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Coupled channel theory of pion-nucleus reactions

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A theory of nuclear reactions induced by pions is presented, which includes scattering and absorption on an equal footing. The theory is consistent with a quantum field theory of mesons, but is developed in an effective Schrödinger equation formulation, to be consistent with conventional multiple scattering methods which are here extended. Methods of application to practical calculations are discussed.

[NUCLEAR REACTIONS Theory of pion scattering and absorption in coupled channel formulation.]

I. INTRODUCTION

The purpose of this paper is to present a theoretical approach to nuclear reactions induced by pions which treats scattering and absorption reactions in a uniform way. On one hand, the theory takes correctly into account the degrees of freedom introduced by the possibility of meson exchange processes in the nuclear system. On the other hand, the method is developed with practical applications in mind, and therefore maintains some of the features of conventional multiple-scattering theory, for which approximation methods are already developed.

The theory presented here was introduced in an earlier paper[1] in the context of the pion-deuteron problem. The formulation was then given in terms of a modified three-body problem, and some details of calculation in terms of the Faddeev equations were given. In the present work we adapt the same general method to the problem of a pion interacting with a complex nuclear target, for which both scattering and absorption reactions are possible. We derive coupled equations for the scattering and absorption channels, and from these equations, the appropriate transition amplitudes. The expressions are similar to those given in Ref. 1 for the πd system. However, for the general nuclear case, we use multiple scattering methods, rather than the Faddeev equations, which are appropriate only for the three body system.

Our coupled channel formulation is based on the use of projection operators to define channels. This method was introduced by Feshbach[2] for treating nuclear reactions with many channels, in which, however, the assumed degrees of freedom are those of the nucleons, and therefore finite in number. We have extended this approach to include an indefinite number of mesons interacting with the nucleons, as would be the case for a meson field theory. This use of projection operators is similar to that adopted by Okubo[3] many years ago, for a discussion of the Tamm-Dancoff method in meson theory. We give more details of the projection method in the present paper.

The interest in a theory of meson reactions with nuclei, that encompasses absorption as well as scattering, is based in features of both the experimental and theoretical situations. It has emerged from a number of recent experimental studies[4] of π-induced reactions at energies below 200 MeV that pion absorption comprises a large fraction of the total cross section, comparable to the integrated elastic or inelastic cross sections. This alone would make it of some importance in understanding the role of absorption in any theory of meson-nucleus scattering.

The theoretical interest arises as follows. Theories that treat scattering of fast projectiles from nuclear targets normally separate, as much as possible, the process of scattering on different elements of the target from the mutual interactions of these elements. In conventional multiple scattering theory, the projectile-nucleus interaction is handled completely independently from the structure of the nuclear target. Once we introduce the possibility of absorption of the projectile—here a pion—the situation changes. Any interaction which can induce nuclear absorption of a meson will have other dynamical effects. It can lead to meson exchange within a nucleus; that is, to nuclear forces. It can also contribute to the nonabsorption scattering as well, by absorption and emission of mesons. Therefore, it is not obvious *a priori* that scattering and nuclear structure may be considered independently, and therefore to what extent one can use consistently any of the tools of multiple scattering theory in the present problem.

A number of different approaches have been

taken. Dover, Hüfner, and Lemmer[5] have developed a diagrammatic expansion method for elastic scattering of pions from nuclear matter which combines some features of multiple scattering theory with some techniques (Green functions) of field theory. The practical development is quite close to that of conventional multiple scattering theory, with additional terms to account for the effects of pion absorption on the optical potential. Some problems of overcounting of diagrams are encountered, and some approximate methods for avoiding these problems are suggested.

Several authors have considered the problem of combining scattering and absorption on an equal footing in a theory applied to the case of $\pi + d$. Here one has a limited number of channels: elastic scattering, $\pi d \to \pi d$; inelastic scattering (breakup) $\pi d \to \pi NN$; and absorption, $\pi d \to NN$. Because of available techniques (e.g., Faddeev equations) which are special to the three-body problem, it is possible to give a more complete treatment in this case than for more complex targets. Thomas[6] has given a method of summing diagrams to get closed expressions for transition amplitudes in this case, and also a calculable model[7] for zero-energy πd reactions. Mizutani[8] has given two derivations of coupled equations for πd reactions, based on field theory. The first is a relativistic method based on reduction techniques. The second (published in Ref. 1), which is equivalent in the nonrelativistic limit, is a coupled channel formulation based on the Feshbach method, from which the present work is also developed. The equations derived in the nonrelativistic case are essentially the same as those obtained by Thomas.[6] Another derivation, based on nonrelativistic reduction techniques, has been given by Rinat,[9] who also made approximate extensions to a theory of the optical potential.[10] Further developments have been given by a number of authors.[11-13]

A rather different method of including absorptive channels in scattering theory is through the Low equation,[14] following the analogy of the Chew-Low theory of πN scattering. This approach has been proposed by Ingraham,[15] and developed further by Cammarata and Banerjee[16] (see also Ref. 17). The resulting nonlinear equations are not amenable to practical solution without rather drastic approximations, so that this method has not been fully employed. Miller[18] has developed a multiple scattering approach which begins with the Low equation, but by successive approximation passes to a linear form of the theory. The treatment of absorption resulting remains rather *ad hoc*.

The isobar-doorway model[19,20] of π-nucleus scattering provides yet another way of introducing meson absorption. In the version of Ref. 20, the effect of pion absorption on scattering reactions shows up in the "spreading width" of a Δ propagating in the nucleus. This is treated phenomenologically in Ref. 20, but has been further developed theoretically by others[21] in terms of diagrammatic approximations. The methods needed are not formally different from the ones we shall consider in this paper. In some cases, the simplifications inherent in the isobar-doorway approach do eliminate some of the over counting problems encountered in a more general theory. This ends our brief review of other recent work on related questions.

The paper is organized as follows. The definitions of projection operators and of channels are introduced and discussed in Sec. II. Coupled equations are derived and solved formally for transition amplitudes in Sec. III. The use of multiple scattering expansions is also discussed here. A more detailed discussion of the important operators which appear in the theory follows in Sec. IV. Methods of application for practical calculation of scattering or absorption are discussed in Sec. V.

II. PROJECTION METHOD

We are interested in nuclear reactions induced by a pion, at energies below the threshold for producing a second meson. The open reaction channels may be classified as elastic scattering, inelastic scattering—including breakup of the target—and meson absorption, in which the target also emits nucleons or nuclear fragments. The degrees of freedom we need to specify all the possible open channels include all the nuclear degrees of freedom of the target and those of the projectile meson. These might be given, for example, in terms of coordinates of all the nucleons. In addition, we need coordinates for the meson, for the elastic and inelastic channels, but not for the absorption channels.

However, the dynamical picture we have of pion-nucleus reactions requires us to introduce more degrees of freedom than those just described for the open reaction channels. Since we will include pion absorption, we must consider interactions which do not conserve the number of pions. Similarly, the theory of the πN interaction (e.g., Chew-Low theory) also includes absorption and emission of pions as a major contribution to scattering. Therefore, a complete dynamical theory must include the full meson degrees of freedom, for example, by a meson field. The theory will then include not only the scattering meson, but also virtual mesons which contribute to the inter-

action among nucleons, as well as to the nucleon self-energies.

