PHYSICAL REVIEW C VOLUME 22, NUMBER 1 JULY 1980

Model for off-shell form factors and application to *NN* scattering

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(Received 23 April 1979)

A model is presented which describes any hadronic form factor with one, two, or three particles off shell. The model is based on the salient most successful features of the dual and Regge models, and its unitarization can be achieved in a straightforward manner. Some of the effects of taking the nucleons off shell in the πNN vertex are discussed. Finally, it is argued that this form factor together with a $\pi + 2\pi + \omega$ dispersion-theoretical nucleon-nucleon model can explain the long standing puzzle of the triplet D-wave phase shifts.

[NUCLEAR REACTIONS Baryon boson form factor, nucleon nucleon scattering, 0–400 MeV NN phase shifts.]

I. INTRODUCTION

There has been a great deal of progress in recent years in understanding the nucleon-nucleon interaction, particularly through the $(\pi+2\pi+\omega)$-exchange dispersion-theoretical potential models of the Stony Brook[1] and Paris[2] groups. However, a curious feature has emerged, i.e., the D-wave fits of the potential models are worse than the P-wave fits. In particular, the predictions for $\delta(^3D_1)$ and $\delta(^3D_2)$ disagree systematically with the data. This is illustrated in Fig. 1 where we plot the predictions of the potential models for $\delta(^3D_1)$, $\delta(^3D_2)$, and $\delta(^3D_3)$ together with the experimental phase shifts[3-5] at 25, 50, 150, 210, 325, 425, and 515 MeV. It is curious that these $^3D_1$ and $^3D_2$ phase shift fits are worse than the P-wave fits since from very general grounds one would have expected precisely the opposite. In fact, nucleons in D waves "impact" at distances of 1.2 fm or greater and the 2π-exchange dispersion-theoretical predictions ought to be fairly good at these distances. It is at the shorter distances, characterizing P-wave interactions, where discrepancies might be expected to arise. One could, of course, argue that there is some remaining freedom in the adjustment of the potentials in the central region, e.g., the ρNN and ωNN coupling constants, as well as the $J=0^+ N\bar{N}\to\pi\pi$ analytically continued amplitude, are somewhat uncertain. However, one of us (B.J.V.) has been unable to greatly improve the D-wave fits of the Stony Brook model after some considerable trial and error variation of these (somewhat) adjustable parameters.[6] The problem here seems to be the large magnitude of the tensor splitting caused by one pion exchange (OPE). Attempts to reduce this effect and bring $\delta(^3D_1)$ and $\delta(^3D_2)$ into agreement with the data require a πNN form factor $F(q^2)$ with a very small cutoff in disagreement with the analyses of Refs. 7 and 8.

However, a point which has been overlooked and might be the solution to this dilemma is the full off-shell behavior of the πNN form factor. Furthermore, if a correct treatment of this vertex function could be capable of explaining the above puzzle, it might also have more far reaching consequences for hadronic physics. For this reason we study in Sec. II a full off-shell dual unitary model for hadronic form factors. We show that the model can describe any hadronic vertex function with one, two, or three particles off-the-mass shell by means of a factorizable analytic function of the square of the four-momenta. For example, calling A, B, and C the three particles of four-momenta P_A, P_B, and P_C in a vertex, the form factor is given by

$$F_{ABC}(P_A{}^2, P_B{}^2, P_C{}^2) = F_A(P_A{}^2)F_B(P_B{}^2)F_C(P_C{}^2)\,,$$

and F_A, F_B, and F_C each contain one free parameter associated with the asymptotic behavior in the spacelike region ($P^2\to-\infty$). The dual model provides unique rules for constructing these functions which exhibit poles in the timelike region ($P^2>0$) and have asymptotic power behavior as $p^2\to-\infty$. Also, unlike dual models for scattering amplitudes, the three-point function can be unitarized in a simple manner without undesirable implications. Except for the location and widths of the poles and the value of the free parameter, the functions F_A, F_B, and F_C above have all the same form. This together with the factorization property has then an important implication, viz., once a function $F_i(P_i{}^2)$ ($i=A$, B, or C) has been fixed in a particular vertex, it should retain its form in a different vertex involving the same hadron.

