Stable phase of nuclear matter

Toshitake Kohmura and Hirokazu Tezuka
Institute of Physics, University of Tsukuba, Ibaraki, Japan
(Received 15 January 1979)

A model of nuclear matter composed of nucleons interacting with neutral scalar mesons and neutral vector mesons, which Walecka has shown in the classical field approximation to reproduce the nuclear saturation property, is studied. The Fermi gas state for the system is shown to be unstable, resulting in the equidistant multilayer structure, a new phase of the system in which nucleons are distributed in multiple layers spaced equidistantly. In this structure, the energy per nucleon is $E/N = -24$ MeV and the nucleon density is $\rho = 0.02\rho_0$ with the normal nucleon density ρ_0. It is pointed out that the saturation property of a nucleon system, as well as the exchange term for nuclear interactions should be of considerable consequence in the discussion of the instability of nuclear states.

[NUCLEAR STRUCTURE Instability of Fermi gas state and stable phase in nuclear matter. Saturation and antisymmetrization considered.]

I. INTRODUCTION

A considerable amount of literature[1] has been devoted to the investigation of bulk properties of an infinite nuclear system. One of the unsolved problems is the saturation property of nuclear matter. Conventionally, starting with plane waves for nucleon wave functions, one modifies them with nucleon-nucleon correlations. The approaches using the correlation, however, might not be efficient for the discussion of bulk properties of nuclear matter. Therefore, other approaches suitable for the consideration of the gross structure might be exploited.

Some papers along this line appeared recently. By scaling the nuclear potential into the potential between ^{3}He atoms, Anderson has predicted that nuclear matter will be solidified[2] as in the case of the ^{3}He liquid, while a possibility of localization of nucleon density or spin-isospin density has been suggested in the course of controversial discussions on meson condensation.[3] Coherent localization of nucleons is found to induce meson condensation, and vice versa.[4]

The conventional theories which favorably predict the localization of nucleons have left out two factors unfavorable to the localization, viz., the saturation property of a nuclear system and antisymmetry between nucleon wave functions. Hence, starting with a Hamiltonian which assures the nuclear saturation, we shall investigate bulk properties of nuclear matter and predict its new phases. Antisymmetry will also be taken into account.

In a nucleon system with spin $S=0$ and isospin $T=0$, the contribution of the one-pion-exchange potential largely averages to zero, while the dominant part of the nuclear interaction is from scalar-meson exchange which represents part of the two-pion-exchange potential. A system of nonrelativistic nucleons interacting through scalar-meson exchanges, however, is not saturated. For a nucleon system to be saturated normally, some relativistic effects of nucleons and vector-meson exchanges have to be taken into account.

We adopt the following Lagrangian density[5] for relativistic nucleons interacting with neutral scalar mesons and neutral vector mesons:

$$L = -\bar{\psi}(\gamma_\mu\partial_\mu + M)\psi - \tfrac{1}{2}(\partial_\mu\phi\partial_\mu\phi + m^2\phi^2) - \tfrac{1}{4}F_{\mu\nu}F_{\mu\nu} - \tfrac{1}{2}m_v^2\phi_\mu\phi_\mu + g\bar{\psi}\psi\phi + ig_v\bar{\psi}\gamma_\mu\psi\phi_\mu \tag{1.1}$$

with a nucleon field ψ of mass M, a scalar-meson field ϕ of mass m, and a vector-meson field ϕ_μ of mass m_v. The field tensor $F_{\mu\nu}$ for ϕ_μ is given by

$$F_{\mu\nu} = \partial_\mu\phi_\nu - \partial_\nu\phi_\mu. \tag{1.2}$$

In Sec. II, we review a formulation for a system of nonrelativistic nucleons interacting with neutral scalar mesons in the random phase approximation as well as in the classical field approximation for the purpose of applying the technique in later discussions.

In Sec. III, we derive an effective mass for nucleons in nuclear matter from a relativistic expression for nucleons. The effective nucleon mass is essential for the nuclear saturation. We also take into account vector-meson exchange effects which modify some properties of nuclear system.

In Sec. IV, we show that the plane wave bases for Hartree-Fock (HF) single nucleon wave functions are unstable in nuclear matter, and predict new phases of nuclear matter where localization of nucleon density develops and gives rise to an equidistant multilayer structure.

A discussion and conclusion will be given in Sec. V.

II. NONRELATIVISTIC NUCLEONS INTERACTING THROUGH SCALAR-MESON EXCHANGES

In this section, we show that a system of nonrelativistic nucleons interacting through only neutral scalar-meson exchanges will collapse. We treat the system in order to sketch the technique which will be applied to a more realistic case later. The Lagrangian density in Eq. (1.1) leads to the Hamiltonian density for the system

$$H=-\frac{1}{2M}\psi^\dagger\nabla^2\psi+\tfrac{1}{2}[\dot{\phi}^2+(\nabla\phi)^2+m^2\phi^2]-g\psi^\dagger\psi\phi\,. \tag{2.1}$$

The fields ψ and ϕ satisfy the equations

$$\left(-\frac{1}{2M}\nabla^2-g\phi\right)\psi=i\dot{\psi}\,, \tag{2.2a}$$

$$(\Box-m^2)\phi=-g\psi^\dagger\psi\,. \tag{2.2b}$$

In the following, the field equations are solved in the classical field approximation and in the random phase approximation as well.

A. Classical field approximation to meson field ϕ

One of the simplest methods in which one may discuss some bulk properties of the nuclear system is the classical field approximation to the meson field ϕ. We separate ϕ into a classical field φ and an additional field η due to quantum fluctuations,[6] i.e.,

$$\phi=\varphi+\eta\,. \tag{2.3}$$

The classical field approximation to ϕ takes into account only φ which is assumed to be a constant over the space and time. In the approximation, we obtain the classical field[5]

$$\varphi=\frac{2g}{3\pi^2m^2}p_F{}^3 \tag{2.4}$$

as a function of the Fermi momentum p_F for the nuclear system.

The energy for the whole system in the ground state is given by

$$\begin{aligned}E&=\int\langle G|H|G\rangle\,d^3r\\&=\sum_{\vec{p}\lambda}^{p_F}\frac{p^2}{2M}+\tfrac{1}{2}m^2\varphi^2\Omega-g\varphi N\\&=\left(\frac{3p_F{}^2}{10M}-\frac{g^2p_F{}^3}{3\pi^2m^2}\right)N\end{aligned} \tag{2.5}$$

with spatial volume Ω, nucleon number N, and the suffix λ representing nucleon spin and isospin. Equation (2.5) indicates that E has no lower bound and that the system will collapse for the values of nucleon density above that corresponding to the Fermi momentum[7]

$$p_F=\frac{3\pi^2m^2}{5g^2M}\,, \tag{2.6}$$

where E is a maximum.

