PHYSICAL REVIEW C VOLUME 22, NUMBER 4 OCTOBER 1980

Superprolate shape of the spontaneous-fission isomer ^{240}Am^{m}

Linus Pauling
Linus Pauling Institute of Science and Medicine, 2700 Sand Hill Road, Menlo Park, California 94025

(Received 13 May 1980)

A superprolate structure for nuclei with $A \sim 240$ proposed in 1965 on the basis of the polyspheron theory leads to the value 0.66 for the deformation parameter β. This value agrees well with a recently reported experimental value, 0.66 ± 0.04 for the spontaneous-fission isomer ^{240}Am^{m}, obtained by Bemis *et al.* from their measurement of the optical isomer shift. This agreement provides additional support for the proposed superprolate structure.

[NUCLEAR STRUCTURE ^{240}Am^{m}; deformation parameter of superprolate shape predicted by cluster theory agrees with experimental value.]

Bemis, Beene, Young, and Kramer[1] have recently observed the optical isomer shift of the spontaneous-fission isomer ^{240}Am^{m}, and have compared the values with values for the ground states of ^{240}Am, ^{241}Am, and ^{243}Am to obtain the value 0.66 ± 0.04 for the quadrupole deformation parameter β. They state that their investigation provides the first direct experimental proof for the postulated large deformations, and that indirect evidence, provided by conversion-electron studies, by rotational-band feeding and decay times using the charge-plunger method, and by delayed-fission fragment angular distributions, had given indirect but consistent evidence for large deformations, with β in the range 0.55 to 0.78. (References are given by Bemis *et al.*[1])

It has been pointed out by Habs, Metag, Specht, and Ulfert[2] that the experimental values are close to values calculated by the Strutinsky procedure.[3] The values, in particular the Bemis value 0.66 ± 0.04, also agree with that given by a superprolate model described in 1965 in connection with a discussion of asymmetric fission on the basis of the polyspheron theory of the structure of nuclei.[4] This was, so far as I know, the first published mention of a superprolate structure, preceding Strutinsky's first paper by a year.[5]

THE POLYSPHERON THEORY

The polyspheron theory[4,6-19] is based on quantum mechanics and involves no fundamental additions to it. Conclusions based on the polyspheron theory are in general less precise and less reliable than those based on detailed quantum mechanical calculations. The valuable aspects of the polyspheron theory are that it is easy to apply in the discussion of a nuclear property and that the nature of the internucleonic interactions important to the property are clearly indicated. The polyspheron theory may suggest detailed quantum mechanical calculations that might profitably be carried out. This theory of nuclear structure is analogous to the chemical valence-bond theory of molecular structure, which is completely compatible with molecular quantum mechanics but is more valuable because it can be applied to more complex molecules.

Localized orbitals in a nucleus can be formed by linear combination of shell-model orbitals.[4,20] Each localized orbital can be occupied by no more than two neutrons and no more than two protons. Such an occupied localized orbital is called a spheron. The most common spherons are the alpha particle and the triton. In most nuclei the number of spherons is half the neutron number N.

Simple quantum mechanical calculations of particles in a container of fixed volume and variable shape demonstrate that the minimum kinetic energy tends to be associated with isotropic shapes of the spherons, such as cubical or approximately spherical. This tendency is abetted by the hard-core or repulsive terms in the internucleonic interactions. The effective radius of a spheron in different directions may vary by 20% or more, with average value also variable—smaller in the core of a nucleus than in the mantle (the outer layer).

A nucleus becomes more stable with increase in the number of spheron-spheron contacts. The structure of the ground state of a nucleus is accordingly the packing of soft spheres with the maximum number of these contacts.

For example, the diameter of the alpha particle, measured to the radius at which the proton density, as determined by electron scattering, is 20% of the maximum, is about 3.2 fm, whereas that of iron is 9.6 fm. With the outer layer of spherons taken to be 3.2 fm thick, there is a hole inside this outer layer 3.2 fm in diameter, which surely is occupied by one spheron (alpha particle). Thus we describe $_{26}$Fe$_{26}$ as a central alpha particle surrounded by a mantle of 12 alpha particles. With these 12 at the corners of an icosahedron, giving the closest packing (tetrahedral), the core alpha particle has an effective radius 10% smaller than the mantle alpha particles. A similar description applies to other nuclei with N between about 18 and 30. Nuclei with N in the range near the magic number 82 have an inner core of one spheron, surrounded by an outer core and a mantle, their diameter being about 5 times the spheron diameter. At $N=126$, with diameter 6 times the spheron diameter, the inner core consists of four spherons (^{16}O), surrounded by an outer core and a mantle.

In tetrahedral packing about a central spheron the icosahedron is surrounded by a rhombic triacontahedron of 32 spherons, the complex thus involving 45 spherons, 90 neutrons. An additional spheron would thus either be held loosely on the surface or be accommodated by a change in structure, involving having an inner core of two spherons, and hence showing prolate deformation. This argument was advanced in 1965 as the explanation of the value $N=90$ at which prolate deformation sets in.[4,9]

THE SUPERPROLATE STRUCTURE

In the spherical nuclei with $N \cong 90$ the icosahedral outer core can be described as two staggered pentagonal rings plus two polar spherons. With two central spherons, as in the prolate rare-earth nuclei, the outer core consists of three staggered pentagonal rings plus two polar spherons. The major diameter, including the mantle, is then 6 spheron diameters, and the minor diameters are 5 spheron diameters. With a correction for the skin by measuring to the center of the outer spherons, the expression

$$\beta = \frac{4}{3}\left(\frac{\pi}{5}\right)^{1/2} \frac{D_z - D_{x,y}}{D_0} = 1.057\,\frac{\Delta D}{D_0}, \qquad (1)$$

with D_0 the diameter of the equivolume sphere, gives the value 0.245 for β; experimental values for rare-earth nuclei lie in the range 0.16 to 0.24.

