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Fluctuations in $^{12}C(^{16}O,\alpha)^{24}Mg$ and $^{12}C(^{16}O,^{8}Be_{g.s.})^{20}Ne$ reactions

R. Singh* and K. A. Eberhard
Sektion Physik, Universität München, D-8046 Garching, Am Coulombwall 1, Federal Republic of Germany

R. G. Stokstad†
Oak Ridge National Laboratory, Oak Ridge, Tennessee 37820
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A statistical model analysis has been performed on the excitation functions for $^{12}C(^{16}O,\alpha)^{24}Mg$ (0^+,g.s.; 2^+,1.37; 4^+,4.12+; 2^+,4.24; 3^+,5.24 MeV) from $E_{c.m.}=13.17$ to $E_{c.m.}=23.09$ MeV at $\theta_L=35°$ and for $^{12}C(^{16}O,^{8}Be_{g.s.})^{20}Ne$ (0^+,g.s.; 2^+,1.63; 4^+,4.25 MeV) from $E_{c.m.}=14.07$ to $E_{c.m.}=23.09$ MeV (for 0^+,g.s.), $E_{c.m.}=14.46$ to $E_{c.m.}=23.09$ MeV (for 2^+,1.63 MeV), $E_{c.m.}=16.65$ to $E_{c.m.}=23.09$ MeV (for 4^+,4.25 MeV) at $\theta_L=32.5°$. The number of effective channels was calculated exactly. The theoretical and experimental distributions of the cross sections agree reasonably well and indicate that the fluctuating component of the experimental cross sections is consistent with statistical model predictions for these quantities.

[NUCLEAR REACTIONS $^{12}C(^{16}O,\alpha)^{24}Mg$, $^{12}C(^{16}O,^{8}Be_{g.s.})^{20}Ne$, statistical model analysis of the excitatton functions at $\theta_L=35°$ and $\theta_L=32.5°$.]

I. INTRODUCTION

The $^{16}O+^{12}C$ system has been studied extensively in both elastic and reaction channels in recent years and several resonances have been reported.[1,2] The $^{12}C(^{16}O,\alpha)^{24}Mg$ and $^{12}C(^{16}O,^{8}Be_{g.s.})^{20}Ne$ reactions exhibit rapid fluctuations in their excitation functions for various states in the residual nuclei. Analyses of these fluctuations show that—on the average—the experimental data for the $^{12}C(^{16}O,\alpha)^{24}Mg$ reaction are compatible with a statistical compound nucleus process.[2-4] On the other hand, some intermediate structures have also been reported.[2,5] Direct processes[6] and intermediate structure effects[7] have been observed in the $^{12}C(^{16}O,^{8}Be_{g.s.})^{20}Ne$ reaction. We have performed a statistical analysis following Ericson,[8] and Brink and Stephen[9] on excitation functions for five states in ^{24}Mg and three states in ^{20}Ne populated at laboratory angles of 35° and 32.5° in the $^{12}C(^{16}O,\alpha)^{24}Mg$ and $^{12}C(^{16}O,^{8}Be_{g.s.})^{20}Ne$ reactions, respectively. This type of analysis can provide a quantitative estimate of the likelihood that the structures observed in the experimental data arise from the interference of strongly overlapping resonances in the compound nucleus or whether they require a nonstatistical explanation such as molecular resonances or intermediate structure resonances.

II. ANALYSIS

A. Data reduction

The data analyzed here are from Ref. 10; they were taken with a target 47.2 $\mu g/cm^2$ thick [which corresponds to an energy loss of about 120 keV (c.m.)]. Excitation functions in steps of 130 keV (c.m.) were taken for the reactions $^{12}C(^{16}O,\alpha)^{24}Mg$ (0^+, g.s.; 2^+, 1.37; (4^+, 4.12+; 2^+, 4.24); 3^+, 5.24 MeV) from $E_{c.m.}=13.17$ to $E_{c.m.}=23.09$ MeV at $\theta_L=35°$ and for $^{12}C(^{16}O,^{8}Be_{g.s.})^{20}Ne$ (0^+, g.s.; 2^+, 1.63; 4^+, 4.25 MeV) from $E_{c.m.}=14.07$ to $E_{c.m.}=23.09$ MeV (for the 0^+ state), from $E_{c.m.}=14.46$ to $E_{c.m.}=23.09$ MeV (for the 2^+ state), and from $E_{c.m.}=16.65$ to $E_{c.m.}=23.09$ MeV (for the 4^+ state) at $\theta_L=32.5°$.

