

Pionic atoms and low energy elastic scattering

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A fit to pionic atom data is used to determine four of the parameters of the low energy pion-nucleus optical potential, while the other parameters are taken from theory. The resulting potential is used to predict elastic scattering from 30–50 MeV pions. The effects of extrapolating the parameters to 50 MeV with a simple energy dependence are examined.

NUCLEAR REACTIONS Optical potential, fit to pi-mesic atoms. Calculated elastic scattering with extrapolated potential at 30, 40, 50 MeV on individual targets.

I. INTRODUCTION

Information about the pion-nucleus optical potential in the low energy region, i. e., pion kinetic energies up to 50 MeV, can be obtained from several sources. In particular, there now exists a body of excellent quality data for pionic atom level shifts and widths and for pion-nucleus elastic differential cross sections for pions of 30–50 MeV kinetic energy. In an earlier paper¹ we presented an optical potential appropriate for low energy pion-nucleus scattering, and related the optical parameters to those from fits² to early pionic atom data. The success of that approach and the improvement in the quality of the pionic atom data have motivated the present investigation. We have redetermined the pionic atom parameters by fitting to recent pionic atom data^{3,4} using an optical potential similar to that of Ref. 1. The energy dependence of the potential has now been included by performing an approximate extrapolation of the parameters from zero to 50 MeV. The resulting optical potential has been used to calculate the differential elastic scattering cross sections, which are then compared to the existing data.

II. THE OPTICAL POTENTIAL

The optical potential form used for this analysis is

$$2\bar{\omega}U_{opt} = -4\pi[b(r) + B(r)] + 4\pi \vec{\nabla} \cdot \{L(r)[c(r) + C(r)]\} \vec{\nabla} - 4\pi \left\{ \frac{p_1 - 1}{2} \nabla^2 c(r) + \frac{p_2 - 1}{2} \nabla^2 C(r) \right\}, \quad (1)$$

where

$$b(r) = p_1 [\bar{b}_0 \rho(r) - \epsilon_r b_1 \delta \rho(r)],$$

$$c(r) = p_1^{-1} [c_0 \rho(r) - \epsilon_r c_1 \delta \rho(r)],$$

$$B(r) = p_2 B_0 \rho^2(r),$$

$$C(r) = p_2^{-1} C_0 \rho^2(r),$$

$$L(r) = \left\{ 1 + \frac{4\pi}{3} \lambda [c(r) + C(r)] \right\}^{-1},$$

and

$$\delta \rho(r) = \rho_n(r) - \rho_p(r).$$

Here p_1 and p_2 are kinematic factors,

$$p_1 = 1 + \frac{\omega}{M}, \quad p_2 = 1 + \frac{\omega}{2M},$$

ω is the pion total energy, ϵ_r the pion charge, and M the nucleon mass. The densities ρ_n , ρ_p , and ρ are normalized to N , Z , and A , respectively.

The parameters b_0 , b_1 , c_0 , and c_1 , referred to as the single nucleon parameters, are taken from a fit to the pion-nucleon scattering amplitude written in the form

$$f_{rN} = b_0 + b_1 \vec{t} \cdot \vec{\tau} + (c_0 + c_1 \vec{t} \cdot \vec{\tau}) \vec{k} \cdot \vec{k}', \quad (2)$$

with \vec{t} and $\vec{\tau}$ the pion and nucleon isospin operators, and \vec{k} and \vec{k}' the incoming and outgoing pion momenta. Note that at zero energy the parameters are purely real.

The effect of correlations has been included to second order in the s -wave via the parameter \bar{b}_0 , where

$$\bar{b}_0 = b_0 - \frac{3k_F}{2\pi} (b_0^2 + 2b_1^2), \quad (3)$$

with k_F the Fermi momentum, $k_F = 1.4 \text{ fm}^{-1}$. The effect of correlations in the p -wave term in the zero energy limit gives rise to the Lorentz-Lorenz Ericson-Ericson (LLEE) effect, giving $\lambda = 1$ in the simplest derivation of that term.⁵

True absorption of the pion, assumed to be dominated by a two nucleon process, is described by the terms in the complex parameters B_0 and C_0 .

The optical potential in Eq. (1) differs from that in Ref. 1 by the inclusion of the p -wave absorption term in the LLEE effect. The proper treatment of this term is, in fact, not clear.

The derivation of the kinematic factors is given in Ref. 1. These have been computed only up to order ω/M , giving rise to the terms in $\nabla^2 c(r)$ as well as the factors p_1 and p_2 . Terms of order $(\omega/M)^2$ and higher are expected to be negligible, with the exception of an induced s -wave term of the form⁶

$$K(r) = -4\pi \frac{(p_1 - 1)^2}{p_1} c_0 \frac{3}{5} \left(\frac{3}{2}\pi\right)^{2/3} \rho^{5/3}, \quad (4)$$

which results from the Fermi averaging of the term $\vec{p} \cdot \vec{p}'$, where \vec{p} and \vec{p}' are nucleon momenta. $K(r)$ is thus proportional to the kinetic energy density in the nucleus and has been evaluated in the Thomas-Fermi approximation. The effect of this term will be discussed in Sec. IV.

