PHYSICAL REVIEW C VOLUME 22, NUMBER 4 OCTOBER 1980

Band structure of odd-A rubidium isotopes in the interacting boson fermion model

U. Kaup, A. Gelberg, and P. von Brentano
Institut für Kernphysik der Universität zu Köln, D-5000 Köln 41, West Germany

O. Scholten
Kernfysisch Versneller Instituut der Rijksuniversiteit, Groningen, Groningen, The Netherlands
(Received 12 May 1980)

Energy spectra and $E2$ matrix elements have been calculated by means of the interacting boson fermion model for ^{79,81,83}Rb. A more schematic description obtained by assuming that the extra proton which is coupled to an even-even Kr core is in a pure $g_{9/2}$ particle state has been compared with calculations within a larger model space including the $d_{5/2}$ and $g_{7/2}$ particle states. We find that it is possible to achieve excellent agreement for both the schematic and the more detailed calculation as far as only level energies are concerned. However, the choice of the model space turns out to have a strong influence on the model parameters. Moreover, we find that $B(E2)$ values differ considerably in both descriptions. We conclude that even if there is a large separation between shells, the mixing due to the strong core-particle quadrupole force does not allow for restricting the model space to a single j shell.

[NUCLEAR STRUCTURE Interacting boson fermion model, ^{79,81,83}Rb.]

INTRODUCTION

An extension of the interacting boson model (IBM)[1] to states in odd mass nuclei has recently been proposed.[2,3] This model has been called interacting boson fermion model (IBFM). The odd nucleus is treated as a system consisting of one fermion coupled to an even-even boson core. The core is described by the IBM version which does not distinguish between proton and neutron degrees of freedom, called IBM-1 in the following.[4-6] The particle core coupling has been given a very general form; one important feature of the Hamiltonian is the presence of a Pauli term. The model is therefore particularly useful for the analysis of transitional nuclei. A preliminary study of ^{81,83}Rb (Ref. 7) has already shown the great importance of the Pauli term[3] in the particle-core coupling.

The model has recently been successfully applied to odd Pd nuclei[8] which show a certain similarity to axial rotors. The low energy positive parity excitations of the odd Rb nuclei may be considered an excellent and more difficult test case for the IBFM. A large number of levels has been found recently in ^{83}Rb by Gast *et al.*[9] using heavy ion reactions. Additional information about low spin levels has been provided by beta decay experiments.[10] Some levels have also been reported for the neighboring odd mass isotope ^{81}Rb (Ref. 11) showing a rather similar structure. For the isotope ^{79}Rb only a few members of the yrast band have been measured recently.[12]

The structure of the odd Rb positive parity spectra resembles somewhat the rotation aligned coupling (RAC) scheme,[13] as it has been suggested in a previous paper on nuclei in the $g_{9/2}$ shell.[14] However, there are some experimental features that cannot be understood in a simple axially symmetric rotor + particle model even if a variable moment of inertia (VMI) is introduced:

(i) the theoretical energies of the so called "unfavored states" are too high, and

(ii) there are several additional states which cannot be explained by the model.[9]

This is obviously caused by neglecting all the sidebands of the core nucleus. In particular, the so called "quasi-gamma-band" based on the second 2^+ state of the core plays a very important role for the nuclei under discussion. The IBM seems therefore to be more appropriate, since it is able to explain all the collective bands of the even-even core.

We will restrict this investigation to states with positive parity. The odd particle, which is coupled to a Kr core, is then mainly in the $1g_{9/2}$ shell model state. Owing to the strong spin orbit force acting on a high-j particle, this state is fairly well separated from other positive parity levels. Hence we first tried to give a description in which the particle is in a pure $g_{9/2}$ particle state, thus restricting the number of parameters as much as possible. A very preliminary version of these calculations has recently been discussed.[7] We further extended this approximation by taking into account up to three shells.

