Effective S-wave πNN interaction in nuclei

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We show that virtual pion rescattering in a nuclear medium gives rise to an effective S-wave πNN interaction Hamiltonian. The first few terms in the density expansion of the strength parameter are estimated and shown to be appreciable in nuclear matter.

[NUCLEAR REACTIONS Pion-nucleus interactions: S-wave interaction.]

Considerable, if inconclusive, attention has been given in recent literature to the "Barnhill" ambiguity in the S-wave πNN interaction in nuclei.[1-5] The ambiguity concerns the strength parameter λ in the Hamiltonian for this interaction:

$$H=\lambda\frac{f}{\mu}\psi^{+}\vec{\sigma}\cdot\vec{\nabla}_N\vec{\tau}\cdot\vec{\phi}\psi\,. \tag{1}$$

Here ψ and $\vec{\phi}$ are the nucleon and isovector pion field operators and $\vec{\nabla}_N$ a symmetrized gradient operator acting on the nucleon fields.[1] In (1) f is the pseudovector pion-nucleon coupling constant ($f^2/4\pi=0.08$) and μ the pion mass.

Straightforward nonrelativistic reduction of the Lorentz invariant pseudovector πNN interaction Hamiltonian leads to $\lambda=\mu/2m$ (m = nucleon mass). The unitary freedom inherent in this reduction leads to the ambiguity in λ.[1] The physical reason for this ambiguity is the lack of a definite treatment of the binding effects in a nuclear medium that force the nucleon under consideration off shell.[3,4] Only in simplified boson exchange models is it possible to make definite predictions for λ.[6] Therefore it has been suggested that the parameter be determined empirically by means of (p,π^+) and (π^+,p) reactions interpreted with a single nucleon stripping model.[3,7] Apart from the obvious criticism that the simple stripping model may not be adequate, we shall in this work show that sizeable density dependent medium corrections to the effective one-body operator (1) are caused by virtual pion rescattering. Thus λ will not have any universal value valid for a range of nuclei, and attempts at empirical determination of λ will be futile.

The main rescattering process that contributes to λ is that involving two nucleons: an incident S-wave pion rescatters off a nucleon (which is ejected) and is absorbed by a particle-hole pair, or it rescatters off a particle-hole pair and is absorbed by a final nucleon (which is ejected). This is illustrated in the diagrams in Fig. 1, which include direct and exchange terms. These processes take place in addition to "true" two-nucleon absorption processes in which two nucleons are ejected from the nucleus.

The diagram in Fig. 1(c) represents distortion of the incident pion wave function and should not be included in the basic πNN interaction, as the distortion can be treated with an optical potential. The corresponding exchange term in Fig. 1(d) should be excluded for the same reason. The amplitude for the diagram in Fig. 2(a) vanishes in spin or isospin 0 nuclei because of the spin-vector isovector nature of the πNN absorption operator. Therefore only the remaining exchange term diagram [Fig. 1(b)] contributes to the S-wave absorption interaction.

In order to construct the amplitude corresponding to Fig. 1(b) we employ the phenomenological zero-range Hamiltonian

$$H=4\pi\frac{\lambda_1}{\mu}\bar{\psi}\vec{\phi}\cdot\vec{\phi}\psi+4\pi\frac{\lambda_2}{\mu^2}\bar{\psi}\vec{\tau}\cdot\vec{\phi}\times\vec{\pi}\psi \tag{2}$$

to describe the S-wave rescattering vertex ($\vec{\pi}=\partial_0\vec{\phi}$). In (2) the coupling constants λ are de-

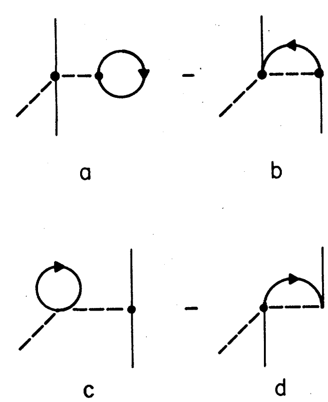

FIG. 1. Pion rescattering contributions to the effective S-wave πNN interaction.

