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Nonlinear meson dynamics and binding corrections in low energy pion-nucleus scattering

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Phenomenological fits to low-energy π-nucleus scattering have uniformly required a greatly enhanced s-wave repulsion in the optical potential, for which no satisfactory theoretical explanation has been given. Evaluation of nonlinear terms in field-theoretic Lagrangians with broken chiral symmetry and inclusion of necessary binding effects leads to a substantial, repulsive $\rho^2(r)$ potential, even on an isospin zero target. This potential may account for the previously unexplained effect.

[NUCLEAR REACTIONS Pion-nucleus elastic scattering; field theoretic optical potential. Binding corrections. ^{16}O and ^{12}C targets.]

Some time ago it was realized that low energy pion-nucleus elastic scattering could not be described using an optical potential derived from the first-order terms of multiple scattering theory [impulse approximation, Fig. 1(a)].[1-3] This difficulty was confirmed by the measurements of Amman *et al.*[4] and was subsequently verified by other groups.[5] For example, the elastic $\pi^+ - {}^{12}C$ and $\pi^+ - {}^{16}O$ data at T (lab) $= 50$ MeV exhibit a minimum at $\theta \sim 60°$, a much smaller angle than would be predicted by the $\theta_{\min} \sim 3.9/kR$ rule of diffraction scattering. The observed minima can be fitted phenomenologically by assuming the optical potential has the form

$$2k^0 V(r) = -4\pi(b_0 + b_1 \vec{\tau}\cdot\vec{T})\rho(r) - 4\pi B_0 \rho^2(r) + 4\pi(c_0 + c_1\vec{\tau}\cdot\vec{T})\nabla\cdot\rho(r)\nabla + 4\pi C_0 \nabla\cdot\rho^2(r)\nabla\,, \tag{1}$$

where the terms involving gradient and divergence operators are called, generically, "p-wave" terms, whereas those lacking such operators are called "s-wave" terms. [For simplicity kinematic corrections and the Lorentz-Lorentz effect are omitted from Eq. (1), but are included in our calculations.] The position of the minimum in the elastic angular distribution arises from the interference between repulsive s-wave and attractive p-wave terms[3,6-9] with a good fit requiring much stronger s-wave repulsion than that predicted in impulse approximation (since the average of π-n and π-p scattering lengths nearly vanishes, compared with their difference). This strong phenomenological repulsion needed to fit low energy scattering data is consistent with that needed to fit the s-orbital level shifts in pionic atoms.[10,11]

The phenomenological analysis of pion-nucleus scattering makes little distinction between terms in the optical potential which are proportional to the nuclear density $\rho(r)$ and those proportional to $\rho^2(r)$.[11] The former are supposed, however, to arise from one-nucleon processes, so that it would be quite hard to understand how the values of these coefficients could be substantially modified from their free-nucleon (impulse approximation) values. If we use this theoretical argument to fix the coefficient b_0, we then obtain a definite value for $\mathrm{Re}(B_0)$, and it is the task of theory to explain the result of the phenomenological analyses. Two sources of nonlinear density dependence have been explored in previous analyses: The first is a double-scattering correction to the optical potential [shown graphically in Fig. 1(b)] required to make the overall double scattering (iterated impulse approximation to the optical potential plus correction) obey the Pauli principle for nuclear intermediate states.[12] In infinite nuclear matter this term is proportional to ρ^2, but for finite nuclei (as shown in the Appendix) it varies more slowly with ρ. Second, the imaginary part of B_0 is popularly though to arise from "true" pion absorption, in which two nucleons share the energy brought in by the pion (kinetic plus rest mass) in such a way that the momentum transfer to the rest of the nucleus is small. Since two nucleons are simultaneously involved, the term is proportional to ρ^2; however, causality demands that to $\mathrm{Im}(B_0)$ there correspond a dispersive real part, given by an unsubtracted dispersion relation

$$\mathrm{Re}[B_0(k^0)] = \frac{2k^0}{\pi} P \int_0^\infty d\nu \frac{\mathrm{Im}B_0(\nu)}{\nu^2 - (k^0)^2}\,. \tag{2}$$

Brueckner[1] seems to have been the first to have pointed this out. Most recent authors who have

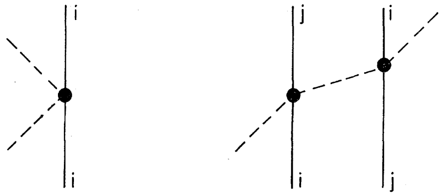

FIG. 1. (a) Impulse approximation for pion-nucleus scattering, and (b) Pauli correction to double scattering in the optical model.

worried about the sign of $\mathrm{Re}(B_0)$ have either hoped that this dispersive part would come out repulsive, or have assumed with Brueckner that it was.[10] Unfortunately, all microscopic calculations of $\mathrm{Im}(B_0)$ lead, via the dispersion relation (2), to an *attractive* dispersive part.[13-16] Since these calculations reproduce reasonably well the magnitude and energy dependence of the absorptive part,[16] it is unlikely that the resulting dispersive part is radically incorrect. When it is added to the (repulsive) second-order Pauli correction described above, the dispersive term increases the disagreement with the phenomenological analyses.

