Test of the isobaric multiplet mass equation from β-delayed proton decay of ^{24}Si

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The highly proton-rich nucleus ^{24}Si has been produced via the $^{24}Mg(^3He,3n)$ reaction. The half-life of ^{24}Si was found to be 103(42) ms, and the energy of the protons de-exciting the $T = 2$ state in the daughter, ^{24}Al, has been measured as 3912.7(37) keV. From detailed consideration of masses in the $A = 24$ isobaric quintet (recently completed), it is concluded that this quintet constitutes a test of the isobaric multiplet mass equation as precise as the mass 9 quartet and that there is, in this case, no significant departure from the equation.

[RADIOACTIVITY ^{24}Si; from $^{24}Mg(^3He,3n)$; measured $T_{1/2}$, E_p (β-delayed protons).]

I. INTRODUCTION

The isobaric multiplet mass equation (IMME) is a result of first order perturbation theory, with the assumption that only two-body forces are responsible for charge-dependent effects in nuclei. The equation predicts that the mass excesses ΔM of analog states of an isobaric multiplet can be determined by a three-parameter quadratic equation

$$\Delta M = a + bT_z + cT_z^2 .$$

Deviations from the quadratic form of the IMME could be expected if there were charge dependent many-body nuclear forces, isospin mixing, or shifts in unbound levels. These deviations are usually parametrized as cubic and quartic terms in T_z (dT_z^3, eT_z^4). If an isobaric quintet ($T = 2$ multiplet) is used to test this equation, both additional terms can be determined, whereas only one can be determined in a quartet ($T = \frac{3}{2}$ multiplet).

There are now 22 complete isobaric quartets, and in one case, the ground state $A = 9$ quartet, a significantly nonzero d coefficient, 5.8(16) keV, is found.[1] However, mass 9 is also the most accurately measured multiplet and one cannot conclude that this deviation is exceptional without obtaining results of comparable accuracy in other multiplets. Isobaric quintets offer the prospect of improved tests of the IMME. This paper describes measurements which yield a test of the IMME in the $A = 24$ quintet of the same level of precision as the mass 9 quartet. There are now four completed isospin quintets, the $A = 8$, 20, 24, and 36 multiplets.[1-7] The $A = 8$ quintet shows a slight deviation from the IMME.

In the $A = 24$ quintet, the mass of the $T_z = 2$ nucleus ^{24}Ne was measured by Silbert and Jarmie[8] using the $^{22}Ne(t,p)^{24}Ne$ reaction. The most accurate measurement of the lowest $T = 2$ state in ^{24}Na was made by Start *et al.*[9] via the $^{22}Ne(^3He,p\gamma)$ reaction, and earlier work has been summarized by Endt and van der Leun.[10] The lowest $T = 2$ level in ^{24}Mg was first observed in the $^{26}Mg(p,t)$ reaction by Garvey, Cerny, and Pehl.[24] It has subsequently been studied as an isospin-forbidden resonance in the $^{23}Na(p,\gamma)$ reaction by Riess *et al.*,[11] by Szücs, Underwood, Alexander, and Anyas-Weiss,[12] and by Heggie and Bolotin.[13]

The $T = 2$ level in ^{24}Al, which is the subject of this paper, has recently been discovered at the Lawrence Berkeley Laboratory. In the experiments of Äystö *et al.*,[4] the reaction $^{24}Mg(^3He,3n)$ was used to produce the parent beta activity ^{24}Si, which was transported using a helium jet to an on-line mass separator system. Mass-24 activity was deposited in front of a ΔE-E telescope. Measurement of the energy of protons emitted in the decay of the $T = 2$ state of ^{24}Al (populated by the β-decay of ^{24}Si) gave a result for the mass of the state accurate to 9 keV.

The quintet has been completed with the observation of ^{24}Si in the $^{28}Si(^4He,^8He)$ reaction by Tribble *et al.*[5] The mass of ^{24}Si was determined to an accuracy of 22 keV.