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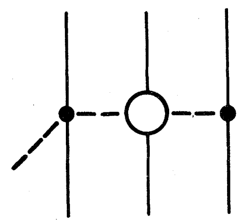

FIG. 2. Three-body contribution to pion absorption with double pion rescattering.

termined from the S-wave pion-nucleon phase shifts to be $\lambda_1 = 0.003$ and $\lambda_2 = 0.05$.[8] This Hamiltonian leads to a satisfactory prediction of the S-wave pion-nucleon phase shifts at low energies and is used in the standard derivations of the first and second order S-wave pion-nucleus optical potential. It has also been used successfully in the two-nucleon model for nuclear pion absorption,[9,10] although it implies a very simple direct off shell extrapolation of the pion-nucleon interaction. For the final absorption vertex we use the standard P-wave πNN interaction. The two-body rescattering amplitude is then

$$T = -4\pi i\left(\frac{f}{\mu^2}\right)\frac{\vec{\sigma}^2\cdot\vec{k}}{\mu^2+k^2}\,[2\vec{\tau}^2_i\lambda_1 - i\lambda_2(\vec{\tau}^1\times\vec{\tau}^2)_i]\,, \quad (3)$$

with $\vec{k}$ being the momentum of the exchanged pion. The isospin index of the initial pion is i.

Taking the matrix element of T over the closed particle-hole line in Fig. 1(b) yields

$$T = i\,\frac{4}{\pi}\,\frac{f}{\mu^2}\,(\lambda_1+\lambda_2)k_F^{\,3}\int d^3r_2\,\frac{j_1(k_F r)}{k_F r} \times\int\frac{d^3k}{(2\pi)^3}\,\frac{\vec{\sigma}\cdot\vec{k}}{k^2+\mu^2}\,e^{-i\vec{r}\cdot\vec{k}}\,. \quad (4)$$

Here k_F is the Fermi momentum and $\vec{r} = \vec{r}_1 - \vec{r}_2$. To reduce (4) to an effective one-body operator we take the 0-range limit $\mu\to\infty$, which leads to

$$T = -i\,\frac{2}{3\pi}\left(\frac{k_F}{\mu}\right)^3\left(\frac{f}{\mu}\right)(\lambda_1+\lambda_2)\vec{\sigma}\cdot(\vec{p}_f+\vec{p}_i)\tau_i\,. \quad (5)$$

Here p_i and p_f are the momenta of the initial and final nucleons, which appear after a partial integration. The employment of the zero-range limit can be expected to overestimate the two-body contribution in Eq. (4). It is, however, a better approximation than that which the static pion propagator in Eq. (4) would imply, because of the self-energy correction to the propagator which reduces the kinetic energy term. This point will be discussed in detail in conjunction with the higher order corrections below.

The amplitude (5) can be generated from the interaction Hamiltonian (1) with the density dependent parameter λ_{II}:

$$\lambda_{II} = \frac{\pi}{\mu^3}\,(\lambda_1+\lambda_2)\rho = \frac{2}{3\pi}\left(\frac{k_F}{\mu}\right)^3(\lambda_1+\lambda_2)\,. \quad (6)$$

In nuclear matter $k_F \simeq 1.35\ \mathrm{fm}^{-1}$ and thus $\lambda_{II} \simeq 0.08$, which is similar in magnitude to that of the usual one-body Galiliean invariance counter term ($\lambda_I = \mu/2m \simeq 0.07$) obtained from the relativistic pseudovector Hamiltonian.

The magnitude for λ_{II} given in Eq. (6) is clearly an overestimate, as the effect of hadronic form factors and nuclear wave function correlations will tend to reduce it. Although the simple zero-range approximation used above will not permit a realistic estimate of these effects, a first estimate of the form factor reduction is possible. Assuming the pion-nucleon vertices to be described by a monopole type form factor $(\Lambda^2-\mu^2)/(\Lambda^2+k^2)$ the expression (1) ought to be multiplied by a factor $(1-\mu^2/\Lambda^2)$. With $\Lambda\sim 1\ \mathrm{GeV}/c^2$, this would lead to a ~5% reduction of the previous estimate for λ_{II} remains the same as that of λ_I.