It should be noted that E in Eq. (2.5) can also be obtained for a Fermi gas system of nucleons interacting through the static one-neutral-scalar-meson exchange potential with the exchange term of the interaction neglected. The system will not be saved from collapsing by inclusion of the exchange term.

B. Random phase approximation

In Sec. II A we have considered only a constant classical field φ for ϕ. In such a treatment, one cannot take into account the nuclear interaction with momentum transfers $k=0$. The interaction gives rise to ground state correlations and will make other nuclear states lie lower than the Fermi gas state. In order to incorporate the effect of the interaction, we consider the quantum fluctuations of the meson field ϕ.

We introduce the random phase approximation (RPA) into the system.[8] The approach can determine whether the Fermi gas state is unstable for an excitation mode of the system. In RPA we obtain the secular equation

$$\begin{aligned}1&=\frac{4g^2}{(\omega_{\vec{k}}^2-\omega^2)\Omega}\sum_{\vec{p}}\frac{(n_{\vec{p}}-n_{\vec{p}+\vec{k}})(\epsilon_{\vec{p}+\vec{k}}-\epsilon_{\vec{p}})}{(\epsilon_{\vec{p}+\vec{k}}-\epsilon_{\vec{p}})^2-\omega^2}\\&=\frac{8g^2}{(\omega_{\vec{k}}^2-\omega^2)\Omega}\sum_{\vec{p}}\frac{n_{\vec{p}}(\epsilon_{\vec{p}+\vec{k}}-\epsilon_{\vec{p}})}{(\epsilon_{\vec{p}+\vec{k}}-\epsilon_{\vec{p}})^2-\omega^2}\,,\end{aligned} \tag{2.7}$$

where $\epsilon_{\vec{p}}$ is the kinetic energy of the nucleon with momentum $\vec{p}$, and $\omega_{\vec{k}}$ is the relativistic energy of the meson with momentum $\vec{k}$. The factor 4 is due to the degrees of freedom for nucleon spin and isospin, and

$$n_{\vec{p}}=\begin{cases}1, & \text{for } p<p_F\\ 0, & \text{for } p>p_F\,.\end{cases} \tag{2.8}$$

The solutions ω deviate from the unperturbed energy of any one-particle–one-hole pair, which indicates that there exist ground state correlations more or less.

A catastrophe occurs at higher nucleon densities. The right hand side of Eq. (2.7) is a continuous and increasing function of ω^2 for $\omega^2<0$. Hence, in the case where the right hand side is larger than unity

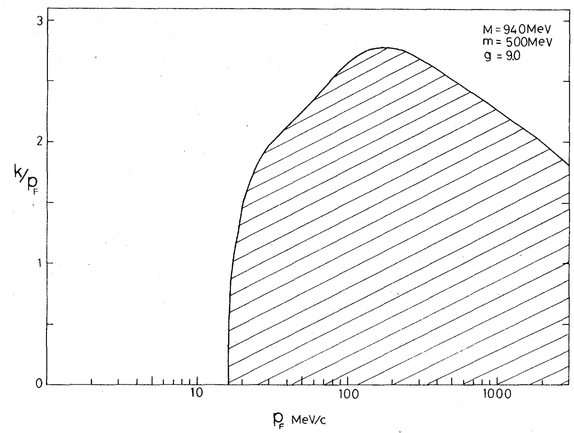

FIG. 1. Hatched area indicates the region where there exist imaginary solutions to the secular equation (2.7) for given k/p_F. The nucleon density is represented by the Fermi momentum p_F. The coupling constant is set at $g=9.0$, the value modified by the exchange term of the nuclear interaction; see Eq. (4.3).

at $\omega^2=0$, i.e.,

$$\frac{8g^2}{\omega_{\vec{k}}^2\Omega}\sum_{\vec{p}}\frac{n_{\vec{p}}}{\epsilon_{\vec{p}+\vec{k}}-\epsilon_{\vec{p}}}>1\,, \tag{2.9}$$

the secular equation has a pair of imaginary solutions, which indicates that the Fermi gas state is unstable for a coherent excitation of particle-hole pairs.

Figure 1 exhibits the region of nucleon density at which there exist imaginary solutions for given k, with the Fermi momentum p_F representing the nucleon density. At higher nucleon densities with

$$p_F>\frac{\pi^2m^2}{2g^2M}\,, \tag{2.10}$$

the Fermi gas state is unstable[9] and particle-hole pairs with momenta k which fall in the hatched area in the figure will be coherently excited.

The instability of the state is closely related to the collapsing property of the model system. In order to discuss bulk properties of stable nuclear matter, it is essential for the nuclear system to be saturated.

III. EFFECTIVE NUCLEON MASS AND VECTOR-MESON EXCHANGES

In order to make the nuclear system saturate, we start with the relativistic Lagrangian density in Eq. (1.1). The fields satisfy the Euler-Lagrange equations for the Lagrangian density:

$$(\gamma_\mu\partial_\mu+M-g\phi-ig_v\gamma_\mu\phi_\mu)\psi=0\,, \tag{3.1a}$$

$$(\Box-m^2)\phi=-g\bar{\psi}\psi\,,$$
$$\partial_\mu F_{\mu\nu}+m_v{}^2\phi_\mu=ig_v\bar{\psi}\gamma_\mu\psi\,. \tag{3.1b}$$

The meson fields ϕ and ϕ_μ are separated into a classical field and an additional field due to quantum fluctuations:

$$\phi=\varphi+\eta,\quad \phi_\mu=\varphi_\mu+\eta_\mu. \tag{3.2}$$

First, we treat the system in the classical field approximation, assuming the system to be translationally invariant. The classical meson fields φ and φ_μ are constants independent of spatial position and time and are obtained from the field equations (3.1b) as

$$\varphi=\frac{g}{m^2}\langle G|\bar{\psi}\psi|G\rangle\,, \tag{3.3a}$$

$$\varphi_\mu=\frac{ig_v}{m_v{}^2}\langle G|\bar{\psi}\gamma_\mu\psi|G\rangle\,, \tag{3.3b}$$

where the space components $\varphi_i\,(i=1,2,3)$ of the vector-meson field vanish in a translationally invariant system since

$$\langle G|\bar{\psi}\vec{\gamma}\psi|G\rangle=0\,. \tag{3.4}$$

The time component is given by

$$\varphi_0=\frac{g_v}{m_v{}^2}\langle G|\psi^\dagger\psi|G\rangle\,. \tag{3.5}$$

