PHYSICAL REVIEW C VOLUME 22, NUMBER 5 NOVEMBER 1980

Hypertriton as a test of theoretical hyperon-nucleon potentials. II

B. F. Gibson
Theoretical Division, Los Alamos Scientific Laboratory, University of California, Los Alamos, New Mexico 87545

D. R. Lehman
Department of Physics, The George Washington University, Washington, D. C. 20052

(Received 2 June 1980)

A separable approximation to meson theoretic hyperon-nucleon potentials is made such that low-energy-scattering parameters are reproduced. Explicit ΛN-ΣN coupling is neglected. The important tensor nature of triplet force is included. Several recently proposed hyperon-nucleon potentials lead to acceptable ${}^3_\Lambda$H binding energy estimates.

[NUCLEAR STRUCTURE ${}^3_\Lambda$H, YN potentials, B_Λ, tensor force separable potential calculation.]

Nagels, Rijken, and deSwart have developed several meson theoretic potentials to describe the available nucleon-nucleon (NN) and hyperon-nucleon (YN) scattering data.[1-4] Mass differences in the various isomultiplets and symmetry breaking exchanges were included in a combined analysis of the NN, Λp, $\Sigma^\pm p$, etc., data. We summarize the resulting low energy scattering parameters (scattering lengths and effective ranges) for several of these models in Table I. In the calculations described below we have used rank-one, s-wave separable potentials designed to reproduce these low energy scattering parameters.

The hypertriton ${}^3_\Lambda$H ($J^\pi = \frac{1}{2}^+$, $T=0$) is the lightest of the bound hypernuclei with a Λ separation energy[5] $B_\Lambda = B({}^3_\Lambda\text{H}) - B({}^2\text{H}) \cong 0.13 \pm 0.05$ MeV. We have previously estimated B_Λ for potential models A and B using a Faddeev-type formalism in an effort to test whether such calculations might help differentiate between the two proposed models of the YN interaction.[6] Because ${}^3_\Lambda$H is lightly bound and therefore a loose structure (essentially a Λ coupled to a deuteron), B_Λ was assumed in Ref. 6 to be insensitive to the short-range (high-momentum) character of the YN force and the tensor nature of the triplet interactions. In addition, it was assumed that *explicit* ΛN-ΣN coupling could be neglected since it is included implicitly in the ΛN low energy scattering parameters of the various potential models. It was later pointed out that, while repulsion in the YN force and explicit ΛN-ΣN coupling were not large effects, the neglect of the tensor nature of the np force was possibly a significant omission.[7] Thus, we report here results for B_Λ of the ${}^3_\Lambda$H using the YN separable potential determined from the scattering lengths and effective ranges listed in Table I along with np triplet potentials[8,9] (Table II) having deuteron d-state percentages of 0%, 4%, and 7%.

Because the average ΛN interaction is $\frac{3}{4}$ singlet and only $\frac{1}{4}$ triplet, we neglect the tensor coupling in the triplet ΛN interaction. This tends to slightly overestimate B_Λ but is compensated for by our neglect of explicit ΛN-ΣN coupling in the triplet channel, which tends to underestimate B_Λ.[10] Since $\frac{3}{4}$ of the ΛN potential (the singlet) suffers neither defect (experimentally, ΛN-ΣN coupling is negligible in the singlet YN channel[3,10]), our estimates should be reasonable.

Because there is only one tensor interaction (that of the np pair) in our calculation, Eqs. (6) of Ref. 6 are modified only slightly; schematically they become

TABLE I. ΛN scattering lengths and effective ranges in fm for YN potential models from Refs. 1–4.

Model	Ref.	$a^s_{p\Lambda}$	$r^s_{p\Lambda}$	$a^t_{p\Lambda}$	$r^t_{p\Lambda}$	$a^s_{n\Lambda}$	$r^s_{n\Lambda}$	$a^t_{n\Lambda}$	$r^t_{n\Lambda}$
A	1	−2.16	2.03	−1.32	2.31	−2.67	2.04	−1.02	2.55
B	2	−2.11	3.19	−1.88	3.16	−2.47	3.09	−1.66	3.33
D	3	−1.77	3.78	−2.06	3.18	−2.03	3.66	−1.84	3.32
F	4	−2.18	3.19	−1.93	3.35	−2.40	3.15	−1.84	3.37

TABLE II. Separable potential parameters and properties for the *np* triplet interaction.