Within the framework of the simple zero-range pi-nucleon T matrix which we use, all Fermi motion, binding effects, and kinematic transformations are given by the terms (which we include) in Ref. 11 [Eq. (5)] proportional to k^0 divided by the nucleon mass M. Hence in this model one needs a new phenomenon to understand the previous phenomenological analyses.

In this paper we propose a new origin for the missing repulsive s-wave potential strength: Nonlinear terms in field-theoretic models incorporating broken chiral symmetry give rise to a real pion-nucleus potential proportional to $G^4\rho^2 m_\pi{}^2$. Here we show that this new effect is sufficiently strong to account for the empirically necessary s-wave repulsion. We consider two field-theoretic Lagrangians which embody broken chiral SU(2) $\times$SU(2) and hence reproduce the low-energy ($\lesssim 1$ GeV) pion-nucleon and pion-pion s-wave scattering lengths. The first, the renormalizable $\sigma+\omega$ model, appends to the Gell-Mann–Lévy[17] σ model a neutral vector meson (ω) which is coupled linearly to the conserved baryon current. The ρ^2 term arises from the quartic couplings inherent in the sigma model [see Figs. 3(b) and 3(c)], as well as the enhancement, due to binding effects, of virtual pair terms in nuclear matter, [see Figs. 3(a) and 3(d)]. The second is the so-called Weinberg + omega ($W\omega$) model, in which the nonlinear pion-nucleon Lagrangian introduced by Weinberg[18] is augmented by the ω meson in the same manner as the $\sigma+\omega$ model. This model is not renormalizable (because of the pseudovector πNN coupling) but can be used in the "tree" approximation. The purpose of introducing the ω into the sigma and Weinberg models is to provide a repulsive interaction between nucleons which would otherwise be too strongly attracted to each other. Recently it was shown that the $\sigma+\omega$ and $W\omega$ models are essentially equivalent for nuclear pion absorption.[19]

The Weinberg model in its usual form [see Figs. 6(a) and 6(b)] predicts negligible scattering of external pions from the nuclear pionic fluctuations.[20,21] However, the effective mass (M^*) of nucleons in nuclear matter differs from the nucleon mass (M) in free space. When M is replaced by M^* in the timelike part of the $W\omega$ model axial current one obtains a repulsive s-wave potential of about the same size as that found in the $\sigma+\omega$ model.

In 1966 Weinberg showed that the requirements of chiral invariance, analyticity, and crossing symmetry lead to the vanishing of the scattering length for a massless pion on a heavy target.[22] If the target is isoscalar, moreover, the coefficient of the "leading" contribution to the scattering length (of order $G^2 m_\pi/M^2$) vanishes. In a recent study of the $\sigma+\omega$ model[23] it was shown that the "leading" $O(G^2)$ contribution to the pion scattering length on a $T=0$ nucleus vanishes even for finite-mass pions, in accordance with the Weinberg theorem, but that in fourth order, the compositeness of the target (anomalous thresholds) yields a substantial, nonvanishing scattering length, approximately

$$a=-\left(\frac{G^2}{4\pi}\right)^2(m_\pi/Mm_\sigma{}^2)k_F{}^3A=-0.036m_\pi{}^{-1}A\ , \quad (3)$$

where A is the nucleon number. (The importance of the anomalous thresholds is manifested by the appearance of k_F, the Fermi momentum of the nucleus.) To recapitulate, the isoscalar $O(G^2)$ scattering amplitude is small in the $\sigma+\omega$ model because the large "pair" graphs (recall the πNN coupling is pseudoscalar), Fig. 2(a), are almost perfectly canceled by the σ-exchange graph, Fig. 2(b). This cancellation of the $O(G^2)$ terms persists in the nucleus. In fourth order, however, a new class of diagrams arises, involving two nucleons at a time, which would have no equivalent if the target were elementary. These are shown in Figs. 3(a)–3(d). The process represented by Figs. 3(a) and 3(d) is enhancement of the "pair" terms by the average nuclear potentials in which the nucleons move—the diagrams shown are meant to stand for all possible orderings of the interactions. The amplitudes represented by Figs. 3(b) and 3(c) describe pion scattering from nuclear σ fluctuations. Such terms were neglected by Huang *et al.*[24] in their current-

