'$ $(k)$ is the initial (final) pion momentum with isospin $\beta$ $(\alpha)=1,2,3$. $p_i$ $(p_f)$ is the initial (final) nucleon momentum. $k$, $p_i$, and $p_f$ all satisfy their own mass shell constraint. The missing initial pion momentum $k'=p_f+k-p_i$ does not necessarily satisfy the relation $k_0'=(m_\pi{}^2+\vec{k}'^2)^{1/2}$. Explicitly, the expansion is

$$\begin{aligned} F_{\beta\alpha}&(p_i,p_f,k) \\ &\equiv F_{\beta\alpha}(k)=\langle p_f s_f|[a_\beta(k),j_\alpha(0)]|p_i s_i\rangle \\ &\quad+\sum_n \frac{\langle p_f s_f|j_\beta(k)|n\rangle_{\rm out\;out}\langle n|j_\alpha(0)|p_i s_i\rangle}{n_0-p_{f0}-k_0-i\eta} \\ &\quad+\sum_n \frac{\langle p_f s_f|j_\alpha(0)|n\rangle_{\rm out\;out}\langle n|j_\beta(k)|p_i s_i\rangle}{n_0-p_{i0}+k_0} . \end{aligned} \qquad (1)$$

Using the definition of the annihilation operator in terms of the field and its derivative,

$$\begin{aligned} F_{\beta\alpha}(k)=\langle p_f s_f|\,i\int d^4x\, e^{ik\cdot x}\delta(x_0) \\ \times[\dot\phi_\beta(x)-ik_0\phi_\beta(x),\, j_\alpha(0)]|p_i s_i\rangle \\ +\sum_n \frac{\langle p_f s_f|j_\beta(k)|n\rangle_{\rm out\;out}\langle n|j_\alpha(0)|p_i s_i\rangle}{n_0-p_{f0}-k_0-i\eta} \\ +\sum_n \frac{\langle p_f s_f|j_\alpha(0)|n\rangle_{\rm out\;out}\langle n|j_\beta(k)|p_i s_i\rangle}{n_0-p_{i0}+k_0} , \end{aligned} \qquad (2)$$

where $j_\beta(k)=\int d^3r\, e^{i\vec{k}\cdot\vec{r}} j_\beta(0,\vec{r})$ and $j_\beta(x)$ is the pion source current $(\Box+m_\pi{}^2)\phi_\beta(x)=j_\beta(x)$.

The equal time commutator in Eq. (2) is called the seagull term. This term is nonvanishing only if the source current $j_\beta(x)$ contains at least one power of the pion field, which, in turn, requires that the interaction Lagrangian must be at least

TABLE I. Scattering volume ($m_\pi{}^{-3}$). A comparison of theoretical and experimental scattering volumes in four $p$ channels. The experimental values are from Ref. 7.

| | $P11$ | $P31$ | $P13$ | $P33$ |
|---|---|---|---|---|
| Experimental | $-0.082\pm0.002$ | $-0.044\pm0.001$ | $-0.032\pm0.001$ | $0.215\pm0.003$ |
| Theoretical | | | | |
| without inelasticity | $-0.131$ | $-0.034$ | $-0.031$ | $0.201$ |
| with inelasticity | $-0.074$ | $-0.033$ | $-0.034$ | $0.207$ |

bilinear in pion fields or its derivatives. Thus, the value of the seagull term depends on the specific choice of the Lagrangian. From microcausality, Lorentz covariance, and crossing symmetry, it follows that the seagull term is of the general form

$$\langle p_f s_f | \Sigma(0)\delta_{\beta\alpha} + i\epsilon_{\beta\alpha\lambda} k\cdot Y_\lambda(0) | p_i s_i \rangle ,$$

where $\Sigma(0)$ is a scalar-isoscalar operator, $Y_\lambda(0)$ is a vector-isovector operator, and $\epsilon_{\beta\alpha\lambda}$ is the Levi-Civita symbol. These terms will arise if the interaction Lagrangian has the structure

$$\mathcal{L}_{\text{int}}(x) = g_\sigma \sigma(x) \sum_\lambda \phi_\lambda{}^2(x) + \frac{i g_v}{2} \sum_{\lambda,\eta,\theta} \epsilon_{\theta\eta\lambda} \phi_\theta \frac{\partial \phi_\eta(x)}{\partial x^\mu}\, \xi^\mu_\lambda(x) , \qquad (3)$$

where $\sigma(x)$ is a scalar-isoscalar field operator and $\xi^\mu_\lambda(x)$ is a vector-isovector field operator. Specifically, if $\sigma(x)$ and $\xi^\mu_\lambda(x)$ are independent fields, one has

$$\Sigma(x) = g_\sigma \sigma(x) , \qquad Y^\mu_\lambda(x) = g_v \xi^\mu_\lambda(x) . \qquad (4)$$

Otherwise, the relationship is slightly more complicated. We stress that here we do not need or make the assumption that $\Sigma(x)$ is necessarily an independent field. It can be a function of the pion field.

Following the spirit of the Gell-Mann–Levy sigma model[5] and that of the previous work on $S$ wave, we assume that the interaction Lagrangian does not contain the vector-isovector term. In other words, $\xi^\mu_\lambda(x) = 0$.

We need to refer to a Lagrangian for the seagull term only. The structure of the remaining two terms of Eq. (2) does not depend on the specifics of a Lagrangian. These two terms have a summation over all admissible outgoing states $|n\rangle_{\text{out}}$. For the rest of this paper, we drop the subscript. The matrix element $\langle p | j_\beta(0) | n\rangle$ can have a disconnected part if the state $|n'\rangle$ which results from the removal of a nucleon $N$ from the state $|n\rangle$ can be produced from the vacuum by the pion source current. Thus,

$$\langle p | j_\beta(0) | n\rangle = \langle p | N\rangle\langle 0 | j_\beta(0) | n'\rangle + \langle p | j_\beta(0) | n', N\rangle_c . \qquad (5)$$