Since we wish to compare the behavior of the experimental cross sections with the predictions of the statistical model, we have removed the energy dependent gross structure from the excitation functions. This was done by dividing the individual data points by the running average of the cross sections $\langle d\sigma(E)\rangle$ taken over an energy interval of $\Delta E=2.45$ MeV (c.m.). The percentage deviations of the reduced data, $d\sigma(E)/\langle d\sigma(E)\rangle$, from unity are shown in Figs. 1 and 2 for the α-channels, and in Fig. 3 for the ^8Be channels. It may be seen that no gross structure remains in the data.

B. Hauser-Feshbach cross sections and the number of effective channels

The cross sections were calculated by the statistical model code STATIS[11] which used the Hauser-Feshbach experession[12] for evaluating energy averaged differential cross sections for the population of specific final states. The exit channels $n+^{27}Si$, $p+^{27}Al$, $d+^{26}Al$, $\alpha+^{24}Mg$, and $^8Be+^{20}Ne$ were included in the calculations. The optical model parameters for $^{16}O+^{12}C$ and $^8Be+^{20}Ne$ were

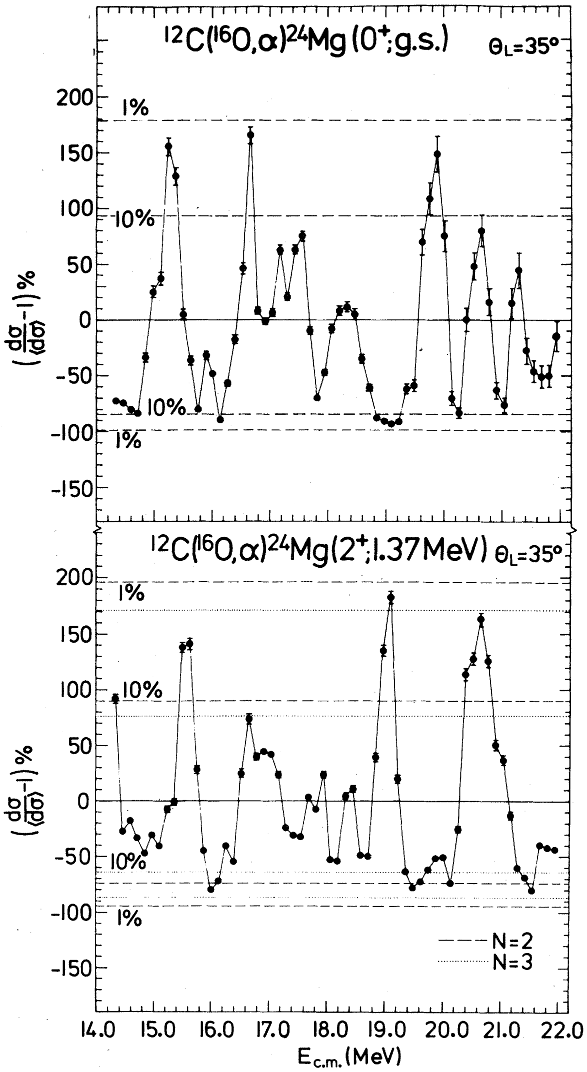

FIG. 1. The percentage deviations from unity of the quantity $d\sigma(E)/\langle d\sigma(E)\rangle$ for ^{12}C(^{16}O, α)^{24}Mg (0^+, g.s.) and ^{12}C(^{16}O, α)^{24}Mg (2^+, 1.37 MeV) at $\theta_L = 35°$. The curves designated as 1% and 10% denote deviations from the average for which the probability of finding a larger deviation is 1% and 10%, respectively (see Sec. IIC).