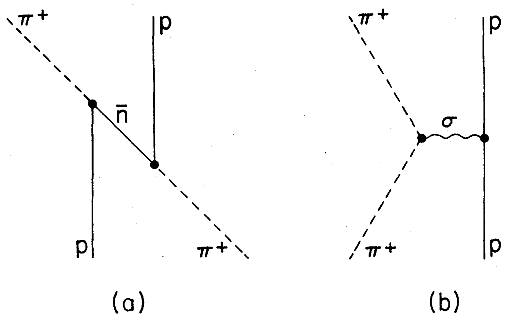

FIG. 2. (a) Virtual pair contribution to s-wave πN scattering, in PS coupling theory (both time orderings are implied), and (b) σ-exchange contribution to s-wave πN scattering in the $\sigma + \omega$ model.

algebraic approach, on the grounds that they should be small owing to the small probability of finding two nucleons within one σ Compton wavelength, m_σ^{-1}, of each other; in fact each of the three terms, Figs. 3(a)–3(c), is extremely large, 3(a) and 3(b) being repulsive, and 3(c) being attractive. In the limit of vanishing pion mass, they sum identically to zero, as they must to preserve chiral symmetry. However, for finite pion mass, the cancellation is spoiled by terms of order $(m_\pi/m_\sigma)^2$, yielding the optical potential [from Figs. 3(a)–3(c)]

$$2k^0 v_\sigma(r) = \tfrac{3}{2}G^4(m_\pi/Mm_\sigma^2)^2(\tfrac{3}{4})\eta\rho^2(r). \tag{4}$$

Similarly, there are $O(G^4)$ terms arising from the average (timelike) ω field [Fig. 3(d)] of the nucleus which cancel to order m_π^2/m_σ^2 and hence contrib-

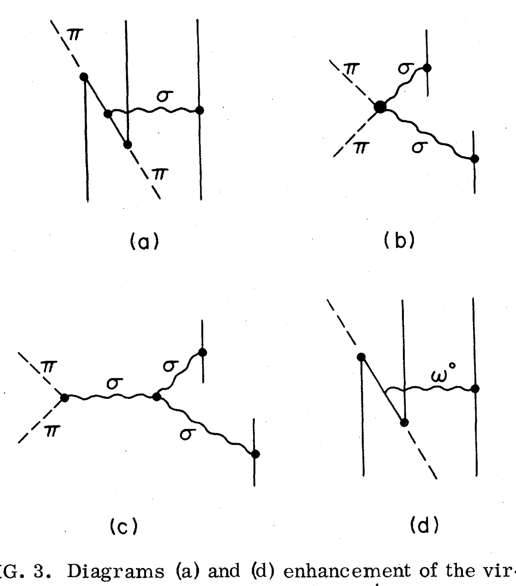

FIG. 3. Diagrams (a) and (d) enhancement of the virtual pair term in nuclear matter, to $O(G^4)$. (All orderings of the σ and ω^0 exchange are implied.) Diagrams (b) and (c) show scattering from the fluctuations of the nuclear σ field, to $O(G^4)$.

ute[25]

$$2k^0 v_\omega(r) = G^2 G_\omega^{\,2}(m_\pi/Mm_\sigma m_\omega)^2(\tfrac{3}{4})\eta\rho^2(r). \tag{5}$$

Finally, the process represented by Figs. 4(a) and 4(b), scattering from the pion fluctuations of the nucleus, yields the optical potential

$$2k^0 v_{\pi\pi}(r) = \tfrac{1}{2}G^2(m_\pi/M)^2\langle A\,|[\pi_0(\vec{r})]^2|A\rangle. \tag{6}$$

In the absence of nuclear Pauli correlations the ground-state expectation value of the squared pion field would vanish; in fact, the contribution from this term is very small ($\sim\frac{1}{22}$) relative to Eq. (4) so we shall neglect it. [In Ref. 23 this term was erroneously given as $\frac{5}{18}$ of Eq. (4).] In arriving at Eq. (4) and Eq. (5), the range of the σ- and ω-meson propagators was taken to be small on the scale of the nuclear wave functions, the effect of the Pauli correlations was included (the factor $\frac{3}{4}$ is the relative probability of finding within the range of these propagators either an np pair or a like pair with opposite spins), and the effect of short-range correlations was included approximately the reduction factor[23]

$$\eta = \tfrac{1}{2}\int_{r_c m_\sigma}^{\infty} dx\, x^2 e^{-x}, \tag{7}$$

where r_c is the nucleon-nucleon hard-core radius. With $r_c = 0.5$ fm and $m_\sigma = 735$ MeV, $\eta = 0.7$. Taking the nuclear central density to be $0.167\ \mathrm{fm}^{-3}$, $G^2/4\pi = 14.2$, and $G_\omega^{\,2}/4\pi = 12.9$, we find the potential depth from Eq. (4) and Eq. (5), at threshold, to be roughly 16 MeV. Since none of the parameters is determined by experiment to better than about 5%, we expect a theoretical uncertainty in $v_\sigma + v_\omega$ of about 20%. We note that the effects of the spacelike parts of ω exchange, shown in Fig. 5, cancel identically.[23]