Equation (6) represents the second term in a density expansion of the parameter λ. The next term involves a secondary pion rescattering, as illustrated in Fig. 2. Here the dominant contribution will come from the P-wave component in the second rescattering amplitude. As the nucleon pole terms in this amplitude represent binding corrections to the two-body amplitude above, we only consider the intermediate states containing the Δ_{33} resonance treated in the sharp resonance

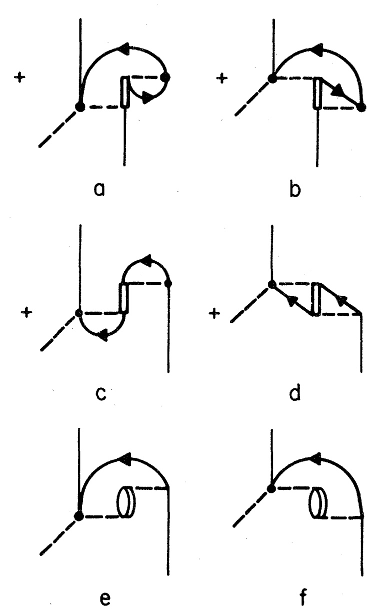

FIG. 3. Three-nucleon contributions with Δ_{33} intermediate states to the effective S-wave πNN interaction.

approximation.

Considering both time orderings for the Δ_{33} resonance intermediate state contributions there will then be a total of 36 three-body diagrams in which two of the final particles remain within the Fermi sea, compared to the four two-body diagrams in Fig. 1. After exclusion of all those diagrams that properly represent distortion of the incident pion wave function, and the corresponding exchange terms, and all those that give no contribution in isospin 0 or spin 0 nuclei, we only need to consider the six diagrams in Fig. 3.

To construct the scattering amplitudes for these diagrams we use the $\pi N\Delta$ coupling Lagrangian

$$\mathcal{L}_{\pi N\Delta}=\frac{f_\Delta}{\mu}\,\vec{\chi}^{\,+}\vec{\xi}^{\,+}\cdot\vec{\nabla}\vec{\phi}\,\xi\chi+\text{H.c.} \tag{7}$$

Here ξ and χ are the spin and isospin field operators for the nucleon and $\vec{\xi}$ and $\vec{\chi}$ the corresponding operators for the Δ_{33} resonance. From the decay width of the resonance one has $f_\Delta^2/4\pi\simeq 0.35$. For the isobar contribution to the general three-nucleon two-pion exchange diagram in Fig. 2 one obtains

$$T=-4\pi i\,\frac{8}{9(m_\Delta-m)}\,\frac{ff_\Delta^2}{\mu^2}\,\frac{\vec{\sigma}^3\cdot\vec{k}_2}{(\vec{k}_1^{\,2}+\mu^2)(\vec{k}_2^{\,2}+\mu^2)}$$
$$\times\{\vec{k}_1\cdot\vec{k}_2[2\lambda_1\tau_i^3-i\lambda_2(\vec{\tau}^1\times\vec{\tau}^3)_i]-\tfrac{1}{4}\vec{\sigma}^2\cdot(\vec{k}_2\times\vec{k}_1)$$
$$\times[2\lambda_1(\vec{\tau}^2\times\vec{\tau}^3)_i-i\lambda_2(\vec{\tau}^1\times(\vec{\tau}^2\times\vec{\tau}^3))_i]\}, \tag{8}$$

with $\vec{k}_1$ and $\vec{k}_2$ being the momenta of the primary and secondary exchanged pions. The mass of the resonance is denoted by m_Δ.

Carrying out the spin and isospin sums over the closed particle-hole lines indicated in the diagrams in Figs. 3(a) and (b) and taking the Fourier transform of the amplitude (8) yields

$$T=-4\pi i\,\frac{8}{9(m_\Delta-m)}\,\frac{ff_\Delta^2}{\mu^2}\int\frac{d^3k_1}{(2\pi)^3}\,e^{i\vec{k}_1\cdot(\vec{r}_3-\vec{r}_2)}e^{i\vec{k}_2\cdot(\vec{r}_2-\vec{r}_1)}\,\frac{1}{k_1^{\,2}+\mu^2}\,\frac{1}{k_2^{\,2}+\mu^2}$$
$$\times(\vec{\sigma}^1\cdot\vec{k}_2\vec{k}_1\cdot\vec{k}_2-\tfrac{1}{3}\vec{k}_2^{\,2}\vec{\sigma}^2\cdot\vec{k}_1+\tfrac{4}{3}\vec{k}_2^{\,2}\vec{\sigma}^1\cdot\vec{k}_1)\,. \tag{9}$$

The same sum for the diagrams in Figs. (3c) and (d) give rise to an identical result.