The structure of the even-even Kr isotopes has been recently discussed[7,15] in the framework of the

IBM-2 model, i.e., the version of the interacting boson model which takes into account proton and neutron bosons separately.[16] It has been found that for ^{82}Kr the collective structure is rather close to the O(6) limit of the model.[6] As has been discussed by Casten and Cizewski[17] and by Meyer-ter-Vehn[18] the O(6) limit of IBM corresponds to the γ-unstable model of Wilets and Jean.[19] There is also a certain similarity with triaxial rotor with $\gamma=30°$. If we go toward the lighter isotopes the structure approaches the SU(3) limit[5] or the axial rotor.

As discussed in Ref. 15, there are some difficulties in the description of the Kr nuclei by means of the IBM due to the influence of shell effects. This regards the phenomenon of backbending and the occurrence of a very low lying $0^+(2)$ excitation which is probably not of collective origin. These features apparently did not affect the calculation of the odd-A spectra.

THE MODEL

The Hamiltonian of the IBFM can be written as[3]

$$H=H(\text{core})+H(\text{particle})+H(\text{coupling})\,, \qquad (1)$$

where H(core) is the Hamiltonian of the IBM-1.[4] The particle Hamiltonian contains the shell model energies of the single particle states:

$$H(\text{particle})=\sum_{jm}\epsilon_j a_{jm}^\dagger a_{jm}\,. \qquad (2)$$

For convenience the energy of the $g_{9/2}$ state has been chosen to be zero. The coupling Hamiltonian takes the form[3]

$$\begin{aligned} H(\text{coupling}) &= \sum_j A_j\sqrt{5}\,(d^\dagger\tilde{d})^{(0)}(\tilde{a}_j^\dagger a_j)^{(0)} \\ &\quad -\sum_{j,j'}\Gamma_{jj'}\sqrt{5}\,\{[(s^\dagger\tilde{d}+d^\dagger s)^{(2)}+\chi(d^\dagger\tilde{d})^{(2)}] \\ &\qquad\qquad \times(a_j^\dagger a_{j'})^{(2)}\}^{(0)} \\ &\quad -\sum_{j,j',j''}\Lambda_{jj'}^{j''}(2j''+1)^{1/2}\!:\;[(\tilde{d}a_j^\dagger)^{(j'')}(d^\dagger\tilde{a}_{j'})^{(j'')}]^{(0)}\!:\,, \end{aligned} \qquad (3)$$

which consists of a monopole-monopole interaction, a quadrupole-quadrupole interaction, and an exchange force taking into account the effect of the Pauli exclusion principle. The coupling constants are the parameters A_j, $\Gamma_{jj'}$, and $\Lambda_{jj'}^{j''}$, respectively. χ is a parameter describing the quadrupole operator of the core. For the case of one single j shell we have only three coupling constants A_j, Γ_{jj}, and Λ_{jj}^{j}, while for many shells their number can be quite large. In order to keep the number of parameters small, one can introduce the following relations between the matrix elements of the coupling Hamiltonian[20]:

$$\Gamma_{jj'}=\Gamma_0[(\alpha_j\alpha_{j'})^{1/2}-(1-\alpha_j)^{1/2}(1-\alpha_{j'})^{1/2}] \times\langle j||Y_2||j'\rangle\,, \qquad (4)$$

$$\Lambda_{jj'}^{j''}=\Lambda_0\beta_{jj''}\beta_{j'j''}\,, \qquad (5)$$

$$\beta_{jj'}=[\alpha_j\alpha_{j'}(1-\alpha_j)(1-\alpha_{j'})]^{1/4}\langle j||Y_2||j'\rangle\,, \qquad (6)$$

where α_j is an occupation number (see below).

We will further discuss the single terms of the coupling Hamiltonian in more detail.

The monopole force renormalizes the boson energy, thus shifting the energies of the core multiplets relatively to each other, without contributing to the splitting of each multiplet. One possible physical interpretation of this term is that the boson energies of the two neighboring even-even nuclei of the odd system under consideration are different. Thus the boson energy of the "true" core may be intermediate between these values.