Although several attempts have been made[24,26] to use partially conserved axial-vector current (PCAC) and analyticity as the dynamical basis of pion-nucleus scattering by analogy with the classical work on πN and $\pi\pi$ scattering lengths, the presence of anomalous thresholds obviates a straightforward

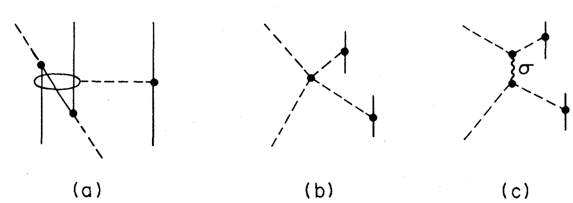

FIG. 4. Diagram (a) shows modification of the "pair" diagram by the nuclear pion field, acting on all three nucleon lines during the external pion scattering and diagrams (b) and (c) show scattering from the nuclear pion field, in the $\sigma + \omega$ model. (Exchange terms implied.)

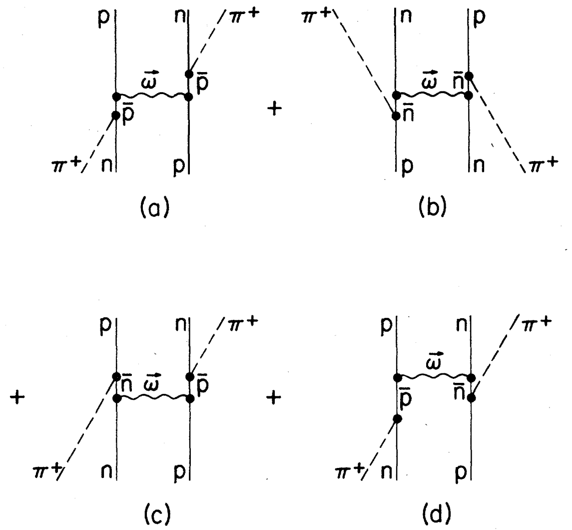

FIG. 5. Diagrams (a)–(d) are contributions to elastic pion-nucleus scattering from spacelike ω exchange [timelike exchange is already included explicitly in Figs. 3(a) and 3(d)].

generalization of soft-pion techniques. We feel that the new dynamical result (which was *not* obtained in the current-algebraic approach) justifies the method used in Ref. 23, of analyzing a concrete field-theoretic model within the Wick[27] formalism, despite its heavy reliance on perturbation theory. In order to explore the sensitivity of our results to the details of the analysis, we repeat the calculation within the framework of the $W\omega$ model. The Lagrangian density of the $W\omega$ model is

$$\begin{aligned}\mathscr{L}_{W\omega} = &-\bar{N}[-i\gamma^\mu D_\mu + M + G_\omega \gamma^\mu \omega_\mu + \frac{G}{2m}\gamma^5\gamma^\mu D_\mu(\vec{\tau}\cdot\vec{\phi}_\pi)]N \\ &+\tfrac{1}{2}g^{\mu\nu}D_\mu\vec{\phi}_\pi\cdot D_\nu\vec{\phi}_\pi - \tfrac{1}{2}m_\pi^2(1+\vec{\phi}_\pi^{\,2}/4f_\pi^2)^{-1}\vec{\phi}_\pi^{\,2} \\ &+\mathscr{L}_\omega(\text{free}),\end{aligned} \tag{8}$$

where ($f_\pi = Mg_A/G$)

$$\begin{aligned} D_\mu\vec{\phi}_\pi &= (1+\vec{\phi}_\pi^{\,2}/4f_\pi^2)^{-1}\partial_\mu\vec{\phi}_\pi, \\ D_\mu N &= [\partial_\mu + i(4f_\pi^2+\vec{\phi}_\pi^{\,2})^{-1}\vec{\tau}\cdot(\vec{\phi}_\pi\times\partial_\mu\vec{\phi}_\pi)]N,\end{aligned} \tag{9}$$

and we are restricted to evaluating all matrix elements in the "tree" approximation. The pion "current" for positive pions is then

$$J_{\vec{k}} = \int \frac{d^3x e^{i\vec{k}\cdot\vec{x}}}{(2\pi)^{3/2}(2k^0)^{1/2}}\left[\frac{\partial\mathcal{H}_{\text{int}}}{\partial\phi_-(\vec{x})} - ik^0\frac{\partial\mathcal{H}_{\text{int}}}{\partial\phi_-^0(\vec{k})}\right]. \tag{10}$$