The method we shall follow is to start with dynamical equations with the full degrees of freedom of the meson-nucleus system, and then to reduce the number of degrees of freedom by a projection method. We begin by assuming a Schrödinger equation for the pion-nucleus system

$$(E-H)\,|\Psi_{\pi A}^{(+)}\rangle = 0\,, \tag{2.1}$$

where the Hamiltonian H includes the full interaction of the target nucleons with each other and with the pion field, including the absorption and emission of mesons. The state vector $\Psi_{\pi A}^{(+)}$ represents the full scattering state for the whole system. The initial channel, denoted by $|\vec{k},0\rangle$, is given by a free pion plane wave of momentum $\vec{k}$ and energy ω_k, and the nuclear target ground state (of A nucleons) of energy E_0. The total energy is $E=\omega_k+E_0$ in the lab system.

We do not need a detailed description of the full Hamiltonian H, since it will not appear directly in our final equations. We understand that it should contain the "bare" kinetic energies for the nucleons and for the pion field, and the interactions between them, which may be of the Yukawa form

$$H_{\pi N}=\int d^3x\, j(\vec{x})\phi(\vec{x})\,,$$

in terms of a nucleon current and the pion field. There may be other meson fields (e.g., vector mesons ρ, ω) interacting with the nucleons and pions. We shall not, however, need to refer to them explicitly.

To reduce the number of degrees of freedom, we introduce vector spaces which correspond, in the asymptotic region, to the open reaction channels discussed above. We generalize a method introduced by Feshbach[2] for many channel nuclear reactions, to include channels with fixed numbers of mesons. (A similar method was introduced in a different context by Okubo.[3]) We define orthogonal projectors P and Q which operate on the state space defined by Eq. (2.1). They satisfy the relations

$$P+Q=1,\quad P^2=P,\quad Q^2=Q,\quad PQ=QP=0\,. \tag{2.2}$$

The P operator is chosen to project into the vector space of interest, as follows. First we separate this space into orthogonal parts by introducing two new projectors P_1 and P_x such that

$$P=P_1+P_x,\quad P_1{}^2=P_1,\quad P_x{}^2=P_x,\quad P_1P_x=P_xP_1=0\,. \tag{2.3}$$

These are chosen to correspond to the scattering reaction channels which have one pion present asymptotically (labeled 1) and the absorption channels which have no asymptotic pions (labeled x).

First consider P_1. We assign a coordinate to each physical nucleon, $\vec{r}_i$ $(i=1,\ldots,A)$, and one to the pion, $\vec{r}_0$. Consider the state space (S) associated with the full Hamiltonian H of Eq. (2.1), and let Ψ be any state vector in that space. Then P_1 is defined to project onto a subspace of S which is given by all functions of the coordinates $\vec{r}_0,\ldots,\vec{r}_A$. That is, the projection $P_1\Psi$ may be represented by

$$\phi_1(\vec{r}_0,\vec{r}_1,\ldots,\vec{r}_A)=\langle\vec{r}_0,\vec{r}_1,\ldots,\vec{r}_A|P_1\Psi\rangle\,. \tag{2.4}$$

We denote the subspace represented by functions of the type (2.4) as the P_1 space.

We want to associate the corrdinate $\vec{r}_i,\vec{r}_0$ with physical nucleons and a physical pion, respectively. We do so as follows. Consider the functions (2.4) for a very large separation of all coordinates, that is, $|\vec{r}_i-\vec{r}_j|\to\infty$ $(i,j=0,1,\ldots,A)$. In this region of configuration space, the $A+1$ particles are noninteracting and move as free particles. The space corresponds to the physical channel in which A nucleons and one pion are asymptotically free and could be detected, individually, in the laboratory. This corresponds to a breakup channel of the target. Hence the assignment of the coordinates to the physical particles is unambiguous, and completely specifies the asymptotic region of this breakup channel. We now *define* the entire P_1 space by continuing the physical channel space so defined, to all values of the coordinates $\vec{r}_0,\vec{r}_1,\ldots,\vec{r}_A$, still associating these coordinates with A physical nucleons and one pion, as in the asymptotic region. By this continuation, the intrinsic structure of the particles in the P_1 space is kept the same for all $\vec{r}_i$, as it is for asymptotically free particles, even in regions where the particles are close enough to interact. This is the kind of vector space normally used for a Schrödinger equation for a system of $A+1$ particles. In particular, this space includes any asymptotic channels that can be reached by scattering (elastic or inelastic) of a pion from the target nucleus (A).

To define P_x we proceed in a similar way: we assign a coordinate to each nucleon, r_i $(i=1,\ldots,A)$. There is no pion coordinate, since there are no pions asymptotically in this space. The operator P_x projects onto a subspace of S which is given by all functions of these coordinates. An element $P_x\Psi$ of this subspace may be represented by

$$\phi_x(\vec{r}_1,\ldots,\vec{r}_A)=\langle\vec{r}_1,\ldots,\vec{r}_A|P_x\Psi\rangle\,. \tag{2.5}$$

We denote the subspace as the P_x space.

Again, we consider the function (2.5) for very large separations of the coordinates, for which the A nucleons move as free physical particles, with positions denoted by the coordinates $\vec{r}_i$. This corresponds to the asymptotic channel in which the target is completely broken up into nucleons, with no asymptotic pion. The P_x space is defined by continuing this asymptotic space to all values of the coordinates $\vec{r}_1, \ldots, \vec{r}_A$, without changing the intrinsic structure of the nucleons so defined. This space includes all asymptotic channels that can be reached by absorption of a pion by the target nucleus (A).

The sum of the two projections, $P_1 + P_x$, defines the P projection and the P space. All open reaction channels of interest to us lie in this P space. The conjugate Q space contains only closed channels, corresponding to the parts of the state-space S with, e.g., A nucleons and n mesons ($n > 1$). Since we are interested in the reaction channels in the P space, we shall eliminate the Q space from our equations.

Before we make use of the projections, we note the following features. First, we have defined what the projectors P_1 and P_x do, but not how they may be constructed explicitly. However, as with the Hamiltonian H of Eq. (2.1), we shall not need explicit forms for P_1 and P_x in the final set of equations. The main point has been the decomposition of the original space S into major parts, so that one part (P space) contains the open reaction channels for scattering and absorption. Second, the nucleons in the P space are "dressed" or physical in the sense that they correspond to measurable, noninteracting particles at large separation, which do not change their intrinsic structure, even in the interaction region.

III. COUPLED EQUATIONS

We now derive dynamical equations which correspond to the degrees of freedom of interest, using the projection operators defined in the previous section. First we project the state $\Psi^{(+)}_{\pi A}$ into the subspaces defined by P and Q:

$$|\Psi^{(+)}_{\pi A}\rangle = P|\Psi^{(+)}_{\pi A}\rangle + Q|\Psi^{(+)}_{\pi A}\rangle . \tag{3.1}$$

Applying the same projections to Eq. (2.1), we may formally solve for the projection $Q\Psi^{(+)}$, which is not of interest, to obtain an effective Schrödinger equation for $P\Psi^{(+)}$, following Feshbach.[2] We obtain

$$[E - \mathcal{H}(E)]P|\Psi^{(+)}_{\pi A}\rangle = 0 , \tag{3.2}$$

where $\mathcal{H}(E)$ is an effective Hamiltonian, given by

$$\mathcal{H}(E) \equiv PHP + PHQ(E^+ - QHQ)^{-1}QHP , \tag{3.3}$$

with $E^+ = E + i\eta$ ($\eta \to 0^+$) corresponding to the usual outgoing-wave boundary conditions. This effective operator must represent all the physics implied by the coupling of the open channels (in P) to the closed channels (in Q) in which any number of mesons may be involved.

