Unified theory of N-N and π-d scattering

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Using a time ordered diagrammatic method, we derive a set of linear integral equations that couple the N-N to the π-d channel and satisfy two- and three-body unitarity. The resultant equations have dressed propagators for the nucleons, and dressed form factors for the πNN vertex. The inclusion of the full π-N amplitude in the P_{11} channel leads to a dressing of both the πNN vertex and N-N propagator without changing the basic structure of the equations.

[NUCLEAR REACTIONS Nucleon-nucleon and pion-deuteron scattering. Derived coupled equations for NN, $N\Delta$, πd with dressed nucleons and πNN vertex.]

I. INTRODUCTION

There has been a growing interest in recent years in the πNN system and in particular its coupling to the N-N channel. This interest has been directed from two distinct but related problems.

(i) The analysis of N-N scattering data above the threshold for pion production. Here we need a model that describes the pion production mechanism, and preferably incorporates two- and three-body unitarity. Such a model allows us to impose a constraint on the imaginary part of the N-N phase shifts from the knowledge of the reaction cross section.[1] Since most of the inelasticity at medium energies (< 1 GeV) is due to single-pion production, and this production is dominated by $NN \to \pi d$ and $NN \to N\Delta$, we need to couple the N-N to the π-d and N-Δ channels. Furthermore, if the recently observed structure in polarized $\vec{p}$-$\vec{p}$ scattering[2] is due to the opening of an inelastic channel[3] dominated by π-N resonances (e.g., the Δ), then we should also couple the N-N to the N-Δ channel.

(ii) The importance of real pion absorption on pion-nucleus scattering. Here there is a strong indication that the discrepancy between experiment and optical potential calculations of low energy pion-nucleus scattering is due to real pion absorption.[4] In that case, the deuteron might be the ideal system in which to study the effect of absorption, as we can sum the multiple scattering series, exactly, using the Faddeev equations.

Both of the above problems can be simultaneously investigated if we can describe the reactions

$$\begin{aligned} N+N &\to N+N \\ &\to \pi+d \\ &\to \pi+N+N \end{aligned} \tag{1a}$$

and

$$\begin{aligned} \pi+d &\to \pi+d \\ &\to N+N \\ &\to \pi+N+N \end{aligned} \tag{1b}$$

using a set of coupled equations for the different physical amplitudes that satisfies two- and three-body unitarity.

The first attempt at such a model was due to Afnan and Thomas[5] who used the Faddeev equations for the πNN system. To couple the πNN to the N-N channel (i.e., to incorporate absorption) they introduced a π-N bound state in the P_{11} channel. Although the model was successful in giving the π-d scattering length and the s-wave pion production cross section at low energies, it has two weaknesses:

(i) Only one of the nucleons could emit the pion in intermediate states. This led to undercounting and was compensated for by taking an unusually large scattering length for the π-N interaction in the P_{11} channel.

(ii) In all of the two-nucleon intermediate states, only one of the nucleons was dressed. Thus the Pauli exclusion principle had to be imposed by proper selection of channel quantum numbers.

These two problems were first overcome by Mizutani and Koltun[6] who used an explicit Hamiltonian with a πNN vertex, a π-N and N-N interaction, and Feshbach projection operators to calculate the contribution of real absorption to π-d scattering. Their result, which was previously derived by Thomas[7] using a diagrammatic method, and later by Rinat[8] using nonrelativistic reduction techniques, gives the contribution of real absorption to π-d scattering in terms of a matrix element which requires the off-shell T matrix for π-d elastic scattering with no absorption. More recently Thomas and Rinat[9] have shown that we can write, using both the diagrammatic[7,10] and reduction[8] techniques, a set of linear integral equations for N-N scattering including the coupling to the

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πNN channel. The resultant equations though similar in form to those of Afnan and Thomas[5] do not have any undercounting or problems with the Pauli exclusion principle. At about the same time Avishai and Mizutani[11] extended the results of Mizutani and Koltun[6] to get a set of linear integral equations for both the N-N and π-d initial states. Finally, we would like to mention the work of Stingl and Stelbovics[12] who started from a Hamiltonian with the only interaction being the πNN vertex, and obtained a set of linear coupled integral equations for π-d scattering which include the effect of real pion absorption.

Here we employ the diagrammatic method used by Thomas[7,10] to derive a set of linear coupled integral equations for the reactions in Eq. (1). In particular, we show that the result of Thomas and Rinat[9] can be extended to π-d scattering with nonseparable π-N and N-N interactions. The resultant equations in the absence of the π-N interaction in the P_{11} channel are identical to those obtained by Avishai and Mizutani.[11] We find that the inclusion of the π-N interaction in the P_{11} channel leads to a dressing of both the form factor for the πNN vertex and the nucleons in the N-N and πNN propagators, with no change in the basic structure of the equations. In this way we ensure that the nucleons in both the N-N and πNN channels are identical, which was not the case in the work of Avishai and Mizutani.[11] By employing the diagrammatic method we are able to establish the connection between the works based on Hamiltonians with the πNN vertex, and the π-N and N-N interaction, on the one hand, and those with a πNN vertex only on the other hand. In this way we show that the π-N interaction in the Hamiltonian should lead to an amplitude in the P_{11} channel with the nucleon pole subtracted.

In Sec. II, we develop the diagrammatic method and show how one can dress both the nucleons and the πNN vertex. In the process we establish which π-N interactions should be included in calculating the π-d multiple scattering series with no absorption. We then derive a set of coupled linear integral equations for the reactions in Eq. (1a). These equations reduce to those of Thomas and Rinat[9] if one drops all dressing from the πNN form factors and nucleon propagators, and assumes separable potentials for the π-N and N-N interactions. In Sec. III we derive a set of coupled linear integral equations for π-d elastic scattering and absorption which have the same kernel as those for N-N scattering. The resultant equations are considerably simpler than those obtained by Mizutani and Avishai after including the nonpole P_{11} interaction. Some concluding remarks are presented in Sec. IV, while in the Appendix we show how one can split the π-N amplitude in the P_{11} channel into a part that leads to multiple scattering and a part that couples to the N-N channel.

II. NUCLEON-NUCLEON SCATTERING

Since we are interested in nucleon-nucleon (N-N) scattering above the threshold for single-pion production, and below the energy where two-pion production becomes important, we need a set of equations that satisfy two- and three-body unitarity at least. From a computational point of view it might be advantageous if the equations couple all the physical amplitudes. In this way we avoid the evaluation of off-shell three-body amplitudes to be used in distorted wave matrix elements. To achieve our goal we need to make two approximations:

(i) We do not include all two-pion intermediate states explicitly and thus lose four-body unitarity. However, some of the effects due to multipion intermediate states may be included through potentials, and thus will not affect two- and three-body unitarity. In spirit, the introduction of a potential to describe more than one-pion intermediate states is similar to N-N scattering below the single-pion production threshold, where one-pion exchange is treated as a static potential.

(ii) The number of nucleons is fixed, which implies that there are no antinucleons in the intermediate state. Therefore, our equations will have no crossing symmetry. At medium energies we should treat both the pion and nucleons using relativistic kinematics. However, since we have excluded antinucleons we cannot use Feynman propagators, but have to resort to a Blankenbeckler-Sugar reduction of the Feynman propagators, which maintains the unitarity cuts. This procedure has been implemented for both π-d (Ref. 13) and N-N (Ref. 14) scattering using the method of Aaron *et al.*[15] An alternative implementation of the reduction which preserves clustering properties[16,17] has been used for calculating Δ components in the deuteron.[18] In either case we can retain relativistic kinematics and a three dimensional equation, but we will lose a certain amount of uniqueness because of the reduction procedure.[19]

The diagrammatic method we employ for deriving our equations, due originally to Zachariasen,[20] was first used by Thomas for the π-d system.[7] It is based on old fashioned time order field theory which allows us to implement the approximation (ii). It also has the advantage of not having to specify the Hamiltonian explicitly. This means we can always write the π-N and N-N interactions in terms of either a basic πNN vertex or in terms of two-body interactions. Finally, and most importantly, it allows us to expose lowest unitarity cuts first. In this case we first expose the two-body and then the three-body unitarity cuts. At a later stage we can expose the higher unitarity cuts. The last two advantages of this method are shared with the covariant approach to N-particle Green's functions developed by Taylor[21] and used by Mizutani[22] for the πNN system.