To order G^4 (we drop divergence terms from the currents since $\vec{k}'$ and $\vec{k}$ are taken to be small near threshold) we find the leading amplitude resulting from the nonlinear couplings to be (note that the nuclear pion field is of order G)

$$\begin{aligned} T^{\pi\pi}_{\vec{k}',\vec{k}} &\simeq \langle A|[J^\dagger_{\vec{k}'}, a^\dagger_{\vec{k}}]|A\rangle \\ &\simeq -\frac{G^2}{2M^2}\frac{1}{(2\pi)^3}\frac{1}{2k^0}\int d^3x\, e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\langle A|\{2m_\pi^2\phi_+(\vec{x})\phi_-(\vec{x}) + [\nabla\vec{\phi}(\vec{x})]^2 + \frac{G}{2M}\vec{\phi}(\vec{x})\cdot \text{div}[N^\dagger(\vec{x})\vec{\sigma}\,\vec{\tau}N(\vec{x})]\}|A\rangle.\end{aligned} \tag{11}$$

Applying the pion field equation

$$(m^2-\nabla^2) = -\frac{G}{2m}\text{div}[N^\dagger(\vec{x})\vec{\sigma}\,\vec{\tau}N(\vec{x})] \tag{12}$$

and using the identity

$$\text{div}[\vec{\phi}\cdot\text{grad}(\vec{\phi})] \equiv \vec{\phi}\cdot\nabla^2\vec{\phi} + \text{grad}(\vec{\phi})\cdot\text{grad}(\vec{\phi}) \tag{13}$$

we reduce Eq. (11) to

$$\begin{aligned} T^{\pi\pi}_{\vec{k}',\vec{k}} = \frac{G^2}{2M^2}m_\pi^2 \int \frac{d^3x}{(2\pi)^3 2k^0} e^{i(\vec{k}-\vec{k}')\cdot\vec{x}} \\ \times\langle A|[\phi_0(\vec{x})]^2|A\rangle,\end{aligned} \tag{14}$$

which is identical to Eq. (6), and is therefore negligible. As Robilotta and Wilkin have pointed out,[21] the leading graphs in nonlinear chiral SU(2)×SU(2) models, shown in Figs. 6(a) and 6(b), cancel each other to a considerable extent. In the limit $m_\sigma\to\infty$, the $\sigma+\omega$ model becomes unitarily equivalent to the $W\omega$ model.[18,28] (Bardeen and Lee[29] have described in detail how the renormalizability of the $\sigma+\omega$ model is lost in this limit.) We have shown that the $O(G^4)$ terms arising from the effects of the nuclear pion field are the same in the $W\omega$ and $\sigma+\omega$ models. However, the (dominant) $v_\sigma(r)$ and $v_\omega(r)$ terms, Eq. (4) and Eq. (5), vanish as $m_\sigma\to\infty$. From Fig. 3 it is easy to see why this has happened: All of the $O(G^4)$ effects involve either pion scattering from

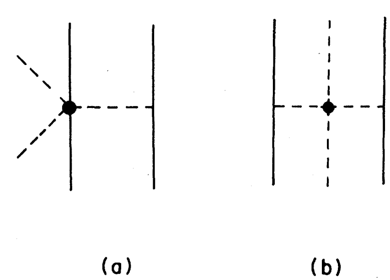

FIG. 6. (a) The point $\pi\pi\pi NN$ process implied by the $W\omega$ model and (b) scattering from the fluctuations of the nuclear pion field in the $W\omega$ model.

internal σ lines, or else modifications of the "pair" terms by the strong average nuclear potentials. The unitary transformation leading from the $\sigma+\omega$ to the $W\omega$ model eliminates explicit reference to such terms. The question which now arises is whether the dependence on the nuclear σ and ω fields is somehow reinstated by this unitary transformation. The answer, we believe, is "yes," since the single-nucleon states in the $W\omega$ model satisfy a Dirac equation[19]

$$\{\vec{\alpha}\cdot\vec{p}+\beta[M+U(r)]+V(r)-E\}\psi=0\,, \tag{15}$$

where $V(r)$ is essentially the nuclear ω field, and $U(r)$ is necessary for reasonable one-body dynamics. That is, the nucleon mass is position dependent. Another way of saying this is that in the $\sigma+\omega$ model, the expectation value of the σ field differs in nuclear matter from f_π, its vacuum value, precisely by $U(r)/G$. In deriving Eq. (11) we neglected the timelike part of the nucleon axial current, which also acts as a pion source. In fact, this extra term is (in the Weinberg model)

$$\Delta J_{\vec{k}}=ik^0\frac{G}{2M}\int\frac{d^3x}{(2\pi)^{3/2}(2k^0)^{1/2}}e^{i\vec{k}\cdot\vec{x}}N^\dagger(\vec{x})\gamma^5\tau_+N(\vec{x})\,. \tag{16}$$

We believe that consistency requires us to replace M by $M^*\equiv M+U$ in (16). This additional piece of the current contributes to the effective pion-nucleus potential through the $N\bar{N}$ parts of the Green's functions in the Wick expression for the T matrix

$$\Delta T=\langle A\,|\Delta J_{\vec{k}'}^\dagger(k^0+E_A+i\eta-H)^{-1}\Delta J_{\vec{k}}|A\rangle + \text{crossed term}\,, \tag{17}$$

giving

$$2k^0\Delta v(r)=\frac{(k^0)^2}{M}\left(\frac{G}{2M}\right)^2\left\langle A\left|N^\dagger(\vec{x})\left[\frac{M}{M^*(\vec{x})}\right]^3N(\vec{x})\right|A\right\rangle . \tag{18}$$