We further decompose Eq. (3.2) by projection with P_1 and P_x separately, to obtain a pair of coupled equations of the form

$$[E - \mathcal{H}_1(E)]|\phi^{(+)}_1\rangle = \Gamma_{1x}(E)|\phi^{(+)}_x\rangle , \tag{3.4a}$$

$$[E - \mathcal{H}_x(E)]|\phi^{(+)}_x\rangle = \Gamma_{x1}(E)|\phi^{(+)}_1\rangle , \tag{3.4b}$$

where

$$|\phi^{(+)}_1\rangle = P_1|\Psi^{(+)}_{\pi A}\rangle, \quad |\phi^{(+)}_x\rangle = P_x|\Psi^{(+)}_{\pi A}\rangle , \tag{3.5a}$$

and

$$\begin{aligned} &\mathcal{H}_1(E) = P_1\mathcal{H}(E)P_1, \quad \mathcal{H}_x(E) = P_x\mathcal{H}(E)P_x , \\ &\Gamma_{1x}(E) = P_1\mathcal{H}(E)P_x, \quad \Gamma_{x1}(E) = P_x\mathcal{H}(E)P_1 . \end{aligned} \tag{3.5b}$$

The wave vectors $\phi^{(+)}_1$ and $\phi^{(+)}_x$, when represented in terms of particle coordinates, as in Eqs. (2.4) and (2.5), respectively, are *channel* wave functions. As discussed in Sec. II, $\phi^{(+)}_1$ includes all the scattering channels, and $\phi^{(+)}_x$ all the absorption channels.

It is useful to consider first only the homogeneous part of Eq. (3.4a), which we may write in the form

$$[E - \mathcal{H}_1(E)]|\chi^{(+)}_1\rangle = 0 . \tag{3.6}$$

The solution $\chi^{(+)}_1$ corresponds to a reduced dynamical problem in which a pion scatters from the nuclear target, but there is no coupling between the scattering channel space P_1 and the absorption channel space P_x. (The P_1 space is still coupled to the Q space, in which the multiple-meson states appear.) Therefore Eq. (3.6) is an effective Schrödinger equation in the space represented by Eq. (2.4) with a projectile pion and A nucleons. This may be treated by standard methods of nuclear scattering theory. We define an effective scattering potential between the pion and the nuclear target (A),

$$V_\pi(E) = \mathcal{H}_1(E) - H_A - h_{\pi A} , \tag{3.7}$$

where H_A is the internal Hamiltonian for the nuclear target and $h_{\pi A}$ the kinetic energy in the c.m. frame. We denote target states by n: $H_A|n\rangle = E_n|n\rangle$, and plane wave pion states by the momentum $\vec{k}$. Then we may write the amplitude for pion scattering corresponding to Eq. (3.6) with a target transition from state 0 to state n, as a t matrix

$$\langle \vec{k}', n|T^s(E)|\vec{k}, 0\rangle = \langle \vec{k}', n|V_\pi(E)|\chi^{(+)}_1\rangle . \tag{3.8}$$

We label the t matrix by s as a reminder that only scattering channels are included in Eq. (3.6); T^s is not the complete physical amplitude for the scattering process.

We may rewrite Eq. (3.6) in the Lippmann-Schwinger form

$$|\chi_1^{(+)}\rangle = |\vec{k},0\rangle + G_A(E)V_\pi(E)|\chi_1^{(+)}\rangle\,, \tag{3.9a}$$

with

$$G_A(E) = (E^+ - H_A - h_{\pi A})^{-1}\,. \tag{3.9b}$$

From Eq. (3.9) we obtain the usual linear equation for the t matrix, extending (3.8) to off-shell values of E, and writing an operator relation

$$T^s(E) = V_\pi(E) + V_\pi(E)G_A(E)T^s(E)\,. \tag{3.10}$$

We now have equations in a form appropriate for multiple scattering expansion, using the approach of Watson and others. However, in the usual Schrödinger treatment of multiple scattering, the projectile-target interaction is assumed to be two body. Our present effective interaction, given in Eq. (3.7), has many-body interactions as well, as we shall explain further in the following section. We therefore write

$$V_\pi(E) = \sum_L V_\pi^L(E)\,, \tag{3.11}$$

where L is a double label $L=(n,\nu)$, with n specifying the number of nucleons interacting simultaneously with the pion ($n=1,2,\ldots,A$), and ν labeling *which* nucleons are involved. For conventional two-body potentials, $L=(1,i)$ with $i=1,\ldots,A$. For details, see Sec. IV.

With this decomposition of the interaction (3.11), we expand $T^s(E)$ in a generalized Watson series,

$$T^s(E) = \sum_L t^L(E) + \sum_{L+L'} t^L(E)G_A(E)t^{L'}(E) + \cdots\,, \tag{3.12a}$$

where

$$t^L(E) = V_\pi^L(E) + V_\pi^L(E)G_A(E)t^L(E)\,. \tag{3.12b}$$

The t matrices of Eq. (3.12b) describe a reduced scattering problem in which the pion interacts only with a fixed set of target nucleons, specified by $L=(n,\nu)$. For the conventional case of two-body interactions, $L=(1,i)$, and $t^L(E)=t_i(E)$ in the t matrix for πN scattering with the ith nucleon. However, this scattering refers to the reduced problem (3.6), from which all coupling to the P_x space has been eliminated. That means that even the process in which a pion is absorbed and then emitted from a single nucleon, as in the usual Born terms for πN scattering, are *not* included in t^L in Eq. (3.12). Therefore, if one wishes to use the free πN scattering amplitude in the multiple scattering expansion, it is necessary first to subtract the direct Born amplitude (see Fig. 1):

$$t^{(1,i)}(E) = t_{\pi N}(E) - t(\text{direct Born})\,. \tag{3.13}$$

In the direct Born amplitude, the intermediate nucleon is partially dressed. For further details, see Appendix A of Ref. 1.

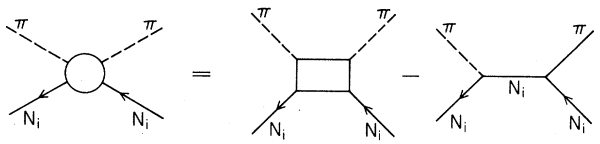

FIG. 1. Illustration of Eq. (3.13). Circle represents $t^{(1,i)}(E)$, square represents complete $t_{\pi N}$, last diagram represents the direct Born amplitude (with partially dressed nucleon).

Given a solution $\chi_1^{(+)}$ of the homogeneous equation (3.6), we can express the solution to the coupled equations (3.4) for the full scattered wave $\phi_1^{(+)}$ in the form

$$|\phi_1^{(+)}\rangle = |\chi_1^+\rangle + G_1(E)\Gamma_{1x}(E)|\phi_x^{(+)}\rangle\,, \tag{3.14a}$$

where

$$G_1(E) = [E^+ - \mathcal{H}_1(E)]^{-1} = G_A(E) + G_A(E)T^s(E)G_A(E)\,. \tag{3.14b}$$

Substitution of (3.14a) into (3.4b) leads to

$$|\phi_x^{(+)}\rangle = G_x(E)\Gamma_{1x}|\chi_1^{(+)}\rangle\,, \tag{3.15a}$$

with

$$G_x(E) = [E^+ - \mathcal{H}_x - \Gamma_{x1}(E)G_1(E)\Gamma_{1x}(E)]^{-1}\,. \tag{3.15b}$$

Finally, substituting (3.15a) back into (3.14a) leads to a formal expression for $\phi_1^{(+)}$ in terms of $\chi_1^{(+)}$,

$$|\phi_1^{(+)}\rangle = [1 + G_1(E)K(E)]|\chi_1^{(+)}\rangle\,, \tag{3.16a}$$

in which we have defined the operator

$$K(E) = \Gamma_{1x}(E)G_x(E)\Gamma_{x1}(E)\,, \tag{3.16b}$$

which includes all the effects of coupling the absorption to the scattering channels.

