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Comparison of proton-proton bremsstrahlung data at 42 and 156 MeV with soft photon calculations

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Proton-proton bremsstrahlung in principle provides information about the off-shell nucleon-nucleon force, but first the dominant on-shell contribution must be understood. Comparisons of purely on-shell soft photon calculations with 200 and 730 MeV data have already been made. In this paper similar comparisons are made for the remaining two modern experiments, those at 42 and at 156 MeV. For 42 MeV soft photon and standard potential model calculations differ by amounts comparable to, or somewhat larger than, the errors in the data which tend to fall in between the theoretical curves, which thus fit the data equally well. For the 156 MeV data the soft photon results are a slight improvement over potential models, but neither fit the data very well.

[NUCLEAR REACTIONS $p+p\rightarrow p+p+\gamma$ $E=42, 156$ MeV; comparison of soft photon approximation calculations with $\sigma(\theta_\gamma)$ data.]

In the past few years there have been several new proton-proton bremsstrahlung ($pp\gamma$) experiments[1-6] which have been designed specifically to explore kinematic regions which might be hoped to be more sensitive than earlier experiments to the off-shell aspects of the nucleon-nucleon force. Thus higher energies and smaller proton angles than before, as well as asymmetric and noncoplanar geometries, have been investigated.

Ultimately, to interpret results of such experiments in terms of off-shell effects one must use a model for the nucleon-nucleon interaction and for the $pp\gamma$ process which correctly reproduces the on-shell information and which allows one to vary the off-shell aspects of the interaction to fit the $pp\gamma$ data. At low energies nonrelativistic potential models are usually used,[7] although only recently have extensive studies been made using transformations which allow variation of off-shell aspects without varying on-shell parts simultaneously.[8] At higher energies potential models no longer apply and other approaches must be found.

First, however, at any energy, before looking at detailed models, it is valuable to see how much of the data can be described by the purely on-shell, Low soft photon approximation (SPA),[9-11] particularly since this approach is simple, satisfies the required general properties of gauge and relativistic invariance, and requires as input only the known elastic phase shifts. Furthermore, it so far seems to fit a surprisingly large amount of the available $pp\gamma$ data.[10,1-3] One hopes, of course, to find a large discrepancy with the SPA, since if off-shell effects are large then, barring some accidental cancellation, SPA should fail, although the converse is not necessarily true since there are on-shell effects not included in SPA.

Such comparisons of data with the SPA have already been made for the 730 MeV UCLA experiment[3] and the 200 MeV TRIUMF experiment.[1,2] To our knowledge, the two other modern experiments, the 42 MeV Manitoba experiment[4,5] and the 156 MeV Orsay experiment,[6] have not been compared systematically with SPA. It is thus the purpose of this paper to present such results and to discuss the common features of such comparisons to all of the modern experiments.

When comparing various experiments performed under different kinematic situations, it is useful to have some criteria for estimating the probabilities of being sensitive to off-shell information. Such analysis has been made recently,[12] and two parameters Δm^2 and (k/E_0) defined, which may help in such estimates. The parameter Δm^2 measures the amount by which some nucleon is off-shell. Its maximum value is set by the available energy. The parameter k/E_0 is related to the region of validity of the soft photon approach. Both parameters must be large if off-shell effects are to be seen, since large Δm^2 indicates that a nucleon can be far off-shell, while large k/E_0 indicates that the amplitude may not be fully determined by on-shell effects via the soft photon theorem. Both depend strongly on the particular geometry chosen, and different geometries tend to emphasize large values of one or the other.