Taking Pauli correlations into account, we have

$$\left\langle A\left|N^\dagger(\vec{x})\left[\frac{M}{M^*(\vec{x})}\right]^3N(\vec{x})\right|A\right\rangle \simeq\rho(\vec{x})\left[1+3\left|\frac{U(\vec{x})}{M}\right|+4\left|\frac{U(\vec{x})}{M}\right|^2+\frac{20}{9}\left|\frac{U(\vec{x})}{M}\right|^3\right], \tag{19}$$

where $U(x)$ is the empirical one-nucleon potential in Eq. (15). The term

$$2k^0v^{(2)}=(k^0)^2(G^2/4M^3)\rho(\vec{x})$$

is just the potential resulting from the (repulsive) s-wave isoscalar pion-nucleon scattering in impulse approximation. This potential is about 5 MeV at the nuclear center. Taking the empirical value $U(0)=-420$ MeV, we find that the rest of Eq. (19) contributes an additional 12 MeV at the nuclear center. Another way of estimating these effects, based on the requirement that the p-wave pseudoscalar and pseudovector couplings be equivalent,[19] gives

$$2k^0\Delta v\simeq(k^0)^2\frac{G^2}{4M^3}\left\langle A\left|N^\dagger(\vec{x})\left[1+\frac{U(x)}{2M}-\frac{V(\vec{x})}{2M}\right]^{-2}\times\left[1+\frac{U(\vec{x})}{M}\right]^{-1}N(\vec{x})\right|A\right\rangle , \tag{20}$$

which leads to an extra repulsive potential of strength 20 MeV at the center of the nucleus.

To recapitulate the results of the preceding paragraph, we have found, in agreement with Robilotta and Wilkin,[20,21] that the Weinberg model in its usual form predicts almost no scattering of external pions from nuclear pionic fluctuations. This agrees identically with the $\sigma+\omega$ model result. We also find that the $W\omega$ model has no explicit terms corresponding to Eqs. (4) and (5). However, by looking at the derivation of the $W\omega$ model from the $\sigma+\omega$ model, under conditions pertaining to nuclear matter described by Eq. (15), we concluded that implicit in the $W\omega$ model are effects which essentially restore Eqs. (4) and (5). In this sense the $\sigma+\omega$ model and the $W\omega$ model are equivalent; however, we must regard the former as being more fundamental, since the additional repulsive potential is missed by naive application of the $W\omega$ model.

Having seen that a new, repulsive, pion-nucleus potential appears in two related theories, giving it a certain credibility, we are in a position to add it to our earlier results on the dispersive real part[16] and the Pauli correction to double scattering (Appendix), for the purpose of comparison with experiment. We emphasize that the results obtained above, together with those of our previous paper[16] constitute a first-principles calculation of the s-wave part of the pion-nucleus optical potential. Our expression for this part is

$$2k^0V=-4\pi\{b_0\rho(r)-\frac{3}{2\pi}k_F(b_0{}^2+2b_1{}^2)(1+\epsilon)^{-1}[\rho(r)/\rho(0)]^{(2+\epsilon)/(1+\epsilon)}\} -4\pi\rho^2(r)[i\,\mathrm{Im}B_0+\mathrm{Re}B_0(\text{dispersive})+\mathrm{Re}B_0(\text{nonlinear})]\,, \tag{21}$$

TABLE I. Prediction of the absorptive and dispersive real parts of B_0 from the $\sigma+\omega$ or $W\omega$ models, as functions of energy, taken from Ref. 16.

$T_\pi = k^0 - m_\pi$ (MeV)	$\mathrm{Re}B_0$(dispersive) $+ i\,\mathrm{Im}B_0$ (fm^4)
0	$0.260 + 0.165i$
20	$0.268 + 0.202i$
40	$0.273 + 0.244i$
50	$0.274 + 0.263i$
60	$0.277 + 0.282i$
80	$0.269 + 0.317i$
100	$0.261 + 0.353i$