The full t matrix for scattering may be obtained by considering (see Appendix)

$$\langle\vec{k}',n|T(E)|\vec{k},0\rangle = \langle\vec{k}',n|U_1(E)P|\Psi_A^{(+)}\rangle\,, \tag{3.17a}$$

where

$$U(E) = \mathcal{H}(E) - H_A - h_{\pi A}\,. \tag{3.17b}$$

A brief calculation yields

$$\langle\vec{k}',n|T(E)|\vec{k},0\rangle = \langle\vec{k}',n|T^s(E)|\vec{k},0\rangle + \langle\chi_1^{(+)}(\vec{k}',n)|K(E)|\chi_1^{(+)}(\vec{k},0)\rangle\,, \tag{3.18}$$

where $\chi_1^{(+)}(\vec{k},0)$ is the solution of the homogeneous equation (3.6), given in Eq. (3.9a), corresponding to the initial plane wave, while $\chi^{(-)}$ is the (adjoint) solution to (3.6) or to

$$\langle\chi_1^{(-)}(\vec{k}',n)| = \langle\vec{k}',n| + \langle\chi_1^{(-)}(\vec{k}',n)|V_\pi(E)G_A(E)\,, \tag{3.19}$$

corresponding to the final plane wave. For all these expressions, E is the energy of the initial pion wave plus target ground state energy.

Equation (3.18) is the principal result for scattering reactions. To calculate the amplitude, we need the scattering waves $\chi_1^{(+)}$ and $\chi_1^{(-)}$ (which are multichannel waves in the P_1 space) and the corresponding t matrix $T^s(E)$. In addition, we need the effective operator $K(E)$, Eq. (3.16b), which gives the contribution to the scattering from coupling to the absorption channels. (We further discuss these operators in the next section.) Similar expressions have been given by Thomas,[6] and more recently by Rinat.[9,10] This result is also similar in form to that given by Feshbach, Kerman, and Lemmer[22] in their "doorway-state" theory of nuclear reactions, which was based, as is the present work, on the Feshbach projection method.[2]

For pion absorption, let us consider a specific reaction channel labeled by $(\alpha,\beta,\dots)$, where $\alpha,\beta,\dots$ are the nuclear fragments in the particular asymptotic channel:

$$\pi + A \to \alpha + \beta + \dots .$$

The fragments $\alpha,\beta,\dots$ might be p, n, d, ^{3}He, etc., and the residual nuclear target. We define the channel Hamiltonian

$$H(\alpha,\beta,\dots) = (H_\alpha + h_\alpha) + (H_\beta + h_\beta) + \cdots , \tag{3.20}$$

where H_α is the internal Hamiltonian for fragment α, and h_α is its kinetic energy, etc. The transition amplitude to this channel may be obtained from the expression [see also Eq. (3.17) and Appendix]

$$\langle \vec{k}_\alpha, \vec{k}_\beta, \dots | T_{\rm abs}(E) | \vec{k}, 0\rangle = \langle \vec{k}_\alpha, \vec{k}_\beta, \dots | U_x(E) P | \Psi_{\pi A}^{(+)} \rangle , \tag{3.21a}$$

where

$$U_x(E) = \mathcal{H}(E) - H(\alpha,\beta,\dots) . \tag{3.21b}$$

With some manipulation, the amplitude may be put in the form

$$\langle \vec{k}_\alpha, \vec{k}_\beta, \dots | T_{\rm abs}(E) | \vec{k}, 0\rangle = \langle \psi^{(-)}(\vec{k}_\alpha, \vec{k}_\beta, \dots) | \Gamma_{x1}(E) | \chi_1^{(+)}(\vec{k}, 0)\rangle . \tag{3.22}$$

We have defined a wave function $\psi^{(-)}$ for the final state fragments 1, which satisfies the scattering equation

$$\langle \psi^{(-)}(\vec{k}_\alpha, \vec{k}_\beta, \dots) | = \langle \vec{k}_\alpha, \vec{k}_\beta, \dots | + \langle \psi^{(-)}(\vec{k}_\alpha, \vec{k}_\beta, \dots) | V_x(E) G_x(E) , \tag{3.23a}$$

with

$$V_x(E) = \mathcal{H}_1(E) + \Gamma_{x1}(E) G_1(E) \Gamma_{1x}(E) - H(\alpha,\beta,\dots) , \tag{3.23b}$$

and

$$G_x(E) = [E^+ - H(\alpha,\beta,\dots)]^{-1} . \tag{3.23c}$$

The amplitude (3.22) is evaluated at the total energy of the initial wave.

We note that the wave $\psi^{(-)}$ in the absorption channel scatters through an interaction $V_x(E)$ which includes the coupling of the absorption and scattering channels. This is explicit in the second term of Eq. (3.23b). In contrast, the scattering waves $\chi_1^{(\pm)}$ in the P_1 space do not reflect coupling to the P_x space. One consequence of this is that $V_x(E)$ will always be a non-Hermitian operator, since E is necessarily above the threshold for production of pions: $\alpha + \beta + \cdots \to \pi + A$. The non-Hermitian property is required by the unitarity of the theory, and therefore must be treated with some care in making approximations. This is discussed further in the following section.

Equation (3.22) is our principal result for the absorption amplitude. The form is similar to that given in Ref. 1 for $\pi d \to NN$. The expression has the form of a distorted wave amplitude, but is in fact considerably more complicated. Both "distorted waves" in the present case are multichannel waves: $\chi_1^{(+)}$ has all inelastic scattering waves in the P_1 space, and $\psi^{(-)}$ has components in all inelastic and rearrangement channels which are reached from the $\alpha + \beta + \cdots$ channel. The effective vertex function $\Gamma_{x1}(E)$ can be decomposed into many-body components, as in the decomposition of $V_\pi(E)$ in Eq. (3.11). This is also discussed in Sec. IV.

We may write the amplitude for π production by taking the adjoint of Eq. (3.22). For a two-body collision $\alpha + \beta \to \pi + A^*$ we may write

$$\langle \vec{k}, n | T_{\rm prod}(E) | \vec{k}_\alpha, \vec{k}_\beta\rangle = \langle \chi_1^{(-)}(\vec{k}, n) | \Gamma_{1x}(E) | \psi^{(+)}(\vec{k}_\alpha, \vec{k}_\beta)\rangle , \tag{3.24}$$

where the residual target A^* is in the state n.