In this section we follow the notation of Thomas and Rinat[9] by labeling the initial and final states by the number of pions. Thus the class of all diagrams with an initial and final state of two nucleons and zero pions is represented by $\langle 0|T|0\rangle$. All diagrams that contribute to $\langle 0|T|0\rangle$ can be divided into two classes: those with at least one pion in every intermediate state, which we represent by $\langle 0|T|0\rangle_1$, and those that have intermediate states of zero pions. The second class can be written using the last (first) cut lemma[21,23] as $\langle 0|T|0\rangle_1 g_0 \langle 0|T|0\rangle$ $(\langle 0|T|0\rangle\, g_0\, \langle 0|T|0\rangle_1)$, where g_0 is the two nucleon free Green's function. Basically the last (first) cut lemma, in this case, exposes the last (first) intermediate state with two free nucleons and zero pions, or the two-body unitarity cut. This classification allows us to write the amplitude for N-N scattering as

$$\langle 0|T|0\rangle^c = \langle 0|T|0\rangle_1^c + (\langle 0|T|0\rangle_1\, g_0 \langle 0|T|0\rangle)^c \qquad (2a)$$

$$= \langle 0|T|0\rangle_1^c + (\langle 0|T|0\rangle\; g_0 \langle 0|T|0\rangle_1)^c\,, \qquad (2b)$$

where the superscript c indicates that we should consider connected diagrams, as only these contribute to N-N scattering. This equation has the form of the two particle Lippmann-Schwinger equation if we take $\langle 0|T|0\rangle_1$ to be the two-body potential. However, we note at this stage that the first term on the right-hand side of Eq. (2) involves only connected diagrams, while the $\langle 0|T|0\rangle_1$ in the second term can involve disconnected diagrams. In Fig. 1 we give an example of diagrams belonging to the second term on the right-hand side of Eq. (2). Such diagrams, which were ignored by Thomas and Rinat,[9] give rise to propagator dressing. Finally in Eq. (2) we do not have any statistical factors since our nucleons at this stage are distinguishable. Later we may perform the antisymmetrization on the final equations. This procedure will simplify the dressing of the πNN form factor.

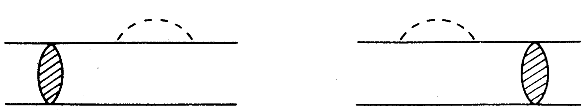

FIG. 1. Examples of diagrams retained in the classification of $\langle 0|T|0\rangle^c$ which lead to the dressing of the N-N propagator.

To get an amplitude that satisfies a Lippmann-Schwinger type equation, we find it necessary to consider the class of diagrams for N-N scattering that excludes the processes such as those shown in Fig. 2. This is achieved in two stages. We first divide all diagrams belonging to $\langle 0|T|0\rangle^c$ into two classes:

(a) Those diagrams which when cut at the first N-N intermediate state give rise to the right-hand part of the diagram being disconnected. We will call these diagrams right (or initial state) reducible. Examples of such diagrams are given in Fig. 2 and arise from the second term on the right-hand side of Eq. (2b). All these diagrams are of the form $\langle 0|T|0\rangle^c\, g_0\, \Gamma_N$ where Γ_N is to be determined later. At this stage we note that diagram 2(d) includes diagram 2(b), and diagram 2(c) may be excluded if we do not take two-pion intermediate states into consideration.

(b) Those diagrams not included in (a) (which we refer to as right irreducible), i.e., those when cut at the first N-N intermediate state give a connected right-hand part, are represented by $\langle 0|T|\tilde{0}\rangle$.

We now have

$$\langle 0|T|0\rangle^c = \langle 0|T|\tilde{0}\rangle + \langle 0|T|0\rangle^c\, g_0\, \Gamma_N \qquad (3)$$

or

$$\langle 0|T|\tilde{0}\rangle = \langle 0|T|0\rangle^c\,(1 - g_0\Gamma_N) \qquad (4a)$$

$$= \langle 0|T|0\rangle^c\, g_0 g^{-1}\,, \qquad (4b)$$

where g, the dressed N-N propagator, is given by

$$g = (g_0^{-1} - \Gamma_N)^{-1}\,. \qquad (5)$$

In the absence of mass renormalization g and g_0 have the same pole. Thus on-shell $g/g_0 = 1$.

We now repeat the above procedure for the left-hand side of $\langle 0|T|\tilde{0}\rangle$ by dividing this class of diagrams according to their final state reducibility.

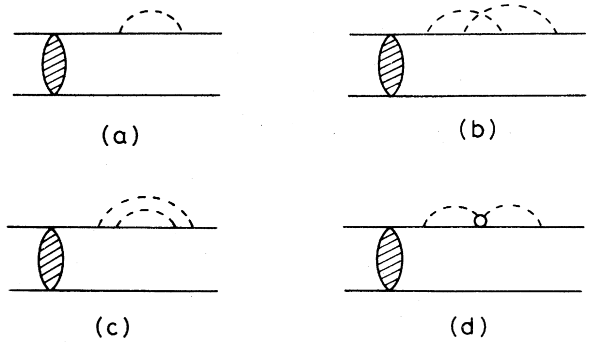

FIG. 2. Examples of right-reducible diagrams that are subtracted from $\langle 0|T|0\rangle^c$ to give $\langle 0|T|\tilde{0}\rangle$.

This allows us to write

$$\langle 0|T|\tilde{0}\rangle = \langle\tilde{0}|T|\tilde{0}\rangle + \Gamma_N g_0 \langle 0|T|\tilde{0}\rangle \tag{6}$$

or

$$\langle\tilde{0}|T|\tilde{0}\rangle = (1-\Gamma_N g_0)\langle 0|T|\tilde{0}\rangle \tag{7a}$$

$$= (1-\Gamma_N g_0)\langle 0|T|0\rangle^c (1-g_0\Gamma_N) \tag{7b}$$

$$= g^{-1} g_0 \langle 0|T|0\rangle^c g_0 g^{-1} . \tag{7c}$$

We observe that on-shell $\langle\tilde{0}|T|\tilde{0}\rangle = \langle 0|T|0\rangle^c$, i.e., the physical amplitude obtained from $\langle\tilde{0}|T|\tilde{0}\rangle$ is identical to that which one gets from $\langle 0|T|0\rangle^c$. We now divide the class of diagrams $\langle\tilde{0}|T|\tilde{0}\rangle$ according to the number of pions in intermediate states: Thus the class of all diagrams with at lease one pion in every intermediate state we represent by $\langle 0|T|0\rangle_1^c$, while the rest of the diagrams belonging to $\langle\tilde{0}|T|\tilde{0}\rangle$, but not to $\langle 0|T|0\rangle_1^c$, have at least one intermediate state of zero pions, and can be written using the last (first) cut lemma as $\langle 0|T|0\rangle_1^c g_0 \langle 0|T|\tilde{0}\rangle = \langle 0|T|0\rangle_1^c g \langle\tilde{0}|T|\tilde{0}\rangle$ ($\langle\tilde{0}|T|0\rangle g_0 \langle 0|T|0\rangle_1^c = \langle\tilde{0}|T|\tilde{0}\rangle g \langle 0|T|0\rangle_1^c$) or

$$\langle\tilde{0}|T|\tilde{0}\rangle = \langle 0|T|0\rangle_1^c + \langle 0|T|0\rangle_1^c g \langle\tilde{0}|T|\tilde{0}\rangle \tag{8a}$$

$$= \langle 0|T|0\rangle_1^c + \langle\tilde{0}|T|\tilde{0}\rangle g \langle 0|T|0\rangle_1^c . \tag{8b}$$

In Eq. (8) we have a two-body Lippmann-Schwinger type equation with the effective N-N potential given by $\langle 0|T|0\rangle_1^c$ and all intermediate states having dressed nucleon propagators. In effect what we have shown above is that if we remove all bubbles from all external nucleon lines then the nucleons in all initial, final, and intermediate states are dressed.