In a previous paper,[14] the description of a cryogenic (liquid-nitrogen cooled) helium jet coupled to a recoil time-of-flight mass analyzer for use in observing short-lived β-delayed particle emitters was presented. This apparatus was used at Princeton University in an attempt to observe ^{24}Si. Results tentatively suggested that protons from the decay of this nucleus had been observed and that the mass of the $T = 2$ state in the daughter nucleus ^{24}Al was consistent with the prediction from the quadratic form of the IMME. With these promising results, an improved apparatus was constructed at Michigan State University with the intent of observing ^{24}Si and other $T_z = -2$ nuclei. Except as noted below, the apparatus is as described in Ref. 14.

It is a feature of this apparatus that mass identification and background reduction can be obtained with only a single Si detector (rather than a counter telescope) to detect the protons. The improved resolution and linearity, as well as the simultaneous recording of strong calibration groups, enable us to report a very precise value for the mass of the $T = 2$ state in ^{24}Al. In addition, we can report the first direct measurement of the half-life of ^{24}Si.

II. EXPERIMENTAL METHOD AND RESULTS

A 1–3 μA beam of 70 MeV ^{3}He particles from the Michigan State University Cyclotron bombarded a 5 mg/cm^2 target of ^{24}Mg, producing ^{24}Si via the ^{24}Mg(^{3}He, $3n$) reaction. The ^{24}Si nuclei recoiling out of the ^{24}Mg target were thermalized in the cold He gas, at a temperature of 77°K and a pressure of 0.7 atm. The atoms were swept into a 1.8 mm-diameter polyethylene capillary tube and transported 2.4 m to the recoil time-of-flight mass analyzer. A skimmer then removed a large fraction of the He gas. The transported atoms passed through the skimmer and were deposited on a 10–12 μg/cm^2 thick Formvar catcher foil. Six catcher foils were mounted on a vertically aligned foil wheel, which was stepped at a rate of 5 steps per second with a high-torque hollow-rotor dc motor. The activity was allowed to collect for 200 ms, minus the measured 20 ms stepping time. The wheel was then moved to its next position, where the newly deposited activity was placed between the particle detectors. After β-decay to particle-unstable states, both the particle emitted and the recoiling ion were observed in coincidence.

Protons (or alphas) passed through the thin catcher foil and were detected in a 150 mm^2, 300 μm deep Si detector. The recoil ions passed through a thin converter foil and secondary electrons were observed by a pair of channel plates placed opposite the Si detector. The 50.8 mm-diameter converter foil served also to isolate the Channel Electron Multiplier Array (CEMA) from the 0.14 Torr pressure in the main detector chamber. This allowed the operation of the channel plates in a vacuum of $10^{-6}-10^{-7}$ Torr. The converter foil was shaped as a section of a sphere with an 80 mm radius of curvature to provide a uniform flight path for the recoil ions. Both the Si detector and converter foil subtended a solid angle of 2.6% of 4π sr. The converter foil consisted of a 30 μg/cm^2 layer of Formvar on a curved wire screen. Onto this Formvar surface a 10 μg/cm^2 layer of gold and a 10 μg/cm^2 layer of CsI were evaporated. The CsI was used because of its supposedly higher secondary emission coefficient.[15,16] A start signal from the proton or alpha in the Si detector and a stop signal from the CEMA gave a value for the time of flight of the recoil ion. This time, combined with the particle energy in the Si detector, was used to derive the recoil mass.

In initial experiments, it was found that the double coincidence data (between the Si detector and the CEMA) showed two different recoil mass groups for the same particle energy. The second (slower) group was delayed by ~9 ns and was attributed to recoil ions directly striking the CEMA but failing to produce secondary electrons in the converter foil. In order to remove these ghost groups, which were producing a background in the region of interest, a gridded electrostatic lens was designed which focused the electrons through an aperture to the central region of the CEMA. A small disk prevented recoil ions traveling in straight lines from hitting the channel plates. A loss of only 7% in solid angle resulted from this arrangement, and a significant reduction in the background was achieved.