The physical motivation of this model and the

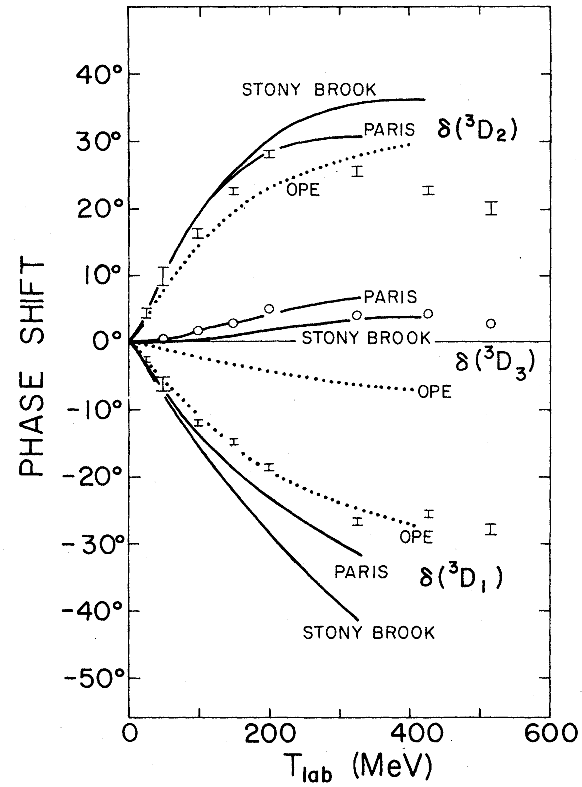

FIG. 1. Theoretical predictions of the Stony Brook and Paris *NN* potentials for $\delta(^3D_1)$, $\delta(^3D_2)$, and $\delta(^3D_3)$ together with some experimental values. The dotted curve is the OPE prediction using geometric unitarization $[\delta(^3D_J)=B^{\pi}(^3D_J)]$ and a form factor at each πNN vertex, $F(M^2,M^2,q^2)$, with $\beta_3\equiv\beta=2$ (see text).

general rules for constructing and unitarizing the analytic functions $F_i(P_i^2)$ are presented in Sec. II. In Sec. III we discuss, as an application of the model, the one-pion exchange contribution to *NN* scattering. The purpose of this is to study the effects of a fully off-shell treatment of the πNN form factor in a simple and unambiguous workable example. Previous treatments of this problem have taken into account only the off-shell behavior of the pion leg without introducing any off-shell nucleon structure in the vertex function. Our main results are the following. When only the pion is allowed to go off-the-mass shell in the πNN vertex, the predictions of our model are essentially identical to those of previous treatments. This is simply a reflection of the fact that $F_{\pi NN}(P_\pi^2)$ in the dual model is not very different from a monopole-like form factor for moderate values of P_π^2. However, the use of our model for the fully off-shell vertex function $F_{\pi NN}(P_\pi^2,P_N^2,P_N^2)$ uncovers new and dramatic effects in the *NN* phase shifts, e.g., $\delta(^3D_2)$ changes by $\simeq 9°$ at 400 MeV, roughly a 25% effect. Although a satisfactory resolution of the triplet-*D*-wave phase shifts puzzle should await a complete one-boson-exchange calculation, we present qualitative arguments indicating that the fully off-shell behavior of the vertex functions may prove to be an essential new ingredient.

II. DUAL UNITARY MODEL FOR HADRONIC FORM FACTORS

The incorporation of hadronic form factors into the calculations of one-particle-exchange potentials has evidenced the need for sound, physically motivated models for three-point functions. Hopefully, a successful model should provide general rules for constructing a vertex function describing any arbitrary types of particles and should also provide general prescriptions for unitarization and off-shell behavior with the least possible number of free parameters. A major step in this direction was taken some time ago[8] with the suggestion that the πNN form factor could be written, in analogy with the dual Veneziano model, as a ratio of two gamma functions, i.e.,

$$F_{\pi NN}(q^2)=C\,\frac{\Gamma[1-\alpha'(q^2-\mu_\pi{}^2)]}{\Gamma[\beta-\alpha'(q^2-\mu_\pi{}^2)]}\,, \tag{1}$$

where C is an overall normalization constant, $\alpha'\simeq 0.8\ \mathrm{GeV}^{-2}$ is the universal slope of Regge trajectories, β is a free parameter, and $F_{\pi NN}(q^2)$ is defined with the pion pole removed. More recently,[9] a general prescription has been given to unitarize Eq. (1) as well as to generalize it to other meson-baryon vertices.