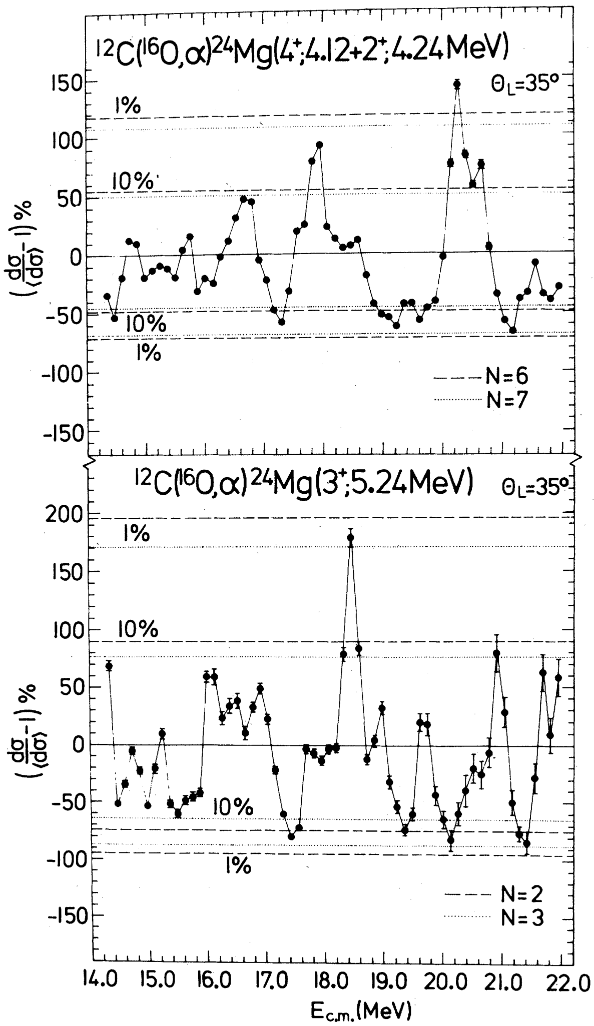

FIG. 2. Same as in Fig. 1, but for ^{12}C(^{16}O, α)^{24}Mg (4^+, 4.12+ ; 2^+, 4.24 MeV) and ^{12}C(^{16}O, α)^{24}Mg (3^+, 5.24 MeV) at $\theta_L = 35°$.

taken from Ref. 13, and for $\alpha + {}^{24}$Mg, $p + {}^{27}$Al, and $d + {}^{26}$Al from Refs. 14–16. The level density parameters were taken from Ref. 17. Table I summarizes the parameters used in the calculations. It is necessary to introduce a maximum value, l_{cut}, for the angular momentum in the entrance channel. This limitation arises because, in a sharp cutoff approximation, angular momenta larger than l_{cut} do not lead to fusion, i.e., to the formation of a compound nucleus. The values of l_{cut} were taken from the fusion cross sections for ^{12}C + ^{16}O as obtained in Ref. 10, and are listed in Table II. (If the grazing angular momentum is used for the maximum value of l leading to fusion, the calculated values of the cross section are of course much larger than the measured values.) Hauser-Feshbach cross sections are compared to experimental angle-integrated cross sections (from $\theta_L = 5°$ to $35°$ for the α channel and from $\theta_L = 7.5°$ to $32.5°$ for the ^{8}Be channel) in Figs. 4, 5, and 6, respectively. From these calculations it appears that the α channel contains little direct contributions. This is in contrast to the ^{8}Be channel where a significant direct reaction contribution is indicated. The structures in the Hauser-Feshbach cross sections result from integral steps in the values of l_{cut} (see Table II).

The code STATIS was also used to calculate the number of effective channels N. This quantity determines the number of statistically indepen-