As in the previous apparatus, an annular plastic scintillator was used to detect the initial β decay of the parent nuclei. This scintillator was placed opposite the Si detector and subtended a solid angle of ~37% of 4π sr. Recoil ions traveled through a conical hole in the scintillator to the converter foil. The triple coincidence data, though useful in reducing the background and improving the mass resolution for the verification of weak groups, was not directly used in obtaining the proton energy because the beta-induced recoil of the daughter had a component of its velocity toward the Si detector. Thus, the particles detected in the Si detector were shifted up in energy. Therefore, only double coincidence (proton-recoil) data were used to obtain the proton energy.

The β-decay recoil determines the mass resolution in these measurements. For ^{25}Si, which was produced in a competing reaction, this β recoil limited the mass resolution to 7%, a value nevertheless adequate for mass identification.

Eight parameters for each event were written on magnetic tape: proton energy, recoil time of flight, energy and time information from the plastic scintillator, CEMA pulse height, leading-edge to crossover time for pulses from the silicon detector (used in discriminating against pileup), position of the foil wheel, and lastly, a ramp initiated by the end of a foil wheel step and strobed by pulses from the silicon detector (used in the measurement of half-lives).

Figure 1 shows a proton energy spectrum gated on recoil mass 23 u with a window width of 2 u. The energy resolution is approximately 25 keV full width at half maximum. In addition to prominent peaks from ^{25}Si which extend partially into this band, a new peak, not seen in lower-energy bombardments,[17] is present at an energy of 3912.7 keV. Mass spectra gated on the new peak, the 4669-keV line from ^{21}Mg and the 4089-keV line from ^{25}Si, are shown in Fig. 2. The peak at recoil mass 23 u (to the right of the peak from broad lines in the ^{21}Mg spectrum) identifies the 3912.7 keV line as originating from ^{24}Si β decay. This transition has been observed[4] by the Berkeley group at high mass resolution, with a proton energy of 3914(9) keV, in good agreement with the present value. The energy is in the vicinity of the IMME prediction for the decay of the $T=2$ state in ^{24}Al.

The proton energy was obtained from the double-coincidence (Si detector and CEMA) spectra using the strong lines from ^{25}Si decay as calibrants. The energies of these lines are determined mainly by the measurement of the excitation energy of the lowest $T=\frac{3}{2}$ state in ^{24}Al carried out by Rogers *et al.*[18] This result, 7901(2) keV, coupled with the $^{24}Mg(p,\gamma)^{25}Al$ Q value,[19] 2271.3(8) keV, and the excitation energy[10] of the first excited state in ^{24}Mg, 1368.59(4) keV, leads to lab proton energies of 4089.0(22) and 5402.2(22) keV. Peak positions were obtained by fitting bivariate Gaussian distributions to the two-dimensional (recoil mass versus proton energy) spectra, using the method of Maximum Likelihood.[20] The use of such distributions with nonzero correlation coefficients gives much improved results over simple projection of recoil bands onto the energy axis because much of the proton line broadening caused by β recoil can be removed, as it appears as a correlation between proton energy and recoil time of flight. In this way the proton energy resolution

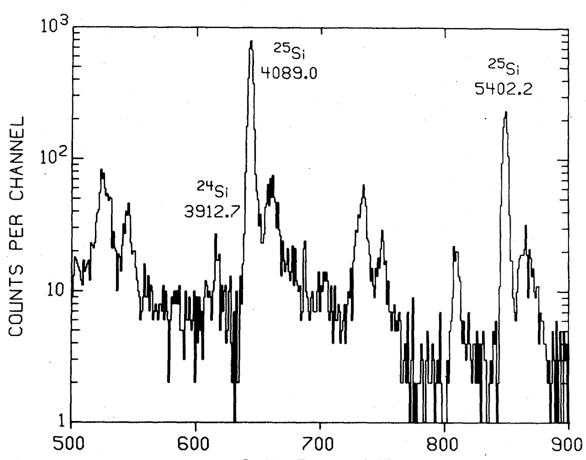

FIG. 1. Proton energy spectrum obtained in coincidence with the CEMA for recoil masses from approximately 22 to 24 u. These data represent a 650 mC bombardment.