The first term is the disconnected part, where a nucleon propagates freely. The second term, with subscript $c$, is called the connected part. The Low expansion can be written in terms of its connected parts. Thus, the second term of Eq. (2) becomes

$$\begin{aligned}
&\sum_n \frac{\langle p_f s_f | j_\beta(k) | n\rangle\langle n | j_\alpha(0) | p_i s_i\rangle}{n_0 - p_{f0} - k_0 - i\eta} \\
&= (2\pi)^3 \sum_n \frac{\langle p_f s_f | j_\beta(0) | n\rangle_{c\,c}\langle n | j_\alpha(0) | p_i s_i\rangle}{n_0 - p_{f0} - k_0 - i\eta}\, \delta(\vec{p}_f + \vec{k} - \vec{n}) \\
&\quad + (2\pi)^3 \sum_n \frac{\langle 0 | j_\beta(0) | n\rangle_{c\,c}\langle n, p_f s_f | j_\alpha(0) | p_i s_i\rangle}{n_0 - k_0 - i\eta}\, \delta(\vec{k} - \vec{n}) + (2\pi)^3 \sum_n \frac{\langle p_f s_f | j_\beta(0) | n, p_i s_i\rangle_{c\,c}\langle n | j_\alpha(0) | 0\rangle}{n_0 - k_0' - i\eta}\, \delta(\vec{k}' - \vec{n}) \\
&\quad - (2\pi)^3 \sum_n \frac{\langle 0 | j_\beta(0) | n, p_i s_i\rangle\langle n, p_f s_f | j_\alpha(0) | 0\rangle}{n_0 + p_{i0} - k_0 - i\eta}\, \delta(\vec{k} - \vec{p}_i - \vec{n}) .
\end{aligned} \qquad (6)$$

Similarly, the last term of Eq. (2) becomes

$$\begin{aligned}
&\sum_n \frac{\langle p_f s_f | j_\alpha(0) | n\rangle\langle n | j_\beta(k) | p_i s_i\rangle}{n_0 - p_{i0} + k_0} \\
&= (2\pi)^3 \sum_n \frac{\langle p_f s_f | j_\alpha(0) | n\rangle_{c\,c}\langle n | j_\beta(0) | p_i s_i\rangle}{n_0 - p_{i0} + k_0}\, \delta(\vec{p}_i - \vec{k} - \vec{n}) + (2\pi)^3 \sum_n \frac{\langle 0 | j_\alpha(0) | n\rangle_{c\,c}\langle n, p_f s_f | j_\beta(0) | p_i s_i\rangle}{n_0 + k_0'}\, \delta(\vec{k}' + \vec{n}) \\
&\quad + (2\pi)^3 \sum_n \frac{\langle p_f s_f | j_\alpha(0) | n, p_i s_i\rangle_{c\,c}\langle n | j_\beta(0) | 0\rangle}{n_0 + k_0}\, \delta(\vec{k} + \vec{n}) - (2\pi)^3 \sum_n \frac{\langle 0 | j_\alpha(0) | n, p_i s_i\rangle\langle n, p_f s_f | j_\beta(0) | 0\rangle}{n_0 + p_{f0} + k_0}\, \delta(\vec{n} + \vec{k} + \vec{p}_f) .
\end{aligned} \qquad (7)$$

In this paper, we retain only a few of the terms of the expansions (6) and (7). The terms in (7) are the crossed terms ($k \leftrightarrow -k'$, $\alpha \leftrightarrow \beta$) of (6). So, we will discuss only the latter and all remarks valid for (6) will apply to (7) also. The first group is retained completely, at least, in principle. It has the following parts:

(i) one nucleon states (nucleon pole),
(ii) one nucleon-one pion states (elastic rescat-

tering, and
(iii) the rest (inelastic rescattering).

The second and the third group are dropped. These involve intermediate states with the quantum numbers of a pion. No such resonance is known. It is likely that the first important candidate for the intermediate state is a $3\pi$ state clustered as a $\rho$ and a $\pi$. Thus, typical values of $n_0$ are $n_0 > 1$ GeV. The other matrix element in the numerator corresponds to the process $N+\pi \to N + 3\pi$. There is no special reason for this amplitude to be large. Finding no compelling reason to the contrary, we drop these two groups. From the fourth group we retain the $N\overline{N}$ intermediate states by using for $\overline{N}$ an antinucleon of momentum $\vec{k}-\vec{p}_i$. This is the term that we call the $Z$ graph. Here the energy denominator is $\sim 2M$ but the numerator is compensatingly huge, viz., $\sim g_\pi{}^2$. We have not retained any other piece of this group. (For $P$-wave interaction, the next candidate is $\overline{N}^*$ with $J^P = \frac{1}{2}^+$.)

Excepting the elastic rescattering term, all the states mentioned above plus the seagull term form the *driving term* of the equation for $F_{\beta\alpha}(k)$. We define the phrase *"elastic" driving term* as the one including everything except the inelastic rescattering terms. The phrase *"inelastic" driving term* applies when the latter are included. The inelastic driving terms (one nucleon multipion states) are of very little consequence to the $P13$, $P31$, and $P33$ channels. The elastic driving terms work quite well in reproducing the low energy phase shifts in these channels. As we will see later, for the $P11$ channel we must use the inelastic driving term.

We now begin the discussion of individual elastic driving terms. First, we discuss the nucleon terms. The Lorentz covariance implies that

$$\langle p's|j_\alpha(0)|ps\rangle = ig_\pi[(p'-p)^2]\overline{u}(p')\gamma_5 u(p)\tau_\alpha . \tag{8}$$

Thus, the nucleon pole terms are

$$\frac{g_\pi[(p_f-p)^2]g_\pi[(p_i-p)^2]}{2(k_0+p_{f0}-M)}\overline{u}(p_f)(1-\gamma_0)\tau_\beta\tau_\alpha u(p_i)$$
$$+\frac{g_\pi[(p_f-l)^2]g_\pi[(p_i-l)^2]}{2l_0(k_0+l_0-p_{i0})}$$
$$\times\overline{u}(p_f)[(l_0-p_{i0}-p_{f0})\gamma_0+M]\tau_\alpha\tau_\beta u(p_i), \tag{9}$$

where the four vectors, $p\equiv(M,0)$ and $l\equiv\{[M^2+(\vec{p}_i+\vec{p}_f)^2]^{1/2}, \vec{p}_i+\vec{p}_f\}$. We follow Ref. 2 and use

$$g_\pi(t)=\frac{g_\pi(0)}{1+t(t-4M^2)/4M^2\mu_N{}^2} \tag{10}$$

for $t<0$, where $\mu_N$ is the form factor mass.