where b_0 and b_1 are the free-nucleon values -0.020 fm and -0.129 fm, respectively, and our calculated values of $\mathrm{Im}(B_0)$ and its corresponding dispersive real part [$\mathrm{Re}B_0$ (dispersive)] are given in Table I. The term proportional to k_F arises from the diagram Fig. 1(b) and is derived in the Appendix, together with the parameter $\epsilon = 4.85\,A^{-2/3}$. The term $-4\pi\rho^2 B_0$ (nonlinear) is the $O(G^4)$ contribution derived in Eq. (4). In Table II we compare our values for the s-wave parameters with those of Stricker *et al.*[11] Attention should be given especially to the last line in Table II, in which the central depth of the s-wave real part of the optical potential is presented. Note that this number is essentially the same in the two empirical fits from Ref. 11, and that our "best" value agrees with the empirical one within the theoretical range of uncertainty. We have calculated the angular distributions for elastic scattering of 50 MeV positive pions from ^{12}C and ^{16}O, taking the s-wave parameters from the last column in Table II, both including and omitting the nonlinear $O(G^4)$ term which is the subject of this paper. We took the p-wave parameters from the analysis of Ref. 11. The computer program PIRK was used for these calculations.[30] The theoretical angular distributions are compared with recent data in Figs. 7 and 8. All effects of angle transformations, recoil, Lorentz-Lorentz damping, etc., are included as in Ref. 11. Considering that

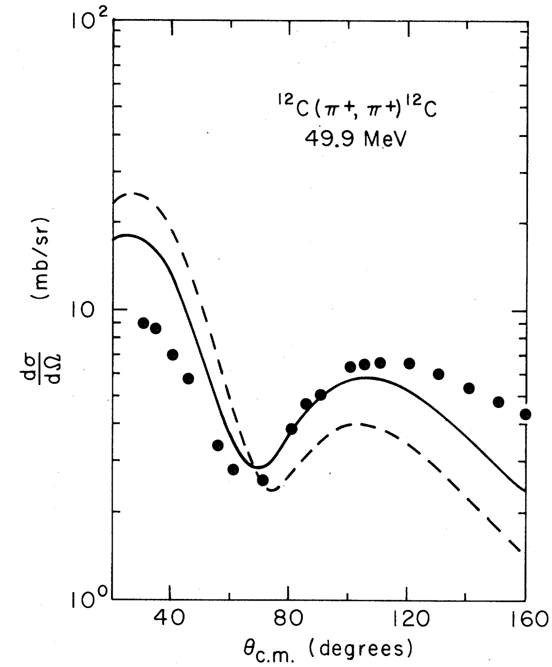

FIG. 7. Elastic π^+ ^{12}C scattering at 50 MeV together with data from Ref. 8. The solid curve uses the parameters from the last column of Table II, the broken curve uses those from the 3rd column.

the p-wave parameters of Ref. 11 were determined by a fit to earlier data, and that no attempt was made to find a best fit by varying the p-wave parameters alone while constraining the s-wave ones to have the values given by theory, we consider the agreement to be quite acceptable. However, we feel the essential point is the comparison between the calculation which lacked the $O(G^4)$ nonlinear term and that which included it. Clearly this effect greatly improves the agreement between theory and experiment.

Use of the repulsive ρ^2 s-wave potential discussed in this paper improves the agreement between theory and experiment in low energy elastic pion-nucleus scattering. The effect has been shown to

TABLE II. Comparison between the s-wave parameters derived in this paper, with two sets derived from empirical fits by Stricker *et al.* (Ref. 11) [referred to as SMC(1) and SMC(2)], for $T_\pi = 50$ MeV and $A = 12$.

Parameter	SMC(1)	SMC(2)	Present calculation [Without $O(G^4)$ term]	Present calculation (Total)
b_0 (fm)	-0.020	-0.20	-0.020	-0.020
Δb_0 (fm) [a]	-0.020	-0.018	-0.011	-0.011
$\mathrm{Im}B_0$ (fm^4)	0.17	0.18	0.25	0.25
$\mathrm{Re}B_0$ (fm^4)	-0.17	-0.18	0.274	-0.056 (± 0.066)
$\mathrm{Re}\,v(r=0)$ (MeV)	14.7	14.6	-3.1	8.64 (± 2.4)

[a] See Appendix for details.

be robust in the sense that it is predicted by two theories related by a unitary transformation. We note, however, that the effect is not simply a consequence of PCAC, since the usual linear PV model of πNN coupling satisfies a form of PCAC,[17] but does not account for low energy $\pi\pi$ and πN scattering. It is not a consequence of chiral SU(2)×SU(2) either, since the "naive" $W\omega$ model has this symmetry, but does not predict the new potential. We must therefore regard the new potential as a consequence either of nonlinear π-σ coupling, combined with dynamical modification of the nucleon "pair" terms by the nuclear potentials ($\sigma+\omega$ description), or as a consequence of the density dependence of the Goldberger-Treiman[31,17] relation in nuclear matter ($W\omega$ description).

Combined with our previous work on the absorptive and dispersive s-wave potentials,[16] the results presented herein indicate that the $\sigma+\omega$ and/or $W\omega$ models describe adequately low energy pion-nucleus interactions.