The attractive features of this model are that it incorporates the salient most successful results of the dual and Regge models, e.g., mass spectrum and asymptotic behavior; it can be easily generalized to describe any hadronic vertex; and the unitarization prescription is reasonably simple. Furthermore, some control can be exercised on the free parameter by using independent theoretical information; e.g., the constituent interchange quark model[10] (CIM) might be used as a guideline in the asymptotic spacelike region to bound β.

In this paper we wish to discuss the generalization of the above model to an arbitrary vertex with all three particles off-the-mass shell, indicate the unitarization prescription, and illustrate some of the effects that result in the πNN form factor when the nucleons are off shell.

Calling p_1, p_2, and p_3 the incoming four-momenta of the three particles in a vertex, kinematic considerations show that the form factor $F=F(p_1^2,p_2^2,p_3^2)$. The model can then be specified by the following assumptions:

(i) Analyticity for $\mathrm{Re}p_1^2<0$, $\mathrm{Re}p_2^2<0$, and $\mathrm{Re}p_3^2<0$.

(ii) $F(p_1^2,p_2^2,p_3^2)$ to satisfy a triple dispersion relation.

(iii) Regge behavior, i.e.,

$$\lim_{|p_i^2|\to\infty} F(p_1^2,p_2^2,p_3^2)\sim(p_i^2)^{-\lambda}$$
$$0<|\arg p_i^2|\leq\pi\,. \tag{2}$$

(iv) Linear Regge trajectories, i.e.,

$$\alpha(p_i^2)=s_i+\alpha'(p_i^2-M_{i,0}^2)\quad(i=1,2,3), \tag{3}$$

where s_i is the spin of the particle with four-momentum p_i and mass $M_{i,0}$.

(v) Veneziano-type mass spectrum, i.e., $F(p_1^2,p_2^2,p_3^2)$ is a meromorphic function with poles at

$$p_i^2=M_{i,n}^2=M_{i,0}^2+n/\alpha'\quad(n=0,1,2\ldots)\,. \tag{4}$$

In the narrow-width approximation [e.g., Eq. (1)], these poles are located on the real axis and become shifted to the second Riemann sheet in the unitarized version of the model. Assumptions (i) and (ii) hardly need justification. Assumption (iii) is supported by the Regge model of a two-body scattering amplitude where $M(s,t)\sim t^{\alpha(s)}$ as $|t|\to\infty$ and $0<|\arg t|\leq\pi$. Besides, there are other models, e.g., the CIM, in which form factors exhibit asymptotic power behavior. Assumption (iv) is well sustained by Regge fits to high energy data[11] and also it holds, together with (v), in a simple dual-resonance model.[12] The mass spectrum Eq. (4) predicts daughter masses in very good agreement with experiment both in the meson sector, e.g., $\rho'(1250)$, $\rho''(1600)$, $K''(1498)$, etc., as well as in the baryon sector, e.g., $N(1470)$, $N(1780)$, $\Delta(1690)$, $\Sigma(1660)$, etc. On the other hand, the existence of daughters is not a peculiarity of dual models but is also expected from a wide class of models, e.g., in the constituent-quark model these daughters correspond to radial excitations of the ground state particles.[13] It should be added that except for the possibility that all three particles might be off shell, the above assumptions are already incorporated into Eq. (1).

The simplest choice for $F(p_1^2,p_2^2,p_3^2)$ may then be written as

$$F(p_1^2,p_2^2,p_3^2)=g\prod_{i=1}^{3}\Gamma(\beta_i-s_i)\times\frac{\Gamma[1-\alpha'(p_i^2-M_i^2)]}{\Gamma[\beta_i-s_i-\alpha'(p_i^2-M^2)]}\,, \tag{5}$$

where the normalization constant g is defined at the point $p_1^2=M_1^2$, $p_2^2=M_2^2$, and $p_3^2=M_3^2$, and satellites have been ignored on account of their nonleading contribution. It is important to point out that the factorization of the form factor into the product of three simple forms of the type Eq. (1) is not merely dictated by simplicity. In fact, drawing a close analogy with the Veneziano four-point function one would have expected *a priori* the following expression to be the most general one:

$$G(p_1^2,p_2^2,p_3^2)=g\Gamma(\lambda)\frac{\prod_{i=1}^{3}\Gamma[1-\alpha'(p_i^2-M_i^2)]}{\Gamma[\lambda-\sum_{i=1}^{3}\alpha'(p_i^2-M_i^2)]}\,. \tag{6}$$

However, it may be verified that there is no value of λ in Eq. (6) that will restore the single-pole approximation. Clearly, Eq. (5) does have this desirable property for $\beta_i=s_i+1$.