Lorentz covariance also tells us that

$$\langle 0|j_\alpha(0)|N(p_i),\overline{N}(\overline{p})\rangle = ig_\pi[(p_i+\overline{p})^2]\overline{v}(\overline{p})\gamma_5\tau_\alpha u(p_i). \tag{11}$$

Thus the $Z$-graph contribution is

$$-\mathrm{Re}\{g_\pi^*[(p_f+p)^2]g_\pi[(p_i+p)^2]\}\frac{\overline{u}(p_f)(\not{p}+\not{p}_f)\tau_\beta\tau_\alpha u(p_i)}{2p_0(k_0+p_{f0}+p_0)}$$
$$-\mathrm{Re}\{g_\pi^*[(p_f+\overline{l})^2]g_\pi[(p_i+\overline{l})^2]\}\frac{\overline{u}(p_f(\overline{l}+\not{p}_i)\tau_\alpha\tau_\beta u(p_i)}{2\overline{l}_0(p_{i0}+\overline{l}_0-k_0)},$$

where

$$\overline{l}\equiv\{[M^2+(\vec{p}_i+\vec{p}_f)^2]^{1/2}, -\vec{p}_i-\vec{p}_f\}. \tag{12}$$

In the $Z$ graph, we encounter $g_\pi(t)$ with $t\geq 4M^2$. The vertex functions in this region are usually unknown. It is a complex number. The expression Re $\{g_\pi^* g_\pi\}$ means that only the real part of the product of these two $g_\pi(t>4M^2)$ can contribute. This reflects the fact that the imaginary part of the amplitude $F_{\beta\alpha}(k)$ of Eq. (2) can come only from the right-hand cut. Furthermore, for the convenience of calculation, we make an *ad hoc* simplification of replacing the real part of the product of two complex functions by a product of two real functions, called $\overline{g}_\pi(t)$:

$$\overline{g}_\pi(t)=\frac{\overline{g}_\pi}{1+(t-4M^2)/4\mu_\pi{}^2}, \quad t\geq 4M^2, \tag{13}$$

where $\overline{g}_\pi\equiv|g_\pi(4M^2)|$ and $\mu_\pi$ is the form factor mass. Our knowledge of $g_\pi(t\geq 4M^2)$ is so limited that a more sophisticated parametrization is meaningless.

Finally, we discuss the seagull term. It has been replaced by the sigma exchange term, i.e., $g_{\sigma\pi\pi}g_{\sigma NN}/(m_\sigma{}^2+q^2)$. As usual, we demand a pair suppression between the sigma exchange and the $Z$-graph terms at threshold ($\vec{k}=\vec{p}_i=\vec{p}_f=0$). The isoscalar part of the $Z$ graph is $-g_\pi{}^2/(2M+m_\pi) - \overline{g}_\pi{}^2/(2M-m_\pi)\approx -\overline{g}_\pi{}^2/M$. This very large amplitude is nearly canceled out by the scalar-isoscalar boson exchange. Accordingly, we set

$$\frac{g_{\sigma\pi\pi}(0)g_{\sigma NN}(0)}{m_\sigma{}^2}=\zeta\frac{\overline{g}_\pi{}^2}{M}; \tag{14}$$

The parameter $\zeta$ must be very close to one. Finally, the isoscalar seagull term is written with a form factor in the following manner:

$$\zeta\frac{\overline{g}_\pi{}^2}{M}\frac{1}{(1+t/2\mu_\sigma)^2}. \tag{15}$$

In the $P$-wave calculation, the sigma exchange term contribution is not very important. The value of $\zeta$ can be chosen to be one. In this paper, we have set $\zeta=0.95$, a value obtained by analysis of the low energy $S$-wave scattering amplitude. This will be described later.

## III. METHOD OF SOLUTION

The half-off-mass-shell amplitude $F_{\beta\alpha}(k)$ may be expanded in partial waves,

$$F_{\beta\alpha}(k)=4\pi\sum_{l,J,I} f^{(l)}_{2I,2J}(|\vec{p}_f|,|\vec{p}_i|)\Pi^I(\beta,\alpha)\times P^J_l(\vec{p}_f s_f,\vec{p}_i s_i), \tag{16}$$

where $l$ and $J$ stand for orbital and total angular momenta and $I$ for isospin. $\Pi^I(\beta,\alpha)$ is the isospin projection operator and $P^J_l$ the spin-angular momentum projection operator. The space momenta and spin components refer to the c.m. frame. It should be remembered that each factor in Eq. (16) is a Lorentz scalar. In this paper, we deal exclusively with the $P$-wave amplitudes, so we drop the superscript on $f_{2I,2J}$. The details of partial wave expansion of the Low equation are given in Ref. 2. Schematically, the Low equation [Eq. (2)] for a partial wave can be written as

$$f_\nu=V_\nu+\int(f_\nu f_\nu)_{\text{direct}}+\sum_\mu\int(f_\mu f_\mu)_{\text{crossed}}, \tag{17}$$

where $V$ is the driving term, and $\nu$ and $\mu$ are the various spin-isospin channel indices.

Since we are trying to develop a low energy theory, our first task will be to try to get a good solution of Eq. (17) in the low energy region. After some trials and errors, we find that a good solution can be achieved in the low energy region $0<T_\pi<400$ MeV by the method of the Padé approximant.[2] We attach an order parameter $\lambda$ on $V$ and generate iteratively a power series in $\lambda$. The method of the Padé approximant consists of matching the first $N+M$ terms of the series with a rational function where the numerator polynomial is of the order $N$ and the denominator polynomial is of the order $M$. This is denoted as the $[N,M]$ Padé approximant.

We find that the [1,1] Padé approximant gives very good solutions for $P11$, $P31$, and $P13$ channels. For the $P33$ channel the solution is not quite as good.

To check how good the solutions are, we substitute the solutions into the right-hand sides of Eq. (17) and compare the output with the left-hand side of Eq. (17), the input amplitude. The imaginary parts are reproduced as an identity. This is because the unitary conditions are explicitly maintained by Eqs. (2). So, the comparison deals with the real parts only. The following quantity $\Delta$

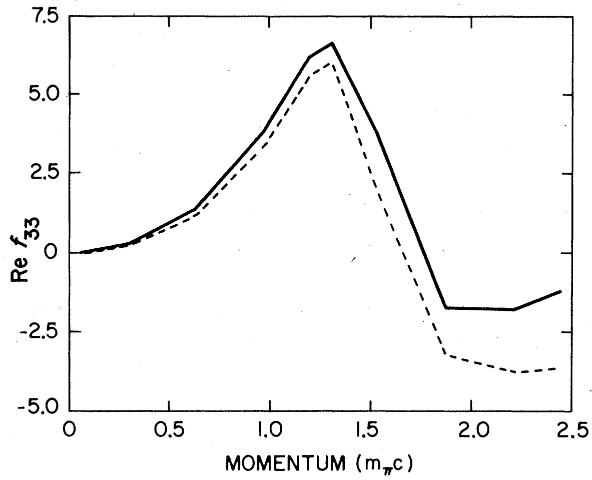


FIG. 2. Check of the [1,1] Padé solution for the $P33$ channel. The solid line is the right-hand side of Eq. (17). The dashed line is the left-hand side of Eq. (17). The curves are drawn by joining the values at successive meshprints with straight lines.

is used as a measure of the accuracy of the solution

$$\Delta\equiv\frac{|[V_\nu+\int(f^*_\nu f_\nu)_{\text{direct}}+\int(f^*f)_{\text{crossed}}]-f_\nu|}{\mathrm{Re}f_\nu}. \tag{18}$$

The value $\Delta$ is calculated at various on- and off-shell momenta up to $2.5\ m_\pi c$. It covers a pion lab kinetic energy region $0<T_\pi<400$ MeV. For $P11$, $P31$, and $P13$ channels, the $\Delta$ values are less than 1% at every momentum mesh point used in the numerical work. For the $P33$ channel, the [1,1] Padé solution has $\Delta$ of about 10% at most mesh points. In Fig. 2 we plot the [1,1] Padé $P33$ amplitudes and the results of using these amplitudes in the right-hand side of Eq. (17). Thus, the two curves represent the two terms in the numerator of Eq. (18). For the three other channels, the two numerator terms are almost identical and we have omitted their plots.