For the 730 MeV UCLA experiment[3] Δm^2 is larger than any other experiment by virtue of the higher energy. However, k/E_0 is relatively small. Thus one finds[3,10] that for photon energies of $\lesssim 100$ MeV SPA, and in fact for $k \lesssim 60$ MeV only the first term of SPA, describes the data quite well. At higher photon energies the data tend systematically above the SPA prediction by amounts reaching several standard deviations, an effect which may be explainable in terms of an intermediate $\Delta(1236)$.[13]

For the TRIUMF experiment[1] at 200 MeV, Δm^2 is somewhat smaller because of the lower energy, although still larger than for all but the UCLA experiment. The parameter k/E_0, however, is quite large; larger than for any other experiment. In the preliminary analysis[2] the data seemed to be in between SPA and potential model predictions. A more refined data analysis,[1] however, has lowered the data somewhat so that now it is in fair agreement with SPA, and in any case definitely below the results calculated in potential models with relativistic corrections,[1,8,14] or in a one-boson exchange model.[15]

Of the remaining two experiments which we discuss here, the 42 MeV Manitoba experiment[4,5] has very small Δm^2 by virtue of its low energy, and so presumedly has relatively smaller off-shell effects. However, k/E_0 is fairly large, and the experiment is one of the most comprehensive with data for a variety of asymmetric and noncoplanar geometries. The Orsay experiment[6] at 156 MeV covers only a very limited kinematic range, with Δm^2 and k/E_0 intermediate between the Manitoba and TRIUMF experiments. It is of interest, though, as it has been impossible to fit using standard potential models.

Before the results are presented it is worthwhile to review briefly the ingredients of the SPA. As is well known, the amplitude for a radiative process can be expanded in powers of the photon momentum k and written as[9,11]

$$M_{pp\gamma}=\frac{A}{k}+B+Ck\,,$$

where A and B are fixed via gauge invariance in terms of on-shell information. C contains the off-shell effects, but also higher order on-shell terms, ambiguities due to the prescription used for choosing the on-shell point at which to evaluate A and B, etc.[10] The cross section then becomes

$$\begin{aligned} d\sigma_{pp\gamma} &\sim k|M_{pp\gamma}|^2 \\ &\sim\frac{A^2}{k}+2\,\mathrm{Re}AB^*+(B^2+2\,\mathrm{Re}AC^*)k+2\,\mathrm{Re}\,BC^*k^2 \\ &\quad +C^2k^3\,. \end{aligned}$$

In the calculations reported here, the amplitudes A and B were evaluated numerically via the explicit formulas of Eq. (2.4) of Ref. 11 using the average energy and momentum transfer prescription for evaluating the elastic amplitudes. The pp elastic phase shifts of Arndt[16] were used; they reproduce the elastic cross section to within a few percent in the necessary energy range. Once obtained, the amplitude $M_{pp\gamma}$ was squared numerically. Thus the SPA cross section given here includes the A^2/k and $2\,\mathrm{Re}AB^*$ terms and also the B^2k part of the $O(k)$ term.

As discussed in detail elsewhere,[10] at 42 MeV the $A^2/k+2\,\mathrm{Re}AB^*$ terms are most important except near $\theta_\gamma=0°$, $\sim 80°$, and 180°, where in the symmetric geometries A vanishes and thus the B^2k term dominates. Off-shell terms and differences between SPA and model calculations first show up in the $\mathrm{Re}AC^*k$ term, except in regions where that term is suppressed by the smallness of A. At the higher energy of the Orsay experiment, the B^2k term dominates, accounting for at least three-quarters of the SPA cross section.

The results derived from this SPA can now be compared with the data and with calculations in potential models. In Fig. 1 such a comparison is shown for the 42 MeV Manitoba data for the complete set of coplanar angle pairs measured. The Hamada-Johnston (HJ) potential model results are taken from Ref. 4 and originate from the work of Liou.[14] The SPA calculations are obtained as described above and in Ref. 10. The data set, taken directly from Ref. 4, is the set for which effects due to the finite size of the angular bins, resolutions, and other experimental effects have been unfolded via a prescription. They are thus the appropriate quantities for comparison with the theoretical point coplanar cross sections evaluated at the center of the bins. This unfolding of experimental effects requires some theoretical input for which the HJ results were used, but is presumedly insensitive to this particular choice of input. Note that these data differ significantly in many cases from comparable preliminary results of Ref. 5 for which the unfolding had not been done as completely.