The splitting of the core multiplets and the mixing of the core coupled states is determined by the competition of the quadrupole and the Pauli force.

If we choose the Pauli term equal to zero, we can investigate the effect of the quadrupole force alone. The model shows, then, some similarity to the intermediate coupling model; the quadrupole term leads to the splitting of the core multiplets. Increasing Γ_0 induces a smooth transition from weak to strong coupling; the decoupled scheme arises as a special case. There is a certain connection between Γ_0 and the deformation parameter β in the geometrical model. As a matter of fact, the quadrupole operator used in IBM has a more general form as the usual one (Bohr-Mottelson).

The relation (4) for the quadrupole force constants of different j shells is apparently in analogy with the treatment given in other nuclear models. The additional factors containing the quantities α_j resemble the BCS attenuation factors; α_j plays the role of an occupation number similar to the V_j^2 in the BCS description of the core.

The parameter χ allows for some reordering of levels in the multiplets. We found that it has a strong influence on the relative position of the fully aligned states and those states with one spin unit less, i.e., the so called "favored" and "unfavored" states. A similar phenomenon happens in the triaxially symmetric model when γ is increased from 0° to 30°. We can say that χ is related to the nuclear shape. In the SU(3) limit which corresponds to a prolate (oblate) shape $\chi=-\frac{-(7)^{1/2}}{2}\,[+\frac{(7)^{1/2}}{2}]$, in the O(6) limit corresponding to the γ-unstable rotor $\chi=0$. Since χ determines the quadrupole operator of the core it can be calculated from the parameters used in the description of the core nucleus. One should, however, keep in mind that the dominant part of the effective nucleon-nucleon interaction which causes the quadrupole force is the proton-neutron interaction. As far as the quadrupole force is concerned, an odd proton should thus be coupled predominantly to the neutrons of the core. The χ in the odd A calculation can therefore not be taken from IBM-1 which averages over protons and neutrons but should be chosen rather close to χ_ν of IBM-2.

The Pauli term describes the effect of the odd particle on the internal structure of the d boson. This may be understood as a kind of blocking, because the particle occupies one orbit which is also contained in the two particle wave function defining the d boson in the even-even system. The occupation of the orbit in the s boson is given by the occupation number α_j. The Pauli term vanishes obviously for $\alpha_j = 0$ or $\alpha_j = 1$, i.e., for a completely empty or a completely full shell. The structural change induced in the odd mass spectrum when varying the α_j is to a certain extent similar to the effect obtained by shifting the Fermi energy, as has been discussed in Ref. 3. However, while the BCS procedure only leads to a renormalization of the quadrupole force, the Pauli term causes an additional strong mixing through large nondiagonal matrix elements. If one expands the Pauli term into multipoles, i.e., rearranging it into pairs of boson operators and fermion operators, respectively, it consists of multipole interactions from zeroth up to fourth order. The dipole, octupole, and hexadecapole obviously cannot be treated as renormalizations of other terms in the coupling Hamiltonian (3).

The importance of the exchange diagrams in the phonon to particle coupling has been stressed by Bohr and Mottelson[21] and by Civitarese, Broglia, and Bes,[22] who showed that it leads to a lowering of the energy of the state $I = j - 1$ as well as that of the band based on this state.

A similar effect has been obtained in a less general way by Alaga and Paar[23] who considered a vibrator coupled to a cluster of three particles with an antisymmetrized wave function.