So far we have exposed the two-particle unitarity cut, and that part of the three-particle unitarity cut that is due to the dressing of the N-N propagator. To get the rest of the three-particle unitarity we need to expose three-particle intermediate states in $\langle 0|T|0\rangle_1$. Here again we classify the diagrams belonging to $\langle 0|T|0\rangle_1$ into two classes—those that have at least two pions in every intermediate state $\langle 0|T|0\rangle_2$, and the rest. To the rest we can apply the last (first) cut lemma to get

$$\langle 0|T|0\rangle_1 = \langle 0|T|0\rangle_2^c + \langle 0|T|1\rangle_2 G_0 \langle 1|T|0\rangle_1 \tag{9a}$$

$$= \langle 0|T|0\rangle_2^c + \langle 0|T|1\rangle_1 G_0 \langle 1|T|0\rangle_2 , \tag{9b}$$

where G_0 is the free Green's function for the πNN system with the nucleons undressed. In writing Eq. (9) we have achieved our aim of exposing three-body intermediate states and thus including three-body unitarity. In the process we have introduced off-shell amplitudes for $NN \leftarrow \pi NN$ ($\pi NN \leftarrow NN$) with all intermediate states having at least two pions, $\langle 0|T|1\rangle_2$ ($\langle 1|T|0\rangle_2$) and the off-shell amplitude for $\pi NN \leftarrow NN$ ($NN \leftarrow \pi NN$) with all intermediate states having at least one pion, $\langle 1|T|0\rangle_1$ ($\langle 0|T|1\rangle_1$). If we don't want to include four-body unitarity then $\langle 0|T|0\rangle_2$, which is the amplitude for $NN \to NN$ with at least two pions in every intermediate state, can be replaced by a static heavy boson exchange potential as suggested by Thomas and Rinat.[9] In a similar manner if we don't want to include the contribution of four-body unitarity through $\langle 0|T|1\rangle_2$ then the simplest representation for $\langle 0|T|1\rangle_2$ is[9]

$$\langle 0|T|1\rangle_2 = \sum_{i=1}^{2} f_0^+(i) = f_0^+ , \tag{10}$$

which is represented diagrammatically in Fig. (3), i.e., $f_0^+(i)$ is the form factor for $N \leftarrow \pi N$ with the ith nucleon absorbing the pion. Note here that f_0^+ is not the fully dressed πNN form factor. Thus we cannot extract f_0^+ from experiment. The dressed πNN form factor has a major contribution from the Feynman diagram in Fig. 4(a)[24] which in time ordered perturbation theory is given in Fig. 4(b), and has an intermediate state of one pion. We will see how the inclusion of the P_{11} interaction gives a dressing of the πNN form factor which includes Fig. 4(b).

We now turn to the amplitude $\langle 1|T|0\rangle_1$ which has intermediate states of at least one pion. The diagrams belonging to $\langle 1|T|0\rangle_1$ can be divided into two classes, those with at least two pions in every intermediate state $\langle 1|T|0\rangle_2$ and the rest, which can be written as $\langle 1|T|1\rangle_1 G_0 \langle 1|T|0\rangle_2$, where $\langle 1|T|1\rangle_1$ includes all diagrams with initial and final πNN state. However, $\langle 1|T|1\rangle_1$ is reducible in that it includes a class of diagrams which when cut at first (last) intermediate state leads to the right- (left-) hand part of the diagram consisting of three disconnected parts. Examples of such diagrams are presented in Fig. 5. As shown above for $\langle 0|T|0\rangle^c$, the right- (left-) irreducible diagrams are given by $\langle\tilde{1}|T|\tilde{1}\rangle_1$ and are related to $\langle 1|T|1\rangle_1$ by

$$\langle\tilde{1}|T|\tilde{1}\rangle_1 = (1-\Gamma_N G_0)\langle 1|T|1\rangle_1 (1-G_0\Gamma_N) \tag{11a}$$

$$= \langle\tilde{1}|T|1\rangle_1 (1-G_0\Gamma_N) . \tag{11b}$$

In a similar manner the class of left-irreduc-

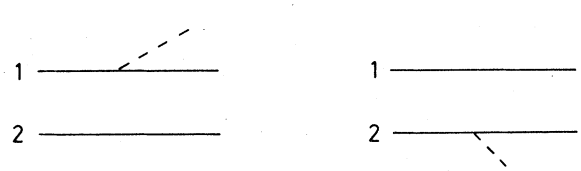

FIG. 3. The NN vertex as the lowest order approximation to $\langle 0|T|1\rangle_2$.

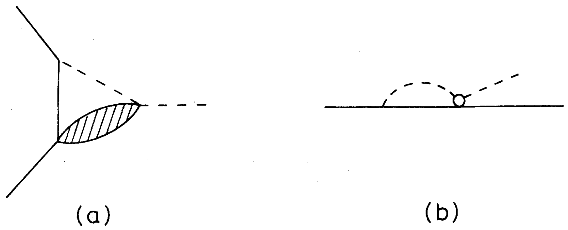

FIG. 4. (a) Feynman diagram that contributes to the πNN form factor. (b) The equivalent time ordered diagram.

ible diagrams belonging to $\langle 1|T|0\rangle_1$ is given by $\langle\tilde{1}|T|0\rangle_1$, where

$$\langle\tilde{1}|T|0\rangle_1=(1-\Gamma_N G_0)\langle 1|T|0\rangle_1 \tag{12a}$$

$$=G^{-1}G_0\langle 1|T|0\rangle_1\ , \tag{12b}$$

where G is the πNN propagator with dressed nucleons, i.e.,

$$G=(G_0{}^{-1}-\Gamma_N)^{-1}\ . \tag{13}$$

Similarly,

$$\langle 0|T|\tilde{1}\rangle_1=\langle 0|T|1\rangle_1 G_0 G^{-1}\ . \tag{14}$$

We now can write Eq. (9) as

$$\langle 0|T|0\rangle_1=\langle 0|T|0\rangle_2+\langle 0|T|1\rangle_2 G\langle\tilde{1}|T|0\rangle_1 \tag{15a}$$

$$=\langle 0|T|0\rangle_2+\langle 0|T|\tilde{1}\rangle_1 G\langle 1|T|0\rangle_2\ , \tag{15b}$$

where we have made use of Eq. (12b) to replace G_0 by the corresponding propagator with dressed nucleons.

Using the classification of the diagrams according to the number of pions in every intermediate state and the last (initial) cut lemma we can write

$$\langle\tilde{1}|T|0\rangle_1=\langle 1|T|0\rangle_2+\langle\tilde{1}|T|1\rangle_1 G_0\langle 1|T|0\rangle_2 \tag{16a}$$

$$=\langle 1|T|0\rangle_2+\langle\tilde{1}|T|\tilde{1}\rangle_1 G\langle 1|T|0\rangle_2\ , \tag{16b}$$

$$\langle 0|T|\tilde{1}\rangle_1=\langle 0|T|1\rangle_2+\langle 0|T|1\rangle_2 G_0\langle 1|T|\tilde{1}\rangle_1 \tag{17a}$$

$$=\langle 0|T|1\rangle_2+\langle 0|T|1\rangle_2 G\langle\tilde{1}|T|\tilde{1}\rangle_1\ . \tag{17b}$$

In replacing $\langle 1|T|1\rangle_1$ by the class of right- (left-)

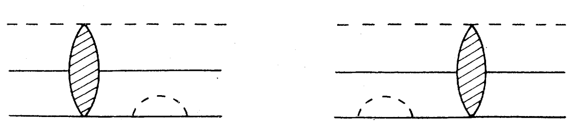

FIG. 5. Examples of right- and left-reducible diagrams belonging to $\langle 1|T|1\rangle_1$; subtraction of these results in $\langle\tilde{1}|T|\tilde{1}\rangle_1$.

irreducible diagrams $\langle\tilde{1}|T|\tilde{1}\rangle_1$ we have replaced the free three-body Green's function with bare nucleons G_0 by the free three-body propagator with dressed nucleons G. In this way we have guaranteed that the nucleons in the πNN channel are identical to those in the N-N channel. This feature which is not present in the formulation of Avishai and Mizutani[11] comes about by including certain two-pion intermediate states. In the process we have included some contribution from four-body unitarity.