III. PIONIC ATOM DATA

Pionic atom level shifts and widths furnish information about the overall strengths of the s - and p -wave parts of the optical potential at zero pion kinetic energy. This provides limiting values for the low energy optical parameters, assuming the pion-nucleus optical potential parameters vary smoothly with energy.

The pionic atom data used in this analysis are given in Table I, where the s -wave data are taken from Ref. 3, and the p -wave data from Ref. 4. The calculated electromagnetic energies of the transitions are also given. The general features of the data are evident from Table I. The level shifts and widths tend to increase smoothly with increasing Z . The s -wave shifts are negative, indicating a repulsive s -wave interaction; the p -wave shifts are positive, indicating an attractive p -wave interaction, but show signs of saturation at high Z .

There is a reasonably close correspondence between measured quantities and terms in the optical potential. That is, the real s -wave part of the optical potential is mostly sensitive to the s -wave shift, the imaginary part to the s -wave width, with similar results for the p wave. There is, however, some mixing, the p -wave parameters being particularly sensitive to the predetermined s -wave parameters, and the $\nabla^2 c(r)$ term is an s -wave term with p -wave coefficients. Thus four parameters may be determined from the analysis of s - and p -wave pionic atom data. In principle, the A dependence of the data might distinguish between a ρ and a ρ^2 term and thus determine more parameters; in practice, the discrimination is

TABLE I. Experimental pionic atom energy shifts and widths in keV.

Nucleus	s wave ^a		
	$(E_p - E_s)_{EM}$	ΔE_s	Γ_s
¹⁰ B	68.714	-2.977 ± 0.085	1.59 ± 0.11
¹² C	99.066	-5.874 ± 0.092	3.14 ± 0.12
¹⁴ N	134.740	-9.915 ± 0.144	4.34 ± 0.24
¹⁶ O	175.413	-15.03 ± 0.24	7.64 ± 0.49
¹⁹ F	220.952	-24.46 ± 0.35	9.4 ± 1.5
²⁰ Ne	270.952	-33.34 ± 0.50	14.5 ± 3.0
²³ Na	327.131	-49.92 ± 0.71	10.3 ± 4.0
Nucleus	p wave ^b		
	$(E_d - E_p)_{EM}$	ΔE_p	Γ_p
²⁷ Al	87.270	0.201 ± 0.009	0.120 ± 0.007
²⁸ Si	101.283	0.308 ± 0.010	0.192 ± 0.009
³² S	132.510	0.635 ± 0.016	0.422 ± 0.018
⁴⁰ Ca	207.674	1.929 ± 0.014	1.590 ± 0.023
⁵⁶ Fe	352.356	4.368 ± 0.088	6.87 ± 0.21
⁶³ Cu	439.016	6.67 ± 0.24	11.4 ± 0.8
⁶⁴ Zn	469.995	6.44 ± 0.33	12.4 ± 1.4

^aReference 3.

^bReference 4.

not good. For example, the combination $(\text{Re}\bar{b}_0)\rho(r) + (\text{Re}B_0)\rho^2(r)$ behaves like $[(\text{Re}\bar{b}_0) + (\text{Re}B_0)\rho_{\text{avg}}]\rho(r)$ with ρ_{avg} independent of A . Thus one of the two parameters $\text{Re}\bar{b}_0$ and $\text{Re}B_0$ must be fixed at some value. The same is true of the p -wave parameters $\text{Re}c_0$ and $\text{Re}C_0$, with the further complication that these are not independent of the parameter λ .

It is clear from the above that a choice must be made concerning which four isoscalar parameters to fit, and values must be assigned to the others. Note that it is possible to fit the isovector parameters b_1 and c_1 as well, using data for nuclei with $N \neq Z$. This is not done in the present analysis; these parameters are fixed at their phase shift values. These parameters have been fitted by others, and we will discuss their importance later.

For the real s -wave potential we choose to fit the parameter \bar{b}_0 , and fix $\text{Re}B_0$ at the value given by Chai and Riska,⁷ with b_1 taken from phase shifts.⁸ The value of \bar{b}_0 is very poorly known near threshold, and therefore is not a good parameter to be held fixed. Unfortunately the value of $\text{Re}B_0$ is also not certain, but the calculations of Chai and Riska indicate that it is small at zero energy. The s -wave width is fitted with the parameter $\text{Im}B_0$. The resulting value can then be compared with various theoretical results.