RESULTS

The ^{83}Rb spectrum has first been calculated in the one shell approximation; the four parameters of the coupling Hamiltonian of Eq. (2) have been fitted to the members of the first core multiplet assuming that a state experimentally found at 1096.5 keV (Ref. 9) is its missing $\frac{9}{2}^+$ number. All other levels seen in the experiments can be explained by the calculation in a quantitative way. The parameters obtained in this way are $A_j = 1.65$ MeV, $\Gamma_{jj} = 1.27$ MeV, $\Lambda^j_{jj} = 1.55$ MeV, and $\chi = 0.3$. The χ is rather different from the χ_ν of IBM-2 for ^{82}Kr ($\chi_\nu = 0.92$).[12] When going to ^{81}Rb we find that we have to vary the monopole and the Pauli force parameters ($A_j = 1.25$ MeV, $\Lambda^j_{jj} = 1.83$ MeV); the change in χ has been taken equal to the change in χ_ν in IBM-2 when going from ^{82}Kr to ^{80}Kr.

Although we have obtained a quite satisfactory description for both Rb isotopes, one could have doubts about the validity of the one shell approximation. In particular, the question arises whether one achieves such a description only at the expense of choosing unrealistic values of parameters. This can be explored by extending the single particle model space to more shells. If the parameters found in the one shell calculation are realistic, the solution should then stay stable. We have carried out this test by including successively the $2d_{5/2}$ and $1g_{7/2}$ shells, the parameters being related by Eqs. (4)–(6). The occupation numbers α_j were 20% for $g_{9/2}$ and 0% otherwise; thus only the quadrupole force gives rise to mixing in the single particle space, while the Pauli term acts only on the $g_{9/2}$ state. In order to get the same parameters for the diagonal $g_{9/2}$ contributions as in the one shell approximation, the "state independent" coupling constants Γ_0 and Λ_0 are then $\Gamma_0 = 2.13$ MeV and $\Lambda_0 = 10.05$ MeV. We find that the solution is not stable if the $d_{5/2}$ state is added to the model space. When doing this the splitting of the first core multiplet is increased by a factor of about 3 on the average, the $\frac{5}{2}^+$ state becoming the ground state. Adding the $1g_{7/2}$ shell has been found to be of minor importance, due to the fact that the quadrupole force favors $\Delta j = 2$ mixing.

In order to find realistic values of the coupling constants a calculation with two shells $g_{9/2}$ and $d_{5/2}$ seems therefore to be appropriate. This has been carried out simultaneously for the isotopes ^{81,83}Rb so as to obtain a consistent set of parameters. As a matter of fact, we can now decrease Γ_0 and Λ_0 by a factor of about 3 compared with the one shell calculation; $\Gamma_0 = 0.71$ MeV, $\Lambda_0 = 3.52$ MeV for both nuclei. The structural change occurring when going from one isotope to the other can be attributed to a change in the occupation numbers alone: $\alpha_{9/2} = 0.18$, $\alpha_{5/2} = 0.0005$ for ^{83}Rb and $\alpha_{9/2} = 0.3$, $\alpha_{5/2} = 0.005$ for ^{81}Rb. Although the effect of $\alpha_{5/2}$ on the level energies is minor, the sensitivity to the extremely small values of this parameter is remarkable. In particular, it provides the lowering of the $\frac{7}{2}^+$ state in ^{81}Rb relative to its position in ^{83}Rb. This feature can be understood if one realizes that in some nondiagonal matrix elements of the Pauli term connecting the $g_{9/2}$ and $d_{5/2}$ states the coefficient $\Lambda_{jj'}^{j''}$ contains the occupation number $\alpha_{5/2}$ only with the power $\frac{1}{4}$ (for instance, the coefficient $\Lambda_{9/2,9/2}^{5/2}$); thus these terms become more comparable with the diagonal $g_{9/2}$ contribution. However, this sensitivity should be the object of further theoretical studies.

The nucleus ^{79}Rb has also been included in these calculations. The parameters Γ_0 and Λ_0 have been taken identical to the values given above for the other isotopes. However, the occupation numbers could not really be fixed, due to the very scarce experimental information.

The values of χ for the three isotopes have now been found equal to the values of χ_ν in the IBM-2 for the Kr cores: $\chi = 0.9$, 0.7, 0.5 for ^{83,81,79}Rb, respectively.