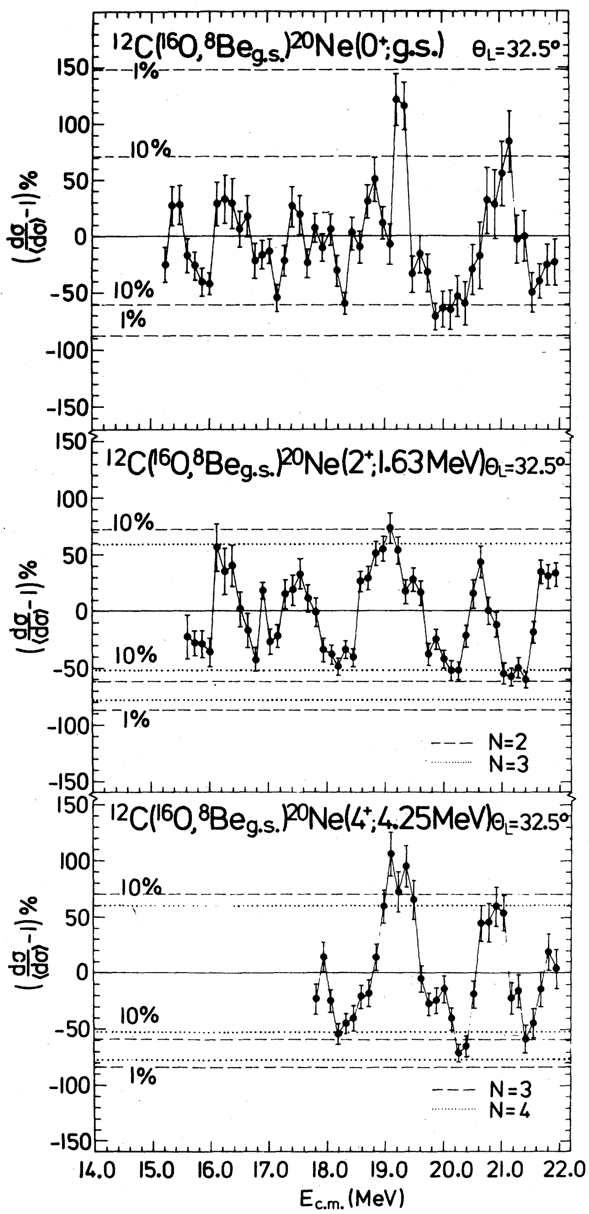

FIG. 3. Same as in Fig. 1, but for the reactions $^{12}C(^{16}O,^{8}Be_{g.s.})\,^{20}Ne$ (0^+, g.s.), $^{12}C(^{16}O,^{8}Be_{g.s.})\,^{20}Ne$ (2^+, 1.63 MeV), and $^{12}C(^{16}O,^{8}Be_{g.s.})\,^{20}Ne$ (4^+, 4.25 MeV) at $\theta_L = 32.5°$. The 1% curves corresponding to positive deviations for $^{12}C(^{16}O,^{8}Be_{g.s.})\,^{20}Ne$ (2^+, 1.63 MeV) and for $^{12}C(^{16}O,^{8}Be_{g.s.})\,^{20}Ne$ (4^+, 4.25 MeV) are not shown here but they lie at 150% (for $N=2$), 124% (for $N=3$), and at 146.5% (for $N=3$), 127.6% ($N=4$), respectively.

dent cross sections which contribute to the measured cross section. The details of the evaluation of N are given in Ref. 18. The variation of N with angle for the states under investigation in both the α and ^{8}Be channels for $E_{c.m.} = 20$ MeV is shown in Fig. 7.

If the fraction Y_D of the observed cross section that proceeds via a direct process is zero, then the number of effective channels is related to the variance of the experimental cross sections through the equation[8]

$$\frac{1}{N} = \frac{\langle\sigma^2\rangle - (\langle\sigma\rangle)^2}{(\langle\sigma\rangle)^2} \equiv V \,, \qquad (1)$$

which is equivalent to

$$\frac{1}{N_{exp}} \equiv \frac{\langle x^2\rangle - (\langle x\rangle)^2}{(\langle x\rangle)^2} \,, \qquad (2)$$

where $x = d\sigma(E)/\langle d\sigma(E)\rangle$. The uncertainty in the variance which results from the finite range of data implies an uncertainty in N_{exp} that is given by[19]

$$N_{exp} = \frac{1}{V}\left[1 \pm \left(\frac{\pi\Gamma}{E_2 - E_1}\right)^{1/2}\right] , \qquad (3)$$

where $(E_2 - E_1)$ is the energy range of the data.

The values of N_{exp} for the 2^+, 1.37 MeV, (4^+, 4.12 + 2^+, 4.24 MeV), and 3^+, 5.24 MeV states in ^{24}Mg as obtained from Eq. (3) are 2.68±0.75, 4.74±1.32, and 3.17±0.89, respectively (assuming $Y_D = 0$). The corresponding predicted values are 2.54–2.8, 6.1–7.0, and 2.3–2.7, respectively, at the higher and lower ends of the energy range. Thus the values of N_{exp}, and N as predicted by the code are in good agreement with each other.

C. Distribution of cross sections

The distributions of the fluctuating cross sections are given by[9,20]

$$P(x) = N(Nx)^{N-1} e^{-Nx}/(N-1)! \,, \qquad (4)$$

$$P(x) = \frac{Nx^{(N-1)/2}}{(1-Y_D)\,Y_D^{(N-1)/2}} \exp\left[-N\left(\frac{x+Y_D}{1-Y_D}\right)\right] \times I_{N-1}\left(\frac{2N\sqrt{xY_D}}{1-Y_D}\right) \qquad (5)$$

for $Y_D = 0$ and $Y_D \neq 0$, respectively, where $x = d\sigma \times (E)/\langle d\sigma(E)\rangle$, N = the number of effective channels, Y_D is the ratio of direct to total cross section, and I_{N-1} is the modified Bessel function of order $N-1$. The values of Y_D were obtained from the formula

$$Y_D = (1 - \alpha_D)^{1/2} \,, \qquad (6)$$

where

$$\alpha_D = \frac{NR(0)}{1 - a - aR(0)} \,,$$

$$a = \frac{2}{m}\tan^{-1}(m) - \frac{1}{m^2}\ln(1+m^2) \,,$$

TABLE I. Optical model parameters for calculating the transmission coefficients and the level density parameters used for the Hauser-Feshbach calculations.