At the $P33$ resonance the real part of the amplitude vanishes. Thus, in the neighborhood of the resonance the measure $\Delta$ is bound to be large. In the subsequent discussion, values of $\Delta$ will refer to regions excluding the neighborhood of resonance.

We have found that we can generate a much better solution for the $P33$ amplitude by the following method. From Fig. 1, where we plotted the experimental $P33$ phase shifts and the results from [1,1] Padé calculation, we can see that the on-shell amplitudes are reproduced quite well. So we retain these. We replace the half-off-shell amplitudes generated by the Padé method with the following factorable form:

$$f_{33}(|\vec{q}|, |\vec{p}|) = \frac{|\vec{p}|}{|\vec{q}|} \frac{\phi(\vec{p}^2)}{\phi(\vec{q}^2)} f_{33}(|\vec{q}|, |\vec{q}|) \tag{19}$$

with

$$\phi(\vec{p}^2) = \left(1 + \frac{\vec{p}^2}{\mu_{33}^2}\right)^{-1}, \tag{20}$$

where $\vec{q}$ is the on-shell momentum and $\vec{p}$ is the off-shell momentum.

Upon choosing a particular value for $\mu_{33}$ we substitute the resulting half-off-shell amplitudes in the right-hand side of Eq. (17) and compare the result with the input amplitudes. In Fig. 3 we plot the input and output amplitudes for $\mu_{33} = \infty$, 20 $m_\pi$, 10 $m_\pi$, and 7 $m_\pi$. We find the unexpected result that the best solution is obtained for $\mu_{33} \to \infty$. For this case the average $\Delta$ is ~5%. For $\mu_{33} = 20$ $m_\pi$, 10 $m_\pi$, and 7 $m_\pi$, the average $\Delta$'s are 6%, 12%, and 28%, respectively. We are forced to conclude that the form factor mass smaller than 10 $m_\pi$ is unacceptable. The conclusion does depend on the reliability of our driving terms. In principle, we have to allow for some degree of uncertainty in the $Z$-graph contributions. The real part of the on-shell amplitude has the maximum value of about 6 $m_\pi^{-1}$ around $q \sim 1.3$ $m_\pi^{-1}$, as one can see from Fig. 2 or Fig. 3. At this point the contribution of the $Z$ graph to the on-shell amplitude is ~0.3 $m_\pi^{-1}$ (see Fig. 7), which is ~5% of the total. In this momentum region the quantity $\Delta$ has values 0.5%, 6%, and 19% for $\mu = 20$ $m_\pi$, 10 $m_\pi$, and 7 $m_\pi$, respectively. If we regard the entire $Z$ graph contribution as a measure of the theoretical uncertainty the conclusion that $\mu < 10$ $m_\pi$ is unacceptable is justified. Discovery of the very hard $P33$ form factor is a major result of our investigations.

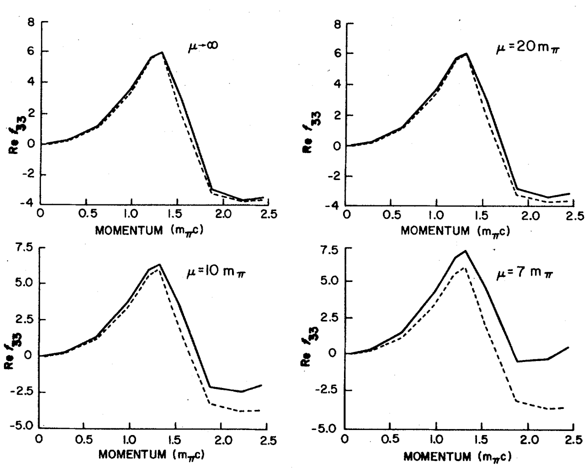


FIG. 3. Check of the $P33$ amplitude constructed by Eqs. (19) and (20) with various values of the form factor mass $\mu_{33}$. The solid line is the right-hand side of Eq. (17). The dashed line is the left-hand side of Eq. (17). The curves are drawn by joining the values at successive meshprints with straight lines.

## IV. DETERMINATION OF VARIOUS PARAMETERS

The most important feature of low energy pion-nucleon scattering is the dominance of the $P33$ channel because of the resonance. Through crossing it plays an important role in determining the amplitudes in the other channels. Conversely, other channels have very little influence on the $P33$ channels. In Figs. 4(a)–4(d) we show the relative contributions of the crossed and the direct rescattering integrals to the on-shell amplitudes as a function of momentum. An inspection of these figures bears out the validity of the foregoing remarks about the roles of the $P33$ amplitude. These results follow primarily from our use of nonstatic kinematics. Some of these features are present in a static calculation, but in a much weaker fashion. We now discuss the evaluation of various parameters.

### A. Parameter of the nucleon pole terms

Since we use a partially conserved axial-vector current (PCAC), the Goldberger-Treiman[9] relation fixes the value of $g_\pi(0)$ from the measured val-

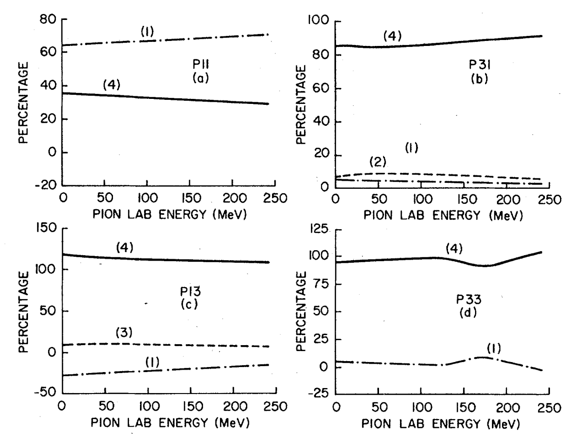


FIG. 4. Decomposition of the rescattering contribution in each channel. Parts (a)–(d) are for the four channels as marked. The curves are labeled (1)–(4). These are contributions of $P11$, $P31$, $P13$, and $P33$ channels, respectively, to a particular channel in question. The contributions are shown as percentages of the total rescattering part of the amplitude, i.e., not including the driving terms. The contribution of any channel to itself is shown by the sum of the right- and the left-hand cuts. The $P31$ channels produce insignificant left-hand cut contributions and the corresponding curves are not included.