The right- (left-) irreducible diagrams for $\pi NN \to \pi NN$ can now be classified according to the number of intermediate pions and the last cut lemma as

$$\langle\tilde{1}|T|\tilde{1}\rangle_1=\langle 1|T|1\rangle_2+\langle 1|T|1\rangle_2 G_0\langle 1|T|\tilde{1}\rangle_1 \tag{18a}$$

$$=\langle 1|T|1\rangle_2+\langle 1|T|1\rangle_2 G\langle\tilde{1}|T|\tilde{1}\rangle_1\ . \tag{18b}$$

The lowest order diagrams belonging to $\langle 1|T|1\rangle_2$ are presented in Fig. 6. Here we observe that Fig. 6(a) and 6(b) correspond to the lowest order N-N and π-N interaction, while Fig. 6(c) is a "three-body force." In the present analysis we have dropped the contribution of Fig. 6(c), since it has problems associated with the fact that it is disconnected. This diagram, which has been discussed by Stelbovics and Stingl, contributes to further dressing of nucleon propagator and πNN vertex. If we now replace Fig. 6(a) and 6(b) by the N-N and π-N potentials then $\langle\tilde{1}|T|\tilde{1}\rangle_1$ becomes the $3\to 3$ amplitude for a pure three-body (πNN) system with no absorption. However, in this three-body system the π-N interaction in the P_{11} channel is not the full interaction since it does not include the diagram in Fig. 7, which has the nucleon pole in the P_{11} amplitude. It is important to note that at this stage we have replaced a Hamiltonian in which the only interaction is through the πNN ver-

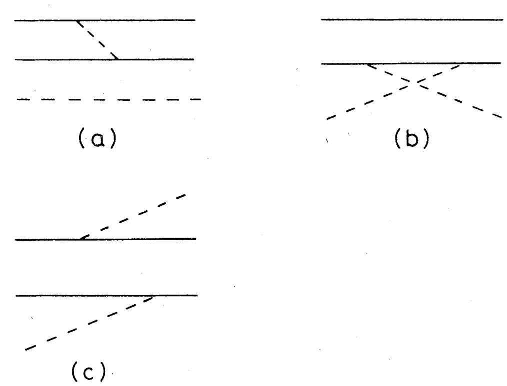

FIG. 6. Lowest order diagrams belonging to $\langle 1|T|1\rangle_2$: (a) represents the lowest order N-N interaction, (b) represents the lowest order π-N interaction, and (c) contributes to the three-body force.

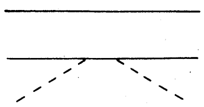

FIG. 7. This type of diagram is due to the pole of the full P_{11} t matrix. In our model we exclude such diagrams from the class $\langle\tilde{1}|T|\tilde{1}\rangle_1$ by including only the nonpole part of the P_{11} t matrix.

tex (as employed by Stelbovics and Stingl[12]) with one which has π-N and N-N interactions as well as the πNN vertex.[6,8,9,11] We now can write $\langle\tilde{1}|T|\tilde{1}\rangle_1$ in terms of the Faddeev amplitude[25] for $3\rightarrow 3$ as

$$\langle\tilde{1}|T|\tilde{1}\rangle_1 \equiv M=\sum_{\lambda\mu}(t_\lambda\delta_{\lambda\mu}+t_\lambda GU_{\lambda\mu}Gt_\mu)\ , \tag{19}$$

where t_λ is the two-body T matrix for either the π-N or N-N subsystems, and $U_{\lambda\mu}$ are the AGS (Alt, Grassberger, and Sandhas) amplitudes[26] which satisfy the equations

$$U_{\lambda\mu}=G^{-1}\bar{\delta}_{\lambda\mu}+\sum_\nu \bar{\delta}_{\lambda\nu}t_\nu GU_{\nu\mu} \tag{20a}$$

$$=G^{-1}\bar{\delta}_{\lambda\mu}+\sum_\nu U_{\lambda\nu}Gt_\nu\bar{\delta}_{\nu\mu}\ , \tag{20b}$$

where $\bar{\delta}_{\lambda\mu}\equiv 1-\delta_{\lambda\mu}$. In Eqs. (19) and (20) G is the free propagator for the πNN system with dressed nucleons.

Using the results of Eq. (19) in Eqs. (16) and (17) we get

$$\langle\tilde{1}|T|0\rangle_1=(1+MG)f_0\ , \tag{21a}$$

$$\langle 0|T|\tilde{1}\rangle_1=f_0^\dagger(1+GM)\ . \tag{21b}$$

Substituting Eqs. (10) and (21) in Eq. (15) we get for $\langle 0|T|0\rangle_1$ from Eq. (9) to be

$$\langle 0|T|0\rangle_1=\langle 0|T|0\rangle_2+f_0^\dagger(G+GMG)f_0\ , \tag{22}$$

where the connected part of $\langle 0|T|0\rangle_1$ is the effective N-N interaction [see Eq. (8)]. To get an explicit expression for $\langle 0|T|0\rangle_1^c$ in the present model we make use of Eq. (19) to write $\langle 0|T|0\rangle_1$ as

$$\langle 0|T|0\rangle_1=\langle 0|T|0\rangle_2+f_0^\dagger\left(G+G\sum_\lambda t_\lambda G\right)f_0 + \sum_{\lambda\mu} f_0^\dagger Gt_\lambda GU_{\lambda\mu}Gt_\mu Gf_0\ . \tag{23}$$

The disconnected part of $\langle 0|T|0\rangle_1$ comes from the second term on the right-hand side of Eq. (23) and corresponds to the diagrams in Fig. 8. Furthermore, it is this disconnected part of $\langle 0|T|0\rangle_1$ which gives rise to the propagator dressing as is clear from Eqs. (2) and (3). This means we can write $\langle 0|T|0\rangle_1$ as a connected part, which is the effective N-N interaction to be used in Eq. (8), and the disconnected part Γ_N involved in dressing the nucleons in both the N-N and πNN propagators. To get an explicit expression for $\langle 0|T|0\rangle_1^c$ and Γ_N we need to introduce particle labels. We will refer to the two nucleons as particles 1 and 2 while the pion is referred to as particle 3. We then let the indices $i,j,\ldots$ run over the two nucleons, while $\lambda,\mu,\nu,\ldots$ run over all three particles. Furthermore we depart from the usual notation by having $\lambda=i$ label the interaction of the pion with the ith nucleon and $\lambda=3$ refer to the N-N interaction. We now can write $\langle 0|T|0\rangle_1$ as a connected plus a disconnected part, i.e.,

FIG. 8. These diagrams make up the disconnected part of $\langle 0|T|0\rangle_1$. They give rise to propagator dressing.

$$\langle 0|T|0\rangle_1=\langle 0|T|0\rangle_1^c+\Gamma_N\ , \tag{24}$$

where

$$\Gamma_N=\sum_{i=1}^{2} f_0^\dagger(i)(G+Gt_iG)f_0(i) \tag{25}$$

and

$$\begin{aligned}\langle 0|T|0\rangle_1^c=\langle 0|T|0\rangle_2^c&+\sum_{ij}\bar{\delta}_{ij}f_0^\dagger(i)Gf_0(j)\\ &+\sum_{ij}\sum_\lambda f_0^\dagger(i)Gt_\lambda Gf_0(j)\\ &-\sum_i f_0^\dagger(i)Gt_iGf_0(i)\\ &+\sum_{ij}\sum_{\lambda\mu} f_0^\dagger(i)Gt_\lambda GU_{\lambda\mu}Gt_\mu G\,f_0(j)\ .\end{aligned} \tag{26}$$

The π-N interaction t_i in Eqs. (25) and (26) is restricted by isospin and angular momentum conservation to the P_{11} channel. Furthermore, it is the nonpole part of the amplitude that is included in t_i.