To complete the evaluation of the particle-hole line matrix elements for these diagrams [Figs. 3(a) and (b)] the expression (9) must be integrated over $\vec{r}_2$ and $\vec{r}_3$, folded with the relevant three-body density

$$\frac{k_F^{\,6}}{4\pi^4}\,\frac{j_1(k_Fr_{23})}{k_Fr_{23}}\,\frac{j_1(k_Fr_{13})}{k_Fr_{13}}\,. \tag{10}$$

The three-body density appropriate for the diagrams in Figs. 3(c) and (d) may be obtained from (10) by the replacement $\vec{r}_{13}\to\vec{r}_{12}$. Finally, because of nuclear correlations the δ function in the bracket in Eq. (9) should be eliminated by the replacement $k_2^{\,2}\to-\mu^2$.

In order to obtain an approximate one-nucleon amplitude we again resort to the zero-range limit of the pion propagators in Eq. (9), i.e., $\mu^2\to\infty$. In that limit the integrations become trivial and the combined result for all the four diagrams in Figs. 3(a)–(d) is

$$T=-i\,\frac{32}{243\pi^3}\left(\frac{k_F}{\mu}\right)^6\frac{ff_\Delta^2}{m_\Delta-m}\,(\lambda_1+\lambda_2)\vec{\sigma}\cdot(\vec{p}_f+\vec{p}_i)\tau_i\,. \tag{11}$$

This amplitude can be obtained from the interaction (1) with $\lambda=\lambda_{III}$:

$$\lambda_{III}=\frac{32\pi^2}{27\mu^6}\left(\frac{f_\Delta^2}{4\pi}\right)\frac{\mu}{m_\Delta-m}\,(\lambda_1+\lambda_2)\rho^2$$
$$=\frac{128}{243\pi^2}\left(\frac{f_\Delta^2}{4\pi}\right)\frac{\mu}{m_\Delta-m}\left(\frac{k_F}{\mu}\right)^6(\lambda_1+\lambda_2)\,. \tag{12}$$

In nuclear matter $\lambda_{III}\simeq 0.024$, which is roughly 30% of the corresponding value for λ_{II}.

We finally consider the diagrams in Figs. 3(e) and (f). These can actually be summed to all orders in the internal isobar-hole propagators and then simply represent the self-energy corrections to the pion propagator in the two-body amplitude (4) corresponding to the diagram in Fig. 1(b). They may thus be taken into account by the replacement

$$\frac{1}{k^2+\mu^2}\to\frac{1}{k^2+\mu^2+\Pi_\Delta} \tag{13}$$

in Eq. (4), with Π_Δ being the isobar-hole contribution to the pion self energy[10]:

$$\Pi_\Delta=-\frac{f_\Delta^2}{4\pi}\left(\frac{32\pi}{9}\right)\frac{\rho k^2}{\mu^2(m_\Delta-m)}\,. \tag{14}$$

This self-energy expression is reduced by taking into account the isobar-hole interaction in the one-pion exchange approximation to[11]

$$\Pi_\Delta\to\Pi_\Delta\left(1-\frac{1}{3}\,\frac{\Pi_\Delta}{k^2}\right)^{-1}, \tag{15}$$

if the nuclear pair correlation function is taken into account to the minimal extent that it eliminates the δ-function terms in the interaction. The main role of this self energy will be to improve the accuracy of the zero-range approximation used in the other many-body diagrams above. Since the magnitude of Π_Δ in (14) and (15) is $\Pi_\Delta \sim -0.6k^2$, it reduces the kinetic energy terms in the pion propagators to less than half of its unmodified value. We therefore feel that the zero-energy approximation should be accurate anough for a first estimate of the medium corrections to the S-wave ΠNN vertex in nuclei.

The net three-body correction to the parameter λ is thus that given in Eq. (12), and it is again necessarily an overestimate as hadronic form factors and nuclear wave function correlations will reduce it. With the same argument as used above to estimate the form factor correction to λ_{II}, we would find λ_{III} reduced by at least ~10% due to the form factors.

The relative smallness of λ_{III} compared to λ_{II} in nuclear matter suggests that higher order rescattering mechanisms will be of little significance. In any case the large value of λ_{II} compared to the "free nucleon" value $\lambda = \mu/2m$ shows that the effective one-body S-wave pion absorption operator (1) will depend strongly on the nuclear density. The interaction strength will thus vary from nucleus to nucleus, and there will be no possibility of determining a universal value for λ by pionic stripping and knockout reactions. It appears that the only realistic approach to the S-wave pion-nucleus interaction will be to take into account its two-nucleon and many-body components explicitly.

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