ue of $g_A(0)=1.25$ and $f_\pi=0.939\ m_\pi^{\ 3}$,

$$g_\pi(0)=\frac{\sqrt{2}M\ m_\pi^{\ 2}}{f_\pi}g_A(0)=12.7\ . \tag{21}$$

In a static theory the nucleon pole terms constitute the full driving term, and therefore the position and width of the $P33$ resonance depend on the choice $g_\pi(0)$ and the form factor. In the preceding paper, we demonstrated that in a nonstatic theory the nucleon pole terms cannot generate the resonance. In fact we will see later than the general features of the resonance are not critically dependent on the nucleon pole terms. Because of this we simply adopt the form factor used in Ref. 2, $\mu_N=8.6\ m_\pi$, and make no effort to explore it further.

### B. Parameters of the $Z$ graphs and the sigma exchange terms

For clarity of discussion, the procedure of searching these parameters is presented in the following three steps.

*Step* (a). As the first trial, we choose to have a total cancellation at threshold between the isoscalar part of the $Z$ graph and the sigma exchange term, i.e., $\xi=1$. Also, we arbitrarily set the form factor masses of these two terms equal: $\mu_\sigma=\mu_z$. We find, by fitting the $P33$ phase shifts, that the value of $\bar{g}_\pi$ can be as low as 9.78 if $\mu_\sigma$ and $\mu_z$ are set to be 14 $m_\pi$, or it can be as high as 10.52 if one sets $\mu_\sigma$ and $\mu_z$ to be 8.6 $m_\pi$.

*Step* (b). One knows from the experimental data that both $P13$ and $P31$ have negative phase shifts and the magnitude of the $P31$ phase shift is about twice as large as that of the $P13$ phase shift in the most part of the energy region we are studying. Our next task is to choose the form factor masses $\mu_\sigma=\mu_z$ to reproduce these qualitative features. No attempt is made to fit the phase shifts exactly.

To appreciate the roles of the various terms of the $P31$ and $P13$ amplitudes we decompose these at a particular energy, $T_\pi=110$ MeV. The decomposition is presented in Table II. We observe several points. First, the most important contribution to $P13$ and $P31$ comes from the nucleon pole terms, which have negative contributions to both channels, but the magnitudes are about the same. The next important contribution comes from the rescattering integrals. These are dominated by the crossed contribution of the $P33$ resonance. This may be seen from Fig. 4 also. Since we do get a very good fit of the $P33$ phase shifts, our evaluation of the rescattering contribution should be reliable. Adding nucleon poles and rescattering contributions, the total magnitude of $P31$ is about 1.4 times larger than the magnitude of $P13$. The observed ratio is ~2.

TABLE II. Decomposition of the real part of $P31$ and $P13$ amplitudes at pion lab energy $T_\pi=110$ MeV.

| | $P31$ ($m_\pi^{-1}$) | $P13$ ($m_\pi^{-1}$) |
|---|---|---|
| Nucleon pole | −0.479 | −0.469 |
| Sigma exchange ($\mu_\sigma=14\ m_\pi$) | −0.042 | 0.069 |
| $Z$ graph ($\mu_z=14\ m_\pi$) | −0.008 | −0.127 |
| Inelastic contribution | 0.007 | −0.041 |
| Rescattering integrals | 0.157 | 0.237 |

Next we explore whether this ratio can be increased with a suitable choice of $\mu_\sigma=\mu_z$. In Fig. 5 we present the sum of the contribution of the sigma exchange and the $Z$ graphs to the driving terms of the two channels as functions of the common form factor mass. It is clear that in order to increase the ratio of $P31$ and $P13$ amplitudes one must select the region $\mu_\sigma=\mu_z>15\ m_\pi$. However, our analysis of the $P33$ phase shifts had established that we cannot increase $\mu_\sigma=\mu_z$ beyond 14 $m_\pi$. So, we simply choose this largest possible value for the form factor masses. With this goes the value $\bar{g}_\pi=9.78$. No attempt is made to vary $\mu_\sigma$ and $\mu_z$ separately. Doing this may help improve the fits for the $P13$ and $P31$ amplitudes. However, being small these amplitudes are particularly sensitive to any other mechanism which we have not included. So, we do not feel that forcing better fits to $P13$ and $P31$ amplitudes is useful.

*Step* (c). Up to now, we have assumed a complete pair suppression. However, with $\bar{g}_\pi=9.78$ and $\mu_\sigma=\mu_z=14\ m_\pi$ one cannot satisfy the $S$-wave Low equation even qualitatively. This difficulty can be avoided by changing $\xi$ slightly from 1 to 0.95 and $\bar{g}_\pi$ from 9.78 to 10.12. The change does

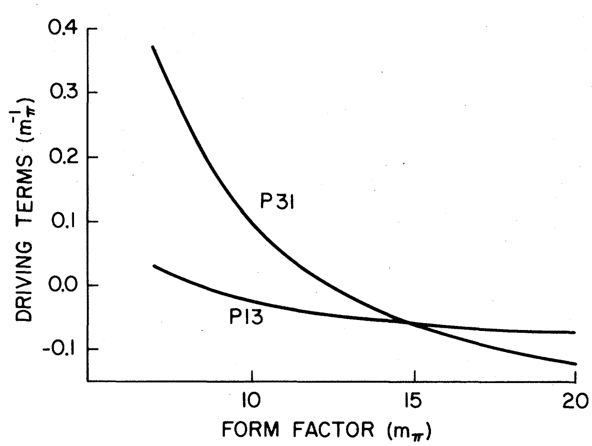


FIG. 5. The sigma exchange and $Z$-graph contributions to the $P13$ and $P31$ driving terms as functions of the common form factor mass $\mu_\sigma=\mu_z$.

not affect the $P$-wave fit in any significant way. At the same time, we should note that we cannot be certain that an unsubtracted equation is reliable for the study of the $S$-wave problem. We do not want to attach too great an importance to the particular value of $\zeta$. Nevertheless, in this work we use $\zeta=0.95$ and $\bar{g}_\pi=10.12$ as our final values.

By now, we have fixed all parameters of the elastic theory. It generates good fits for the $P31$, $P13$, and $P33$ channels. But the $P11$ case is unsatisfactory. The experimental $P11$ phase shifts, as shown in Fig. 1, start out as negative and then turn over, going through zero at $T_\pi\sim150$ MeV. In marked contrast to this, the elastic theory is well known to produce phase shifts which remain negative and increase in magnitude with increasing energy.