The analytic structure of the form factor, Eq. (5), can be best exhibited by series expanding the gamma functions; i.e.,

$$F(p_1^2,p_2^2,p_3^2)=g\prod_{i=1}^{3}\frac{1}{\alpha'}\Gamma(\beta_i-s_i)\times\sum_{n=0}^{\infty}\frac{(-)^n}{n!}\frac{1}{M_{i,n+1}^2-p_i^2}\times\frac{1}{\Gamma[\beta_i-s_i-1-n]}\,. \tag{7}$$

Equation (7) shows that $F(p_1^2,p_2^2,p_3^2)$ is a meromorphic function with a finite or infinite number of poles according to whether β_i-s_i is an integer or noninteger number, respectively. These poles are located on the real axis as may be seen by taking the imaginary part of Eq. (7): i.e.,

$$\mathrm{Im}F(p_i^2,p_2^2,p_3^2)=g\pi^3\prod_{i=1}^{3}\Theta(p_i^2-p_{i,th}^2)\frac{1}{\alpha'}\Gamma(\beta_i-s_i)\times\sum_{n=0}^{\infty}\frac{(-)^n}{n!}\frac{\delta(M_{i,n+1}^2-p_i^2)}{\Gamma(\beta_i-s_i-1-n)}\,. \tag{8}$$

The unitarization of the form factor can be achieved by the substitution

$$\delta(M_n^2-p^2)\to\frac{1}{\pi}\frac{\Gamma_nM_n}{(M_n^2-p^2)^2+\Gamma_n^2M_n^2} \tag{9}$$

and by introducing in Eq. (8) the appropriate threshold behavior. The widths Γ_n in Eq. (9) are expected *a priori* to be functions of the daughter masses and should be fixed by experimental data or else by independent theoretical considerations. For instance, for vector mesons (ρ,ω,ϕ) it is known[14] that $\Gamma_n=\Gamma_0M_n$, where M_n is given by Eq. (4), while nucleon data[15] seem to indicate that Γ_n are mass independent. The threshold behavior of $F(p_1^2, p_2^2, p_3^2)$ may be determined by considering the first daughter particle as a resonance in the respective two- or three-body channel. For example, the ρ', π', and N' may be viewed as resonances in the two-pion, three-pion, and pion-nucleon system. From a knowledge of the threshold behavior of the corresponding four- or five-point function one can thus infer the threshold behavior of the form factor for each separate leg. We shall not pursue this matter any further here since the narrow-width approximation will suffice for the purposes of the present paper.[16]

In order to illustrate some of the effects of off-shell extrapolations, let us consider the πNN form factor with the pion and one nucleon on-the-mass shell, i.e., $F(p_1^2, M^2, \mu_\pi^2)$. In this case, Eq. (5) becomes

$$F_{\pi NN}(p_1^2, M^2, \mu_\pi^2) = g\,\frac{\Gamma(\gamma - \frac{1}{2})\Gamma[1 - \alpha'(p_1^2 - M^2)]}{\Gamma[\gamma - \frac{1}{2} - \alpha'(p_1^2 - M^2)]}, \tag{10}$$

where M is the nucleon mass and we have set $\beta_1 \equiv \gamma (\gamma \geq \frac{3}{2})$. For $\gamma = \frac{3}{2}$ Eq. (10) becomes the single nucleon-pole approximation, i.e., $F_{\pi NN} = g$. Figure 2 shows some of the effects to be expected from Eq. (10) (with $g = 1$, for simplicity). The broken line corresponds to $\gamma = \frac{3}{2}$, the solid line is for $\gamma = \frac{5}{2}$, and the dot-bar line corresponds to $\gamma = \frac{7}{2}$. Although any value of $\gamma \geq \frac{3}{2}$ is *a priori* possible (γ is a free parameter of the model) the CIM[10] favors[17] $\gamma \simeq \frac{5}{2}$. Furthermore, half integer values of γ reduce Eq. (10) to very simple forms,