The above expression for the effective N-N interaction $\langle 0|T|0\rangle_1^c$ can be further simplified, and in particular the πNN form factor $f_0(i)$ can be replaced by the corresponding dressed form factor $f(i)$ given by

$$f(i)=f_0(i)+t_iGf_0(i)\ . \tag{27}$$

Here again the only contribution from t_i is the nonpole part of the π-N interaction in the P_{11} channel. In the Appendix we give a precise procedure, first suggested by Mizutani and Koltun,[6] for dividing the P_{11} amplitude into its pole and nonpole part. Furthermore, this procedure leads to a dressing of both the πNN form factor and nucleon propagator that is identical to the results of Eqs. (5), (13), and (27).

Using Eq. (20) for the AGS amplitudes in Eq.

(26) and the definition of the dressed πNN form factor, Eq. (27), allows us to rewrite $\langle 0|T|0\rangle_1^c$ as

$$\langle 0|T|0\rangle_1^c = V_{NN} + \sum_{ij}\sum_{\lambda\mu} f^+(i)\bar{\delta}_{i\lambda}Gt_\lambda G(\delta_{\lambda\mu}+U_{\lambda\mu}Gt_\mu G)\bar{\delta}_{\mu j}f(j)\,, \tag{28}$$

where

$$V_{NN}=\langle 0|T|0\rangle_2+\sum_{ij}\bar{\delta}_{ij}f^+(i)Gf(j)\,. \tag{29}$$

If we assume $\langle 0|T|0\rangle_2$ to be the contribution of heavy boson exchange[9] to the N-N potential then V_{NN} is the one boson exchange potential with dressed form factor for the πNN vertex and the threshold for pion production built into the potential. In other words V_{NN} becomes complex above the pion production threshold to account for the opening of an inelastic channel. The rest of the contribution to the effective N-N interaction $\langle 0|T|0\rangle_1^c$ comes from the diagrams in Fig. 9 and their iterates. We now can write the Lippmann-Schwinger equation for N-N scattering as

$$\langle\tilde{0}|T|\tilde{0}\rangle \equiv T_{NN}=\left[V_{NN}+\sum_{ij}\sum_{\lambda\mu} f^+(i)\bar{\delta}_{i\lambda}Gt_\lambda G(\delta_{\lambda\mu}+U_{\lambda\mu}Gt_\mu G)\bar{\delta}_{\mu j}f(j)\right](1+gT_{NN})\,. \tag{30}$$

Although $\langle 0|T|0\rangle_1^c$ gives us an effective N-N interaction that includes the effect of pion production and the final result satisfies two- and three-body unitarity, the evaluation of such a potential is by no means simple. This is mainly due to the fact that we need the fully off-shell πNN amplitudes $U_{\lambda\mu}$. To overcome this problem we eliminate the $U_{\lambda\mu}$ in Eq. (30) by formally solving the AGS equations. We then can recast Eq. (30) into a set of coupled integral equations for the physical amplitudes for N-N elastic scattering and pion production. To get the coupling to the pion production channel we need to examine the amplitude for $\pi NN \leftarrow NN$, $\langle\tilde{1}|T|\tilde{0}\rangle$. Employing our classification scheme according to the number of pions in every intermediate state and the last cut lemma we have

$$\langle\tilde{1}|T|\tilde{0}\rangle=\langle\tilde{1}|T|0\rangle_1+\langle\tilde{1}|T|0\rangle g_0\langle 0|T|\tilde{0}\rangle \tag{31a}$$

$$=\langle\tilde{1}|T|0\rangle_1(1+gT_{NN})\,. \tag{31b}$$

Before we proceed any further we need to write $\langle\tilde{1}|T|0\rangle_1$ in terms of the dressed πNN form factor. We have that

$$\langle\tilde{1}|T|0\rangle_1=(1+MG)f_0 = \left(1+\sum_\lambda t_\lambda G+\sum_{\lambda\mu} t_\lambda GU_{\lambda\mu}Gt_\mu G\right)\sum_j f_0(j)\,. \tag{32}$$

Using the AGS equations we can simply rewrite this as

$$\langle\tilde{1}|T|0\rangle_1=\sum_j f(j) + \sum_{j\lambda\mu} t_\lambda G(\delta_{\lambda\mu}+U_{\lambda\mu}Gt_\mu G)\bar{\delta}_{\mu j}f(j)\,. \tag{33}$$

We note here that the disconnected part of $\langle\tilde{1}|T|0\rangle_1$ is nothing but the dressed πNN form factor. Equation (31) can now be written in terms of the dressed πNN form factor as

$$\langle\tilde{1}|T|0\rangle=\left[\sum_j f(j) + \sum_j\sum_{\lambda\mu} t_\lambda G(\delta_{\lambda\mu}+U_{\lambda\mu}Gt_\mu G)\bar{\delta}_{\mu j}f(j)\right]\Omega_N \tag{34}$$

with

$$\Omega_N=1+gT_{NN}\,. \tag{35}$$

To get the physical amplitude for $N\Delta \leftarrow NN$ or $\pi d \leftarrow NN$ we need to take the left-hand residue of $\langle\tilde{1}|T|\tilde{0}\rangle$ at the appropriate resonance or bound state pole. To achieve this we have to write the two-body T matrix as

$$t_\lambda(E)=\frac{|\phi_\lambda\rangle\langle\phi_\lambda|}{E-\epsilon_\lambda}+t'_\lambda(E)\,, \tag{36}$$

where ϵ_λ and ϕ_λ are the position and form factor associated with the pole, while $t'_\lambda(E)$ is the rest of the T matrix, which has no poles in the physical energy plane. Using this result and the definition of M in Eq. (19) we get

$$^{\lambda R}\langle\tilde{1}|T|\tilde{0}\rangle=\langle\phi_\lambda|G\sum_{j\mu}(\delta_{\lambda\mu}+U_{\lambda\mu}Gt_\mu G)\bar{\delta}_{\mu j}f(j)\Omega_N|\chi_N\rangle \equiv\langle\phi_\lambda|GT_{\lambda N}|\chi_N\rangle\,, \tag{37}$$

where $^{\lambda R}\langle\tilde{1}|$ means taking the left-hand residue in channel λ, and

$$T_{\lambda N}=\sum_{j\mu}(\delta_{\lambda\mu}+U_{\lambda\mu}Gt_\mu G)\bar{\delta}_{\mu j}f(j)\Omega_N\,. \tag{38}$$

FIG. 9. The effective N-N potential $\langle 0|T|0\rangle_1$ consists of a one boson exchange part together with these diagrams and their iterates.