Some qualitative results can be immediately obtained from Fig. 1, without any quantitative analysis. First observe that there is a significant difference between HJ potential model results and SPA calculations in many cases, and that this difference becomes relatively larger as the final proton angles θ_3 and θ_4 become smaller. This difference is presumedly an indication of contributions of the Ck and higher terms in the amplitude. Calculations by Bohannon[8] using phase equivalent potentials have shown, however, that there is little off-shell sensitivity at 42 MeV provided the long range portions of the interaction are fixed. Thus these Ck terms must be primarily from higher order on-shell effects, or perhaps differences in the elastic amplitudes obtained or used in HJ and SPA calculations, rather than true off-shell effects.

A second qualitative observation is that the data

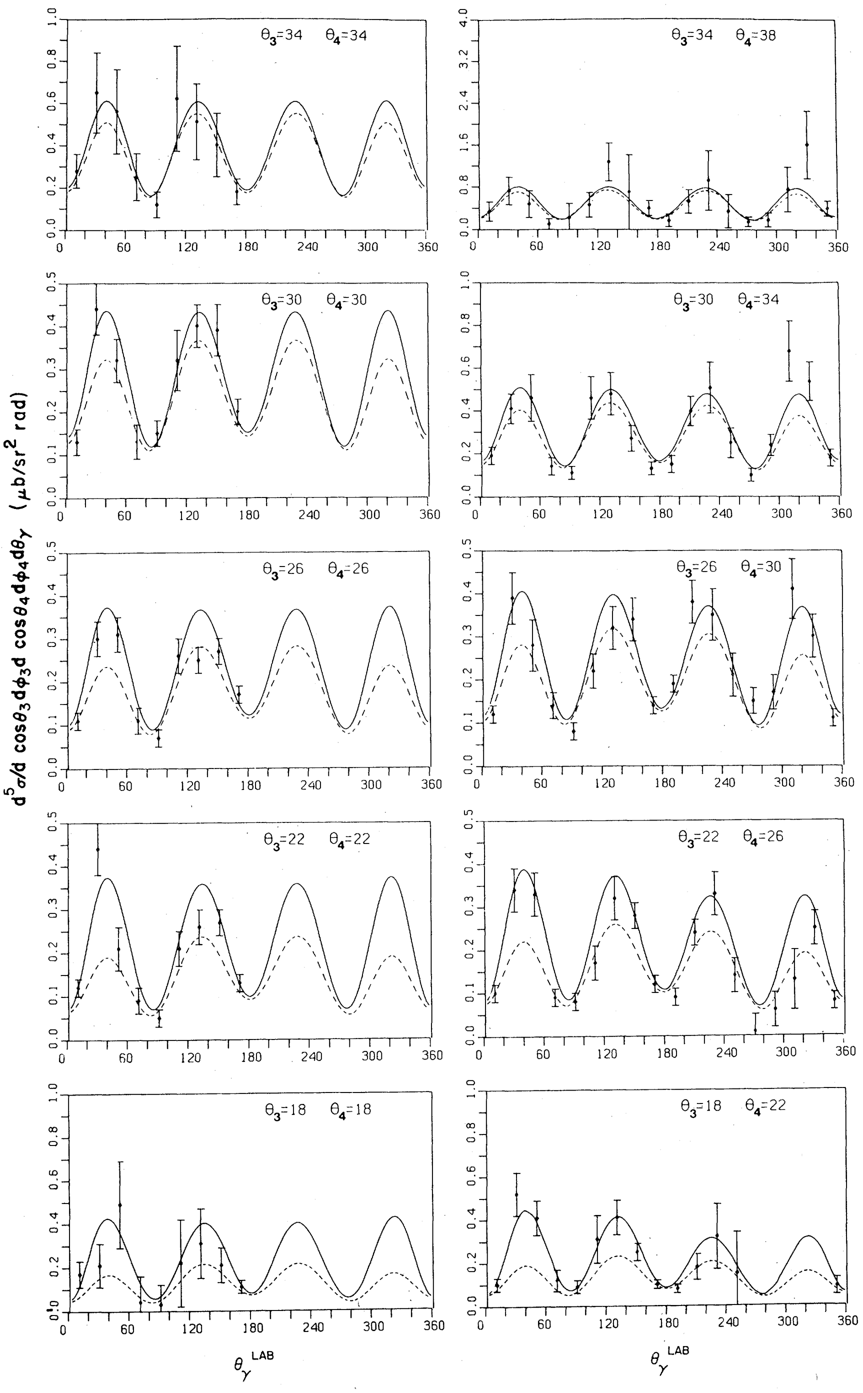

FIG. 1. Comparison of SPA calculations (dashed curve) and Hamada-Johnston potential model calculations of Ref. 4 (solid curve) with the 42 MeV data of the Manitoba experiment for all of the coplanar angle pairs measured. The data have had binning and resolution effects unfolded, and so are appropriate for comparison with the point coplanar theoretical cross sections given.

tend to fall in between the two curves, or in any case lower than the HJ predictions, but have error bars which are often larger than, and which span, the difference between the two calculations. This observation can be backed by quantitative calculations. In particular, a simple estimate of the chi-square per degree of freedom gives for comparison of data with HJ calculation $\chi_{\mathrm{HJ}}^2 \simeq \frac{393}{252} = 1.56$, and with