This model lends itself readily to increase in length by integrals of one spheron diameter. In my 1965 paper on symmetrical and asymmetrical fission[4] I proposed the next structure, with major axis increased by one spheron diameter, as the superprolate model for symmetrical fission in the range of N around 126. The value of β calculated by Eq. (1) with $\Delta D=2$ and $D_0=4.58$ is 0.46. No experimental value has been reported.

The next structure, with $\Delta D=3$ and $D_0=4.82$, gives $\beta=0.658$. Another value for β was calculated from measurements of a model that I made of cork balls in 1965 and measurements of a published photograph of it (Ref. 4, Fig. 12). These measurements also gave the value $\beta=0.66$, the decreased size of the inner spherons, which affects the major axis more than the minor axes, apparently compensating for the better packing into triangular interstices along the latter than the former.

These calculated values of β agree with the experimental value 0.66 ± 0.04. The agreement provides additional evidence of the usefulness of the polyspheron description of nuclei.

As mentioned above, this description has a quantum mechanical basis, and it is not surprising that conclusions drawn from it agree not only with experiment but also with the results of detailed theoretical calculations. In fact, Strutinsky has found values of β equal to 0.6 ± 0.1 in the range $A=230$ to $A=240$ by his macroscopic-microscopic calculation, in which single-particle shell effects are superimposed on a liquid-drop (macroscopic) treatment,[5] and other calculations for the second potential well by the Strutinsky procedure have given the β values 0.55 to 0.78 (Q_0 from 34.3 to 38.2 *e*b). (References are given by Habs *et al.*[2])

In the polyspheron model the normal states of nuclei in the region near $A=240$ have prolate structures with major diameters 6, 6, and 7 spheron diameters (5, 5, and 6 to the centers of the outer spherons). The calculation of β with $\Delta D=1$ and $D_0=5.31$ gives the value 0.20, in reasonable agreement with the observed value, reported as 0.24 for the isotopes of americium.[1]

In 1965 I predicted that symmetric fission would begin to be evident at about $N=163$, and that $^{266}_{103}\mathrm{Lw}_{163}$ would show both symmetric and asymmetric fission.[4] It can be predicted that the superprolate structure for symmetric fission in this region will have the value $\beta=0.79$, correponding to $\Delta D=4$, $D_0=5.04$.

I point out again that there is no incompatibility between the polyspheron theory and nuclear quantum mechanics. Two neutrons and two protons are permitted by the Pauli exclusion principle to occupy the same orbital, which makes it sensible to consider spherons, rather than individual nucleons, in discussing the nature of the spatial packing in nuclei. In applying the polyspheron theory the possibility of resonance among two or more structures (configuration mixing) must be taken into consideration, as it is in other mechanical treatments. In the quantum mechanical attack on problems of nuclear structure the polyspheron theory may be helpful as a guide; for example, in suggesting the form of the zeroth-order wave function.

[1] C. E. Bemis, Jr., J. R. Beene, J. P. Young, and S. D. Kramer, Phys. Rev. Lett. 43, 1854 (1979).

[2] D. Habs, V. Metag, H. J. Specht, and G. Ulfert, Phys. Rev. Lett. 38, 387 (1977).

[3] V. M. Strutinsky, Nukleonika 20, 67 (1975).

[4] L. Pauling, Science 150, 297 (1965).

[5] V. M. Strutinsky, Yad. Fiz. 3, 614 (1966) [Sov. J. Nucl. Phys. 3, 449 (1966)]; Nucl. Phys. A95, 420 (1967).

[6] L. Pauling, Nature (London) 208, 174 (1965).

[7] L. Pauling, Phys. Rev. Lett. 15, 499 (1965).

[8] L. Pauling, Proc. Nat. Acad. Sci. USA 54, 989 (1965).

[9] L. Pauling, Phys. Rev. Lett. 15, 868 (1965).

[10] L. Pauling, Rev. Roum. Phys. 11, 825 (1966).

[11] L. Pauling, Proc. Nat. Acad. Sci. USA 58, 2175 (1967).

[12] L. Pauling, *Maria Sklodowska-Curie: Centenary Lectures* (IAEA, Vienna, 1968).

[13] L. Pauling, Phys. Rev. 182, 1357 (1969).

[14] L. Pauling, Phys. Today 22, 9 (1969).

[15] L. Pauling, Proc. Nat. Acad. Sci. USA 64, 807 (1969).

[16] L. Pauling and J. Blethen, Proc. Nat. Acad. Sci. USA 71, 2905 (1974).

[17] L. Pauling and A. B. Robinson, Can. J. Phys. 53, 1953 (1975).

[18] L. Pauling, Phys. Rev. Lett. 35, 1480 (1975).

[19] L. Pauling, Phys. Rev. Lett. 36, 162 (1976).

[20] P. K. Nesbit, Phys. Rev. 100, 228 (1955). Nesbit was the first to suggest the use of localized orbitals in the discussion of the structure of nuclei.