Here χ_N is the plane wave for the two nucleon system which we have so far suppressed. We now make use of Eq. (38) to extend the definition of $T_{\lambda N}$ to all two-body channels λ for which there is no physical amplitude. With the help of Eq. (38) we can write Eq. (30) as

$$T_{NN}=V_{NN}(1+gT_{NN})+\sum_{i\lambda} f^+(i)\bar{\delta}_{i\lambda}Gt_\lambda GT_{\lambda N}\,. \quad (39)$$

Although Eqs. (38) and (39) form a set of coupled integral equations for the elastic N-N and pion production amplitudes, their solution is complicated by the presence of $U_{\lambda\mu}$, the three-body amplitude for the πNN system. This problem is easily overcome by solving Eq. (20) for $U_{\lambda\mu}$ and substituting in Eq. (38). This is most simply achieved by writing Eq. (20) in matrix form as

$$\underline{U}=G^{-1}\underline{\mathcal{g}}+\underline{\mathcal{g}}\,\underline{t}G\underline{U}\,, \quad (40)$$

which has a formal solution of the form

$$\underline{U}=(\underline{I}-\underline{\mathcal{g}}\,\underline{t}G)^{-1}\underline{\mathcal{g}}G^{-1}\,, \quad (41)$$

where $\underline{I}$ is the unit matrix,

$$[\underline{\mathcal{g}}]_{\lambda\mu}=\bar{\delta}_{\lambda\mu} \quad\text{and}\quad [\underline{t}]_{\lambda\mu}=\delta_{\lambda\mu}t_\lambda\,. \quad (42)$$

We now can write Eq. (38) as

$$\underline{T}_N=(\underline{I}+\underline{U}G\underline{t}G)\underline{F}\Omega_N\,, \quad (43)$$

where $\underline{T}_N$ and $\underline{F}$ are column matrices defined by

$$[\underline{T}_N]_\lambda=T_{\lambda N}, \quad\text{and}\quad [\underline{F}]_\lambda=\sum_{j=1}^{2}\bar{\delta}_{\lambda j}f(j)\,. \quad (44)$$

Using Eq. (41) in Eq. (43) and multiplying from the left by $(\underline{I}-\underline{\mathcal{g}}\,\underline{t}G)$ we get

$$\underline{T}_N=\underline{F}\Omega_N+\underline{\mathcal{g}}\,\underline{t}G\underline{T}_N \quad (45)$$

or

$$T_{\lambda N}=\sum_j \bar{\delta}_{\lambda j}f(j)(1+gT_{NN})+\sum_\mu \bar{\delta}_{\lambda\mu}t_\mu GT_{\mu N}\,. \quad (46)$$

Equations (39) and (46) are a set of linear coupled integral equations for T_{NN} (the elastic N-N amplitude) and $T_{\lambda N}$ (production amplitude). The input to the equations are the dressed πNN form factor and the π-N and N-N amplitudes. These equations reduce to those of Thomas and Rinat,[9] if we (i) assume the two-body π-N and N-N interactions are separable, (ii) exclude the nonpole part of the π-N interaction in the P_{11} channel, and (iii) ignore all propagator and form factor dressing. On comparing our results with those of Avishai and Mizutani[11] we find our equations reduce to theirs in the absence of the nonpole P_{11} amplitude and the dressing of the nucleons in the πNN propagator. It is important to note that in our present formulation the nucleons in the πNN channel are identical to those in the N-N channel, which is not the case in the work of Avishai and Mizutani.[11]

III. PION-DEUTERON SCATTERING

We now turn to the π-d system and apply the same procedure and approximations as was the case for N-N scattering. Here we consider the class of right- (left-) irreducible diagrams for $\pi NN\to\pi NN$, $\langle\tilde{1}|T|\tilde{1}\rangle$. These diagrams can be split into two classes—those with at least one pion in every intermediate state $\langle\tilde{1}|T|\tilde{1}\rangle_1$, and the rest, to which we can apply the last (first) cut lemma to expose the two-body unitarity cut. This gives

$$\langle\tilde{1}|T|\tilde{1}\rangle=\langle\tilde{1}|T|\tilde{1}\rangle_1+\langle\tilde{1}|T|0\rangle_1 g\langle\tilde{0}|T|\tilde{1}\rangle \quad (47a)$$

$$=\langle\tilde{1}|T|\tilde{1}\rangle_1+\langle\tilde{1}|T|\tilde{0}\rangle g\langle 0|T|\tilde{1}\rangle_1\,, \quad (47b)$$

where we have made use of the fact that $g\langle\tilde{0}|T|\tilde{1}\rangle=g_0\langle 0|T|\tilde{1}\rangle$. As in Sec. II we take $\langle\tilde{1}|T|\tilde{1}\rangle_1$ to be the $3\to 3$ amplitude for the πNN system, while for $\langle\tilde{1}|T|0\rangle_1$ and $\langle\tilde{1}|T|\tilde{0}\rangle$ we use the results of Eqs. (33) and (34). This allows us to write Eq. (47) as

$$\langle\tilde{1}|T|\tilde{1}\rangle=M+\left[f+\sum_i\sum_{\lambda\nu}(\delta_{\lambda\nu}t_\lambda+t_\lambda GU_{\lambda\nu}Gt_\nu)G\bar{\delta}_{i\nu}f(i)\right](g+gT_{NN}g)\left[f^++\sum_j\sum_{\rho\mu}f^+(j)\bar{\delta}_{j\rho}G(\delta_{\rho\mu}t_\mu+t_\rho GU_{\rho\mu}Gt_\mu)\right]. \quad (48)$$

To get the physical π-d elastic and rearrangement amplitudes we need to take right and left residues at the appropriate poles. Thus

$${}^{\lambda R}\langle\tilde{1}|T|\tilde{1}\rangle^{\mu R}\equiv\langle\phi_\lambda|GT_{\lambda\mu}G|\phi_\mu\rangle\,, \quad (49)$$

where

$$T_{\lambda\mu}=U_{\lambda\mu}+\sum_{i\nu}(\delta_{\lambda\nu}+U_{\lambda\nu}Gt_\nu G)\bar{\delta}_{i\nu}f(i)(g+gT_{NN}g)\times\sum_{j\rho}f^+(j)\bar{\delta}_{j\rho}(\delta_{\rho\mu}+Gt_\rho GU_{\rho\mu})\,. \quad (50)$$

Introducing a matrix notation for $T_{\lambda\mu}=[\underline{T}]_{\lambda\mu}$ and the definitions of Eqs. (42) and (44) we can write Eq. (50) as

$$\underline{T}=\underline{U}+(\underline{I}+\underline{U}\underline{G}t\underline{G})\underline{F}(g+gT_{NN}g)\underline{F}^{+}(\underline{I}+\underline{G}t\underline{G}\underline{U}) \tag{51a}$$

$$=\underline{U}+(\underline{I}+\underline{U}\underline{G}t\underline{G})\underline{F}g\underline{T}_N^{+} \tag{51b}$$

$$=\underline{U}+\underline{T}_N\, g\, \underline{F}^{+}\ (\underline{I}+\underline{G}t\underline{G}\underline{U})\ , \tag{51c}$$

where $\underline{T}_N$ is given in Eq. (43). Using Eq. (41) we can write Eq. (51.b) as

$$\underline{T}=(\underline{I}-\underline{\mathfrak{g}}\,\underline{t}\,\underline{G})^{-1}\,\underline{\mathfrak{g}}\,\underline{G}^{-1}+[\underline{I}+(\underline{I}-\underline{\mathfrak{g}}\,\underline{t}\underline{G})^{-1}\underline{\mathfrak{g}}\,\underline{t}\underline{G}]\underline{F}\ g\ \underline{T}_N^{+}\ . \tag{52}$$

Multiplying from the left by $(\underline{I}-\underline{\mathfrak{g}}\,\underline{t}\,G)$ we get

$$\underline{T}=\underline{\mathfrak{g}}\,\underline{G}^{-1}+\underline{F}\ g\ \underline{T}_N^{+}+\underline{\mathfrak{g}}\underline{t}\underline{G}\underline{T} \tag{53}$$

or

$$T_{\lambda\mu}=\bar{\delta}_{\lambda\mu}\ G^{-1}+\ \sum_j \bar{\delta}_{\lambda j}f(j)\ g\,T_{N\mu}$$
$$+\sum_{\nu}\bar{\delta}_{\lambda\nu}\ t_{\nu}\ G\,T_{\nu\mu}\ . \tag{54}$$