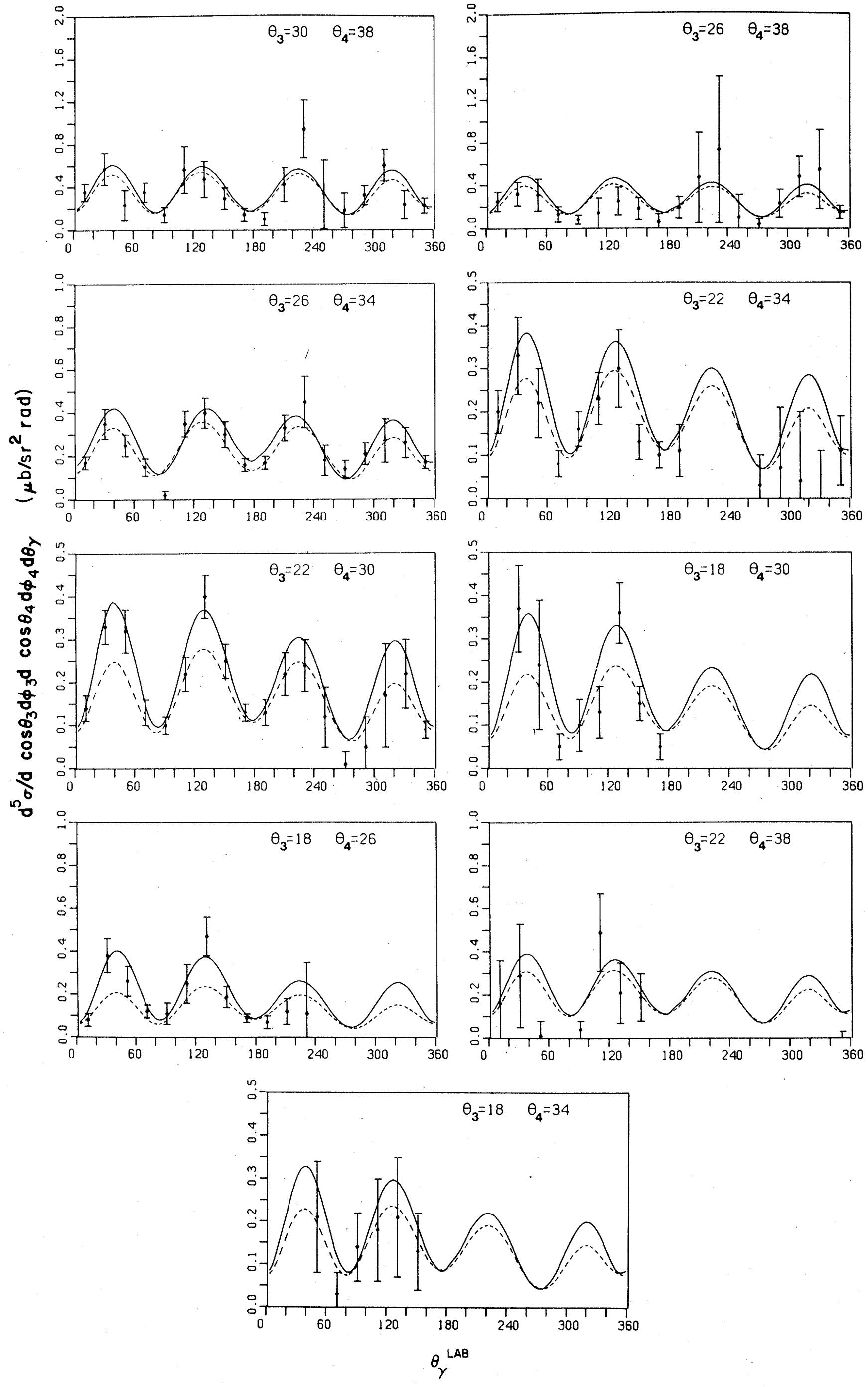

FIG. 1. *(Continued)*.

SPA calculation $\chi_{\mathrm{SPA}}^2 \simeq \frac{402}{252} = 1.60$. Thus the goodness of fit to the two calculations is essentially the same. While the overall fit is identical, there are some differences once one starts looking at details. In general, as a group, the symmetric angle pairs tend to fit the HJ theory somewhat better than the SPA theory, while the fit to HJ gets worse, and that to SPA better, as the angle pair becomes more and more asymmetric. This effect was observed for the HJ case before[4,5] using a more sophisticated fitting analysis. Note that some angle pairs, e.g., 18°–22°, are exceptions to this trend.

Another question arises from looking at Fig. 1. Is it possible to renormalize the data to achieve a better fit? For the SPA, reducing all data by a factor 0.95 gives $\chi_{SPA}{}^2 = 1.54$, which is only marginally better. For the HJ calculation a significant improvement can be obtained by increasing all data by a factor 1.18, which gives $\chi_{HJ}{}^2 = 1.07$. Such a large renormalization is greater, however, than the overall renormalization uncertainty of 4%, which is given for the experiment and so is presumedly ruled out. Clearly any kind of systematic renormalization effect which increased asymmetric angles more than symmetric ones could improve the fit to HJ even more, but there is no known reason for any such effect.

A similar comparison of data with potential model results and with SPA can be made for the Orsay 156 MeV experiment. Results for one angle pair are shown in Fig. 2. The potential model calculations are taken from Brown[17] and use a nonrelativistic approach based on a HJ potential. Relativistic corrections have been considered by Celenza *et al.*[18] and tend to reduce the curve by roughly half the difference between HJ and SPA curves. The data fall below the HJ results and in overall normalization are in better agreement with the SPA curve. The shape of both HJ and SPA curves is similar, but in neither case corresponds very well to the data.

Figure 3 shows, following Celenza *et al.*,[18] the results for the other angle pairs measured, averaged over the relatively large bin size in θ_γ, and plotted as a function of θ_4. Again the data fall below the potential model results, and at the larger values of θ_4, even below the SPA curve. Clearly the trend of the data with increasing θ_4 is just not reproduced by either potential calculations or SPA.

Thus to summarize for the Orsay data, the SPA gives a "better" fit as far as normalization is concerned since it, like the data, falls significantly below the potential calculations. However, the dependence on θ_γ or on θ_4 is similar in all theoretical calculations and does not reproduce the data very well.