The constant of the monopole force has been fitted essentially to the states of the yrast bands and taken equal for both shells; $A_j = 0.5$, 0.3, 0.3 MeV which is again a factor of 3 smaller than in the one shell calculation.

The $d_{5/2} - g_{9/2}$ energy difference has been chosen equal to about 3.0 MeV for $A = 80$. This is taken from the single particle energies in a Woods-Saxon well as given in Ref. 23. In order to take into account the mass dependence of the potential, the actual values were 3.2, 3.1, 3.0 MeV for ^{83,81,79}Rb; this energy has not been used as a free parameter.

The computer code ODDA[25] written by Scholten has been used for the IBFM calculations.

A comparison of experiment and calculation is shown in Fig. 1 for ^{83}Rb and in Fig. 2 for ^{81}Rb. In

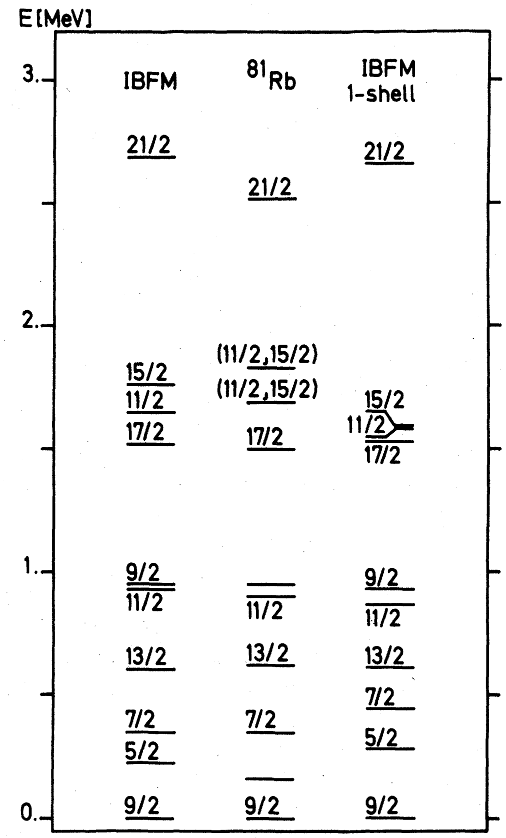

FIG. 2. Comparison of the IBFM calculations with two shells (left) and one shell (right) with the experiment (middle).