Channel	V_{Re} (MeV)	$V_{Im(Vol)}$ (MeV)	$V_{Im(surf)}$ (MeV)	R_{Re} (fm)	a_{Re} (fm)	R_{Im} (fm)	a_{Im} (fm)
$^{12}C+^{16}O$	$7.5+0.4E_{c.m.}$	$0.4+0.125E_{c.m.}$		6.49	0.45	6.49	0.45
$^{20}Ne+^{8}Be$	$7.5+0.4E_{c.m.}$	$0.4+0.125E_{c.m.}$		6.36	0.45	6.36	0.45
$^{24}Mg+\alpha$	125.3	30.7		4.47	0.54	4.6	0.39
$^{27}Si+n$	$56.3-0.32E_{c.m.}-24.0\frac{N-Z}{A}$	$0.22E_{c.m.}-1.56$	$13.0-0.25E_{c.m.}-12\frac{N-Z}{A}$	3.51	0.75	3.78	0.58
$^{27}Al+p$	$54.0-0.32E_{c.m.}+24.0\frac{N-Z}{A}+0.4\frac{Z}{A^{1/3}}$	$0.22E_{c.m.}-2.7$	$11.8-0.25E_{c.m.}+12\frac{N-Z}{A}$	3.51	0.75	3.96	0.53
$^{26}Al+d$	117.0		18.9	3.11	0.86	4.71	0.54

Level density parameters

	^{16}O	^{24}Mg	^{27}Al	^{27}Si	^{20}Ne	^{26}Al	^{28}Si
a/A	0.136	0.149	0.137	0.137	0.152	0.152	0.116
Δ_p	5.13	5.13	1.80	2.09	5.13	0.00	3.89
E_{cut}^{a} (MeV)	12.71	9.97	5.25	3.54	13.66	3.68	

[a] E_{cut} = energy up to which explicit levels were used in calculation of the denominator.

and $m=\Delta E/\Gamma$ is the sample size. The expression for α_D was obtained from Eq. (B1) of Ref. 21. The value of $R(0)$ was appropriately corrected for the finite energy resolution of ~120 keV (c.m.) and for the effect of averaging to remove the gross structure from the excitation functions. The latter correction to $R(0)$ was made by using Fig. 15(b) of Ref. 21 with the assumptions that the $^{12}C+^{12}C$ gross structure at 90°, assumed in synthetic excitation functions, is similar to $^{16}O+^{12}C$ gross structure, and that Y_D is approximately the same for $^{12}C+^{12}C$ and $^{16}O+^{12}C$ systems. The theoretical distributions have been calculated for both the maximum and minimum values of N predicted over the energy range of the data under consideration. Figures 8 and 9 show the theoretical and experimental distributions. The predicted distributions are in reasonable agreement with the experimental ones.

The values of Y_D and N are in principle energy dependent. We have, however, used the average of the Y_D values obtained for $N=2.78$ and 2.5 for the 2^+ state of ^{20}Ne at 1.63 MeV since these values of N are so close to each other. For the 4^+, 4.25 MeV state of ^{20}Ne, the value of N varies little in the energy range of interest and, therefore, the value of Y_D is practically constant at 0.45. The values of Y_D obtained from Eq. (6) for α and ^{8}Be channels are in accordance with the qualitative indications given by the Hauser-Feshbach cross sections in Fig. 4–6.

The probability of observing a cross section fluctuation which is larger than certain value x' is given by[18]

$$Q(x')=\int_{x'}^{\infty} P(x)\,dx\ . \tag{7}$$

The probability of observing a fluctuation which is less than some value x' is simply $1-Q(x')$.

TABLE II. Maximum values of the entrance channel angular momentum used in the Hauser-Feshbach calculations.