For the p -wave potential we fix $\text{Re}c_0$, c_1 , and $\text{Re}C_0$, and fit λ and $\text{Im}C_0$. $\text{Re}c_0$ is fixed at the value given by Rowe, Salomon, and Landau,⁸ which

is extrapolated from their smooth fits to the π - N phase shifts. This eliminates the noise in $\text{Re}c_0$ below 80 MeV and allows a smooth extrapolation from threshold to higher energies. We have chosen two different values of $\text{Re}c_0$, the first from the calculation of Chai and Riska,⁷ which results in parameter set A, and the second from Oset, Weise, and Brockmann,⁹ which results in parameter set B. These two values are quite different; the second includes the effects of higher order diagrams. These two sets A and B are used to illustrate the sensitivity of the calculation to variations in these theoretical parameters. There have been a wide range of values predicted for λ , from 1.2 to 2.3,¹⁰ while earlier predictions had $0 < \lambda < 1.0$ (Ref. 11); it is to be hoped that some guidance on this matter can be obtained by the fitting of λ to pionic atom data.

The pionic atom calculations were performed using the computer code MATOM, written by Seki,¹² and modified to include the full optical potential in Eq. (1). This program sets up and solves the Klein-Gordon equation for the bound pion, determining the complex eigenvalue corresponding to a given orbital. The fitted parameters were varied to minimize the χ^2 . The parameters were determined to no more than two significant figures, as the ambiguity in the fixed parameters makes a determination of the fitted parameters to great accuracy rather meaningless. Care was taken to ensure that parameter sets A and B produced the same calculated shifts and widths so that the parameters of the two sets could be compared.

A third parameter set, labeled C, has been defined. For this set the parameters B_0 and C_0 have been taken from Riska *et al.*⁷; b_1 , c_0 , and c_1 from the phase shift analysis of Rowe *et al.*⁸; and λ was chosen to be 1.6. The value of \bar{b}_0 was chosen to give good agreement with the low energy scattering data. This is therefore a "theoretically" determined set with the exception of \bar{b}_0 .

IV. RESULTS

The results of the fitting procedure with parameter sets A and B are shown in Figs. 1 and 2. The solid line connects the values calculated from both sets; the differences between them are too small to be distinguished on this scale. The dash-dot lines are the calculations from set C. The parameters of the three sets are given in Table II. The fitted parameter sets produce results that reproduce well the A dependence of the data. The results of set C clearly show a lack of sufficient absorption in the optical potential parameters of this set.

The fact that parameter sets A and B produce the

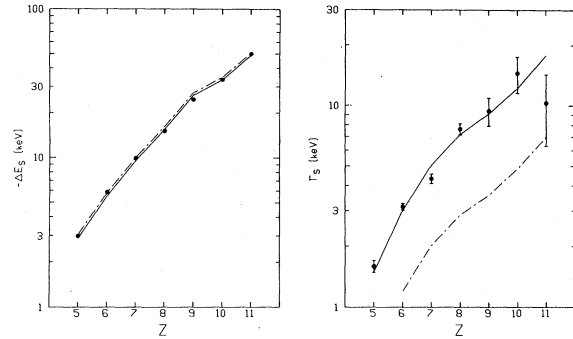


FIG. 1. The s -wave shift and width in keV plotted as a function of Z . Note that the s -wave shift is negative. The solid curve is the fit from set A; the dash-dot curve shows a calculation with set C. The data are from Ref. 3.

same A dependence is an illustration of the statement that the pionic atom data are mainly sensitive to overall s - and p -wave strengths. For example, the effective real p -wave strength (the coefficient of ρ) can be written, over some range of parameter space, as

$$c_{\text{eff}} = \text{Re} \left[\frac{p_1^{-1} c_0 + p_2^{-1} C_0 \rho_{\text{eff}}}{1 + \frac{4\pi}{3} \lambda (p_1^{-1} c_0 \rho_{\text{eff}} + p_2^{-1} C_0 \rho_{\text{eff}}^2)} \right]. \quad (5)$$

With $\rho_{\text{eff}} = 0.12 \text{ fm}^{-3} = 0.7\rho_0$, set A gives $c_{\text{eff}} = 0.434 \text{ fm}^3$; set B gives $c_{\text{eff}} = 0.434 \text{ fm}^3$. Thus, within broad limits, those sets of p -wave parameters which produce the same c_{eff} will yield the same results. A similar and simpler expression gives the effective real s -wave strength,

$$b_{\text{eff}} = p_1 \bar{b}_0 + p_2 \text{Re} B_0 \rho_{\text{eff}}. \quad (6)$$

As sets A and B differ only in the assumption for the p -wave parameters, which have very little effect on the calculated s -wave strength, the s -wave parameters are nearly identical, giving

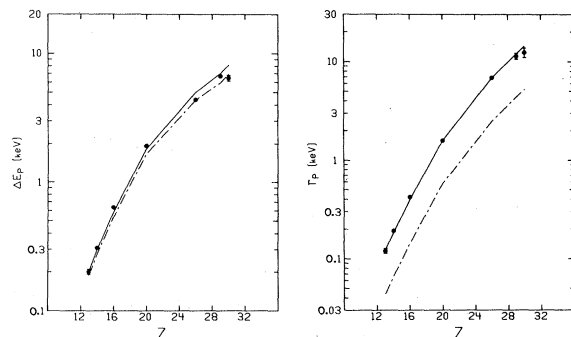


FIG. 2. The p -wave shift and width in keV plotted as a function of Z . The two curves use the same convention as in Fig. 1. The data are from Ref. 4.