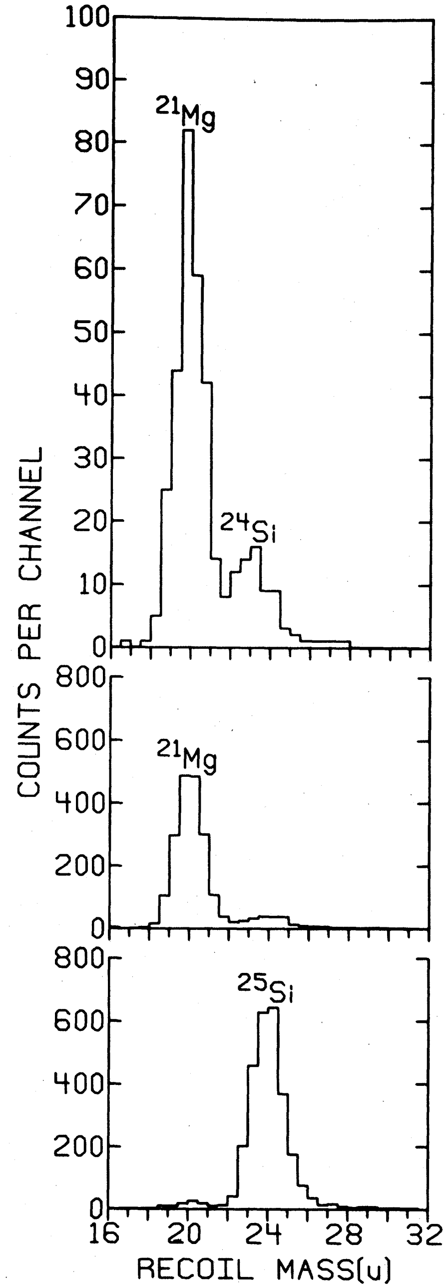

FIG. 2. Recoil mass spectra for proton energies within ± 20 keV of 3911 keV (top), 4669 keV (middle), and 4089 keV (bottom). These energies correspond to proton groups from ^{24}Si, ^{21}Mg, and ^{25}Si decay, respectively.

was improved from 25 to 15 keV. The shapes of these distributions were fixed using the strong ^{25}Si lines as standards. In general, the recoil distributions are a function of β^+ end point energy, but the end points for ^{24}Si and ^{25}Si are so similar (within 200 keV) that the same recoil distribution parameters were used for the two isotopes. There is a background underlying the ^{24}Si peak which was fitted making various assumptions about its dependence on recoil mass and proton energy. The result for the centroid of the ^{24}Si peak was insensitive to these assumptions, and the final form of the background chosen was linearly varying in the recoil dimension and constant in the energy dimension. Integration of the maximum likelihood function provided an estimate for the uncertainty in the centroid position.

A possible systematic effect arises from the β^+-proton angular distribution. Two sources of proton energy centroid shift can be distinguished; first, the presence in the angular distribution of kinematic terms with a dependence on the cosine of the angle between the lepton momentum and proton momentum, and, second, the loss of events in which both the proton and the β^+ pass through the Si detector. Both of these effects were analyzed approximately using the formalism of Holstein[23] and were found to cause a centroid shift of the order of +0.4 keV in the ^{24}Si peak. This small shift is, furthermore, almost exactly canceled by a commensurate shift in the ^{25}Si calibration lines which arise from a quite pure Fermi transition of similar energy. Thus no correction is needed.

A linear energy calibration was used to extract the energy of the ^{24}Si proton group. The latter group lies sufficiently close to the 4089 keV line that only small effects would be expected from neglecting higher-order terms; however, this assumption was tested by extracting the energy of the 4669(4)-keV group[17] from ^{21}Mg decay for several different experimental arrangements, involving different analog-to-digital converter gains, amplifier gains and amplifier manufacturers. Analysis of these data suggested that an excess error (beyond statistical uncertainties) of about 1.5 keV was present. Combination of the calibration uncertainties with the statistical uncertainty in the ^{24}Si peak position (2.5 keV) gives E_p = 3912.7(37) keV, in good agreement with the value 3914(9) obtained by the Berkeley group.[4] The precision of this result permits in principle a very stringent test of the IMME.