$E_{c.m.}$ (MeV)	13.16	13.55 –14.97	15.48	16.00 –20.00	20.51 –21.55	22.06 –22.84
$l_{cut}(\hbar)$	9	10	11	12	13	14

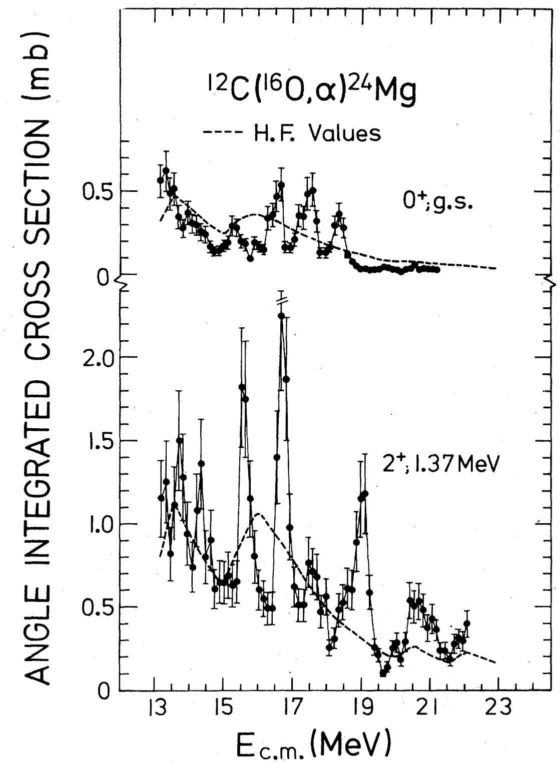

FIG. 4. The angle-integrated experimental cross sections for $^{12}C(^{16}O,\alpha)^{24}Mg$ (0^+, g.s.; 2^+, 1.37 MeV) and corresponding Hauser-Feshbach cross sections (shown by dashed lines). The errors on the data include a 20% error which results from the relatively large experimental angular step size of $\theta_L = 5°$. This was added to the statistical error.

The curves designated as 10% (1%) in Figs. 1–3 for different values of N correspond to the deviations from the average value for which the probability of finding a larger deviation is 10% (1%), respectively. It is clear that nearly all of the deviations occur with a probability larger than 1% and over half of the deviations occur with probability larger than 10%.

D. Coherence widths

An empirical estimate of the coherence width can be obtained from the formula[20,22]

$$\Gamma = 14\exp(-4.69\sqrt{A/E_x})\ \text{MeV}\ , \qquad (8)$$

where A is the mass number and E_x is the excitation energy in MeV of the compound nucleus. The empirical widths thus obtained vary between about 150 and 270 keV from $E_{c.m.} = 13$ to $E_{c.m.} = 23$ MeV. These estimates are slightly lower than the experimental values which vary between 180 and 320 keV and are obtained by counting the number of maxima M in the energy range $(E_2 - E_1)$ and employing the relation[9] $\Gamma_{exp} \approx 0.83\,(E_2 - E_1)/2M$.