In the last few years, several authors[10,11,12] have included the inelastic contribution in their theories of pion-nucleon interaction. Mostly the attention has been on the $P33$ channel. The $P11$ case is discussed in Ref. 12 only. These authors did not succeed in reproducing the turnover of the phase shift. In the next section we will demonstrate one possible way to fit the $P11$ phase shift by including the inelastic contribution.

## V. INELASTIC CONTRIBUTION AND $P11$ PHASE SHIFT FIT

A study of Sec. II of the preceding paper will show that the contribution of the inelastic channel from both the right- and the left-hand cuts has the following general structure[3]:

$$\int \frac{d(W^2+\vec{L}^2)^{1/2}}{(W^2+\vec{L}^2)^{1/2}-\epsilon}\left\{\sum_{s,\gamma_1\gamma_2\cdots\gamma_N}\int\frac{d\vec{p}}{(2\pi)^3}\frac{M}{P_0}\frac{d\vec{q}_1}{(2\pi)^3}\frac{1}{2q_{10}}\cdots\frac{d\vec{q}_N}{(2\pi)^3}\frac{1}{2q_{N0}}\right.$$
$$\times\langle p_f s_f|j_{\beta'}(0)|\vec{p}s,\vec{q}_1\gamma_1,\ldots,\vec{q}_N\gamma_N\rangle_{\text{out out}}\langle\vec{p}s,\vec{q}_1\gamma_1,\ldots,\vec{q}_N\gamma_N|j_{\alpha'}(0)|\vec{p}_i s_i\rangle$$
$$\left.\times\,\delta\left[p_0+\sum_{j=1}^{N}q_{j0}-(W^2+\vec{L}^2)^{1/2}\right]\delta\left(\vec{p}+\sum_{j=1}^{N}\vec{q}_j-\vec{L}\right)\right\}. \quad (22)$$

The four vector $L\equiv[W^2+\vec{L}^2)^{1/2},\vec{L}]$ stands for the total energy and momentum of an intermediate state. Thus $\vec{L}=0$ for the right-hand cut and $\vec{L}=\vec{p}_i+\vec{p}_f$ for the left-hand cut, $W$ is the total energy of the intermediate state in its rest frame. $\epsilon=p_{f0}+k_0$ for the right-hand cut and $\epsilon=p_{i0}-k_0$ for the left-hand cut. The intermediate state considered here contains one nucleon $(\vec{p},s)$ and $N\geq2$ pions. The point to note is that the quantity in the curly bracket in Eq. (22) is a Lorentz scalar and it is related to the total reaction cross section. Indeed, for forward scattering $\vec{p}_i=\vec{p}_f$; if the four-momentum $L-p_f$ is a valid four-momentum for a physical pion the curly bracket equals

$$\{\ \}=\frac{W}{Mq}(1-|\eta_{2I,2J}|^2)$$
$$=\frac{Wq}{\pi M}\sigma_{2I,2J}^{(\text{inel})}, \quad (23)$$

where $\eta_{2I,2J}$ is the modulus $S$-matrix element for the given channel. The quantity $q$ is the c.m. momentum of the pion and the nucleon, i.e.,

$$W=(M^2+\vec{q}^2)^{1/2}+(m_\pi^2+\vec{q}^2)^{1/2}. \quad (24)$$

It should be noted that $q$ refers to the initial and final pion-nucleon states and not to any individual particle in the intermediate state. Of course, it determines $W$, the total c.m. energy of the intermediate state.

When the condition for the pion being on the mass shell is not met we need a conjecture to relate the curly bracket to the expression (23). The conjecture is best stated in terms of the variables of the intermediate rest frame, i.e., the frame obtained by boosting $\vec{L}$ down to zero with a boost velocity of $-\vec{L}/(W^2+\vec{L}^2)^{1/2}$. The boost produces the following changes:

$$|\vec{p}_f|\to|\vec{p}_f'|=\left(\frac{(p_f\cdot L)^2}{W^2}-M^2\right)^{1/2},$$
$$|\vec{p}_i|\to|\vec{p}_i'|=\left(\frac{(p_i\cdot L)^2}{W^2}-M^2\right)^{1/2}. \quad (25)$$

We make the ansatz that

$$\{\ \}=|\vec{p}_f'|\,|\vec{p}_i'|\,\theta(\vec{q}^2,\vec{p}_i'^2)\theta(\vec{q}^2,\vec{p}_f'^2)\frac{W}{M|\vec{q}|^3}(1-|\eta_{2I,2J}|^2), \quad (26)$$

where $\theta(\vec{q}^2,\vec{p}_i'^2)$ is a form-factor-like function with the condition that

$$\theta(\vec{q}^2,\vec{q}^2)=1. \quad (27)$$

In Fig. 6 the quantity $(W/M|\vec{q}|^3)(1-|\eta_{2I,2J}|^2)$ is plotted against the pion lab energy. One can see that the important inelastic contribution comes mainly from the $P11$ channel. Therefore, we include the inelastic contribution in the $P11$ channel only.

At this stage we note that our low energy elastic

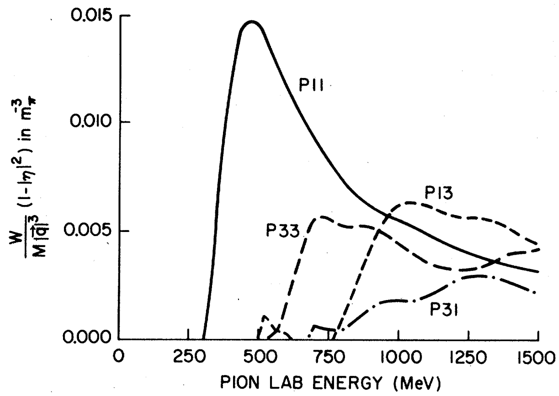


FIG. 6. Plots of expression (23) as a function of pion lab energy. These are pion-nucleon data magnetic tape produced by Karlsruhe and Helsinki collaboration. See Ref. 7.

theory may not be very good in the energy region where inelasticity comes into play. We felt that it is more sensible to adopt a procedure where we avoid calculating these high energy elastic amplitudes. This can be done by separating the energy region into two parts: (I) $0 \leq T_\pi \leq 320$ MeV, and (II) $T_\pi > 320$ MeV. We make the simplifying (and innocuous) approximation that the inelastic cross section is exactly zero in region I and starts out as a step function in region II. We then lump the elastic rescattering contribution with the inelastic contribution in region II. Thus, the sum in Eq. (22) includes $N=1$ also. Accordingly, Eq. (23) changes to

$$\{\ \}_{\text{total}} = \frac{2W}{M|\vec{q}|}(1-\eta_{2I,2J}\cos 2\delta_{2I,2J}) = \frac{Wq}{M\pi}\sigma^{(\text{total})}_{2I,2J}\,. \tag{28}$$