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APPENDIX

Here we calculate the contribution to the s-wave optical potential of the exchange correction to second-order multiple scattering. From the work of Ericson and Ericson[12] we find

$$2k^0\Delta v = 4\pi(b_0^2+2b_1^2)\tfrac{1}{4}\int \frac{d^3s}{s}C^2(k_Fs)\rho(r)\times\rho(|\vec{r}+\vec{s}|)\,, \tag{A1}$$

corresponding to the diagram, Fig. 1(b). The factor of 2 weighting b_1^2 relative to b_0^2 is actually $\vec{t}\cdot\vec{t}=t(t+1)=2$, where $\vec{t}$ is the pion isospin. The function $C(k_Fs)$ is given by

$$C(x)=3j_1(x)/x\,. \tag{A2}$$

The integral in Eq. (A1) has usually been evaluated in the constant density approximation (infinite nuclear matter) or in the zero-range approximation, which comes to the same result, using

$$\int\frac{d^3s}{s}C^2(k_Fs)=\frac{9\pi}{k_F^2}\,. \tag{A3}$$

In order to better include surface effects, we employ approximate forms of $\rho(r)$ and $C(x)$ which lead to simple integrals:

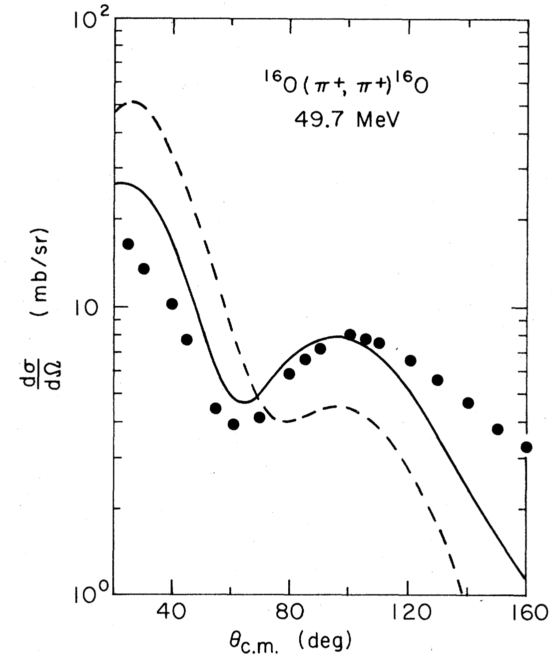

FIG. 8. As in Fig. 6 except for ^{16}O target. Data from Ref. 9.

$$\rho(r)=\rho_0\exp(-\tfrac{5}{2}r^2/R^2)\,, \tag{A4}$$

$$C^2(k_Fs)=\exp(-s^2/\lambda^2)\,. \tag{A5}$$

In (A4) R is the nuclear radius, so Eq. (A4) gives the correct nuclear rms radius; and in (A5) we adjust λ so as to give the correct value for the integral (A3):

$$\int\frac{d^3s}{s}e^{-s^2/\lambda^2}\equiv 2\pi\lambda^2=9\pi/k_F^2\,, \tag{A6}$$

or

$$\lambda=3/k_F\sqrt{2}\,.$$

The value of λ chosen this way also gives a reasonably good approximation (within 10%) for the volume integral of the term of (A2)

$$\int d^3sC^2(k_Fs)\equiv\frac{6\pi^2}{k_F^3}\simeq 27(\pi/2)^{3/2}/k_F^3\,. \tag{A6$'$}$$

Then using (A4) and (A5) in (A1) we obtain

$$\int\frac{d^3s}{s}\rho(r)\rho(|\vec{r}+\vec{s}|)C^2(k_Fs)\simeq\frac{9\pi}{k_F^2}\frac{\rho^2(r)}{1+\epsilon}2\int_0^\infty dx\,e^{-x^2}\frac{\sinh(\eta x)}{\eta}\,, \tag{A7}$$

where

$$\epsilon=\tfrac{5}{2}\lambda^2/R^2=\frac{45}{(9\pi A)^{2/3}}=4.85\,A^{2/3}\,,\qquad \eta=\frac{5r\lambda}{R^2}/(1+\epsilon)^{1/2}\,. \tag{A8}$$

The integral may be evaluated approximately to get

$$2\int_0^\infty dx\, e^{-x^2}\frac{\sinh(\eta x)}{\eta} \simeq e^{(\eta/2)^2},$$

so that Eq. (A7) becomes

$$\int\frac{d^3s}{s}\rho(r)\rho(|\vec{r}+\vec{s}|)C^2(k_F s) \simeq \frac{9\pi}{k_F^2}\frac{\rho^2(0)}{1+\epsilon}[\rho(r)/\rho(0)]^{(2+\epsilon)/(1+\epsilon)}. \quad \text{(A9)}$$

In comparison with the phenomenological analysis of Ref. 11, Table II, the Pauli correction is expressed as a change in b_0, so one power of $\rho(0)$ must be removed from (A9). Thus

$$\Delta b_0 = -(b_0^2 + 2b_1^2)\frac{3k_F}{2\pi}(1+\epsilon)^{-1}. \quad \text{(A10)}$$

The values in Ref. 11 were computed with $\epsilon = 0$.

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