With φ and φ_0, the classical equation for the nucleon field

$$[-i\vec{\alpha}\cdot\vec{\nabla}+\beta(M-g\varphi)]\chi_{\vec{p}\lambda}^{(\pm)}=(\epsilon_{\vec{p}}-g_v\varphi_0)\chi_{\vec{p}\lambda}^{(\pm)} \tag{3.6}$$

has plane wave solutions with energy

$$\epsilon_{\vec{p}}=\pm[\vec{p}^2+(M-g\varphi)^2]^{1/2}+g_v\varphi_0 \tag{3.7}$$

as eigenstates. The double sign $\pm$ represents "positive" and "negative" energy solutions. The nucleon field ψ is expanded in terms of $\chi_{\vec{p}\lambda}^{(\pm)}$ as

$$\psi=\sum_{\vec{p}\lambda}c_{\vec{p}\lambda}\chi_{\vec{p}\lambda}^{(+)}+\sum_{\vec{p}\lambda}d_{\vec{p}\lambda}^\dagger\chi_{-\vec{p}-\lambda}^{(-)}\,, \tag{3.8}$$

where $c_{\vec{p}\lambda}$ and $d_{\vec{p}\lambda}$ are operators. The ground state $|G\rangle$ is composed of N nucleons occupying the lower lying states with "positive" energies:

$$|G\rangle=\prod_{\lambda p\leq p_F}c_{\vec{p}\lambda}^\dagger|0\rangle\,.$$

Hence Eq. (3.5) leads to

$$\varphi_0=\frac{g_v}{m_v{}^2}\rho \tag{3.9}$$

with nucleon density $\rho=N/\Omega$. The scalar-meson field φ is determined by the equation[5]

$$\varphi=\frac{4g}{m^2\Omega}\sum_{p\leq p_F}\frac{M-g\varphi}{[\vec{p}^2+(M-g\varphi)^2]^{1/2}}\,, \tag{3.10}$$

which is derived from Eq. (3.3a).

Now the energy of the whole system is given by

$$E=\sum_{\vec{p}\lambda}^{p_F}(M^{*2}+\vec{p}^2)^{1/2}+\tfrac{1}{2}(m^2\varphi^2+m_v{}^2\varphi_0{}^2)\Omega \tag{3.11}$$

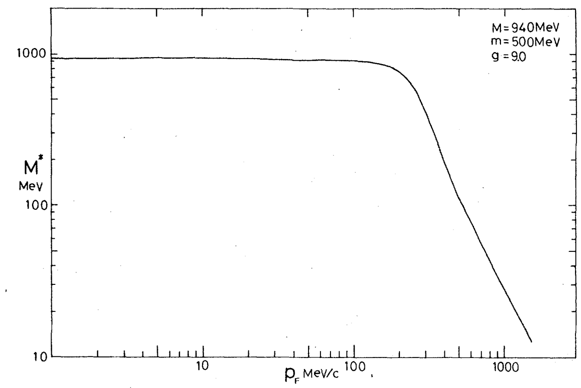

FIG. 2. Effective nucleon mass M^* as a function of Fermi momentum p_F. The coupling constant is $g = 9.0$.

with an effective nucleon mass

$$M^* = M - g\varphi . \tag{3.12}$$

In the classical field approximation, Walecka shows that the Fermi gas state for the nuclear system exhibits the saturation property for the coupling constants[5]

$$g = 8.6, \quad g_v = 11.4 . \tag{3.13}$$

The effective mass M^* is shown as a function of Fermi momentum p_F in Fig. 2. At the normal nucleon density $p_F = 270$ MeV, we obtain

$$M^*/M = 0.6 . \tag{3.14}$$

A nonrelativistic expression for the energy[10] is

$$E = \sum_{\vec{p}\lambda}^{p_F} \frac{p^2}{2M^*} - g\varphi N + \tfrac{1}{2}(m^2\varphi^2 + m_v^2\varphi_0^2)\Omega , \tag{3.15}$$

where, compared with Eq. (2.12), the nucleon mass has been modified into M^* and the effect of the vector-meson field φ_0 has been added.

Next, we take account of the interaction of nucleons with mesons with momenta $k \neq 0$ which has been neglected in the classical field approximation. The Hamiltonian is then given by[11]

$$\begin{aligned} H = {} & \sum_{\vec{p}\lambda} \frac{p^2}{2M^*} c^\dagger_{\vec{p}\lambda} c_{\vec{p}\lambda} - g\varphi N \\ & + (\tfrac{1}{2} m^2\varphi^2 + \tfrac{1}{2} m_v^2 \varphi_0^2)\Omega + \sum_{\vec{k} \neq 0} \omega_{\vec{k}} a^\dagger_{\vec{k}} a_{\vec{k}} \\ & - \sum_{\vec{p}\lambda\vec{k} \neq 0} \frac{g}{(2\omega_{\vec{k}}\Omega)^{1/2}} c^\dagger_{\vec{p}+\vec{k}\lambda} c_{\vec{p}\lambda} (a_{\vec{k}} + a^\dagger_{-\vec{k}}) \\ & + \sum_{\vec{p}\vec{p}'\lambda\lambda'\vec{k}} \frac{g_v^2}{2\omega^{(v)2}_{\vec{k}}\Omega} c^\dagger_{\vec{p}+\vec{k}\lambda} c^\dagger_{\vec{p}'-\vec{k}\lambda'} c_{\vec{p}'\lambda'} c_{\vec{p}\lambda} \end{aligned} \tag{3.16}$$

with energy for vector meson

$$\omega^{(v)}_{\vec{k}} = (m_v^2 + \vec{k}^2)^{1/2} . \tag{3.17}$$

In the Hamiltonian, the spin-dependent interaction due to the quantum fluctuations of vector-meson fields $\phi_i (i = 1, 2, 3)$ is not taken into account, since it will not greatly affect bulk properties of nuclear matter with spin $S = 0$.