As before, when the pion is not on its mass shell, we assume that the total curly bracket has the form

$$\{\ \}_{\text{total}} = |\vec{p}_f'|\,|\vec{p}_i'|\,\psi(\vec{q}^{\,2},\vec{p}_f'^{\,2})\psi(\vec{q}^{\,2},\vec{p}_i'^{\,2}) \times \frac{2W}{M|\vec{q}|^3}(1-\eta_{2I,2J}\cos 2\delta_{2I,2J})\,, \tag{29}$$

where $\psi(\vec{q}^{\,2},\vec{p}_f'^{\,2})$ is another form-factor–like function similar to $\theta(\vec{q}^{\,2},\vec{p}_f'^{\,2})$. Accordingly,

$$\psi(\vec{q}^{\,2},\vec{q}^{\,2})=1\,. \tag{30}$$

A certain amount of care is necessary in choosing a form for $\psi(\vec{q}^{\,2},\vec{p}_f'^{\,2})$. For example, one must avoid the temptation of choosing a factorable form. $\psi(\vec{q}^{\,2},\vec{p}^{\,2})=\phi(\vec{p}^{\,2})/\phi(\vec{q}^{\,2})$, where $\phi(\vec{p}^{\,2})$ is a suitably chosen function which continuously decreases. Since $\phi(\vec{q}^{\,2})$ appears in the denominator it will amplify the contribution of the large $q$ region to the right- and left-hand cut integrals and tend to make the integrals very large, if not unbounded. A large left-hand cut integral will mean large crossed contribution to other channels, which may upset the satisfactory state of affairs in these channels. This is easily avoided by choosing a form where $\psi(\vec{q}^{\,2},\vec{p}^{\,2})$ remains bounded as $\vec{q}^{\,2}$ tends to infinity. We find it adequate to choose the form

$$\psi(\vec{q}^{\,2},\vec{p}^{\,2}) = \exp\frac{\vec{q}^{\,2}-\vec{p}^{\,2}}{\lambda_1+\lambda_2|\vec{q}|^3} \tag{31}$$

for our region of interest where $\vec{q}^{\,2} > \vec{p}^{\,2}$. The asymptotic behavior

$$\psi(\vec{q}^{\,2},\vec{p}^{\,2}) \xrightarrow[\vec{q}^{\,2}\to\infty]{} 1$$

is sufficient to keep the left-hand cut integrals under control. After some search we settle on the values

$$\lambda_1 = 4\ m_\pi^{\,2} \text{ and } \lambda_2 = 0.2\ m_\pi^{\,-1}\,. \tag{32}$$

The choice amplifies the contribution of inelasticity from the region where the inelastic cross section is largest. Without this amplification it is difficult to fit the $P11$ phase shifts.

In Fig. 7 we have plotted the various pieces of the on-shell driving terms for the four channels as functions of the on-shell momentum. The contributions of inelasticity (only $P11$ included) to $P13$, $P31$, and $P33$ are negligible.

By the procedure described above we are able to fit the $P11$ phase shifts satisfactorily without

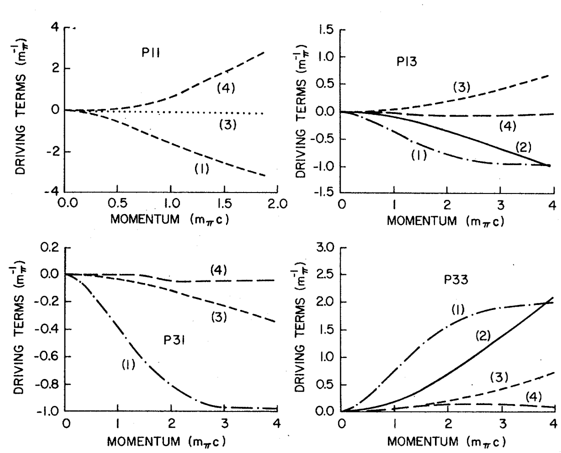


FIG. 7. Decomposition of the on-shell driving terms in each channel. The curves are labeled (1)–(4). These are the contributions of the nucleon pole term, the $Z$-graph term, the sigma exchange, and the $P11$ inelasticity, respectively, to the particular channel as marked on each graph. The $Z$ graph contributes little to the $J=\frac{1}{2}$ channels; the corresponding curves are not shown.

spoiling the fits in the other channels. This may be seen from Fig. 1.

It should be stressed that the success of our approach depends critically on the role of nonstatic kinematics on the spin crossing matrix. The constant crossing matrix elements of the static theory would have produced large and unpleasant contributions from the left-hand cut.

We should also note that we offer no explanations why the ansatz Eq. (29) should be valid nor why the form factor should be as given by Eq. (31). We are led to these by the simple fact that we cannot invent any other reasonable model to explain the curious behavior of the $P11$ phase shifts.

We have also analyzed the $P11$ amplitudes to see if these are factorable. If the amplitude $f_{11}(k,k')$ is factorable, one can write it as

$$f_{11}(k,k')=\theta(k)\phi(k')kk' , \tag{33}$$

where one expects $\theta(k)$ and $\phi(k')$ to be relatively slowly varying functions. A test of factorability is provided from the analysis of the ratio

$$R_k(k')=\frac{k}{k'}\frac{f_{11}(k,k')}{f_{11}(k,k)} . \tag{34}$$

If $f_{11}$ is factorable we will have

$$R_k(k')=\frac{\phi(k')}{\phi(k)} . \tag{35}$$

So, if we plot $\log R_k(k')$ as a function of the off-shell momentum $k'$ for various fixed values of the on-shell momentum $k$ we should have curves which are parallel to each other. In Fig. 8 we show the curves $\ln R_k(k')$ as functions of $k'$ for $k=0.96\ m_\pi$, $1.19\ m_\pi$, and $1.88\ m_\pi$. We see that there is no hint of parallelism among the curves. We conclude that our half-off-mass-shell $P11$ amplitudes are not factorable. We have not attempted to fit the $P13$ and $P31$ phase shifts with any degree of precision. We are not in a position to comment on the structure of these amplitudes. We may remind the reader that these are small amplitudes.