We now have to get an equation for T_N^{+} which completes the coupling between the elastic and absorption amplitudes. From Eq. (43) we have that

$$\underline{T}_N^{+}=\Omega_N^{+}\,\underline{F}^{+}(\underline{I}+\underline{G}\,\underline{t}\,g\,\underline{U})\ . \tag{55}$$

Substituting for $T_{NN}g$ from Eq. (39), which in matrix form is given by

$$T_{NN}=V_{NN}\ \Omega_N+\underline{F}^{+}\ \underline{G}\underline{t}\underline{G}\underline{T}_N\ , \tag{56}$$

into Eq. (55), and making use of Eq. (51c), we get

$$\underline{T}_N^{+}=\underline{F}^{+}+\underline{F}^{+}\underline{G}\underline{t}\underline{G}\underline{T}+\underline{V}_{NN}g\,\underline{T}_N^{+}\ , \tag{57}$$

or

$$T_{N\mu}=\ \sum_j\ \bar{\delta}_{j\mu}f^{+}(j)+\sum_{j\nu}f^{+}(j)\,\bar{\delta}_{j\nu}\ G\,t_{\nu}\ G\,T_{\nu\mu}$$
$$+V_{NN}\,g\,T_{N\mu}\ . \tag{58}$$

In Eqs. (54) and (58) we have a set of coupled linear integral equations for πd elastic scattering and pion absorption. Here again our results reduce to those of Avishai and Mizutani if we exclude the nonpole P_{11} interaction and drop the dressing in the πNN propagator.

We can combine the results of Eqs. (45), (52), (56), and (57) into a single 4×4 matrix equation of the form

$$\begin{bmatrix} T_{NN} & \underline{T}_N^{+} \\ \underline{T}_N & \underline{T} \end{bmatrix}=\begin{bmatrix} V_{NN} & \underline{F}^{+} \\ \underline{F} & \underline{G}^{-1}\underline{\mathfrak{g}} \end{bmatrix}\left\{1+\begin{bmatrix} g & \underline{0} \\ \underline{0} & \underline{G}\underline{t}\underline{G} \end{bmatrix}\ \begin{bmatrix} T_{NN} & \underline{T}_N^{+} \\ \underline{T}_N & \underline{T} \end{bmatrix}\right\}. \tag{59}$$

This demonstrates that the kernels of the integral equations for the reactions in Eqs. (1a) and(1b) are identical. Although we have included the nonpole part of the P_{11} channel, the kernel of the integral equation is connected and the equations can be used for practical calculations.[27]

We now assume separable π-N and N-N interactions for the subsystem, i.e.,

$$t_{\lambda}=|\phi_{\lambda}>\tau_{\lambda}<\phi_{\lambda}|\ , \tag{60}$$

and define the physical amplitudes as

$$X_{\lambda\mu}=<\phi_{\lambda}|G\,T_{\lambda\mu}\,G|\phi_{\mu}>\ ,$$
$$X_{N\mu}=<\chi_N|T_{N\mu}\ G|\phi_{\mu}>\ , \tag{61}$$
$$X_{\lambda N}=<\phi_{\lambda}|G\,T_{\lambda N}|\chi_N>\ ,$$

where χ_N is a plane wave for the two nucleons, and is an eigenstate of g. We can then write Eqs. (39) and (46) for NN scattering and pion production as

$$X_{NN}=V_{NN}\ (1+gX_{NN})+Z_{Nd}\ \tau_d X_{dN}$$
$$+\sum_j Z_{N\Delta_j}\ \tau_{\Delta_j}\ X_{\Delta_j N}\ , \tag{62a}$$

$$X_{\Delta_i N}=Z_{\Delta_i N}\ (1+gX_{NN})+Z_{\Delta_i d}\ \tau_d\ X_{dN}$$
$$+\sum_j\ Z_{\Delta_i\Delta_j}\ \tau_{\Delta_j}\ X_{\Delta_j N}\ , \tag{62b}$$

$$X_{dN}=Z_{dN}\,(1+gX_{NN})+\ \sum_j\ Z_{d\Delta_j}\ \tau_{\Delta_j}\ X_{\Delta_j N}\ , \tag{62c}$$

where (i,j) run over the two nucleons, Δ_i labels the appropriate π-N channel, and $Z_{\alpha\beta}$ are given by

$$Z_{Nd}=\sum_i\ \langle f(i)|G|\phi_d\rangle\ , \tag{63a}$$

$$Z_{N\Delta_j}=\ \sum_i\ \bar{\delta}_{ij}\ \langle f(i)|G|\phi_{\Delta_j}\rangle\ , \tag{63b}$$

$$Z_{\Delta_i d}=\langle\phi_{\Delta_i}|G|\phi_d\rangle\ , \tag{63c}$$

$$Z_{\Delta_i\Delta_j}=\bar{\delta}_{ij}\ \langle\phi_{\Delta_i}|G|\phi_{\Delta_j}\rangle\ . \tag{63d}$$

Similarly Eqs. (54) and (58), for π-d scattering and absorption, reduce to

$$X_{dd}=\ \sum_j Z_{d\Delta_j}\ \tau_{\Delta_j}\ X_{\Delta_j d}+Z_{dN}gX_{Nd}\ , \tag{64a}$$

$$X_{\Delta_i d}=Z_{\Delta_i d}\ \ (1+\tau_d\ X_{dd})+\sum_j\ Z_{\Delta_i\Delta_j}\ \tau_{\Delta_j}\ X_{\Delta_j d}$$
$$+Z_{\Delta_i N}\ g\ X_{Nd}\ , \tag{64b}$$

$$X_{Nd}=Z_{Nd}\ \ (1+\tau_d\ X_{dd})+\ \sum_j\ Z_{N\Delta_j}\ \tau_{\Delta_j}\ X_{\Delta_j d}$$
$$+V_{NN}\ g\ X_{Nd}\ . \tag{64c}$$

Although Eqs. (63) and (64) are similar in form to the equations of Afnan and Thomas,[5] they have the advantage that both nucleons can emit the pion, and the nucleons in the intermediate state are identical and satisfy the Pauli exculsion principle. Thus there are no undercounting problems. They differ from the equations of Thomas and Rinat in the following ways:

(i) They include π-d scattering as well as N-N scattering.

(ii) We do not have to assume separability of the two-body interactions in the π-N and N-N channels.

(iii) We have dressed the πNN form factor and nucleons in the N-N and πNN propagators. The latter will allow us to extract the form factor for the πNN vertex from experimental analyses of other systems.[28]

(iv) We have incorporated the full π-N interaction in the P_{11} channel, which is of importance in π-d elastic scattering.[27]

A comparison of our results with those of Avishai and Mizutani[11] shows that in the absence of the nonpole P_{11} interaction, the two sets of equation are identical, even though the derivations are quite different. However, in our case the inclusion of the nonpole P_{11} gives rise to dressing of the nucleons in both the NN and πNN propagators and the form factor for the πNN vertex, without a change in the basic structure of the equations other than the inclusion of the nonpole P_{11} as another π-N channel. This considerably simplifies the equations that include the full P_{11} as compared to those of Avishai and Mizutani.[11]

IV. CONCLUSION

In the present investigation we have shown that we can write a set of coupled linear integral equations for the physical amplitudes corresponding to the reactions in Eq. (1). These equations, similar in form to the Faddeev equations, include two- and three-body unitarity. The input to these equations are the dressed form factor for the πNN vertex, and the π-N and N-N interactions in the form of potentials, or two-body amplitudes. If we determine the πNN vertex from n-p and $\bar{p}$-p data,[28] then the only uncertainty in the input is the off-shell behavior of the π-N and N-N amplitudes. If needed, we can include the effect of multipion intermediate states by introducing potentials. Thus to include the effect of $(\rho, \omega, \ldots)$ mesons in N-N scattering one can replace $\langle 0|T|0\rangle_2$ by a potential that describes the exchange of these mesons. One can also introduce ρ exchange due to the diagrams in Fig. 10 which might be important in pion production. If one considers the ρ as a two-pion state term then Fig. 10(a) will be part of $\langle 1|T|0\rangle_2$, which at the present time we have taken to be the πNN vertex, while Fig. 10(b) comes as a three-body force in the analysis of $\langle 1|T|1\rangle_2$ in Eq. (18).