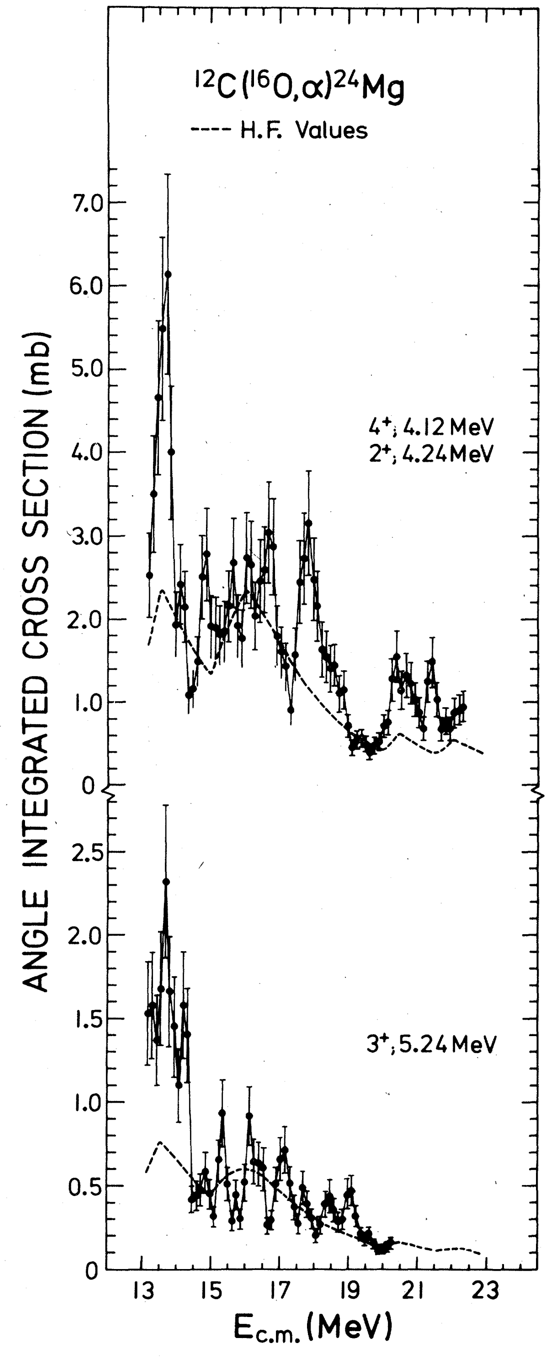

FIG. 5. Same as in Fig. 4, but for $^{12}C(^{16}O,\alpha)^{24}Mg$ (4^+, 4.12+ ; 2^+, 4.24; 3^+, 5.24 MeV).

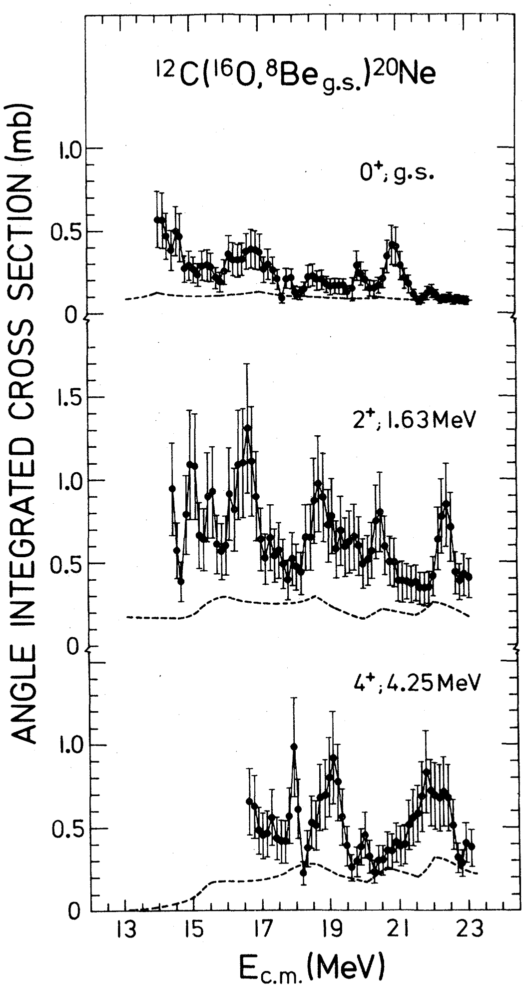

FIG. 6. Same as in Fig. 4 but for the reaction $^{12}C(^{16}O, {}^{8}Be_{g.s.})\,{}^{20}Ne$ (0^+, g.s.; 2^+, 1.63; 4^+, 4.25 MeV). The errors on the data include a 30% error which results from the relatively large experimental angular step size of $\theta_L = 5°$. This was added to the statistical error.

The factor 0.83 includes a correction for target thickness and the finite spacing of experimental points.[23]

III. CONCLUSION

Angle-integrated cross sections predicted with the Hauser-Feshbach theory were found to be in good overall agreement with the experimental data provided a maximum angular momentum in the entrance channel is introduced. This maximum is deduced directly from the measured cross section for fusion. The comparison of the angle-integrated experimental and theoretical cross sections indicates a larger direct reaction contribution for the ^8Be channel than for the α channel.