Since corrections due to relativistic effects of nucleons other than the effective mass are small at the normal density, we use the Hamiltonian for nonrelativistic nucleons with the effective mass M^* which is determined by the scalar-meson field as in Eq. (3.12). For the Hamiltonian which makes the nuclear system saturated, we have performed an RPA calculation,

$$[H, O^\dagger_{\vec{k}}] = \omega O^\dagger_{\vec{k}} \tag{3.18}$$

with

$$\begin{aligned} O^\dagger_{\vec{k}} = {} & \alpha a^\dagger_{\vec{k}} + \sum_{\vec{p}\lambda}{}' \beta_{\vec{p}} c^\dagger_{\vec{p}+\vec{k}\lambda} c_{\vec{p}\lambda} + \gamma a_{-\vec{k}} \\ & + \sum_{\vec{p}\lambda}{}' \delta_{\vec{p}} c^\dagger_{\vec{p}\lambda} c_{\vec{p}-\vec{k}\lambda} , \end{aligned} \tag{3.19}$$

where the summations $\sum'_{\vec{p}}$ run over $\vec{p}$ with $p < p_F$ and $|\vec{p} \pm \vec{k}| > p_F$, and found that for the coupling constants given in Eq. (3.13) the Fermi gas state is unstable at certain nucleon densities. In Fig. 3, we show the region of nucleon density where the Fermi gas state is unstable for certain $\vec{k}$. It should be noted that the state is unstable at the normal density.

It was discussed by Chin that the quantum fluctuations of the scalar-meson field renormalize the nucleon mass and the coupling constant g,[12] to which the instability of the Fermi gas state is supposed to be sensitive. We have found, however, that such effects do not affect our conclusion that the Fermi gas state is unstable in nuclear matter.

The exchange term[12] for the nuclear interaction influences the energy of nuclear matter,[13] as will

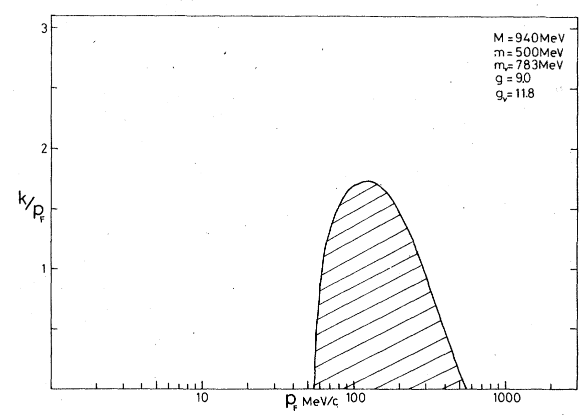

FIG. 3. Hatched area indicates the region where there exist imaginary solutions for given k/p_F in RPA calculation for the Hamiltonian in Eq. (3.16).

be discussed later. In the calculation above, the exchange term for the vector-meson exchange interaction, however, has not been taken into account, since that for the scalar-meson exchange is not effective in the RPA calculation.

Figure 3 indicates that the stability of the Fermi gas state is recovered at relatively high densities owing to the saturation property due to the decreasing effective mass M^*. The stability is not recovered, by contrast, in the system of nucleons with mass independent of the density, as shown in Fig. 1.

IV. STABLE PHASE OF NUCLEAR MATTER

Let us investigate the possible structure of nuclear matter in the case where the Fermi gas state becomes unstable. For this purpose, we consider a system of nucleons interacting through the static meson exchange potentials

$$v(r) = -\frac{g^2}{4\pi r} e^{-mr} + \frac{g_v^{\,2}}{4\pi r} e^{-m_v r}, \qquad (4.1)$$

where nucleons have the effective mass M^* due to exchange effects of scalar mesons with momentum $\vec{k}=0$. The Hamiltonian for the system is given by

$$H = \sum_{\vec{p}\lambda} \frac{p^2}{2M^*} c^\dagger_{\vec{p}\lambda} c_{\vec{p}\lambda} + \sum_{\vec{p}\vec{p}'\vec{k}\lambda\lambda'} \left(-\frac{g^2}{2\omega_{\vec{k}}^{2}\Omega} + \frac{g_v^{\,2}}{2\omega_{\vec{k}}^{(v)2}\Omega} \right) c^\dagger_{\vec{p}+\vec{k}\lambda} c^\dagger_{\vec{p}'-\vec{k}\lambda'} c_{\vec{p}'\lambda'} c_{\vec{p}\lambda} . \qquad (4.2)$$

For this Hamiltonian, the exchange term for the nuclear interaction can be taken into account. In the RPA calculation one may average the exchange term.[14]

Since properties of nuclear matter are phenomenologically deduced only in an extrapolation from finite nuclei, little information has been available as to the possible structure of nuclear matter. The average energy per nucleon[1] $E/N = -15.7$ MeV has been obtained from the term linear in nucleon number in the nuclear mass formula. The extrapolation may suggest that the average energy is attributable to nucleons in an independent particle field, which are described by a Fermi gas. Hence introducing the coupling constants g and g_v so that the saturation property for nuclear matter can be reproduced by the Fermi gas state, we set out to search for a stable phase of nuclear matter. The exchange term for the nuclear interaction modifies the energy for the whole system. Setting the Fermi gas state to be saturated with energy per nucleon $E/N = -15.7$ MeV at $p_F = 270$ MeV, we obtain coupling constants

$$g = 9.0, \quad g_v = 11.8, \qquad (4.3)$$

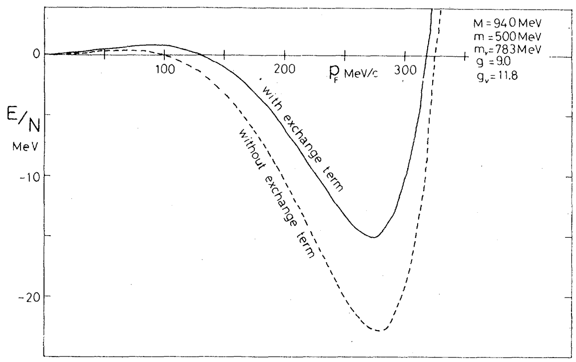

FIG. 4. Energy per nucleon E/N in the Fermi gas state obtained for the revised set of coupling constants in Eq. (4.3). The exchange term for the nuclear interaction is taken into account.

deviating from that of Walecka's, owing to the presence of the exchange term in the present calculation. We show in Fig. 4 that the coupling constants give rise to the saturation property of the nuclear system. We shall hereafter set the coupling constants at the values in Eq. (4.3).