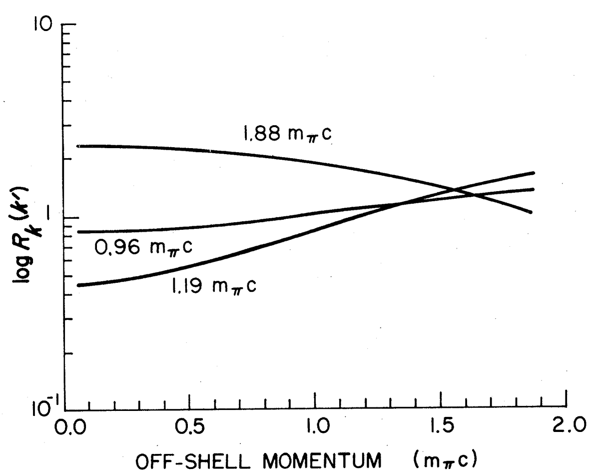


FIG. 8. Plots of the log of expression (34) as a function of the off-shell momentum. The on-shell momentum is marked on each curve.

## VI. SUMMARY AND DISCUSSION

We have developed a theory of $P$-wave pion-nucleon scattering in the spirit of the Chew-Low theory with the following three significant differences:

(i) inclusion of nonstatic kinematics for the nucleon,
(ii) inclusion of the $Z$ graphs, and
(iii) inclusion of the $P11$ channel inelastic contribution.

In the preceding paper, we show that with nonstatic kinematics the nucleon pole terms fail to produce a resonance in the $P33$ channel. Inclusion of the $Z$ graphs, as was done, immediately produces a resonance. Getting the position and the width correctly becomes a matter of detail. As long as the $Z$ graphs are included the contribution of a scalar-isoscalar boson exchange is also included keeping in mind the pair suppression mechanism in the $S$ wave. However, in the $P$ wave the scalar-isoscalar boson has a relatively small role.

The $Z$ graphs make important contributions in the $J=\frac{3}{2}$ states only. Fig. 7 shows this and the size of the other driving terms in the various channels.

To appreciate the role of the $Z$ graphs in producing the resonance we refer to the form of the full amplitude as given by $[1,1]$ Padé approximant. The form is

$$f_{33}(q,q)=\frac{V_{33}{}^2(q,q)}{V_{33}(q,q)-[\text{second order iterate}]} , \tag{36}$$

where $V$ is the driving term and the second order iterate comes from the integrations of Eq. (17). If the second order iterate is small there can be no resonance. In view of this it is instructive to examine the contribution of the various pieces of the driving term to the second order iterate for $0\leqslant q\leqslant 2.5\ m_\pi$, shown in Fig. 9. The second order $Z$-graph contribution is the largest single part and accounts for about 35% of the whole. The second order nucleon pole contribution is less than 15% throughout.

A very important result of our studies is the discovery that the $P33$ amplitudes are factorable, but the form factor is very hard. With the choice of a monopole form the mass parameter $\mu_{33}$ is

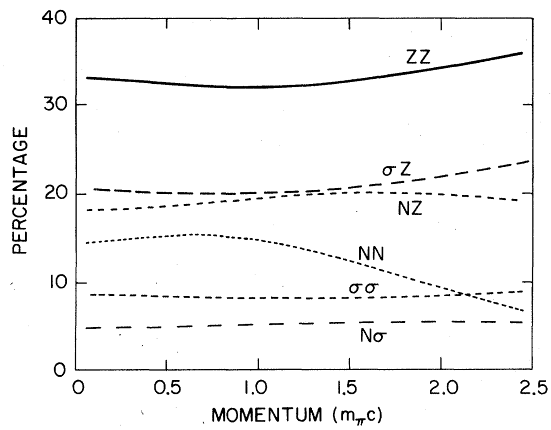


FIG. 9. Decomposition of the on-shell second order iterate of Eq. (17) for the $P33$ channel. The labels, $Z$, $N$, and $\sigma$ stand for the $Z$ graph, the nucleon pole, and the $\sigma$ exchange mechanisms. The curve labeled $ZZ$ shows the contribution of the square term from $Z$ graphs as percentages of the total. The curve labeled $\sigma N$ shows the cross product of the $\sigma$ exchange and the nucleon pole and so on.

bound to be at least as large as 10 $m_\pi$. The amplitudes in the other channels are not factorable.

The change of sign of the $P11$ phase shift at $T_\pi \sim 150$ MeV is an intriguing feature of the pion-nucleon scattering problem. Ever since the days of the Chew-Low theory (1956) it has been known that the normal driving terms produce phase shifts which are negative and which increase in magnitude with increasing energy. So, a new type of driving term is needed which is important for the $P11$ channel only. Inelasticity meets the requirement. However, we do find that the contribution of inelasticity has to be suitably amplified by an appropriate choice of form factors. Ours is given by Eqs. (31) and (32). We have made no effort to try to understand this in terms of a theory of pion production.

For the $Z$ graphs we find the values $\bar{g}_\pi \equiv |g_\pi(4M^2)| = 10.12$ and $\mu_\pi = 14\ m_\pi$. The $S$-wave study of Ref. 2 reported $\bar{g}_\pi = 11.7$ and $\mu_\pi = 8.6\ m_\pi$. The differences are significant in the sense that the $S$-wave work would fail with the $P$-wave parameters and vice versa. But it must be understood that while the $S$-wave amplitudes depend mainly on the size of the $Z$-graph contribution, the $P$-wave amplitudes depend on the rate of variation of $Z$ graphs with momentum transfer. We picked a monopole form for the $t$ dependence of $g_\pi(t)$ in the range $t \geq 4M^2$. This is, of course, a purely arbitrary choice. The simplicity reflects our ignorance. Thus, the difference between the $S$-wave and $P$-wave parameters may not be as irreconcilable as it may appear at first glance. We should also stress that among the various driving terms we have neglected, the one where the antinucleon is replaced with $\bar{N}^*$ by $J^P = \frac{1}{2}^+$ may play an important role in the $P$-wave dynamics. At present our $Z$ graphs and the inelasticity terms have to make up for these and other terms not included by us.

Also, we feel that a subtraction is not required for the $P$-wave calculation. On the other hand, it is very difficult to use this unsubtracted equation to calculate $S$-wave phase shifts. Since the $S$-wave amplitudes have cancellation of two terms, the sigma exchange and $Z$ graph, a cancellation at threshold as we demanded in our theory is not enough to guarantee continued cancellation beyond threshold. A noncanceled $Z$ graph and sigma exchange driving terms can build up a huge contribution to an $S$-wave amplitude through the rescattering processes. This contribution depends very much on what off-shell form factor one wants to choose. One can get almost any value of the amplitude by a clever choice of the form factor. Thus, an approach to the $S$-wave problem with an unabstracted equation is inherently unreliable. The works of Refs. 2 and 3 used a soft pion limit to produce once-subtracted equations.

## ACKNOWLEDGMENTS

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