Finally, we may write transition operators, corresponding to the amplitudes (3.18), (3.22), and (3.24), respectively, as follows:

$$T(E) = T^s(E) + \tilde{\Omega}_1(E) \Gamma_{1x}(E) G_x(E) \Gamma_{x1}(E) \Omega_1(E) , \tag{3.25a}$$

$$T_{\rm abs}(E;\ \alpha,\ \beta\dots) = \tilde{\Omega}_{\alpha,\beta,\dots}(E) \Gamma_{x1}(E) \Omega_1(E) , \tag{3.25b}$$

$$T_{\rm prod}(E;\ \alpha,\beta) = \tilde{\Omega}_1(E) \Gamma_{1x}(E) \Omega_{\alpha,\beta}(E) . \tag{3.25c}$$

We have introduced the Møller operators for the

waves $\chi_1^{(\pm)}$:

$$\Omega_1(E)=1+G_A(E)T^s(E)\,,\qquad \tilde{\Omega}_1(E)=1+T^s(E)G_A(E)\,, \tag{3.26a}$$

and for the waves $\psi^{(\pm)}$:

$$\Omega_{\alpha,\beta,\ldots}(E)=1+g_x(E)V_x(E)\,,\qquad \tilde{\Omega}_{\alpha,\beta,\ldots}(E)=1+V_x(E)g_x(E)\,, \tag{3.26b}$$

where

$$g_x(E)=[E^+-H(\alpha,\beta,\ldots)-V_x(E)]^{-1}\,. \tag{3.26c}$$

IV. EFFECTIVE OPERATORS

We give a brief discussion of some of the effective operators which appear in Sec. III, and which must be specified in order to calculate the scattering amplitude of Eq. (3.18) and the absorption amplitude of Eq. (3.22). The particular forms of these operators depend on the details of the assumed underlying dynamics [e.g., the pion-nucleon coupling, etc., in Eq. (2.1)], but there are general features common to any dynamics.

We consider first the effective Hamiltonian $\mathcal{H}_1(E)$ that gives the interaction in the pion plus A-nucleon space (P_1), and which is defined in Eq. (3.5b). In Eq. (3.7) we have decomposed $\mathcal{H}_1(E)$ into parts involving the target nucleus, the pion-nucleus interaction, and the c.m. kinetic energy. We now separate the target Hamiltonian H_A into the kinetic energy of the nucleons and the mutual interaction of the nucleons. We write

$$\mathcal{H}_1=H_0+V_N(E)+V_\pi(E)\,, \tag{4.1}$$

where H_0 is the total kinetic energy of pion plus nucleons, $V_N(E)$ is the nuclear interaction, and $V_\pi(E)$, is, as before, the pion-nucleus interaction. We discuss these in order.

The kinetic energy operator H_0 is given by the sum of the operators for A free physical nucleons plus one pion, that is,

$$H_0=\sum_{i=1}^{A}\frac{p_i^{\,2}}{2M}+(k^2+m^2)^{1/2}\,, \tag{4.2}$$

where $\vec{p}_i$ is the momentum operator for the ith nucleon with mass M (treated nonrelativistically here), and $\vec{k}$ is the momentum operator of the pion (of mass m). This form follows from the method of constructing the P_1 projection operator given in Sec. II. We have defined the particle coordinates $\vec{r}_0,\vec{r}_1,\ldots,\vec{r}_A$ and therefore the conjugate momenta $\vec{k}$, $\vec{p}_1,\ldots,\vec{p}_A$, for physical particles, by starting in the asymptotic region of configuration space in which all $A+1$ particles are well separated [see Eq. (2.4) and the following discussion]. In this region, H_0 is well defined, and coincides with the total Hamiltonian. The operator H_0 is then continued to all points of P_1 space, along with the particle coordinates.

The nuclear interaction $V_N(E)$ may be defined as the part of $\mathcal{H}_1(E)$ which does not involve the pion variables $(\vec{r}_0,\vec{k})$ at all. We may decompose $V_N(E)$ into parts of different rank n, involving n-nucleon interactions, as follows:

$$V_N(E)=\sum_L V_N^L(E)\,, \tag{4.3}$$

where $L=(n,\nu)$ is a double label, as in Eq. (3.11), n specifying the number of interacting nucleons ($n=2,3,\ldots,A$), and ν labeling the specific nucleons. This is illustrated in Fig. 2. Some specific examples are shown in Fig. 3, where the interaction is mediated by the exchange of one or more more mesons ($\pi,\rho,\omega,\ldots$). This clearly involves coupling of the P_1 space to the Q space as in Eq. (3.3). We note that the n-body interaction may be connected, as in (a) and (b), or disconnected, as in (c). The occurrence of energy-dependent, many-body, and even disconnected interactions among the target nucleons is of formal interest, but is not likely to affect practical calculation of scattering. In most multiple scattering approximations, the purely nuclear interactions are treated in terms of target states and energies, e.g., the closure approximation for high energy projectiles. An exception is for a two-body target in the Faddeev approach, in which the NN interaction is treated in greater detail. (See also Ref. 1.)

The pion-nucleus interaction $V_\pi(E)$, which was defined in Eq. (3.7), may also be decomposed into parts of different nucleon rank,

$$V_\pi(E)=\sum_L V_\pi^L(E)\,, \tag{4.4}$$

with $L=(n,\nu)$, $n=1,2,\ldots,A$, as in Eq. (3.11); the projectile pion always interacts. This is illustrated in Fig. 4; some specific examples are shown in Fig. 5. Again, the interaction diagrams may be connected, as in (a) and (b), disconnected, as in (c).

In a similar way, the channel coupling interactions $\Gamma_{x1}(E)$ [and $\Gamma_{1x}(E)$] of Eq. (3.4) may be de-

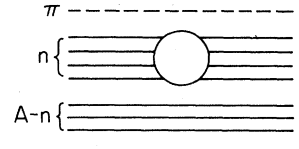

FIG. 2. Representation of $V_N^L(E)$ of Eq. (4.3). Solid lines are nucleons, dashed line is pion, circle is n-nucleon interaction.

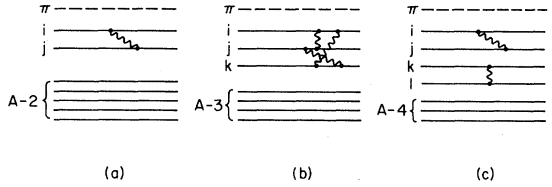

FIG. 3. Examples of V_N^L: (a) $L=(2,ij)$; (b) $L=(3,ijk)$; (c) a disconnected diagram with $L=(4,ijkl)$. Wavy lines are exchanged mesons ($\pi, \rho, \omega \ldots$).

composed by nucleon rank:

$$\Gamma_{x1}(E)=\sum_L \Gamma_{x1}^L(E)\,, \tag{4.5}$$

with $L=(n,\nu)$, $n=1,2,\ldots,A$. This is illustrated in Fig. 6, and some simple examples are shown in Fig. 7.

