Comments

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Comments on testing the Pauli principle

R. D. Amado and H. Primakoff
Department of Physics, University of Pennsylvania, Philadelphia, Pennsylvania 19104
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Particle identity in quantum mechanics makes it impossible to test the Pauli exclusion principle by looking for "forbidden" x rays or γ rays. Such experiments do, however, test particle stability. Evidence for the indistinguishability of electrons and nucleons is discussed.

[NUCLEAR STRUCTURE Particle identity and the Pauli exclusion principle, particle stability.]

I. INTRODUCTION

There have recently been two experiments reported that purport to test the validity of the Pauli principle, one for nucleons[1] and one for electrons.[2] Other experiments have also been proposed.[3] In this work we show that in a strict sense these experiments cannot test the Pauli principle, but do test particle (nucleon or electron) stability.

The archetypal experiment to "test" the Pauli exclusion principle (PEP) is to look for K x rays from a block of material. On the basis of a seemingly plausible argument it would appear that electrons in higher orbits are only prohibited from falling into the K shell by the PEP, and if that principle is violated "a little" they should fall in "a little." Hence by setting a limit on the intensity of the K x rays one seems to be putting a limit on that "little." The purpose of the present note is to show that this interpretation is not correct. Nuclear experiments to detect Pauli-forbidden γ rays are conceptually the same as the atomic experiment and subject to the same objections.

II. THE PAULI PRINCIPLE

At the level of nonrelativistic quantum mechanics, the PEP is a statement about the permutation symmetry of wave functions for identical particles. The fact that the particles are identical is crucial, since if they are not, the wave function need have no particular permutation symmetry. How does that identity manifest itself in the formalism? Or better, how does the formalism reflect that identity? The answer is that the Hamiltonian must treat the identical particles completely symmetrically; that is, the Hamiltonian commutes with all permutations. On the basis of the Hamiltonian alone, one cannot tell to what symmetry class the particles must belong. In fact, one can only say that, since the Hamiltonian is symmetric, it is possible to classify the states into (orthogonal) symmetry classes. In this connection, it is important to realize, for example, that the parastatistics discussed a few years ago[4] do not correspond to putting particles into states with arbitrary mixtures of symmetric and antisymmetric pairs, but correspond to particular mixtures that are orthogonal to both symmetric and antisymmetric states.

Suppose we now consider a particular atomic state, a "normal" state, in which the electrons (assumed to be all identical) do satisfy the PEP; that is, the wave function is completely antisymmetric. Since the full Hamiltonian of the system, including the interaction Hamiltonian with the radiation field, is completely symmetric in the electrons, transitions are *only* possible to states of this same antisymmetric character. Thus a symmetric Hamiltonian forbids transition from a normal antisymmetric atomic state to a "forbidden" state with mixed symmetry. No transitions to the occupied K shell are therefore possible from a normal state.

If all electrons are identical, transitions to the K shell are only possible if *all* electronic states have a component of mixed symmetry. Since we know that electronic states are mostly totally antisymmetric (think, for example, of the Fermi sea in metals) these admixtures must be small. But if the Hamiltonian is totally symmetric and the PEP is not imposed, there is nothing to keep the component of mixed symmetry small; that is, there is nothing to set the scale of "small" for the mixed component since there is no small "wrong" symmetry piece of the Hamiltonian. If the mixed component can be large, it will be, since it is energetically favored; this is in total contradiction to experiment.

Even if some principle permitted small mixed symmetry components in wave functions that are primarily antisymmetric, and kept them small, the symmetric world Hamiltonian would only connect mixed symmetry states to mixed symmetry states, just as it connects only antisymmetric states to antisymmetric states. Hence the symmetry class of the state is conserved by a superselection rule and different symmetry classes *do not mix*. Under these circumstances it would be a miracle if any, let alone all, electron states of mixed type were degenerate with the antisymmetric states. But only degenerate states can be mixed, and even then they can only be formally superposed, since they cannot mix in any physical process, just as the conservation of charge prevents mixing between states of different charge.

In summary, identical particles in nonrelativistic quantum mechanics can be classified according to *unmixable* symmetry types, and worlds of different symmetry type do not mix. It is an important experimental question to decide to what symmetry type a particular particle belongs, but nuclear physics and atomic and solid state physics seem to give a very clear answer for nucleons and electrons. Having decided on a symmetry type, one cannot mix them. The essential difference between this situation and other symmetry laws is that if the particles are truly identical, one cannot then add a "small" nonidentical particle violation to the Hamiltonian. If the particles are identical, they are identical. Furthermore, as we now show, there seems to be no way compatible with experiment to make some or a few a little different.

It is clear that the entire above argument is based on standard quantum mechanics. In fact we do not know how to state the PEP rigorously outside of quantum mechanics.