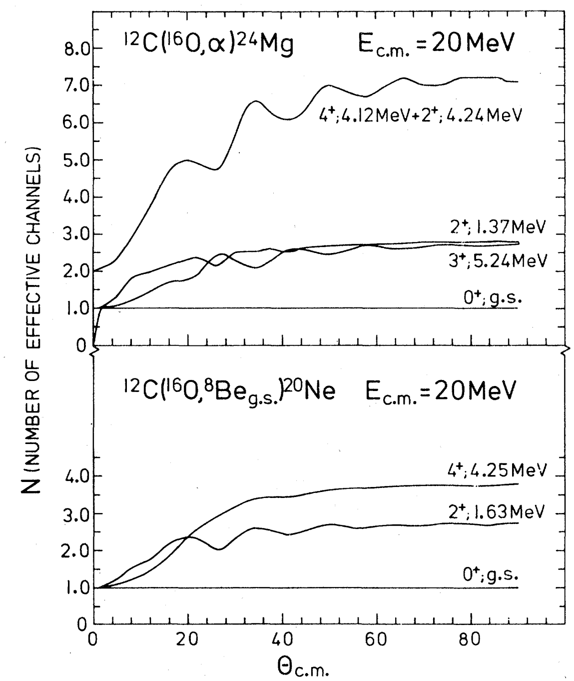

FIG. 7. The variation of the effective number of channels N with center of mass angle at $E_{c.m.} = 20$ MeV for the reactions indicated in the figure.

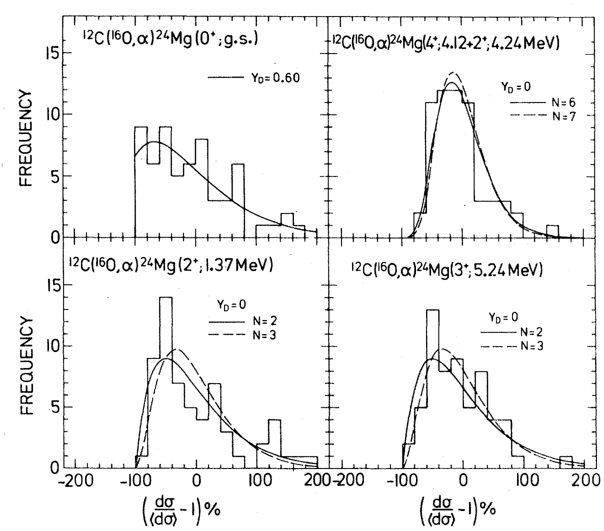

FIG. 8. The distribution of the experimental cross sections, shown in Figs. 1 and 2, about the average value. The curves show the theoretical distributions for different values of Y_D and the number of effective channels N, as indicated in the figure [$N=1$ for $^{12}C(^{16}O, \alpha)^{24}Mg$ (0^+, g.s.)].

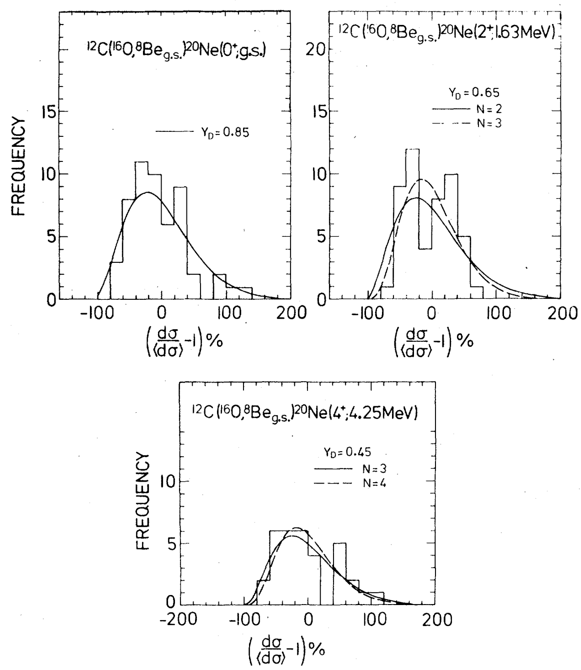

FIG. 9. Same as in Fig. 8 but for the cross sections shown in Fig. 3 [$N=1$ for $^{12}C(^{16}O, ^{8}Be_{g.s.})^{20}Ne$ (0^+, g.s.)].

The theoretical and experimental probability distributions in Figs. 8 and 9 agree rather well with each other. Almost all the deviations occur with more than 1% probability and a majority of them with more than 10% probability.

The present analysis shows only that the fluctuating features of the experimental data are consistent with statistical model expectations. This, however, does not rule out the possibility that intermediate structure may be found by other types of statistical tests. In fact, a detailed cross-channel and angle correlation analysis is required on a larger body of experimental data in order to isolate any intermediate structure or nonstatistical effects in these reactions. Such an analysis on states up to excitation energies of 6.43 MeV in ^{24}Mg and 7.17 MeV in ^{20}Ne will be reported separately.

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*Permanent address: Department of Physics, School of Physical Sciences, North-Eastern Hill University, Shillong-793003, India.

†Present address: Lawrence Berkeley Laboratory, Berkeley, California 94720.

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