Model	Ref.	a_t	r_t	P_D	Q	λ_t	β_c	ξ_T	β_T
GL	8	5.423	1.761	0	0	0.381 5	1.406	0	
P_4	9	5.397	1.727	0.04	0.282	0.243 10	1.3134	1.6894	1.5283
P_7	9	5.397	1.722	0.07	0.283	0.142 97	1.2412	4.4949	1.9476

$$\left(1-\lambda_t\int\frac{g_c^{\,2}+g_T^{\,2}}{\Delta}\right)F_\Lambda^C=-\frac{\sqrt{3}}{2}\Lambda_s^n\int g_c h_s^n F_s^p\Big/\Delta-\tfrac{1}{2}\Lambda_t^n\int g_c h_t^n F_t^p\Big/\Delta-\frac{\sqrt{3}}{2}\Lambda_s^p\int g_c h_s^p F_s^n\Big/\Delta-\tfrac{1}{2}\Lambda_t^p\int g_c h_t^p F_t^n\Big/\Delta\,,$$

$$\left(1-\lambda_t\int\frac{g_c^{\,2}+g_T^{\,2}}{\Delta}\right)F_\Lambda^T=-\frac{\sqrt{3}}{2}\Lambda_s^n\int g_T h_s^n P_2 F_s^p\Big/\Delta-\tfrac{1}{2}\Lambda_t^n\int g_T h_t^n P_2 F_t^p\Big/\Delta-\frac{\sqrt{3}}{2}\Lambda_s^p\int g_T h_s^p P_2 F_s^n\Big/\Delta-\tfrac{1}{2}\Lambda_t^p\int g_T h_t^p P_2 F_t^n\Big/\Delta\,,$$

$$\left(\frac{\mu_{p\Lambda}}{\mu}-\Lambda_s^p\int\frac{(h_s^p)^2}{\Delta}\right)F_s^n=-\frac{\sqrt{3}}{2}\lambda_t\int h_s^p(g_cF_\Lambda^c+g_TP_2F_\Lambda^T)+\frac{\sqrt{3}}{2}\Lambda_t^n\int h_s^p h_t^n F_t^p\Big/\Delta+\tfrac{1}{2}\Lambda_s^n\int h_s^p h_s^n F_s^p\Big/\Delta\,,$$

$$\left(\frac{\mu_{p\Lambda}}{\mu}-\Lambda_t^p\int\frac{(h_t^p)^2}{\Delta}\right)F_t^n=-\tfrac{1}{2}\lambda_t\int h_t^p(g_cF_\Lambda^c+g_TP_2F_\Lambda^T)-\tfrac{1}{2}\Lambda_t^n\int h_t^p h_t^n F_t^p\Big/\Delta+\frac{\sqrt{3}}{2}\Lambda_s^n\int h_t^p h_s^n F_s^p\Big/\Delta\,,$$

$$\left(\frac{\mu_{n\Lambda}}{\mu}-\Lambda_s^n\int\frac{(h_s^n)^2}{\Delta}\right)F_s^p=-\frac{\sqrt{3}}{2}\lambda_t\int h_s^n(g_cF_\Lambda^c+g_TP_2F_\Lambda^T)+\frac{\sqrt{3}}{2}\Lambda_t^p\int h_s^n h_t^p F_t^n\Big/\Delta+\tfrac{1}{2}\Lambda_s^p\int h_s^n h_s^p F_s^n\Big/\Delta\,,$$

$$\left(\frac{\mu_{n\Lambda}}{\mu}-\Lambda_t^n\int\frac{(h_t^n)^2}{\Delta}\right)F_t^p=-\tfrac{1}{2}\lambda_t\int h_t^n(g_cF_\Lambda^c+g_TP_2F_\Lambda^T)-\tfrac{1}{2}\Lambda_t^p\int h_t^n h_t^p F_t^n\Big/\Delta+\frac{\sqrt{3}}{2}\Lambda_s^p\int h_t^n h_s^p F_s^n\Big/\Delta\,.$$

Here the triplet *np* potential is of the usual Yamaguchi-Yamaguchi form

$$V_t=-\frac{\lambda_t}{2\mu}g_t(\vec{k})g_t(\vec{k}')\,,$$

where we have

$$g_t=g_c+\frac{S_{ij}}{\sqrt{8}}g_T\,,$$

$$g_c=(k^2+\beta_c^{\,2})^{-1}\,,$$

$$g_T=\xi_T k^2(k^2+\beta_T^{\,2})^{-2}\,,$$

$$S_{ij}=3\vec{\sigma}_i\cdot\hat{k}\vec{\sigma}_j\cdot\hat{k}-\vec{\sigma}_i\cdot\vec{\sigma}_j\,.$$

The F_Λ^C and F_Λ^T describe the spectator Λ particle when the interacting *np* pair have $l=0$ and $l=2$, respectively; P_2 is the 2nd order Legendre polynomial of argument $\cos\theta=\hat{k}_{np}\cdot\hat{p}_{\text{spectator}}$ (see Ref. 11).