III. NONIDENTICAL PARTICLES

The only way out of our selection rule is to give up identity, but in the sense we have already described, we are then no longer discussing the PEP at all. If all electrons were distinguishable, for example, by charge differences (however small) among them, there would no longer be any symmetry requirements on the wave function and the allowed states of lowest energy would no longer be antisymmetric. There would then be no periodic table of the elements, no Fermi gas model of metals, no stability of matter in the usual sense, etc. Clearly then, the evidence is overwhelmingly against the notion that every electron is somehow different from every other. A remaining possibility is that only a few "electrons" are different, but still "nearly" like ordinary electrons. Such an electron would already have jumped from an upper shell into the occupied K shell in a typical single-particle transition time. Thus the K x ray would not be seen in the archetypal experiment. However, the best evidence against another electronlike particle comes from quantum electrodynamics. (In fact, quantum field theory in general places many more restrictions on possible symmetries than does nonrelativistic quantum mechanics; for example, by the connection between spin and statistics, but to this point we have not needed these restrictions.) Thus, if there were a new particle with a mass and charge near the electron mass and charge, it would be pair produced with about the same probability as electron-positron pairs. Therefore the pair production total cross section would be twice the QED value in dramatic contradiction with experiments. Similar contradictions would occur through the virtual effects of pair production (vacuum polarization) on the very well tested predictions of QED, e.g., $g-2$ for the muon. The point here is that even if in ordinary matter extraordinary electronlike particles are rare, their charge will couple them to the electromagnetic field in the normal way and their effects in pair production will be strong. The only way to avoid this is to make the coupling of these particles to the electromagnetic field very weak. That coupling is their charge, and if it is small, the particles are no longer extraordinary electrons.

Similar remarks can be made about extraordinary "nucleons", specially charged ones, since they too would be pair produced; for example, in e^+e^- experiments. Neutral extraordinary nucleons may be somewhat more difficult to rule out, particularly since detailed strong interaction calculations are not yet possible. The best test for such particles is probably a search for "Pauli-forbidden" stable nuclei as "^{5}He" or "^{8}Be", or other "wrong mass" nuclei, which would contain one "extraordinary nucleon".[5]

IV. ELECTRON DECAY

We have seen that a search for K x rays from an ordinary piece of matter does not test the PEP or even test for unusual electrons. It does test for electron stability, however. If a K electron decays, there will be a genuine K hole and a subsequent K x ray. This may be the only signature of the decay, since if charge is not conserved the electron decay products could be effectively invisible (e.g., $e^- \to 3\nu$).

Since it seems to be only electron decay that the archetypal K x ray experiment tests, we will make a few remarks about it. Suppose first that we imagine electron decay that does not violate charge conservation. Suppose the electron decays to a charged particle with the full electron charge (e.g., $e^- \to X^- + \gamma$ or $e^- \to Y^- + \nu$), where X or Y have charge e. Since X or Y must be lighter than the electron, we can once again use the pair production arguments to rule out existence of X or Y, no matter how weak the electron decay coupling. Similar arguments hold for decays to particles carrying any substantial fraction of the electron's charge. A possibility that does seem difficult to rule out is what we call electron fracture $e^- \to Nx$, where the x carries a charge e/N, and N is a large integer. Energy conservation also requires $m_x < m_e/N$. For N sufficiently large, pair production of x will be small, as will the vacuum polarization effects. The small mass enhances these, but only logarithmically. There may be cosmological arguments against the existence of these small x particles, but we have not been able to exclude them on the basis of terrestrial experiments so long as N is large.

If it is suspected that charge conservation is violated in electron decay, the best model-independent test seems to be the K x rays, since the decay products may well be invisible (e.g., $e^- \to 3\nu$). We have nothing to add to the works that have already been written[6] on this alternative, except to note that the x rays associated with electron decay would have an energy corresponding to that of normal K x rays, whereas x rays associated with the fallacious view of PEP violation would have a slightly different energy.

V. CONCLUSION

The identity of electrons and of nucleons together with the general principles of quantum dynamics makes it impossible to test the Pauli Exclusion Principle for them in the usual sense. There is also very strong evidence against the existence of nonidentical electrons or protons, but not such good evidence against nonidentical neutrons. Searches carried out so far for (Pauli forbidden) x rays or γ rays actually test electron or nucleon stability, and for electrons may be the only practical test.

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[2]F. Reines and H. W. Sobel, Phys. Rev. Lett. 32, 954 (1974).
[3]R. Steinberg, private communication.
[4]For a review see M. Dresden, in *Brandeis Summer Institute Lectures on Astrophysics and Weak Interactions*, edited by K. W. Ford (Brandeis University, Waltham, Mass., 1963), Vol. 2, p. 379.
[5]Such a search is underway at the University of Pennsylvania; W. E. Stephens, private communication.
[6]Cf. L. B. Okun and Ya. B. Zeldovich, Phys. Lett. 78B, 597 (1978) and references therein. See also A. Ya. Ignatiev, V. A. Kuzmin, and M. E. Shaposhnikov, Phys. Lett. 84B, 315 (1979).