TABLE II. Parameter sets used for pionic atom calculations described in the text.

	Set A	Set B	Set C	Set D
b_0 (fm)	-0.046 ^a	-0.047 ^a	-0.049	-0.046 ^a
b_1 (fm)	-0.134	-0.134	-0.134	-0.134
$\text{Re}B_0$ (fm ⁴)	0.007	0.007	0.007	0.007
$\text{Im}B_0$ (fm ⁴)	0.19 ^a	0.19 ^a	0.08	0.19 ^a
c_0 (fm ³)	0.66	0.66	0.66	0.66
c_1 (fm ³)	0.428	0.428	0.428	0.428
$\text{Re}C_0$ (fm ⁶)	0.287	1.0	0.287	0.287 ^b
$\text{Im}C_0$ (fm ⁶)	0.93 ^a	1.2 ^a	0.34	0.46 ^{a,b}
λ	1.4 ^a	1.8 ^a	1.6	1.45 ^a

^aThese values were fitted to the pionic atom data.

^bThe $C_0\rho^2$ parameter was outside the LLEE term.

$b_{\text{eff}} \cong -0.0515$ fm. In this way we can find a simple Kisslinger potential, using the parameters b_{eff} and c_{eff} , which will be at least representative of the original potential. Table III lists these effective strengths, along with those corresponding to other analyses.

There is general agreement in the results of different investigators, with all of the strengths falling within 10% of each other. Much of these differences are due to the fitting procedure—the choice of which nuclei and which parameters to fit. We took an average fit to the nuclei, while others concentrated on the isovector terms and their effect on high Z nuclei. The analysis of Batty *et al.*⁴ produces a larger effective p -wave strength, $c_{\text{eff}} = 0.48$, with the isovector terms causing the saturation at high Z . All cases show the same qualitative effect: c_{eff} is reduced and b_{eff} is enhanced from the free space values.

The density dependence introduced by the LLEE effect introduces an additional A dependence into the theory, so that “equivalent” potentials do not produce equivalent fits. A pionic atom calculation of the p -wave shift using the effective strength for set A does well, but looks more like the dash-dot curve on the left side of Fig. 2. Even if the data are fitted giving $b_{\text{eff}} = -0.051$ and $c_{\text{eff}} = 0.46$, the

shape of the curve is such that agreement with the p -wave shift data is less satisfactory than when the full potential is used. Hence, although these effective strengths allow a convenient ground for comparison of parameters of similar potentials, it must be kept in mind that the density dependent form of the potential, e. g., the LLEE effect, produces results that are different in important details from other forms of the potential.

As noted in Sec. II, a second order kinematic term, $K(r)$, proportional to the nucleon kinetic energy density, has not been included in the optical potential used for this analysis. This term is an attractive s -wave term; its inclusion in the optical potential causes a change in the value of the fitted parameter \bar{b}_0 in set A from -0.046 to -0.053 fm. The difference between the fitted value and the theoretical value for \bar{b}_0 is between -0.024 and -0.017 fm, depending on whether $K(r)$ is included or not. The defect is thus about the same size as the second order correction already included. Although, in principle, some of the defect can be eliminated by a large negative value for $\text{Re}B_0$, the calculations of Chai and Riska⁷ indicate that this is not the case; they consider their calculated value a lower bound, with further refinements having the effect of making it more positive. The missing s -wave strength is at present an unsolved problem, although recent results¹⁵ suggest it can be explained by medium corrections to the s -wave scattering.

As the value of c_0 at zero energy is uncertain, with different analyses giving somewhat different values, we investigated the effect of fixing λ at some reasonable value, $\lambda = 1.6$, and fitting c_0 with the other parameters as in set A. The result was $c_0 = 0.69$ fm³. This indicates the degree of uncertainty in λ that corresponds to a change in c_0 , so increasing λ by 0.2 compensates for increasing c_0 by 0.03.

The removal of the p -wave absorption term $C_0\rho^2$ from the LLEE effect causes a substantial reduction in the value of $\text{Im}C_0$ required to repro-

TABLE III. Effective s - and p -wave strengths for pionic atom potentials.

	Set A	Set B	Set C ^a	Set D	Batty ^b	Batty ^c	Powers ^d	Friedman ^e
$\text{Re}b_{\text{eff}}$ (fm)	-0.051	-0.052	-0.055	-0.051	-0.051	-0.051	-0.047	-0.053
$\text{Im}b_{\text{eff}}$ (fm)	0.024	0.024	0.010	0.024	0.024	0.024	0.024	0.023
$\text{Re}c_{\text{eff}}$ (fm ³)	0.434	0.434	0.415	0.443	0.482	0.493	0.460	0.477
$\text{Im}c_{\text{eff}}$ (fm ³)	0.051	0.051	0.017	0.051	0.046	0.040	0.049	0.039

^aThese “theoretical” parameters were not fitted to the pionic atom data.