Using the ΛN separable potentials determined by the low energy scattering parameters listed in Table I and the *np* triplet potentials defined in Table II, we have calculated the Λ separation energies listed in Table III. The results for the $P_D=0$ *np* force model are included for completeness and as a comparative measure of the importance of the tensor nature of the *np* triplet interaction, even in such a weakly bound system as the hypertriton.

Model A clearly overbinds ${}^3_\Lambda$H, regardless of the *np* triplet force used. This is a result of the comparatively small values ($\leqslant 2.5$ fm) for the effective ranges of the ΛN potentials in that model, as noted in Ref. 6. Although the value of B_Λ differs among models B, D, and F by 0.1–0.2 MeV, none of these models is obviously incorrect. (B_Λ for $P_D=0$ is not considered to be realistic, and we do not consider B_Λ for model D to lie significantly outside the experimental limits.) The B_Λ from model D are systematically smaller than those of models B and F, because the average ($\frac{3}{4}$ singlet plus $\frac{1}{4}$ triplet) effective range is larger; $r_0>r_0'$ implies $B_3<B_3'$.[11] Models B and F produce very similar values of B_Λ because their average singlet scattering lengths and effective ranges are similar; they would produce different values of ΔB_Λ in the ${}^4_\Lambda$He-${}^4_\Lambda$H isodoublet system where the differences in the Λp and Λn triplet scattering lengths and effective ranges are significant.[12] A recent estimate of B_Λ using a sum of local Yukawa forms (including short range repulsion) to represent the

TABLE III. ${}^3_\Lambda$H Λ-separation energy B_Λ in MeV for YN models A–F as a function of P_D in the *np* triplet interaction.

np model	GL	P_4	P_7
YN model A	0.90	0.56	0.35
YN model B	0.37	0.22	0.13
YN model D	0.12	0.06	0.03
YN model F	0.37	0.23	0.13

model F ΛN interaction and the Reid soft core (RSC) potential for the np triplet interaction by Narumi, Ogawa, and Sunami gave a value of 0.14 MeV.[13] This agrees very well with our 0.13 MeV for model F using an np potential model with $P_D = 7\%$.

In summary, only the meson theoretic potential model A appears to produce hypertriton B_Λ estimates which are inconsistent with the experimental value. However, the ${}^3_\Lambda$H ground state is primarily sensitive to the YN singlet interactions. Proper attention to the tensor nature of the np triplet interaction is necessary if realistic estimates of $B({}^3_\Lambda\mathrm{H})$ are to be obtained, as is the case in treating the ^{3}H bound state.

The work of B.F.G. was performed under the auspices of the U. S. Department of Energy, and that of D.R.L was supported in part by the U. S. Department of Energy.

---

[1] M. M. Nagels, T. A. Rijken, and J. J. deSwart, Ann. Phys. (N.Y.) 79, 338 (1973).

[2] M. M. Nagels, T. A. Rijken, and J. J. deSwart, in *Few Particle Problems in the Nuclear Interaction*, edited by I. Slaus, S. A. Moszkowski, R. P. Haddock, and W. T. H. van Oers (North-Holland, Amsterdam, 1972), pp. 41–45.

[3] M. M. Nagels, T. A. Rijken, and J. J. deSwart, Phys. Rev. D 15, 2547 (1977).

[4] M. M. Nagels, T. A. Rijken, and J. J. deSwart, Phys. Rev. D 20, 1633 (1979).

[5] M. Juric, G. Bohm, J. Klabuhn, U. Krecker, F. Wysotzki, G. Coremans-Bertrand, J. Sacton, G. Wilquet, T. Cantwell, F. Esmael, A. Montwill, D. H. Davis, D. Kielczewska, T. Pniewski, T. Tymieniecka, and J. Zakrzewski, Nucl. Phys. B52, 1 (1973).

[6] B. F. Gibson and D. R. Lehman, Phys. Rev. C 10, 888 (1974); 14, 2346 (1976).

[7] B. F. Gibson and D. R. Lehman, Phys. Rev. C 11, 2092 (1975); L. H. Schick, *ibid.* 11, 2089 (1975).

[8] B. F. Gibson and D. R. Lehman, Phys. Rev. C 11, 29 (1975).

[9] A. C. Phillips, Nucl. Phys. A107, 209 (1968).

[10] J. Dabrowski and E. Fedorynska, Nucl. Phys. A210, 509 (1973).

[11] B. F. Gibson and G. J. Stephenson, Jr., Phys. Rev. C 11, 1448 (1975).

[12] B. F. Gibson and D. R. Lehman, Nucl. Phys. A329, 308 (1979).

[13] H. Narumi, K. Ogawa, and Y. Sunami, Lett. Nuovo Cimento 26, 294 (1979). A later publication [Prog. Theor. Phys. 63, 533 (1980)] quotes a slightly different value.