The half-life of ^{24}Si has been extracted from the data using the well-known half-lives of ^{20}Na, ^{21}Mg, and ^{25}Si as calibrations. Events in the ^{24}Si proton peak were binned into four equal time groups. The decay of ^{24}Si, shown in Fig. 3, occurs with a half-life of 103(42) ms. This result agrees well with both the predicted value of 115 ms from a shell-model calculation,[21] and the estimate of 100^{+90}_{-40} ms made by Äystö *et al.*[4]

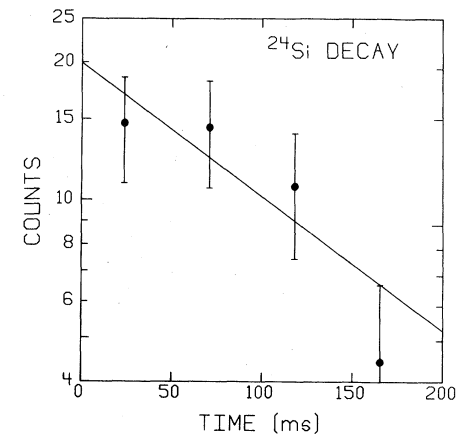

FIG. 3. Decay of ^{24}Si. The half-life derived is 103(42) ms.

III. DISCUSSION

In this multiplet some of the mass excesses are derived from measurements of nuclear reaction Q values, some from excitation energies, and some from proton resonance or decay energies. Considered independently, each of these $T = 2$ state mass excesses includes in its uncertainty the uncertainty in a ground state mass excess. However, mass differences in a local group of nuclides are frequently known more precisely (by direct measurement) than are the absolute mass excesses themselves, and it is these mass differences which are relevant in testing the IMME, because the higher-order terms are all expressible as mass differences. In other words, the uncertainties in the ground state masses are strongly correlated, and a proper evaluation of the uncertainties in the coefficients of the IMME requires considera-

TABLE I. Reaction Q values in local mass adjustment.

Reaction	Q value (keV)	Reference
^{23}Na$(p,\gamma)^{24}$Mg	11 691.2(11)	19
^{23}Na$(n,\gamma)^{24}$Na	6 959.41(12)	19
^{23}Na$(p,n)^{23}$Mg	−4 839.1(26)	19
^{24}Mg$(p,d)^{23}$Mg	−14 307.5(15)	19
^{24}Na$(\beta^-)^{24}$Mg	5 514.8(20)	19
^{24}Mg$(T=2) \rightarrow {}^{23}$Na + p	3 741.2(23)	11, 12
^{24}Mg$(T=2) \rightarrow {}^{24}$Mg + γ	15 436.3(6)	12, 13

TABLE II. Mass excesses obtained using local mass adjustment.

Nuclide	Ground state (keV)	$T=2$ state (keV)
^{23}Na	−9 527.7(9)	
^{23}Mg	−5 470.2(13)	
^{24}Ne	−5 949(10)	−5 949(10)
^{24}Na	−8 415.6(9)	−2 445.7(12)
^{24}Mg	−13 930.6 [a]	1 505.5(6)
^{24}Al		5 902.2(40)
^{24}Si	10 782(22)	10 782(22)

[a] Assigned reference mass. Quoted masses in ^{23}Na, ^{23}Mg, ^{24}Na, ^{24}Mg, and ^{24}Al are based directly on this value.

tion of these correlations.