^bCalculated from fit using procedure (a) in Ref. 4.

^cCalculated from fit using procedure (b) in Ref. 4.

^dCalculated from six-parameter fit with $\lambda = 2.25$ in Ref. 13.

^eCalculated from form I in Ref. 14.

duce the p -wave widths. In this case fitting gives $\lambda = 1.45$ and $\text{Im}C_0 = 0.46 \text{ fm}^6$, with the other parameters as in set A. This set will be referred to as set D. We note that the value of $\text{Im}C_0$ given here is much smaller than that quoted in Ref. 1, $\text{Im}C_0 = 0.79 \text{ fm}^6$, and more near that given in set 2 of that reference. The difference is due to the great improvement in the p -wave data since that analysis was made.

V. ELASTIC SCATTERING

With several sets of pionic atom parameters determined, we turn now to the elastic scattering calculations. The effect on the cross sections of changes in various terms of the optical potential is not as easy to predict as the effect on pionic atom shifts and widths. As was illustrated in Ref. 1, the scattering is chiefly sensitive to the relative strengths of the s - and p -wave parts of the optical potential. The presence of imaginary parts in the potential complicates this picture somewhat. One must be careful in assessing the quality of agreement between calculation and experiment because the sensitivity to small changes in certain parameters is very great. The difference between a good fit and one that appears rather poor may be a relatively small change in one parameter of the calculation.

The elastic scattering calculations are performed with a modified version of the program PIRK,²³ using the optical potential Eq. (1). Above threshold the single nucleon parameters b_0 , b_1 , c_0 , and c_1 acquire imaginary parts to account for quasi-elastic processes. These are calculated from the π - N phase shifts and modified by a factor to account for the Pauli exclusion principle, as described in Ref. 1. We will illustrate the effects of extrapolation in the next three figures, using 50 MeV π^+ scattering from ^{16}O as the test case.

As a first approximation we take the parameter sets directly from the pionic atom analysis, using the assumption of Ref. 1 that the energy dependence can be neglected. The results of elastic scattering calculations with set A (solid curve), set B (dashed curve) and set D, with the p -wave absorption outside the LLEE term (short-dashed curve), are shown in Fig. 3. Although the correspondence of the curves with data is not particularly good, their agreement among themselves supports the statement that elastic scattering is sensitive to overall strengths, not individual terms. As the three sets are more or less equivalent, we will concentrate on set A in the remainder of this discussion.

Next we consider the energy dependence of the parameters. The energy dependence of the π - N

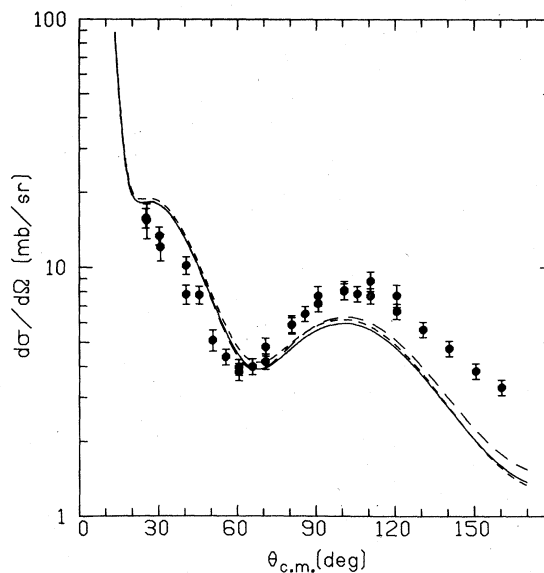


FIG. 3. Elastic scattering of 50 MeV π^+ from ^{16}O . The curves are calculated using the parameters fit to pionic atoms given in Table II. The solid curve is for set A, the dashed curve is for set B, and the short-dashed curve is for set D. The data are from Refs. 16 and 17.

phase shifts is assumed to give the energy dependence for b_1 , c_0 , and c_1 ; the calculations of Riska and collaborators give the dependence for the absorption parameters. In other words, the parameters fitted to pionic atoms are assumed to have the same slope as a function of energy as their theoretically determined counterparts. Thus for \bar{b}_0 , for example, the defect is assumed to be the same for all energies up to 50 MeV. The parameter λ is assumed to be energy independent. The corresponding parameters are shown in Table IV for 50 MeV scattering using sets A, B, and C.

Figure 4 shows the effect of this procedure for set A. The solid curve is the calculation with pionic set A and no extrapolation; the short-dashed curve is the same set extrapolated to 50 MeV. The dashed curve is a calculation with the same parameters as for the short-dashed curve except $\text{Re}c_0$, which has been kept at its zero energy value. This reduction in the p -wave strength improves the fit somewhat in the s - p interference region at forward angles.