In the HF approach, wave equations for single nucleons are given by

$$\left[-\frac{1}{2M^*}\nabla^2 + u(\vec{r}) \right]\psi_i(\vec{r}) - \int \tilde{u}(\vec{r},\vec{r}\,')\psi_i(\vec{r}\,')d^3r' = \epsilon_i \psi_i(\vec{r}), \qquad (4.4)$$

where the HF potentials are

$$u(\vec{r}_i) = \int \sum_j v(r_{ij}) |\psi_j(\vec{r}_j)|^2 d^3 r_j , \qquad \tilde{u}(\vec{r}_i,\vec{r}_k) = \sum_j \psi_j^*(\vec{r}_k) v(r_{ik}) \psi_j(\vec{r}_i) . \qquad (4.5)$$

For nuclear matter, the set of plane waves

$$\psi_i(\vec{r}) = \frac{1}{\sqrt{\Omega}} e^{i\vec{p}_i\cdot\vec{r}} \chi_{s_z I_z}, \qquad (4.6)$$

with $p_i \leq p_F$, are solutions to the HF wave equations. We note that for these solutions the expectation value of the Hamiltonian is stationary, i.e., it is an extremum, while it is not necessarily the minimum.

In the case where the nuclear system has other states lying at lower energies than the Fermi gas state, HF wave functions for single nucleons in one of the nuclear states will have to be modified from plane waves; they may be expanded as

$$\psi_i = \alpha_i \xi_{\vec{p}_i} + \sum_{j\neq i} \beta_{ij} \xi_{\vec{p}_j} \qquad (4.7)$$

in terms of plane waves

$$\xi_{\vec{p}} = \frac{1}{\sqrt{\Omega}} e^{i\vec{p}\cdot\vec{r}} \chi_{s_z I_z}. \qquad (4.8)$$

Let us first consider, for simplicity, wave functions for single nucleons composed of two plane waves

$$\psi_i = \alpha_i \xi_{\vec{p}_i} + \beta_i \xi_{\bar{\vec{p}}_i}, \quad |\alpha_i|^2 + |\beta_i|^2 = 1, \tag{4.9}$$

where $\bar{\vec{p}}_i$ are to be determined by $\vec{p}_i$ on a one-to-one correspondence basis. The interaction energy between two nucleons expressed by

$$\psi = \alpha \xi_{\vec{p}} + \beta \xi_{\bar{\vec{p}}}, \quad \psi' = \alpha' \xi_{\vec{p}'} + \beta' \xi_{\bar{\vec{p}}'}, \tag{4.10}$$

is

$$\begin{aligned} \langle \psi(\vec{r}_i)\psi'(\vec{r}_j) - \psi'(\vec{r}_i)\psi(\vec{r}_j) | v(r_{ij}) | \psi(\vec{r}_i)\psi'(\vec{r}_j) \rangle = {} & v_0 + v_{\vec{p}-\bar{\vec{p}}}(\delta_{\vec{p}-\bar{\vec{p}},\vec{p}'-\bar{\vec{p}}'} + \delta_{\vec{p}-\bar{\vec{p}},\bar{\vec{p}}'-\vec{p}'})2\alpha\beta\alpha'\beta' \\ & - [v_{\vec{p}-\vec{p}'}\alpha^2\alpha'^2 + v_{\vec{p}-\bar{\vec{p}}'}\alpha^2\beta'^2 + v_{\bar{\vec{p}}-\vec{p}'}\beta^2\alpha'^2 + v_{\bar{\vec{p}}-\bar{\vec{p}}'}\beta^2\beta'^2 \\ & + (v_{\vec{p}-\vec{p}'}\delta_{\vec{p}-\vec{p}',\bar{\vec{p}}-\bar{\vec{p}}'} + v_{\vec{p}-\bar{\vec{p}}'}\delta_{\vec{p}-\bar{\vec{p}}',\bar{\vec{p}}-\vec{p}'})2\alpha\beta\rho'\beta'] \, \delta_{s_z s_z'}\delta_{I_z I_z'}, \end{aligned} \tag{4.11}$$

where

$$v_{\vec{q}} = \frac{1}{\Omega}\int v(r) e^{i\vec{q}\cdot\vec{r}} d^3r. \tag{4.12}$$

Terms with a positive (negative) sign in Eq. (4.11) are due to the direct (exchange) term for the nuclear interaction.

Equation (4.11) indicates that a gain in the interaction energy will result when single nucleon wave functions for a pair of nucleons are composed of two plane waves, i.e., $\alpha\beta \neq 0$, and that the gain will be achieved only for a pair of nucleons whose wave functions satisfy $\bar{\vec{p}} - \vec{p} = \pm(\bar{\vec{p}}' - \vec{p}')$. Since

$$\tfrac{1}{2} \geq \alpha\beta \geq -\tfrac{1}{2}, \tag{4.13}$$

the interaction is most effective for a pair of nucleons with ($\alpha = \beta$, $\alpha' = \beta'$) or ($\alpha = -\beta$, $\alpha' = -\beta'$) and is most defective for a pair with ($\alpha = \beta$, $\alpha' = -\beta'$) or ($\alpha = -\beta$, $\alpha' = \beta'$).

Accordingly, let us relate momenta $\bar{\vec{p}} = \vec{p} + \vec{k}$ to $\vec{p}$ with a given $\vec{k}$ and assign (+) and (−) states to

$$\begin{aligned} \psi_{\vec{p}}^{(+)} &= \frac{1}{\sqrt{2}}(\xi_{\vec{p}} + \xi_{\bar{\vec{p}}}), \\ \psi_{\vec{p}}^{(-)} &= \frac{1}{\sqrt{2}}(-\xi_{\vec{p}} + \xi_{\bar{\vec{p}}}), \end{aligned} \tag{4.14}$$

respectively. Then we see from the foregoing discussion that a pair of nucleons occupying states with like signs interact constructively with each other, while a pair of nucleons of opposite signs interact destructively. Thus, as far as the interaction energy is concerned, all nucleons are likely to occupy states with an equal sign. Wave functions for the (+) and (−) states in Eq. (4.14) are expressed explicitly as

$$\begin{aligned} \psi_{\vec{p}}^{(+)} &= \left(\frac{2}{\Omega}\right)^{1/2} e^{i(\vec{p}+\vec{k}/2)\cdot\vec{r}} \cos\frac{\vec{k}\cdot\vec{r}}{2}\, \chi_{s_z I_z}, \\ \psi_{\vec{p}}^{(-)} &= \left(\frac{2}{\Omega}\right)^{1/2} i e^{i(\vec{p}+\vec{k}/2)\cdot\vec{r}} \sin\frac{\vec{k}\cdot\vec{r}}{2}\, \chi_{s_z I_z}, \end{aligned} \tag{4.15}$$

which indicate that probability density for any (+) state undulates with maxima at $\vec{k}\cdot\vec{r} = 2n\pi$ while that for any (−) state with maxima at $\vec{k}\cdot\vec{r} = (2n+1)\pi$, and wave functions for any pair of states of opposite signs scarcely overlap each other.