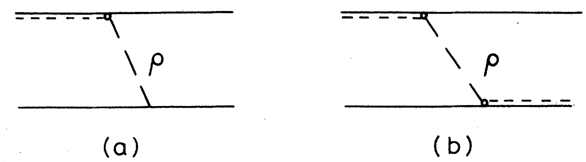

FIG. 10. If we assume that the meson is a two-pion system, then the contribution of ρ exchange to $\langle 1|T|0\rangle_2$ and $\langle 1|T|1\rangle_2$ is given by (a) and (b), respectively.

Furthermore, we have shown how one can relate the π-N and N-N interaction, involved in calculating the pure three-body πNN amplitude, with the more fundamental Hamiltonian involving the πNN vertex only. Finally by dressing the nucleons in both the N-N and πNN channel we have guaranteed that all nucleons are identical.

These equations are presently being used to study the following:

(i) The importance of pion production in N-N scattering, and in particular the determination of the imaginary part of the N-N phase shifts above the production threshold;

(ii) the effect of real absorption on pion-deuteron and possibily pion-nucleus scattering; and

(iii) the role of ρ-exchange in pion production and π-d elastic scattering.

Finally, since we have not specified the form of the two- and three-body Green's functions, we can include relativistic kinematics for both the pion and nucleons by proper choice of the propagator.

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APPENDIX

In this appendix we show how we can divide the π-N amplitude in the P_{11} channel into a pole part and a nonpole part. The result was previously obtained by Mizutani and Koltun[6] using Feshbach projection operators. We would like to rederive this decomposition of the P_{11} amplitude using our diagrammatic method in order to establish the connection with our results in Sec. II.

For the π-N system we restrict all states to just one nucleon and label the state by the number of pions. Thus the amplitude for π-N scattering is $\langle \tilde{1}|t|1\rangle$. We now use the classification scheme used previously to expose the zero-pion intermediate state, i.e.,

$$\langle\tilde{1}|t|\tilde{1}\rangle=\langle\tilde{1}|t|\tilde{1}\rangle_1+\langle\tilde{1}|t|0\rangle_1 g_0\langle 0|t|\tilde{1}\rangle, \tag{A1}$$

where we have used g_0 in Sec. II to label the single nucleon free propagator. Since the π-N amplitude will be used in a calculation which involves two nucleons we can think in terms of the other nucleon being a spectator at all times in our discussion of π-N scattering. In that case, g_0 is the free two nucleon propagator. In Eq. (A1), $\langle\tilde{1}|t|0\rangle_1$ is the amplitude for $\pi N\leftarrow N$ with at least one pion in every intermediate state, while $\langle 0|t|\tilde{1}\rangle$ is the full amplitude for $N\leftarrow\pi N$. This latter amplitude can be written using our classification scheme as

$$\langle 0|t|\tilde{1}\rangle=\langle 0|t|\tilde{1}\rangle_1+\langle 0|t|0\rangle g_0\langle 0|t|\tilde{1}\rangle_1 , \tag{A2}$$

where we have exposed the lowest unitarity cut. We now can write Eq. (A1) as

$$\langle\tilde{1}|t|\tilde{1}\rangle=\langle\tilde{1}|t|\tilde{1}\rangle_1+f\,gf^{+} , \tag{A3}$$

where the dressed propagator g and the dressed form factor f are given by

$$g=g_0+g_0\langle 0|t|0\rangle g_0 , \tag{A4}$$

$$f=\langle\tilde{1}|t|0\rangle_1 . \tag{A5}$$

Before we proceed to derive explicit expressions for f and g let us analyze the first term on the right-hand side of Eq. (A3), i.e., $\langle\tilde{1}|t|\tilde{1}\rangle_1$. This is the amplitude for $\pi N\leftarrow\pi N$ with at least one pion in every intermediate state, and can be written as

$$\langle\tilde{1}|t|\tilde{1}\rangle_1=\langle 1|t|1\rangle_2+\langle 1|t|1\rangle_2\, G\,\langle\tilde{1}|t|\tilde{1}\rangle_1 . \tag{A6}$$

In this way we have exposed the one-pion intermediate state. Here again we can think of G as the πNN free Green's function with one nucleon spectator at all times. Since we are not interested in exposing two-pion intermediate states, we will replace $\langle 1|t|1\rangle_2$ in Eq. (A6) by a static potential (e.g., that due to a ρ exchange). Thus for the present investigation, $\langle\tilde{1}|t|\tilde{1}\rangle_1$ can be considered as an amplitude obtained by solving a two-body equation for a potential.

We now turn to the form factor f. This we can write using the classification scheme as

$$\langle\tilde{1}|t|0\rangle_1=\langle 1|t|0\rangle_2+\langle\tilde{1}|t|\tilde{1}\rangle_1\, G\,\langle 1|t|0\rangle_2 , \tag{A7}$$

where we have exposed the one-pion intermediate state. The amplitude $\langle 1|t|0\rangle_2$ is identical to that defined in Eq. (9) which we referred to as the undressed form factor f_0. We thus can write the dressed form factor as

$$f=f_0+t^{NP}Gf_0 , \tag{A8}$$

where $t^{NP}\equiv\langle\tilde{1}|t|\tilde{1}\rangle_1$. We observe here that only the P_{11} amplitude contributes to the form factor dressing, and in fact, it is not the full P_{11} amplitude, but the part that can be described by potential scattering (t^{NP}).

Finally we need to get an equation for the dressed propagator g, which requires exposing the zero-pion states in $\langle 0|t|0\rangle$, i.e.,

$$\langle 0|t|0\rangle=\langle 0|t|0\rangle_1+\langle 0|t|0\rangle_1\, g_0\,\langle 0|t|0\rangle . \tag{A9}$$

Using this result in Eq. (A4) and iterating the equation once, we get an integral equation for the dressed nucleon propagator which is

$$g=g_0+g_0\langle 0|t|0\rangle_1\, g . \tag{A10}$$

The interaction that gives the dressing is $\langle 0|t|0\rangle_1$, which can be written as

$$\begin{aligned}\langle 0|t|0\rangle_1&=\langle 0|t|0\rangle_2+\langle 0|t|1\rangle_2\, G\langle\tilde{1}|t|0\rangle_1\\&=\langle 0|t|0\rangle_2+f_0^{+}Gf_0+f_0^{+}Gt^{NP}Gf_0 .\end{aligned} \tag{A11}$$

In writing the second line of Eq. (A11) we have made use of Eq. (A5), and the definition of f_0 and t^{NP}. In Eq. (A11) the amplitude $\langle 0|t|0\rangle_2$, which corresponds to diagrams with at least two pions inevery intermediate state, will be ignored in the present investigation. In this approximation Eq. (A11) takes the form

$$\langle 0|t|0\rangle_1\equiv\Gamma_N=f_0^{+}Gf_0+f_0^{+}Gt^{NP}Gf_0 \tag{A12}$$

and the dressed nucleon propagator becomes

$$g=g_0+g_0\Gamma_N\, g=(g_0^{-1}-\Gamma_N)^{-1} . \tag{A13}$$

We thus see that the π-N T matrix in the P_{11} channel is the sum of two terms [Eq. (A3)]. The first term can be represented by a t matrix that comes from a potential and has no pole at the nucleon mass, i.e., $t^{NP}=\langle\tilde{1}|t|\tilde{1}\rangle_1$. The second term has the nucleon pole in the dressed propagator g, and the residue at this pole is the dressed form factor f. It is the sum of these two terms (i.e., $\langle\tilde{1}|t|\tilde{1}\rangle$) that should fit the experimental π-N data as was originally pointed out by Mizutani and Koltun.[6]

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