As the values of the absorption parameters $\text{Im}B_0$ and $\text{Im}C_0$ calculated by Riska *et al.* are appreciably smaller at zero energy than those required by the pionic atom analysis, we have investigated the effect on scattering of using these smaller values. Shown in Fig. 5 is the calculation with the extrapolated parameters of set A (solid curve), to which is compared the calculation with the same

TABLE IV. Optical parameter sets for 50 MeV scattering, extrapolated from the sets given in Table I.

	Extrapolated set A	Extrapolated set B	Extrapolated set C
b_0 (fm)	$-0.057 + 0.006i$	$-0.057 + 0.006i$	$-0.060 + 0.006i$
b_1 (fm)	$-0.134 - 0.002i$	$-0.134 - 0.002i$	$-0.134 - 0.002i$
B_0 (fm ⁴)	$-0.02 + 0.25i$	$-0.02 + 0.25i$	$-0.02 + 0.14i$
c_0 (fm ³)	$0.75 + 0.03i$	$0.75 + 0.03i$	$0.75 + 0.03i$
c_1 (fm ³)	$0.428 + 0.014i$	$0.428 + 0.014i$	$0.428 + 0.014i$
C_0 (fm ⁶)	$0.36 + 1.2i$	$1.2 + 1.45i$	$0.36 + 0.59i$
λ	1.4	1.8	1.6

parameter set but with the calculated absorption values $\text{Im}B_0 = 0.14 \text{ fm}^4$ and $\text{Im}C_0 = 0.59 \text{ fm}^6$, denoted set A' (dashed curve). Also shown is the result using the extrapolated set C (short-dashed curve), which also has the smaller calculated absorption values. Clearly the reduction in absorption improves appreciably the quality of the fit, so the nearly theoretical set C fits the 50 MeV scattering data very well. However, set C has too little absorption to explain the pionic atom data. Other adjustments of parameters can also improve the fit, so this must be studied further, perhaps using partial cross section measurements. The difference between the dashed and short-dashed curves is due to the different ratios of s - and p -wave strengths of the two parameter sets. Since sets A' and C are qualitatively similar, we will

use A', rather than C, to show the effect of using "theoretical parameters" for the absorption in the calculations that follow.

Figures 6, 7, 8, and 9 are included to illustrate the energy dependence, A dependence, and pion charge dependence, respectively, of the calculations. In each case the solid curve is the calculation with the unextrapolated set A, the dashed curve is that with extrapolated set A, and the short-dashed curve is that with set A', described in the previous paragraph.

Figure 6 shows π^+ scattering from ^{12}C at 30, 40, and 50 MeV. Experimentally the character of the interference pattern in the forward direction changes with energy, reflecting the increase in the p -wave attraction with energy relative to the s -wave repulsion. The solid curve, for which the

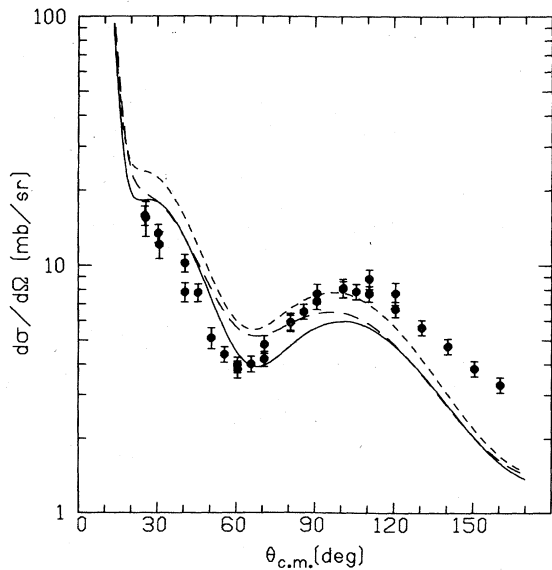


FIG. 4. Elastic scattering of 50 MeV π^+ from ^{16}O with extrapolated parameters. The short-dashed curve is for extrapolated set A from Table IV. The dashed curve is extrapolated set A with $c_0 = 0.66 \text{ fm}^3$, while the solid curve is for the (unextrapolated) set A.

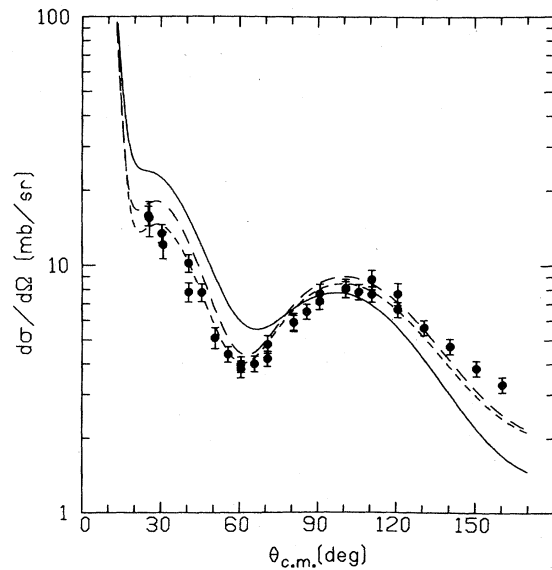


FIG. 5. Elastic scattering of 50 MeV π^+ from ^{16}O showing the effect of decreased absorption. The short-dashed curve is for extrapolated set C and the dashed curve is for set A' described in the text. The solid line is for extrapolated set A for comparison.