The appearance of disconnected terms in the effective interactions $V_N(E)$, $V_\pi(E)$, and $\Gamma_{x1}(E)$ $[\Gamma_{1x}(E)]$ as illustrated in Figs. 3(c), 5(c), and 7(c) follows from the projection method we have used to define channels. It is connected to the fact that these interactions are energy dependent and non-Hermitian. For $V_N(E)$ in particular, these formal peculiarities have long been known in dealing with diagrammatic methods for many-body bound systems, as for spectra of complex nuclei. One technique which has been introduced to eliminate both the energy dependence and the disconnected terms is the method of *folded diagrams*, introduced by Brandow[23] and further developed by others.[24] This method is presumably directly applicable to $V_N(E)$, which represents the excited spectrum of the target. The same technique has also been applied to NN scattering,[25] and presumably could be extended to remove the E dependence and disconnected parts from $V_\pi(E)$ and $\Gamma_{x1}(E)$ as well. A different method which removes these problems by transformation to an orthogonal basis in which the effective interactions are Hermitian was proposed long ago by Okubo.[3] This has been used by Gari and Hyuga[26] for removing disconnected terms in the calculation of mesonic exchange currents, which is a closely related problem to ours. Therefore, the disconnected parts appear to present no formal difficulty in our approach, and presumably

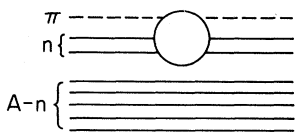

FIG. 4. Representation of V_π^L of Eq. (4.4); lines as in Fig. 2; circle represents interaction of pion with n nucleons.

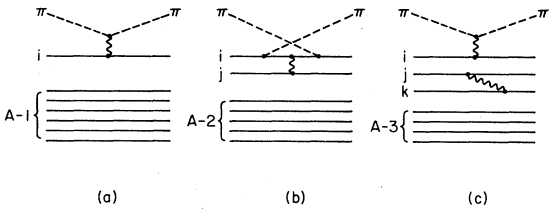

FIG. 5. Examples of V_π^L: (a) $L=(1,i)$, given by meson exchange between π and N_i; (b) $L=(2,ij)$, given by meson exchange between nucleons N_i, N_j, with crossed pion lines; (c) a disconnected diagram with $L=(3,ijk)$.

can be removed by one of these methods. As already mentioned, some of these features are lost in common approximations in multiple scattering theory.

A somewhat different problem arises in connection with the effective interaction $V_x(E)$, defined in Eq. (3.23), that gives the dynamics of the target in the absorption (P_x) channel. This interaction includes coupling to the open scattering channels, through the term

$$\Gamma_{x1}(E)G_1(E)\Gamma_{1x}(E) \tag{4.6}$$

in Eq. (3.23b). Because of the channel coupling, the nonhermiticity of $V_x(E)$ is *essential*, reflecting the loss of flux back into the scattering channels (as in any optical potential). Similarly, the energy dependence cannot be removed simply by transformation to an orthogonal basis, but is connected to the essential nonhermiticity (see Ref. 2), and is required to maintain the unitarity of the theory.

This property introduces a difficulty, which is illustrated in Fig. 8. The pion that propagates between particles i and j in Fig. 8(a) may be on-energy-shell or off-energy-shell, since there is always sufficient total energy for an on-shell pion to propagate. One cannot replace 8(a) by a real (Hermitian) and static one-pion exchange (OPE) potential of finite range, since the on-shell pion can propagate indefinite distance. The contribution of this term will not only be long range but complex and energy dependent as well. This same behavior will also occur in the nucleon self energy terms, as shown in Fig. 8(b).

In Ref. 1 we were able to avoid the explicit ener-

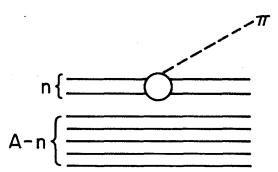

FIG. 6. Representation of the absorption operator $\Gamma_{X1}^L(E)$, Eq. (4.5).

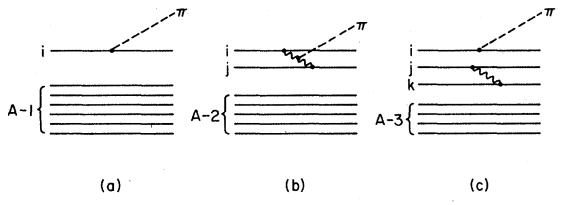

FIG. 7. Examples of $\Gamma^{L}_{X1}(E)$: (a) $L=(1,i)$, given by a one-nucleon absorption vertex; (b) $L=(2,ij)$, given by absorption by an exchanged meson; (c) a disconnected diagram with $L=(3,ijk)$.

gy dependence and nonhermiticity of $V_x(E)$ approximately by working at threshold, where the channel coupling is weak and V_x is real. For higher energy and for stronger coupling, these non-Hermitian properties should be kept explicitly.

V. APPLICATIONS

The main results of Sec. III are the expressions (3.18) for the scattering amplitude and (3.22) for the absorption amplitude. We now show how these formulas might be evaluated, using standard methods of multiple-scattering theory. We separately discuss the treatment of elastic scattering, inelastic scattering, and specific absorption transitions.

A. Elastic scattering

The elastic scattering amplitude is given in Eq. (3.18) as the sum of two terms:

$$\langle\vec{k}'|T_{el}|\vec{k}\rangle=\langle\vec{k}'|T^{s}_{el}|\vec{k}\rangle+\langle\chi_1^{(-)}(\vec{k}')|K|\chi_1^{(+)}(\vec{k})\rangle, \tag{5.1}$$

where we suppress the target (ground) state label, and the energy variable. The first term is given by Eq. (3.8), which is obtained in turn from a solution of the uncoupled effective Schrödinger Eq. (3.6). Here one may use standard techniques for solving the elastic scattering problem approximately, for example, based on a multiple scattering expansion of the amplitude, as in Eq. (3.12), or the optical potential. However, unlike the usual case of the Watson expansion the effective scattering interaction $V_\pi(E)$ is not simply a two-body interaction (pion-nucleon), but also contains pion-many nucleon contributions as well, as already noted in connection with Eq. (3.11). Similarly, the t matrices on which a multiple scattering expansion are based, include many-body contributions, as in Eq. (3.12) and Fig. 5(b).

To calculate the second term of (5.1) we need the solutions $\chi_1^{(\pm)}$ of Eqs. (3.6) or (3.19), as well as the operator $K(E)$, which was defined in Eq. (3.16b). The scattering solutions are quite complicated, since they contain all the nuclear channels excited by the pion through the interaction $V_\pi(E)$. There is no reason to assume that the elastic channel will dominate here, since absorption can occur as easily after an inelastic collision as before. Therefore one cannot approximate $\chi_1^{(\pm)}$ by elastic waves, i.e., solutions of the optical wave equation used to calculate $\langle\vec{k}'|T^{s}_{el}|k\rangle$, unless inelastic scattering is very weak. Since a complete inelastic coupled channel solution is usually impractical, a suitable approximation might be based on selecting the dominant inelastic channels, as follows. We write $\chi_1^{(\pm)}$ in terms of the Møller operators (3.26) acting on plane wave states:

$$\begin{aligned}&|\chi_1^{(+)}(\vec{k})\rangle=\Omega_1|\vec{k},0\rangle,\\&\langle\chi_1^{(-)}(\vec{k}')|=\langle\vec{k}',0|\hat{\Omega}_1.\end{aligned} \tag{5.2a}$$

The strongly excited channels are denoted by n. These might include, say, the elastic wave ($n=0$) and the one-nucleon knockout channels ($n=1$), and perhaps more. Then the wave $\chi^{(+)}$ of (5.2a) could be approximated by a (finite) series of terms

$$|\chi^{(+)}(\vec{k})\simeq\sum_n\Omega(n)|\vec{k},0\rangle, \tag{5.2b}$$

with $\Omega(n)$ the distortion in the appropriate channel n, and similarly for $\hat{\Omega}_1$. (This is still a formidable problem for continuum channels.) Another possible approximation would be the fixed scatterer assumption, in which $\chi_1^{\pm}$ are given for every position of each target nucleon: $\chi^{\pm}_{\vec{k}}(\vec{r}_1,\ldots,\vec{r}_A)$,

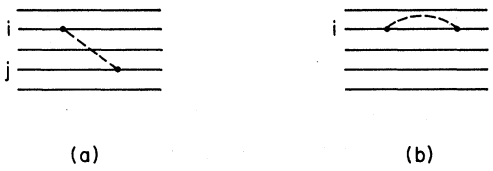

FIG. 8. Examples of contributions to the effective interaction in Eq. (4.6), generated by the emission and reabsorption of one pion. The pion can be on-energy-shell, making the contribution of Eq. (4.6) complex and energy dependent.