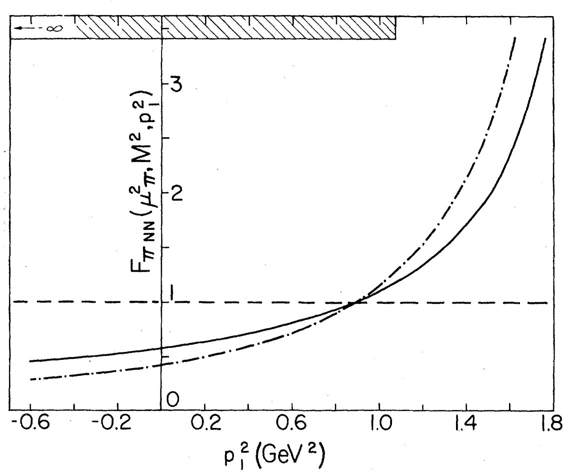

FIG. 2. The πNN form factor Eq. (10) with only one nucleon off shell. The broken line corresponds to $\gamma = \frac{3}{2}$ (single-nucleon-pole approximation), the solid curve is for $\gamma = \frac{5}{2}$, and the dot-bar line is for $\gamma = \frac{7}{2}$. The shaded line corresponds to the integration range in the Blankenbecler-Sugar equation for $T_{1ab} = 400$ MeV (see Sec. III).

e.g., if $\gamma = \frac{5}{2}$, Eq. (10) becomes a monopole, i.e.,

$$F_{\pi NN}(p_1^2, M^2, \mu_\pi^2, \gamma = \tfrac{5}{2}) = g\,\frac{1}{1 - \alpha'(p_1^2 - M^2)}. \tag{11}$$

Unitarization of the form factor will cause a slight further reduction of $F_{\pi NN}$ near the origin and will smooth out its behavior near the first nucleonic pole located at $p_1^2 = M_1^2 \approx 2.13$ GeV^2.

Strong off-shell variations of $F_{\pi NN}$ have been advocated by Nutt and Shakin[18] and challenged recently by Epstein.[19] At this point it should be stressed that the pole model discussed here may be viewed as an approximation to a dispersion integral. Testimonies indicating that this is in fact an excellent approximation[20] include predictions for the electromagnetic form factors of the pion, nucleon, $\Delta(1236)$, the axial-vector form factor of the nucleon, the πNN form factor itself, and the kaon-baryon vertex.

III. APPLICATION TO NUCLEON-NUCLEON SCATTERING

In this section we discuss some of the effects of our Eq. (5), for the case of the πNN vertex, on nucleon-nucleon scattering. Figure 3 defines the standard kinematics with p_3 in Eq. (5) replaced by q, the pion four-momentum.

To this end we have calculated $L = 0, 1, 2, \ldots 5$ NN phase shifts using the OPE potential together with the Blankenbecler-Sugar (BbS) equation. We show in Fig. 4 the critical triplet-D-wave phase shifts over the range 0–400 MeV. The solid curve is the result obtained with the fully off-shell πNN form factor $F(p_1^2, p_2^2, q^2)$ as given by Eq. (5) with $\beta_1 = \beta_2 \equiv \gamma = \frac{5}{2}$ and $\beta_3 \equiv \beta = 2$. In this case Eq. (5) reduces to the following simple form:

$$F(p_1^2, p_2^2, q^2) = \frac{1}{1 - \alpha'(q^2 - \mu_\pi^2)}\,\frac{1}{1 - \alpha'(p_1^2 - M^2)} \times \frac{1}{1 - \alpha'(p_2^2 - M^2)}, \tag{12}$$

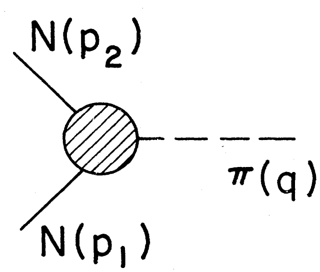

FIG. 3. The πNN vertex.