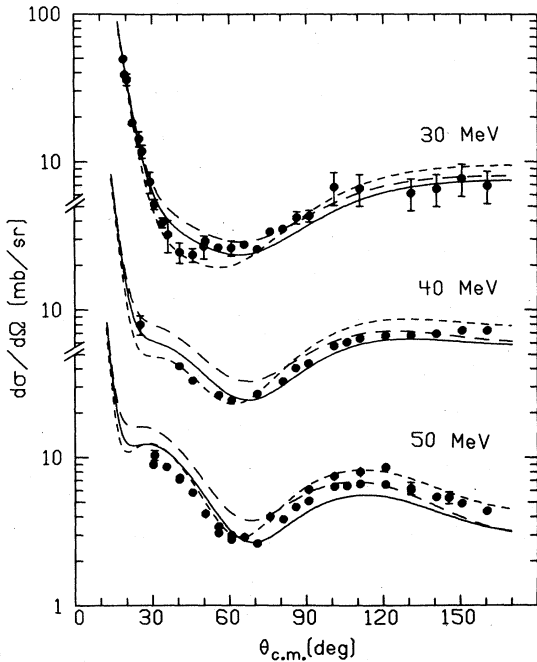


FIG. 6. Elastic scattering of 30, 40, and 50 MeV π^+ from ^{12}C . In all cases, the solid curve is for set A, the dashed curve is for extrapolated set A, and the short-dashed curve is for set A'. The data are from Refs. 18, 19, 16, and 17.

parameters are held fixed at their pionic atom values, reproduces the trend quite well. So, the most important energy dependence in the low energy region is the form of the p -wave interaction itself rather than the energy dependence of the parameters. The extrapolated set A (dashed curve) gives cross sections that are consistently too high at forward angles. Set A' (short-dashed curve) with lowered absorption parameters gives the best fits at 40 and 50 MeV.

The A dependence of the calculation is illustrated in Fig. 7, which shows 40 MeV π^+ scattering from a variety of elements from ^{12}C to ^{208}Pb . Pionic atom values (solid curves) for the optical potential parameters give qualitative agreement with data. Using the extrapolated set A (dashed curve) improves agreement with the data for heavy elements. The best overall fit is given by set A' (short-dashed curve) with reduced absorption parameters.

30 MeV π^+ and π^- scattering from ^{12}C is shown in Fig. 8. The best fit is given by pionic atom values (solid line). The scattering of 30 MeV π^+ and π^- from ^{208}Pb is shown in Fig. 9. Here the best fit is extrapolated set A (dashed line). Set A' (reduced absorption, short-dashed line) gives a definitely poorer fit to the π^- data. Thus extrapolated set A, with the larger absorption para-

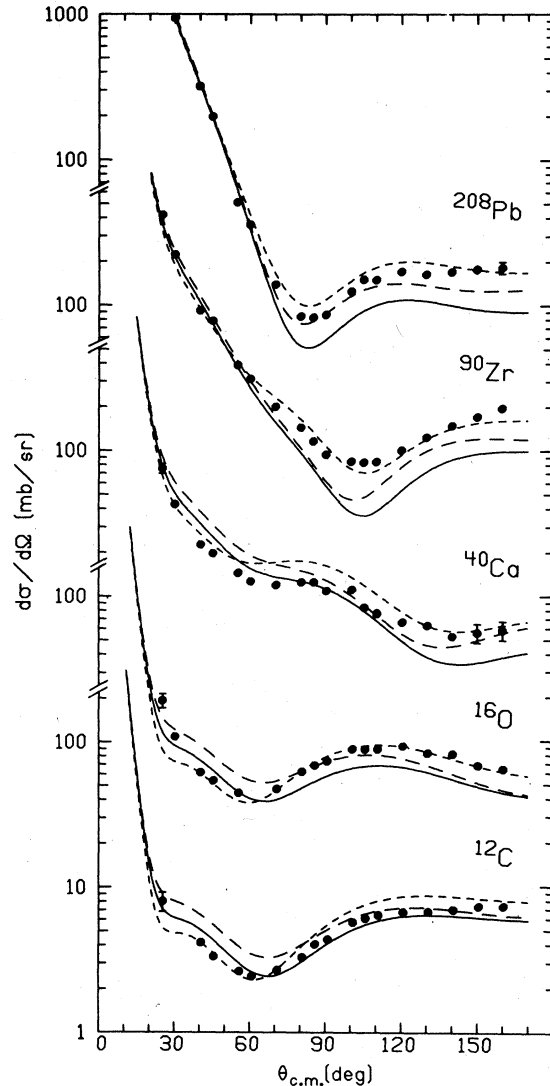


FIG. 7. Elastic scattering of 40 MeV π^+ from ^{12}C , ^{16}O , ^{40}Ca , ^{90}Zr , and ^{208}Pb . The curves follow the convention in Fig. 6, and the data are from Ref. 19.

eters, seems consistent with the data up to 30 MeV.