The gain in the interaction energy due to the composition of (±) states will be balanced with an increase of the kinetic energy of nucleons, which suppresses some of the nucleons from composing states with an equal sign. It requires twice as large a momentum space as that for the Fermi gas state that all nucleons occupy states with an equal sign.

Hence, for nuclear matter in the neighborhood of the critical density to undergo a phase transition, nucleons around the Fermi surface only compose (±) states with an equal sign. As a result, the Fermi surface is softened. At higher nuclear densities, the phase transition will be achieved, while one of the two nucleons occupying states $\vec{p}$ and $\vec{p}+\vec{k}$ inside the Fermi sphere in the Fermi gas state forms a state $\psi_{\vec{p}}^{(\pm)}$ and the other leaves the sphere to form a new (±) state, resulting in deformation of the Fermi surface.

The exchange term in the nuclear interaction suppresses nucleons from forming (±) states. From the dependence of $v_{\vec{q}}$ on $\vec{q}$, however, it is seen that for small $\vec{k}$ the direct term is dominant and that a pair of nucleons with larger momentum difference $\vec{p} - \vec{p}'$ will be favorable to the formation of (±) states.

The discussion above can be generalized to many-wave approaches. In a $2n$-wave approach, single nucleon wave functions are given by

$$\psi_{\vec{p}} = \sum_{i=-n+1}^{n} \alpha_i \xi_{\vec{p}+i\vec{k}}, \quad \sum_i |\alpha_i|^2 = 1. \tag{4.16}$$

The interaction energy between a pair of nucleons described by

$$\begin{aligned} \psi &= \sum_i \alpha_i \xi_{\vec{p}+i\vec{k}}, \\ \psi' &= \sum_i \alpha'_i \xi_{\vec{p}'+i\vec{k}} \end{aligned} \tag{4.17}$$

is

$$\langle \psi\psi' | v | \psi\psi' \rangle = v_0 + \sum_l v_{lk} \left(\sum_{i=-n+1}^{n-l} \alpha_i^* \alpha_{i+l} \right) \times \left(\sum_{j=-n+1}^{n-l} \alpha'^*_j \alpha'_{j-l} \right), \tag{4.18}$$

where the interaction energy for the direct term only has been shown for simplicity. Since the inequality

$$\left| \sum_i \alpha_i^* \alpha_{i+l} \right| < 1 \tag{4.19}$$

is satisfied, the composition of many plane waves for single nucleon wave functions gives rise to a limited gain in the interaction energy, and for a large n a pair of nucleons occupying states with $\alpha_i \cong (\pm 1)^i 1/\sqrt{2n}$ will have the maximum gain. As a generalization, let us assign to ($\pm$) states, single nucleon wave functions

$$\psi_{\vec{p}}^{(\pm)} = \sum_{i=-n+1}^{n} \frac{(\pm 1)^i}{\sqrt{2n}} \xi_{\vec{p}+i\vec{k}} = \begin{cases} \dfrac{\sin n\vec{k}\cdot\vec{r}}{\sqrt{2n}\,\sin(\vec{k}\cdot\vec{r}/2)} e^{i(\vec{p}+\vec{k}/2)\cdot\vec{r}} \chi_{s_z I_z} \Big/ \sqrt{\Omega}, & \text{for } (+) \\[2ex] \dfrac{i\sin n\vec{k}\cdot\vec{r}}{\sqrt{2n}\,\cos(\vec{k}\cdot\vec{r}/2)} e^{i(\vec{p}+\vec{k}/2)\cdot\vec{r}} \chi_{s_z I_z} \Big/ \sqrt{\Omega}, & \text{for } (-), \end{cases} \tag{4.20}$$

for which probability density is localized and which undulate with a wavelength $2\pi/k$. Any pair of wave functions of opposite signs overlap each other less than those in the two-wave approach.

In the limit of n going to infinity, the probability density for the wave functions is distributed in multifold layers spaced equidistantly as

$$|\psi|^2 \to \frac{k}{N_z S} \sum_{m=0}^{N_z-1} \delta(\vec{k}\cdot\vec{r} - 2\pi m), \tag{4.21}$$

with N_z the number of layers aligned along the $\vec{k}$ direction. In this limit the interaction between nucleons will become most effective. In actual nuclear matter the interaction energy is balanced with the kinetic energy of nucleons, so that the layers would be diffused.

Let us introduce the following three models of the nucleon system in an attempt to search for a stable phase of nuclear matter.

(a) Two-wave approach on plane wave bases in the Fermi sphere (TWAS). This approach provides a model of the nucleon system around the critical density for the system to turn from the Fermi gas state into a new phase. For single nucleon wave functions $\psi_{\vec{p}}$, we take (+) states

$$\psi_{\vec{p}}^{(+)} = \frac{1}{\sqrt{2}} (\xi_{\vec{p}} + \xi_{\tilde{\vec{p}}}), \quad \text{for } p < p_F,\ \tilde{p} > p_F \tag{4.22}$$

with $\tilde{\vec{p}} = \vec{p} \pm \vec{k}$ for $\vec{p}\cdot\vec{k} \gtrless 0$ and take plane waves $\xi_{\vec{p}}$ for $p < p_F$ and $\tilde{p} < p_F$. The signs in front of $\vec{k}$ in $\tilde{\vec{p}} = \vec{p} \pm \vec{k}$ have been determined to make the p space for $\psi_{\vec{p}}^{(+)}$ larger, which is essential for making the energy for the nuclear system lower when the Fermi surface is little deformed. In the ground state of the system, the single nucleon states with p in the Fermi sphere are occupied: The state is described by

$$|G\rangle = \prod_{\substack{\vec{p}\lambda \\ (\tilde{p}<p_F)}}^{p_F} c_{\vec{p}\lambda}^\dagger \prod_{\substack{\vec{p}\lambda \\ (\tilde{p}>p_F)}}^{p_F} c_{\vec{p}\lambda}^{(+)\dagger} |0\rangle \tag{4.23}$$

with creation operators $c_{\vec{p}\lambda}^{(+)\dagger}$ for nucleons forming (+) states. The nucleon effective mass M^* is determined by Eqs. (3.12) and (3.10).