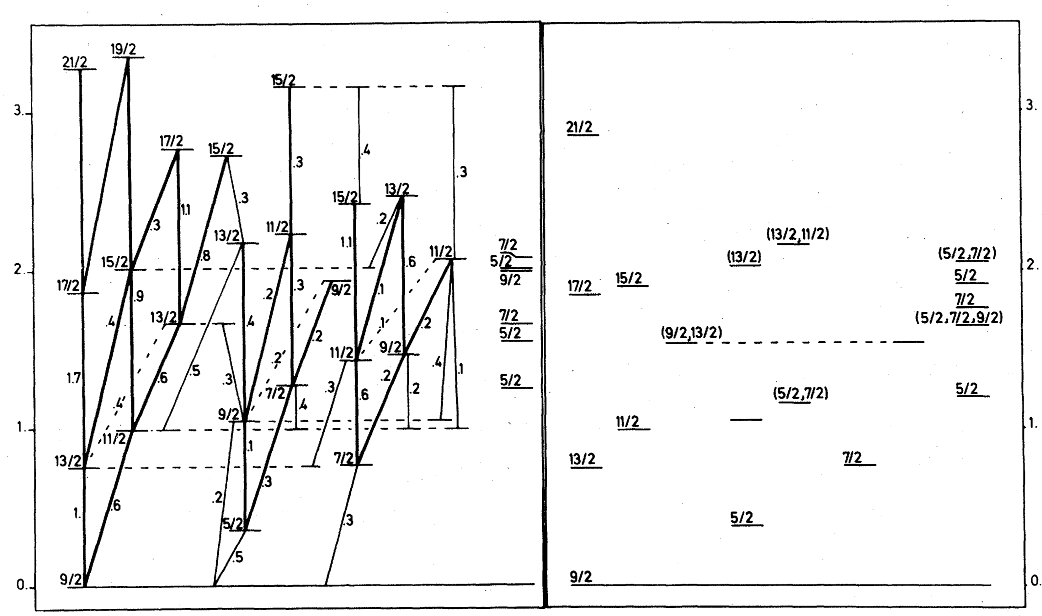

FIG. 1. Comparison of the IBFM calculation with two shells (left) and the experiment (right) for ^{83}Rb. The allowed transitions of the unperturbed multiplet structure of a system $g_{9/2} \otimes$ O(6) are indicated by thick lines. Numbers along lines are $E2$ reduced matrix elements. There are no $E2$ matrix elements known from experiment.

order to make the band structure transparent we give $E2$ matrix elements in the figure for ^{83}Rb. The levels linked by strong matrix elements form families of states which roughly correspond to representations of O(6). It is of some interest to notice that the underlying $g_{9/2}\otimes O(6)$ structure which is indicated in the figure is strongly perturbed. This is due to the admixture of $d_{5/2}$ and to the occurrence of some terms in the Hamiltonian which are not generators of O(6), i.e., $(d^+d)^{(2)}$ and $(d^+d)^{(4)}$. In the unperturbed scheme there would be no strong transitions between different multiplets. For ^{81}Rb we give also a comparison with the one shell calculation in Fig. 2.

The results of the one and two shell calculations are essentially identical, as far as level energies are concerned. There does not exist any meaningful structural difference. Hence the conclusion that the restriction of the model space leads to unphysical results is not correct. On the other hand, by comparing the two different calculations we have learned that the coupling constants of the effective core-particle interaction are sensitive to the choice of the model space. Restricting the model space requires a renormalization of the interaction. Therefore we can say that the coupling constants we have found in the calculation with one shell only are effective coupling constants, while the values obtained by calculating with two shells are closer to the real values of the parameters of the particle boson interaction. It is obvious that the sensitivity of the coupling constants to the model space is due to the structure of the odd-A wave functions. Although the $d_{5/2}$ state is 3 MeV apart from the $g_{9/2}$ state, the quadrupole non-spin-flip matrix elements are large enough to cause some mixing between the single particle states. Thus the results of the one- and two-shell calculations are no longer similar if we compare, for instance, $E2$ matrix elements which are by far more sensitive to the wave functions than are the level energies. In some cases, in particular for weak transitions, $B(E2)$ values differ by factors of 100; even for the strongest transitions we find in some cases factors of ten or more.

In particular, if we compare only transitions in the yrast band, there is a systematic trend, the $B(E2)$ values of the one shell calculation being about 50% larger than those of the two shell calculation. This could be interpreted as an effect of core polarization, due to the very strong quadrupole force between core and particle.

We want to stress therefore that the calculation with many shells may contain considerably more physics than a more schematic one within a restricted model space. If the relations between parameters given in Eqs. (4)–(6) are accepted, a multishell calculation does not require a larger number of parameters, apart from the occupation numbers. Thus the complication introduced by extending the model space is only minor.

A similar phenomenon happened in the description of odd Pd nuclei mentioned above.[8] The extremely large coupling constants given in Ref. 8 can be understood on the basis of our investigation to be due to restricting the model space to the $1h_{11/2}$ shell. By including the $2f_{7/2}$ shell one probably could reduce drastically the values of the parameters.

CONCLUSIONS

In conclusion we can say that the interacting boson fermion model is able to reproduce the quite complex structure of odd-A $g_{9/2}$ nuclei in a way which is consistent with the description of the cores in the same model. We showed that a description with a pure $g_{9/2}$ particle state is possible for these nuclei. Some anomalies in the values of the coupling constants could be understood as renormalization effects due to truncating the model space. Even for the case of large separation between shells, the mixing due to the $\Delta j = 2$, non-spin-flip matrix elements of the quadrupole force turns out to have a strong influence and cannot be neglected if one wishes to achieve an accurate description of the odd-A wave functions. We believe that this feature will be important for further applications of the model.