We therefore return to the original measurements which link the nuclei of interest. Those measurements constitute a practically uncorrelated body of data, although there are small correlations which arise from the use of common calibration lines, e.g., the 6129.17 keV line of ^{16}O. The uncertainties in these calibrations are small enough that their influence can be neglected. We may confine our attention to masses which affect the $T=2$ states in ^{24}Al, ^{24}Mg, and ^{24}Na because the uncertainties in the masses of ^{24}Si and ^{24}Ne, 22 keV and 10 keV respectively, are so large. Furthermore, the masses of the $T=2$ states in ^{24}Al and ^{24}Na are, for all practical purposes, correlated only with the masses of ^{23}Mg and ^{24}Na, respectively, while in ^{24}Mg the gamma decay energy to the ground state and the proton resonance energy in ^{23}Na(p,γ) are known to comparable precisions. In earlier mass evaluations these two independent measures of the mass of the ^{24}Mg $T=2$ state were consistent, but the 1977 Wapstra-Bos tabulation[19] leads to values obtained in the two types of experiment[11-13] which differ by 5.0(24) keV. The origin of this discrepancy is difficult to identify exactly because of the global nature of mass adjustments, but it appears to lie outside of the local group of masses needed for the present purposes, masses interrelated by several precise and consistent experimental measurements.[22] The ground state masses required in the analysis are ^{23}Mg, ^{23}Na, ^{24}Mg, and ^{24}Na. In addition, the mass of the $T=2$ state in ^{24}Mg should be included on the same footing as the ground state masses because it links ^{24}Mg and ^{23}Na.

The data base given in Table I forms the input for a local least-squares mass adjustment. Each mass is linked to at least two others, and 5 masses are related by 7 mass differences. Normalized χ_ν^2 for the fit is 1.1 (2 degrees of freedom). To obtain actual numerical values for masses we arbitrarily assign ^{24}Mg a mass excess of −13930.6 keV—all the derived masses are then based on (and fully correlated with) this mass. The resulting ground state and $T=2$ state mass excesses are listed in Table II. This table is then used to derive the coefficients of the IMME (the covariances are negligible with the exception of ^{23}Na-^{24}Na). It should be borne in mind that the masses in Table II do not include the ^{24}Mg mass uncertainty (0.7 keV), and that it is this distinction which eliminates the otherwise strong correlation between them. Coefficients of the IMME, with their uncertainties, are listed in Table III. The coefficient a is subject to the additional uncertainty contributed by the uncertainty in the mass of ^{24}Mg.

Examination of Table III shows that the quadratic IMME gives a reasonably good fit to the data, with $\chi_\nu^2=1.1$ per degree of freedom. Addition of either a cubic or a quartic term by itself does not improve the quality of fit. The uncertainty in the d coefficient, 1.6 keV in the cubic fit, may be compared directly with other d coefficients in both quintets and quartets. Only in mass 9 has such a small uncertainty in d been obtained. This result increases confidence that departures from the IMME are exceptional. Further experimental work, especially on the inner members of multiplets, will be required in order to subject the IMME to tests at the level of precision now obtained in mass 9 and mass 24.

TABLE III. Coefficients in the IMME (keV).

a	b	c	d	e	χ_ν^2
1505.5(6)	−4171.0(34)	221.1(31)	−2.9(21)	1.7(12)	
1505.4(6)	−4174.3(25)	224.4(18)	−1.0(16)		1.8
1505.5(6)	−4174.9(20)	223.0(27)		0.5(9)	1.9
1505.4(6)	−4175.3(18)	224.2(17)			1.1

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[22]The problem in the middle of the *s*-*d* shell has been noted by Wapstra and Bos (Ref. 19). Wapstra (private communication) has carried out an adjustment in which the direct mass spectroscopic data is omitted and the results show reduced closure errors. Furthermore, Wapstra and Bos have published tables of mass differences which have been calculated taking correct account of the correlations between the masses concerned. However, these tabulations do not list the correlations between mass differences, nor do they include the linkage through the $T=2$ state of ^{24}Mg, i.e., the proton resonance energy and the γ-decay energy, which significantly constrains the $^{23}\mathrm{Na}-{}^{24}\mathrm{Mg}$ mass difference. For these reasons we have adopted a local mass adjustment.

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