(b) Two-wave approach on plane wave bases in the Fermi cylinder (TWAC). This approach provides a model of the nucleon system above the critical density for the phase transition. For the system the Fermi surface will be deformed. Single particle wave functions are taken to be (+) states

$$\psi_{\vec{p}}^{(+)} = \frac{1}{\sqrt{2}} (\xi_{\vec{p}} + \xi_{\tilde{\vec{p}}}) \tag{4.24}$$

with $\tilde{\vec{p}} = \vec{p} \pm \vec{k}$ for $\vec{p}\cdot\vec{k} \lesseqgtr 0$. The signs in front of $\vec{k}$ in $\tilde{\vec{p}}$ have been determined to make the kinetic energy of nucleons smaller, which is essential for making the energy for the nuclear system lower when the Fermi surface is deformed. The momenta $\vec{p}$ for occupied states $\psi_{\vec{p}}^{(+)}$ are confined in a cylinder with length k, which is aligned along the $\vec{k}$ direction. With the volume of the cylinder determined by the nucleon density, its radius p_0 satisfies

$$\pi p_0^2 k = 4\pi p_F^3/3\,. \tag{4.25}$$

The ground state of the system is given by

$$|G\rangle = \prod_{\substack{\vec{p}\lambda \\ (p_z<k,\, p_\perp<p_0)}} c_{\vec{p}\lambda}^{(+)\dagger} |0\rangle, \tag{4.26}$$

and the nucleon effective mass by $M^* = M - g\varphi$ with

$$\varphi = \frac{4g}{m^2\Omega} \sum_{\substack{\vec{p} \\ (p_z<k,\, p_\perp<p_0)}} \frac{M - g\varphi}{[\vec{p}^2 + (M-g\varphi)^2]^{1/2}}\,. \tag{4.27}$$

(c) Equidistant multilayer structure (EMULS). This is a model of the nucleon system in a new phase in which nucleons enjoy the nuclear interaction most effectively. Instead of composing many plane waves for single nucleon wave functions, we take wave functions in the form

$$\psi_{n,\vec{p}} = \left(\frac{1}{a\sqrt{\pi}} \right)^{1/2} e^{-(z-nd)^2/2a^2} e^{i\vec{p}\cdot\vec{\rho}} \Big/ \sqrt{S}\,, \tag{4.28}$$

which have a large probability density in multilayers aligned equidistantly along the z direction, with an interval $d=2\pi/k$ and a diffuseness parameter a. In Eq. (4.28), we use the cylindrical coordinates with cross section S of the nuclear system and nucleon coordinates $\vec{\rho}$ perpendicular to the z direction. The two-dimensional momenta $\vec{p}$ for the occupied nucleon states are confined in the Fermi cylinder given in Eq. (4.25), and the ground state of the nuclear system is described by

$$|G\rangle=\prod_{\substack{n\vec{p}\lambda \\ (p\leq p_0)}} c^{\dagger}_{n\vec{p}\lambda}|0\rangle\,, \tag{4.29}$$

with creation operators $c^{\dagger}_{n\vec{p}\lambda}$ for nucleons with quantum numbers n, $\vec{p}$, and λ. For this state, we take the effective nucleon mass $M^*=M-g\varphi$ with the scalar-meson field φ averaged over the nucleon density:

$$\begin{aligned}\varphi &= \int \varphi(\vec{r})\langle G|\bar{\psi}(\vec{r})\psi(\vec{r})|G\rangle\, d^3r/N \\ &= g\int \langle G|\bar{\psi}(\vec{r})\psi(\vec{r})|G\rangle \frac{e^{-m|\vec{r}-\vec{r}\,'|}}{4\pi|\vec{r}-\vec{r}\,'|}\langle G|\bar{\psi}(\vec{r}\,')\psi(\vec{r}\,')|G\rangle\, d^3r\, d^3r'/N \\ &\doteq g\rho\sum_{l}\frac{1}{m^2+l^2k^2}e^{-l^2a^2k^2/2}\,.\end{aligned} \tag{4.30}$$

In each of the three model states described above, we have evaluated the expectation value $E=\langle G|H|G\rangle$ of the Hamiltonian (4.2) with the effective nucleon mass M^* adequate for the state, in order to determine a nuclear state which has the lowest energy. In the evaluation, we have taken into account the exchange term for the nuclear interaction. Most probable values of k and a are obtained in the variational method. The energy E of the system and the ratio k/p_F thus obtained are shown to be functions of p_F in Fig. 5 and Table I.

The TWAS state lies below the Fermi gas state at nucleon densities where the latter state is unstable. On account of a gain in the interaction energy, some nucleons in the former state compose two-wave states on the Fermi sphere bases. In contrast, the TWAC state lies above the Fermi gas state, since an increase in the kinetic energy of nucleons in this state is larger than a gain in the interaction energy. In the region from $p_F=50$ to 230 MeV, the EMULS state lies lowest owing to a large gain in the interaction energy. At $p_F>230$ MeV, on the other hand, the EMULS state lies above the Fermi gas state, since the effective nucleon mass M^* becomes smaller in the former state. It is seen in Fig. 5 that the EMULS state is stable with the energy per nucleon $E/N=-24$ MeV at the nucleon density corresponding to $p_F=60$ MeV.

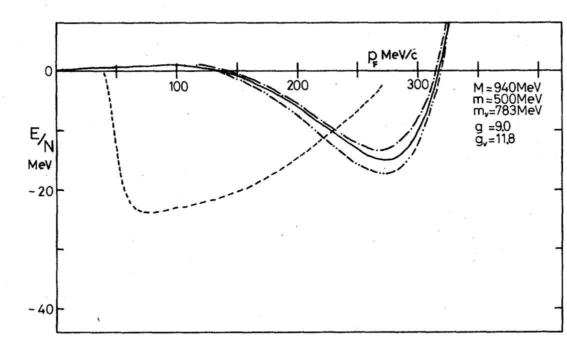

FIG. 5. Energy per nucleon E/N in Fermi gas (FG) ——, in TWAS — — · — —, in TWAC —·—·—, and in EMULS------.