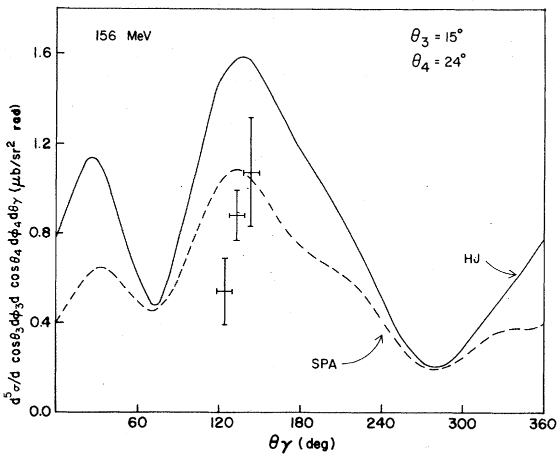

FIG. 2. Comparison of the 156 MeV Orsay data for one angle pair, $\theta_3 = 15°$ and $\theta_4 = 24°$, with SPA calculations (dashed curve) and with Hamada-Johnston nonrelativistic potential model calculations (solid curve) of Brown (Ref. 17).

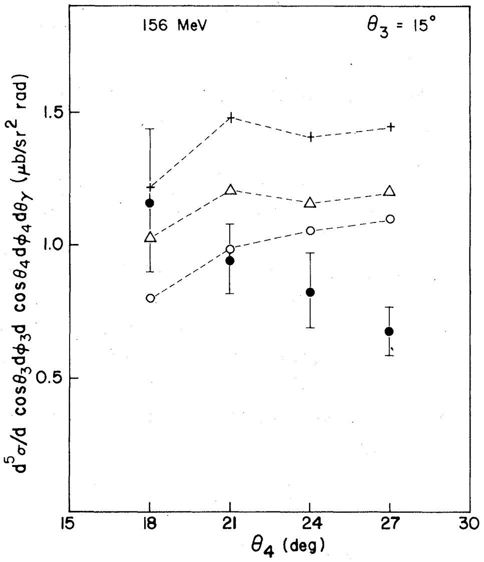

FIG. 3. Comparison of the 156 MeV Orsay data as a function of proton angle θ_4, with $\theta_3 = 15°$ fixed, with various theoretical calculations averaged over the appropriate angular bins. The upper, middle, and lower curves correspond, respectively, to the nonrelativistic potential model calculations of Brown (Ref. 17), a relativistically corrected version of such model by Celenza *et al.* (Ref. 18), and the SPA results.

From the above discussion and the analysis elsewhere of the TRIUMF[1,10] and UCLA[3,10] experiments, one may begin to see a general picture emerging at least for the region ≤ 200 MeV. In the first place, in this region all data seem to fall below the standard nonrelativistic potential models using the Hamada-Johnston potential. Such relativistic corrections as have been made[14,18] reduce the potential results, but not sufficiently to agree with the data. Somewhat surprisingly, the purely on-shell SPA seems to agree with the data as well or better than the potential model calculations.

At both 200 and 156 MeV, it is now clear that relativistic corrections can be important, and thus the whole concept of potential model calculations may be suspect. Within such an approach, however, investigations of phase equivalent potentials have shown[8] that there is a fair sensitivity to off-shell effects, at least at 200 MeV, although it is difficult to find any transformation, i.e., choice of off-shell behavior, which can reduce the HJ cross section sufficiently to fit the data. Thus it remains a puzzle why the SPA works as well here as it does.

At 42 MeV one should be able to do potential calculations essentially exactly, as relativistic corrections should be small, and there seems to be essentially no dependence on off-shell effects.[8] Thus it is quite disturbing that the data are not in better agreement with the potential calculations.

Clearly a number of further things need to be done. Potential model calculations should be reexamined and a "good" calculation made combining all of the various effects now known, e.g., relativistic corrections, Coulomb effects, double scattering terms, etc. At the same time, some modern potential, such as that of the Paris group,[19] should be used to make sure that difficulties with the potential calculations are not just the fault of an inaccurate potential which does not reproduce elastic data sufficiently well. Finally, additional data would be useful, particularly asymmetry data as proposed earlier,[12,20,21] as that would give different combinations of amplitudes against which to test the theoretical calculations.

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