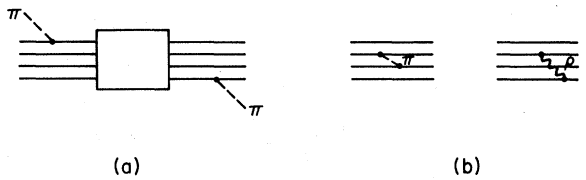

FIG. 9. (a) Representation of the operator $K(E)$ defined in Eq. (3.16b). (b) Examples of contributions to the box in (a).

at which the matrix element of $K(E)$ is calculated, the entire expression subsequently averaged over the target ground state density. This method is equivalent to keeping all channels and ignoring the differences in excitation energies, i.e., assuming closure on nuclear states. This might work best for high energy pions.

The operator $K(E)$ can be symbolized by Fig. 9(a), where the box indicates all interactions among the nucleons between the first absorption and last emission of the pion. Examples of simple processes in the box are shown in (b) and (c): they include both processes usually assigned to the static nuclear interactions, and also *retarded* (i.e., nonstatic) interactions involving propagation of pions (or other mesons). It is not clear what approximations will suffice here, or whether closure approximations will work for the box. (In the case of the πd scattering length, closure was not an accurate approximation. See Ref. 1.)

Clearly the second term of Eq. (5.1) is more complicated to calculate than the first term. For the latter, one needs a conventional method of dealing with multiple scattering for the elastic channel; for example, an optical potential. For the second term, one needs information on multiple scattering in inelastic channels as well, and in addition, a theory of the absorption mechanism. A natural question is: How important is the second term in elastic scattering? It is clear that if the total absorption is weak, the absorption-emission term should be small, but the converse need not also be true. There are circumstances for which the second term in Eq. (5.1) could be small, even though the total absorption cross section is large. In such a case, the amplitude T_{el}^{S} must be a good approximation to T_{el}. This can happen, for example, if there are many inelastic channels open, strongly coupled to the elastic channel, as is the case at high energy. The total reaction cross section is then dominated by geometric considerations, tending to the "black-sphere" limit for strong coupling. The elastic amplitude reflects this behavior, through unitarity (or the optical theorem). Now for T_{el}^{S}, all nonelastic channels are *inelastic*, while for T_{el}, the absorption channels are also included. In the high energy, strong-coupling limit, however, the effect of the nonelastic channels on the elastic amplitude should be similar for T_{el}^{S} and T_{el}. Then, even with a substantial cross section for absorption, the second term of Eq. (5.1) will be small. Examples of this kind of behavior have been presented[27] in terms of models of pion absorption at high energy.

The subject of the effects of unitarity on the elastic channel is more appropriately discussed in terms of an optical potential in which case it has been called the *reactive content* of the optical potential.[27-29] We reserve an extended discussion of the theory of the optical potential for pion elastic scattering for a future publication. For the present, it is useful to point out that one may always reformulate the calculation of elastic scattering, in terms of such a potential, $\mathcal{U}(E)$, for which

$$T_{el}(E) = \mathcal{U}(E) + \mathcal{U}(E) g_0(E) T_{el}(E) . \tag{5.3}$$

We define $g_0(E)$ to be the propagator for the pion with the target restricted to its ground state, which we obtain by taking the ground state expectation value of $G_A(E)$, Eq. (3.9b):

$$g_0(E) = \langle G_A(E) \rangle . \tag{5.4}$$

Now suppose the amplitude $T_{el}^{S}(E)$ is also calculated from an optical potential $U(E)$, based on a multiple scattering solution of (3.8–3.10) in the elastic channel, with

$$T_{el}^{S}(E) = U(E) + U(E) g_0(E) T_{el}^{S}(E) . \tag{5.5a}$$

This optical potential can be written as the target ground state expectation value $U(E) = \langle R^S(E) \rangle$, where R^S obeys the integral equation

$$R^S(E) = V_\pi(E) + V_\pi(E) G_A(E)(1 - \Lambda) R^S(E) , \tag{5.5b}$$

and where Λ projects onto the target ground state. One can find an expression for $\mathcal{U}(E)$, for which Eq. (5.3) is formally equivalent to (3.1), by manipulation of the equations of Sec. III and Eqs. (5.3) and (5.5). The result may be written in the form

$$\begin{aligned} &\mathcal{U}(E) = \mathfrak{U}(E) + \mathfrak{z}(E) , \\ &\mathfrak{z}(E) = \mathfrak{K}(E)[1 + g_s(E)\mathfrak{K}(E)]^{-1} , \end{aligned} \tag{5.6}$$

where

$$\mathfrak{K}(E) = \langle \tilde{\Omega}_1(E) K(E) \Omega_1(E) \rangle$$

is the target ground state expectation of the second term of Eq. (3.25a), giving an operator in the elastic channel. Similarly,

$$g_s(E) = \langle G_1(E) \rangle , \tag{5.7}$$

where $G_1(E)$ is defined in Eq. (3.14b). This gives us a formal expression for the optical potential, in terms of the quantities calculated in our approach. We postpone a more complete discussion of the calculation and use of optical potentials for a future publication.

B. Inelastic scattering

The amplitude for inelastic scattering to a specific final state n is given by Eq. (3.18), which we rewrite in the form

$$\langle \vec{k}' | T_{n0} | \vec{k} \rangle = \langle \vec{k}' | T^S_{n0} | \vec{k} \rangle + \langle \chi_1^{(-)}(\vec{k}', n) | K(E) | \chi_1^{(+)}(\vec{k}, 0) \rangle . \tag{5.8}$$

As for elastic scattering, the first term $\langle \vec{k}' | T^S_{n0} | \vec{k} \rangle$ may be handled by conventional multiple scattering methods, with the modifications mentioned following Eq. (5.1). For many cases in which the final channels may be directly coupled to the elastic channel by a one-step πN scattering process, some form of distorted-wave impulse approximation (DWIA) may be appropriate (for example: charge exchange, giant resonance excitation). The second term in Eq. (5.8) would be handled in a manner analogous to the second term of Eq. (5.1), except that the final wave $\chi_1^{(-)}(\vec{k}', n)$ has a pion scattering from the excited state n of the final target.

It is interesting to ask whether the second term of Eq. (5.8) may play a more important role in inelastic scattering, than the analogous term does in the elastic case. This may well be the case whenever $\langle \vec{k}' | T^S_{n0} | \vec{k} \rangle$ is small (as a direct reaction), for example, for double charge exchange to analog states.