V. DISCUSSION AND CONCLUSION

Single nucleon wave functions in nuclear matter are conventionally taken to be plane waves, i.e., eigenstates of HF wave equations, and they are modified by nuclear correlations. The HF calculation, however, cannot take into account effectively the nuclear tensor force which is supposed to be essential for the nuclear saturation property. The tensor force affects HF single nucleon energies only through the exchange term. In the conventional calculation, one cannot ignore higher order corrections to the energy for the

TABLE I. Energy per nucleon E/N and ratios k/p_F and a/d for EMULS.

	FG	EMULS		
p_F (MeV/c)	E/N (MeV)	E/N (MeV)	k/p_F	a/d
50	+0.526	−13.9	0.040	0.002
75	+0.798	−23.5	0.060	0.005
100	+0.735	−22.7	0.11	0.011
125	+0.222	−21.5	0.17	0.022
150	−1.03	−19.7	0.25	0.038
175	−3.14	−17.4	0.37	0.066
200	−6.18	−14.3	0.52	0.107
225	−9.74	−10.4	0.68	0.16
250	−13.4	−6.4	0.93	0.27

nuclear system due to the tensor force, since at present no HF expression for single nucleons diagonalized with respect to the tensor force is available.

Some higher order effects of the tensor force may be reproduced by an exchange of scalar meson which represents two-pion exchange. Neutral scalar-meson exchange by itself, however, does not correctly reproduce the nuclear saturation property. Hence we have considered an infinite system of nucleons interacting with neutral scalar mesons and neutral vector mesons instead, and found that the system reproduces the saturation property.[5] It is brought about by the scalar-meson field dependent on the nucleon density, which is expressed as an effective mass M^* of nucleon. Our result may imply that the scalar part in higher order terms of the tensor force is essential for the nuclear saturation.

Appearance of layered structure, or solidification of nuclear matter, has been discussed in the literature,[2,4] where, however, much importance has not been attached to the saturation property of the nuclear system and the antisymmetry between nucleons, in spite of the fact that these properties suppress nucleons from being localized. Discussion based on the direct term only for a nuclear attractive interaction has predicted the localization of nucleons to be too favorable. In our discussion we have demonstrated a nuclear system to reproduce the saturation property by introducing an effective nucleon mass that varies with scalar-meson field dependent on nucleon density. In the discussion we have taken account of the exchange term for the nuclear interaction and the nuclear repulsive force due to vector-meson exchange which affect the saturation property to a considerable extent. All three factors tend to make nucleon localization unfavorable.

We have started with the Hamiltonian for which the coupling constants g and g_v are so determined for the Fermi gas state to reproduce the saturation property. The antisymmetry between nucleons has led to a set of the coupling constants deviated from those obtained by Walecka. For the Hamiltonian the RPA calculation indicates the Fermi gas state is unstable and EMULS, a nuclear state in which nucleons are distributed in multilayers spaced equidistantly, is lying lower. We have also examined a three-dimensional solid phase, which has an energy larger than EMULS.

We have searched for a stable state for nuclear matter, considering nucleon excitation modes with one definite $\vec{k}$ and neglecting coupling between different $\vec{k}$'s. This is in accordance with the fact that the nuclear interaction favors coherent excitations of nucleons with one definite $\vec{k}$, as is shown in Eq. (4.11). In contrast to the conventional discussion of pion condensation from the point of view that the Fermi gas state is unstable for a pion condensate mode, we have shown that nucleons in nuclear matter are distributed in layers and that instability for pion condensation should be discussed on this basis.

[1]See, for example, H. A. Bethe, Annu. Rev. Nucl. Sci. 21, 93 (1971). R. B. Wiringa and V. R. Pandharipande, Nucl. Phys. A299, 1 (1978); B. D. Day, Rev. Mod. Phys. 50, 495 (1978).

[2]P. W. Anderson and R. G. Palmer, Nature Phys. Sci. 231, 145 (1971); J. W. Clark and N.-C. Chao, *ibid.* 236, 37 (1972); D. Schiff, *ibid.* 243, 130 (1973); S. Chakravarty, M. D. Miller, and C.-W. Woo, Nucl. Phys. A220, 233 (1974).

[3]A. B. Migdal, Zh. Eksp. Teor. Fiz. 61, 2209 (1972) [Sov. Phys.—JETP 34, 1184 (1972)]; Nucl. Phys. A210, 421 (1973); R. F. Sawyer, Phys. Rev. Lett. 29, 382 (1972); D. J. Scalapino, *ibid.* 29, 386 (1972).

[4]R. G. Palmer, E. Tosatti, and P. W. Anderson, Nature Phys. Sci. 245, 119 (1973); T. Takatsuka and R. Tamagaki, Prog. Theor. Phys. 55, 624 (1976); T. Matsui, K. Sakai, and M. Yasuno, *ibid.* 57, 1453 (1977).

[5]J. D. Walecka, Ann. Phys. (N.Y.) 83, 491 (1974).

[6]S. Chang, Phys. Rev. D 12, 1071 (1975).

[7]L. I. Schiff, Phys. Rev. 84, 1 (1951); R. Serber, Phys. Rev. C 14, 718 (1976); T. Negishi and T. Kohmura, *ibid.* 19, 253 (1979).

[8]A. Suzuki, Y. Futami, and Y. Takahashi, Prog. Theor. Phys. 53, 1204 (1975).

[9]K. Sawada and N. Fukuda, Prog. Theor. Phys. 25, 653 (1961); C. B. Dover and R. H. Lemmer, Phys. Rev. 183, 908 (1969).

[10]For simplicity, we use the nonrelativistic expression with the effective nucleon mass M^*, since corrections due to relativistic effects are small at the normal density as is usually supposed to be.

[11]Quantized vector mesons are not considered here.

[12]S. A. Chin, Ann. Phys. (N.Y.) 108, 301 (1977). The renormalizability of the nucleon self-energy requires the counterterms for the scalar-meson field

$$L_c = \alpha\phi + \frac{\beta}{2!}\phi^2 + \frac{\gamma}{3!}\phi^3 + \frac{\lambda}{4!}\phi^4$$

in the Lagrangian density. Although the renormalization modifies the instability criterion of the Fermi gas state to some extent, it does not affect our conclusion on the instability.

[13]The stability of the Fermi gas state in neutron matter at high densities is discussed by S. A. Chin, Ref. 12.

[14]J. Hubbard, Proc. R. Soc. London A243, 336 (1957).