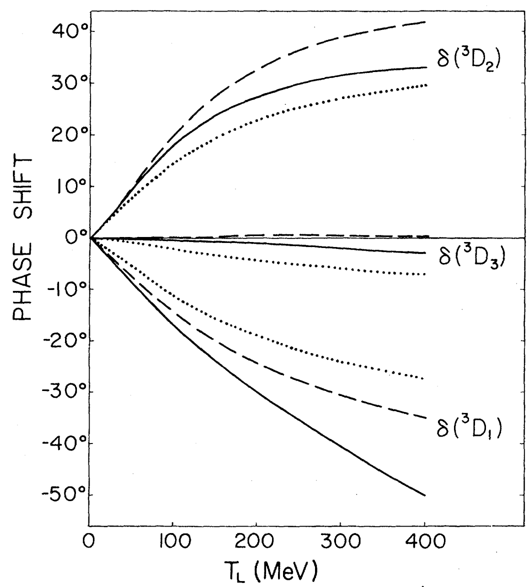

FIG. 4. OPE predictions for the triplet-D-wave phase shifts. Dotted curve is the cutoff OPE prediction using geometric unitarization $\delta({}^3D_J)=B({}^3D_J)$ with a form factor $F(M^2,M^2,q^2)$. Dashed curve corresponds to the Blankenbecler-Sugar OPE prediction using the same form factor. Solid curve is the Blankenbecler-Sugar OPE prediction using a fully off-shell form factor $F(p_1^2,p_2^2,q^2)$.

where we have defined the form factor without the coupling constant for simplicity. It should be noted that $\gamma\approx\frac{5}{2}$ and $\beta\approx 2-3$ are to be expected from the power counting rules of the CIM.[10] The dashed curve in Fig. 4 is the result when only the pion is allowed to go off shell and $\beta=2$, in which case Eq. (5) reduces to

$$F(M^2,M^2,q^2)=\frac{1}{1-\alpha'(q^2-\mu_\pi^{\,2})}\,. \tag{13}$$

Finally, we show the Born approximation curves of Fig. 1 to provide physical insight as well as to allow an easy comparison between the information contained in Figs. 1 and 4.

One can see from Fig. 4 that the effect of the fully off-shell form factor is to lower the phase shift in the case of each triplet D wave. In fact, our calculations show that our form factor lowers the phase shifts of *every* partial wave. In the case of the uncoupled states, the reason for this behavior can be understood as follows: First of all, our version of OPE includes a form factor $F(M^2,M^2,q^2)$ which is used together with geometric unitarization to obtain the prediction shown in Fig. 4 (dotted line). Next, the Blankenbecler-Sugar iteration, with the same form factor $F(M^2,M^2,q^2)$, raises the phase shift over the pure geometrically unitarized Born phase shift because the second Born term (which dominates the T-matrix Born series) is always positive at these (sufficiently) low energies. Finally, replacing the form factor of this potential by the fully off-shell form $F(p_1^2,p_2^2,q^2)$ results in a smaller second-Born contribution to the T matrix, and hence in a lower phase shift (the first-Born term is of course unaffected). The second-Born term is lowered because for most of the range of integration over intermediate-state nucleons in the Blankenbecler-Sugar equation the πNN form factor is less than unity. The actual integration range for $T_{\rm lab}=400$ MeV is indicated by the shaded line at the top of Fig. 2, which shows that the upper limit in the timelike region ($p^2>0$) is well below the position of the first pole ($p^2\approx 2.13$ GeV). This justifies the use of the zero-width approximation for the form factor; the effects of unitarizing Eq. (5) are expected to be minimal in this region of momentum transfer.

In the case of the ${}^3D_1$ phase shift, the analysis is more complicated due to the strong coupling of the ${}^3D_1$ state to the ${}^3S_1$ state. However, the net effect of using in the BbS equation $F(p_1^2,p_2^2,q^2)$ instead of $F(M^2,M^2,q^2)$ is to reduce $\delta({}^3D_1)$ as may be seen from Fig. 4.

At this point we wish to discuss the relevance of the above OPE-BbS form factor calculations for the $\pi+2\pi+\omega$ BbS potential model fits to NN data. The use of restricted form factors $F(M^2,M^2,q^2)$ together with such complete potentials should yield triplet-D-wave phase shifts similar to the Paris phase shifts. [We note that in the Stony Brook model, use of both u- and t-channel form factors (of the eikonal type) lowers $\delta({}^3D_1)$ considerably below the result that would be obtained with only a t-channel vertex function.] The significance of replacing $F(M^2,M^2,q^2)$ by our fully off-shell form $F(p_1^2,p_2^2,q^2)$ is that it lowers $\delta({}^3D_2)$ by $\simeq 9°$ at 400 MeV in the OPE-BbS model. Moreover, it will probably also lower $\delta({}^3D_2)$ in *any* realistic $\pi+2\pi+\omega$ model. The magnitude of this decrease in $\delta({}^3D_2)$ is precisely that required to fit the data. Furthermore, no permissible variation in any other parameter in NN models seems to be able to reduce $\delta({}^3D_2)$ by the above amount; e.g., variation of $f_{\rho NN}$ leaves $\delta({}^3D_2)$ practically unchanged and variation of the $J=0N\bar{N}\to\pi\pi$ amplitude is severely restricted by the very precise $\delta({}^1D_2)$ data.