C. Absorption transitions

The amplitude for pion absorption leading to a specific final channel denoted by $(\alpha, \beta, \ldots)$ is given by Eq. (3.22), which we rewrite in slightly condensed notation,

$$\langle \alpha, \beta \ldots | T_{abs} | \vec{k}, 0 \rangle = \langle \psi^{(-)}_{\alpha,\beta\ldots} | \Gamma_{x1}(E_k) | \chi_1^{(+)} \rangle . \tag{5.9}$$

The wave $\chi_1^{(+)}$ is the solution of the uncoupled scattering Eqs. (3.6) and (3.9a), which we have already discussed in Eq. (5.2), in connection with the elastic amplitude (5.1). The absorption operator $\Gamma_{x1}(E)$ couples the one-pion and no-pion channels. As discussed in Sec. IV, the effective operator $\Gamma_{x1}(E)$ can be decomposed into operators for one, two, or more nucleons. The one-body operator [see Fig. 7(a)] is conventionally taken to be of a simple form, e.g.,

$$\Gamma_{x1} \propto \sum_{i=1}^{A} \int d^3k \, j^{(i)}(\vec{k}) a(\vec{k}) , \tag{5.10}$$

where $a(\vec{k})$ absorbs a pion of momentum $\vec{k}$, and $j^{(i)}(\vec{k})$ is a nonrelativistic vertex function, for the ith nucleon. For example, for the static model, $j^{(i)}(\vec{k}) \propto \vec{\sigma}^{(i)} \cdot \vec{k} \tau_\mu^{(i)} F(k)$ in terms of the nucleon spin $\vec{\sigma}^{(i)}$ and isospin $\tau_\mu^{(i)}$ operators (for a pion of charge μ), and a form factor (cutoff function) $F(k)$. More complicated processes, involving, e.g., rho-meson exchange, are illustrated in Fig. 7(b).

The final channel wave functions $\psi^{(-)}_{\alpha,\beta\ldots}$ are given by solutions of the effective wave equation (3.23a) in the no-pion channel. This is in general a very complicated problem involving highly excited channels of the final nucleus, with various numbers of particles in the continuum. For example, for the reaction (π, p), the final channel state $\psi_p^{(-)}$ involves a proton (of energy $E_p \gtrsim m_\pi$) scattering from a nuclear target, which would be in an excited state. The wave $\psi_p^{(-)}$ may strongly couple to other final channels $[(\pi, 2N), (\pi, 3N)]$ by interaction of the proton with target nucleons. In addition, $V_x(E)$ is energy dependent and non-Hermitian, as discussed in Sec. IV.

The amplitude (5.9) expresses the absorption amplitude in a generalized distorted wave form, but, as we have noted, there may be considerable coupling of channels in both the entrance and exit channels. Are there circumstances for which one may use something like conventional distorted-wave Born approximation (DWBA), in which the initial and final waves are simply optically distorted by single channel potentials? For the initial channel that does not seem likely, since for most absorption reactions that have been examined, nonelastic scattering of the incoming pion seems to play a role, as discussed in connection with Eq. (5.2). Even for pions absorbed at rest, virtual scattering is important. It may be useful to try to include the initial-state scattering together with the absorption operator (if only one or two nucleons are involved). Then one can define a new effective absorption operator

$$\Gamma_{eff} = \Gamma_{x1} \Omega^{(+)}_{inel} , \tag{5.11}$$

where the wave operator $\Omega_1^{(+)}$ of (5.2a) is written as a product of inelastic and elastic (optical) wave operators:

$$\Omega_1^{(+)} = \Omega^{(+)}_{inel} \cdot \Omega^{(+)}_{el} , \tag{5.12a}$$

where

$$\Omega^{(+)}_{el} = 1 + G_A(E) \Lambda T^S_{el}(E) , \tag{5.12b}$$

and

$$\Omega^{(+)}_{inel} = 1 + G_A(E)(1 - \Lambda) R^S(E) , \tag{5.12c}$$

where T^S_{el}, Λ, and R^S are defined in Eq. (5.5). Then $\Omega^{(+)}_{el} | \vec{k}, 0 \rangle$ is the optically distorted incoming wave.

For the final state $(\alpha, \beta \ldots)$ a distorted-wave approach might work in two cases: first for an *exclusive* reaction to a particular final state for which the nuclear structure favors the direct transition. For example, in $^6\mathrm{Li}(\pi^-, 2n)^4\mathrm{He}_{g.s.}$ it might be safe to ignore channel coupling through the outgoing neutrons of the ground and excited states of ^{4}He. The final distortion is still a three-body problem with two neutrons and the residual ^{4}He target. The second case might be for an *inclusive*

reaction, like $(\pi, 2N)$, where only the two (fast) nucleons are observed, and a range final states is reached. Since the details of the final channel are summed over, optical distortion of the outgoing waves may be an adequate description.

VI. CONCLUSION

We have given a theoretical method of treating nuclear reactions induced by pions, which includes scattering and absorption on an equal footing. The theory is consistent with the meson degrees of freedom of the nuclear system itself, and therefore with quantum field theory. However, we have developed the formulation to be consistent with the conventional Schrödinger treatment of nuclei, and with multiple scattering theory. The idea has been to see how to generalize existing multiple scattering approaches to include absorption in a consistent way. We have discussed the form that such extensions might take, and have explored some details of the effective operators in our approach. There is considerable work required to flesh out a complete theory for calculation. This remains true because, although there has been considerable progress in recent years in our ability to handle the multiple scattering aspects of pion reactions, we do not have more than a rudimentary theory of absorption, particularly for energetic mesons. The one exception here is for the case of πd reactions, in which there has been progress both in the scattering and absorption sides of the problem (see Refs. 6–13). We have provided a consistent framework on which such a theory for complex nuclear targets might be built.

In a future publication, we intend to apply the present methods to the theory of the optical potential for pion-nucleus scattering.

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APPENDIX

The expressions for the scattering amplitude (3.17) and for the absorption amplitude (3.21) may be obtained from the formal theory of scattering for a system of many channels, following the discussion of Goldberger and Watson[30] (see also Ref. 31). We begin with a formal expression based on the exact Eq. (2.1) for the inelastic amplitude (on-shell)

$$\langle \vec{k}', n | T(E_k) | \vec{k}, 0 \rangle = \lim_{\eta \to 0^+} \langle \vec{k}', n | H - E_k | \Psi_A^{(\eta)} \rangle , \tag{A1}$$

where the limit $\eta \to 0^+$ is taken on the entire matrix element, and where $\Psi_A^{(\eta)}$ is defined, in terms of wave packets, in Ref. 20, with the usual limit $\Psi^{(\eta)} \to \Psi^{(+)}$. Now writing

$$(H - E_k) = (H - E_k)(P + Q) , \tag{A2}$$

and using Eq. (3.6) we find

$$\begin{aligned} \langle \vec{k}', n | T(E_k) | \vec{k}, 0 \rangle &= \lim_{\eta \to 0^+} \langle \vec{k}', n | (\mathcal{K}(E_k) - E_k) P | \Psi_A^{(\eta)} \rangle \\ &= \langle \vec{k}', n | (\mathcal{K}(E_k) - H_A - h_{\pi A}) P | \Psi_A^{(+)} \rangle . \end{aligned} \tag{A3}$$

The last line gives Eq. (3.17); Eq. (3.21) may be obtained by a similar argument.

Alternatively, one may derive Eq. (3.18) directly from Eq. (3.16) by forming the amplitude $\langle \vec{k}', n | \phi_1^{(+)} \rangle$. This can be reexpressed as a matrix element of G_A, by use of Eqs. (3.9), (3.10), (3.14b), and (3.16a). The residue at the pole of G_A may be expressed as in Eq. (3.18).

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