VI. CONCLUSION

Pionic atom data were fitted with an optical potential of the form used in scattering. There is a whole family of equivalent potentials, as basically the data determine only four-parameters. The potentials are almost, but not quite, equivalent to a four-parameter simple Kisslinger potential. Expressing them this way enables different potentials and different analyses to be compared. For the real parts of the optical potential, all analyses agreed in demanding a rather strong s -wave repulsion, with parameter $b_{eff} \cong -0.05$

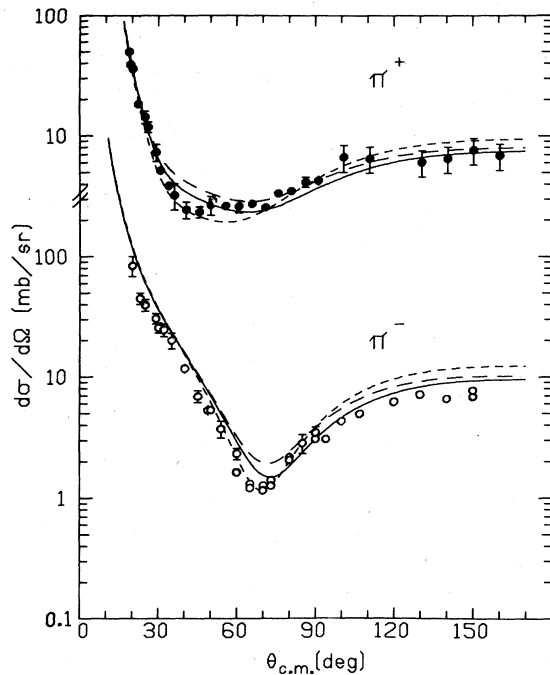


FIG. 8. Elastic scattering of 30 MeV π^+ and π^- from ^{12}C . The curves follow the same convention as in Fig. 6, and the data are from Refs. 18 and 20.

about twice as large as simple estimates. They also require a weakened p -wave attraction, with c_{eff} decreased from its phase shift value of 0.58 fm^3 to a value ranging from 0.43 to 0.49 fm^3 , the precise value depending on the procedure used to fit the data and in particular on how the isovector parameters are fitted. These values for c_{eff} are consistent with a large LLEE effect with $1 < \lambda < 2$. The imaginary parts of the optical potential, corresponding to absorption, are considerably larger than theoretical estimates, such as those of Chai and Riska,⁷ derived from two-body operators adjusted to fit pion absorption on the deuteron. A recent calculation of the p -wave absorption parameter by Weise⁹ is much nearer the mark, falling about halfway between the values of set A and set C.

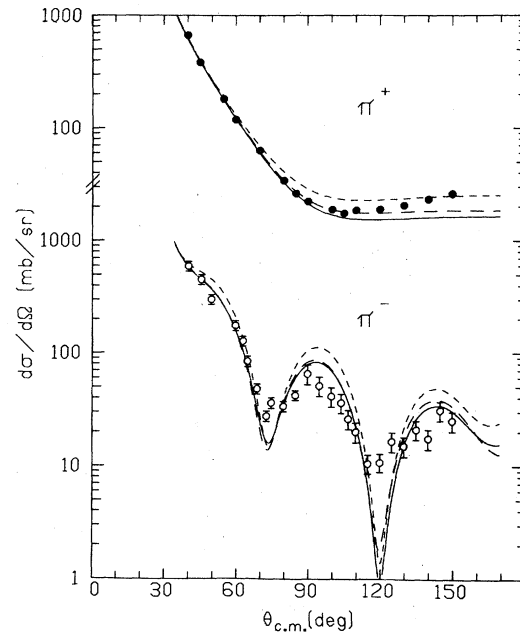


FIG. 9. Elastic scattering of 30 MeV π^+ and π^- from ^{208}Pb . The conventions are the same as in Fig. 6, and the data are from Refs. 21 and 22.

The elastic scattering calculations confirm the conclusions of our earlier paper, that an optical potential fitted to pionic atoms produces qualitative agreement with low energy pion scattering, and in fact fits quite well for 30 MeV pions. A simple extrapolation of the parameters also does well at 30 and 40 MeV, but not at 50 MeV. Similarly, a potential originally fitted to the 50 MeV data does poorly when extrapolated in the same way down to pionic atoms. The main discrepancy is in the absorptive parameters, so a study of absorption cross sections may help clarify the energy dependence of these parameters.

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