At this point it could be objected that although $F(p_1^2,p_2^2,q^2)$ decreases $\delta({}^3D_2)$ by the right amount, it also lowers $\delta({}^3D_1)$, a seemingly undesired feature. However, we have found that an increase in $f_{\rho NN}$, within experimentally allowable limits, produces a significant increase in $\delta(^3D_1)$, enough to more than compensate for the decrease brought about by $F(p_1^2, p_2^2, q^2)$. It is important to point out here that our form factor does not lower $\delta(^3D_1)$ below the Stony Brook curve in Fig. 1.

It is, therefore, our opinion that our fully off-shell form factor, together with some allowable variations in other parameters of the model, will bring the $(\pi + 2\pi + \omega)$-exchange NN theory into agreement with experiment for D waves in the region 200-400 MeV. This corresponds to the critical central region of the nucleon-nucleon force (1–2 fm).

One might inquire if these modifications which solve the D-wave puzzle could have detrimental effects for NN fits to the higher partial waves. The answer to this appears to be no. In fact, our calculations show that use of $F(p_1^2, p_2^2, q^2)$ instead of $F(M^2, M^2, q^2)$ changes the $L \geqslant 3$ OPE-BbS phase shifts only slightly; also, permissible variations in the ρ and ω coupling constants will have only minor effects in F waves and higher, because of the inherently short range of the ρ- and ω-exchange potentials.

We conclude that use of our vertex function $F(p_1^2, p_2^2, q^2)$ in place of previously used forms will allow for agreement between dispersion-theoretical $(\pi + 2\pi + \omega)$-exchange calculations and experiment for D waves and higher. It is more difficult to predict the effect in P and S states, but uncertainty in the ωNN coupling constant allows for considerable readjustment of the critical triplet-P-wave phase shifts (to name just one possibility) and, of course, the $^1S_0$ and $^3S_1$ scattering lengths are not predicted by any meson-exchange theory, but are to be fixed in the manner of subtractions in dispersion theories. Thus it is quite possible that good fits will be possible in $L = 0$ and 1 states as well.[21]

IV. CONCLUSIONS

We have discussed here a simple model for an off-shell hadronic form factor that incorporates the salient most successful features of the dual and Regge models. Furthermore, the unitarization of this form factor may be achieved in a straightforward manner. Another attractive feature is that the single free parameter of the model (in each leg) has a transparent physical interpretation; i.e., it is related to the asymptotic behavior of the form factor in the spacelike region. Independent theoretical information, e.g., the CIM, may then be used in order to estimate this parameter.

An application of this model to the πNN vertex shows that the effects of taking the nucleons off shell turn out to be rather dramatic. In fact, as seen in Fig. 2, a reduction of $\approx 50\%$ in the πNN form factor is to be expected at zero-momentum transfer. This feature might prove essential in explaining some long standing problems in NN phase shifts.

It goes without saying that the model presented here is not restricted to describe just the πNN form factor. Equation (5) or its corresponding unitarized version can describe any hadronic vertex function with one, two, or three particles off shell. Therefore, it should be possible to use this model in conjunction with one-boson-exchange potential calculations, $\Delta(1232)$ production, three-body force calculations, etc. In view of the results obtained so far, we foresee some radical changes in results obtained under the assumption of perfectly smooth behavior of $F(p_1^2, p_2^2, p_3^2)$ as two or three particles go off shell.

This work was supported in part by the Department of Energy under Contract No. EY-76-S-05-5223.

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*On sabbatical leave from Centro de Investigacion y de Estudios Avanzados del I. P. N., Mexico.

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[21]A more detailed and quantitative treatment of this as well as other related problems is currently under investigation; R. Bryan, C. A. Dominguez, and B. J. VerWest (unpublished).