ACKNOWLEDGMENTS

The authors would like to express their gratitude to F. Iachello and R. F. Casten for numerous stimulating discussions. One of us (U. K.) wishes to express his gratitude to the KVI Groningen for the kind hospitality during his stay there. This work has been supported by BMFT.

[1]A. Arima and F. Iachello, Phys. Rev. Lett. 35, 1069 (1975).
[2]A. Arima and F. Iachello, Phys. Rev. C 14, 761 (1976).
[3]F. Iachello and O. Scholten, Phys. Rev. Lett. 43, 679 (1979).
[4]A. Arima and F. Iachello, Ann. Phys. (N.Y.) 99, 253 (1976).
[5]A. Arima and F. Iachello, Ann. Phys. (N.Y.) 111, 201 (1978).
[6]A. Arima and F. Iachello, Phys. Rev. Lett. 40, 385

(1978).
[7]A. Gelberg and U. Kaup, in *Interacting Bosons in Nuclear Physics*, edited by F. Iachello (Plenum, N. Y., 1979); U. Kaup, A. Gelberg, W. Gast, and P. von Brentano, in *Proceedings of the International Conference on the Structure of Medium-Heavy Nuclei, Rhodos, 1979* (The Institute of Physics, Bristol, 1980).
[8]R. F. Casten and G. J. Smith, Phys. Rev. Lett. 43, 337 (1979).
[9]W. Gast, K. Dey, A. Gelberg, U. Kaup, F. Paar, R. Richter, K. O. Zell, and P. von Brentano, Phys. Rev. C 22, 462 (1980).
[10]R. Broda, A. Z. Hrynkiewicz, and J. Styczen, Nucl. Phys. A216, 493 (1973).
[11]H. G. Friedrichs, A. Gelberg, B. Heits, K. O. Zell, and P. von Brentano, Phys. Rev. C 13, 2247 (1976).
[12]J. S. Clements, L. R. Medsker, L. H. Fry, Jr., L. V. Theisen, and L. A. Parks, Phys. Rev. C 20, 164 (1979).
[13]F. S. Stephens, R. M. Diamond, and S. G. Nilsson, Phys. Lett. 44B, 164 (1973).
[14]P. von Brentano and B. Heits, in *Problems of Vibrational Nuclei*, edited by G. Alaga, V. Paar, and L. Sips (North-Holland, Amsterdam, 1975).
[15]U. Kaup and A. Gelberg, Z. Phys. A 293, 311 (1979).
[16]A. Arima, T. Otsuka, F. Iachello, and I. Talmi, Phys. Lett. 66B, 205 (1977); T. Otsuka, A. Arima, F. Iachello, and I. Talmi, *ibid.* 76B, 139 (1978).
[17]R. F. Casten and J. A. Cizewski, Nucl. Phys. A309, 477 (1978).
[18]J. Meyer-ter-Vehn, in *Interacting Bosons in Nuclear Physics*, edited by F. Iachello (Plenum, N. Y., 1979).
[19]L. Wilets and M. Jean, Phys. Rev. 102, 788 (1956).
[20]O. Scholten, Ph.D. thesis, University of Groningen, 1980 (unpublished).
[21]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, Reading, Mass., 1975), Vol. II.
[22]O. Civitarese, R. A. Broglia, and D. R. Bes, Phys. Lett. 72B, 45 (1977).
[23]G. Alaga and V. Paar, Phys. Lett. 61B, 129 (1976).
[24]A. Bohr and B. R. Mottelson, *Nuclear Structure* (Benjamin, New York, 1969), Vol. I.
[25]O. Scholten, computer code ODDA.