$\bar{K}$-nucleus scattering in the isobar-doorway model

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The isobar-doorway model is formulated for $\bar{K}$-nucleus scattering. A $\bar{K}$-nucleus optical potential is derived. Using the closure approximation, binding energies and widths of the $\Lambda(1405)$ and $\Lambda(1520)$ in nuclear matter are obtained from $\bar{K}$-atomic data and elastic scattering.

[NUCLEAR REACTIONS Meson induced reactions. Properties of hyperon resonances.]

INTRODUCTION

These early years of experiments with the new meson factories have already led to important new information about pion-nucleus interactions. A most striking aspect is the development of a program of study of the interaction of baryon resonances (N^*'s) with nucleons and nuclei. This has been accomplished by using nuclear states with an N^* as one of the constituents in the formulation of the many body theory. Once N^*'s have been explicitly introduced, one can study the N-N^* interaction, the propagation and decay of N^*'s in nuclei, and so forth. Such a quantitative study of these fundamental dynamic properties can only be done with nuclear targets.

Kaon-nucleon dynamics, like pion-nucleon dynamics, is dominated by baryon resonance (Y^*) formation at medium energy. The purpose of the present paper is to give the basic theoretical formulation, to survey the present experimental situation with regard to information about Y^* dynamics, and to discuss possibilities for gaining more information in this regard with future experimental and theoretical development. It will be shown that Y^*-nucleon and Y^*-nucleus interactions can be investigated, thereby further extending our knowledge of strong interactions.

The physics of Y^*'s in nuclei is a part of hypernuclear physics. The Y^* isobar-doorway states, discussed below, are important aspects of nuclei with one unit of strangeness such as Λ hypernuclei. The conventional nuclear structure aspects of kaonic scattering and reactions will not be discussed here, but it is evident from the section on the kaon-nucleus optical potential that there are interesting structure features also.

The starting point of the theoretical development is the concept of isobar-doorway states, introduced for pion-nucleus physics.[1] Since the resonant meson-nucleon interaction can be viewed as the absorption of a meson by a nucleon to form an N^* or Y^*, all such meson interactions with an A-nucleon target are mediated by states of A-1 nucleons and an N^* or Y^*. We refer to these as isobar-doorway states. If there are relatively few of them, one can diagonalize the doorway space and obtain a convenient alternative form for the meson-nucleus T matrix and optical potential which includes all orders in the conventional multiple scattering theory.[2] The essential parameters of the phenomenological isobar-doorway model are basic interaction and propagation parameters for the N^* or Y^* in nuclei.

For the Δ, several groups[3-5] have now made extensive microscopic studies of pion-nucleus scattering using a basis of Δ-nucleon-hole states. They each have shown that there is a doorway state dominance of the type described above. In each partial wave a single Δ-hole state dominates the scattering so one could treat the entire process as the projection of an isobar doorway onto the partial wave channels. This provides a strong theoretical motivation for the phenomenological model and some guidance for the choice of parameters.

The isobar-doorway model is formulated for kaon-nucleus scattering and reactions in Sec. II. There are no essential new theoretical developments, so that it is mainly an overview of the situation with regard to the Y^*'s. It is shown that with regard to isobars, the kaon-nucleus system is richer and more promising than the pion-nucleus system. In contrast to the single broad Δ resonance which has been studied with pions, there are several narrower Y^*'s which can be treated fruitfully. In Sec. III the $\bar{K}$-atomic data is used to extract information about the $\Lambda(1405)$. In Sec. IV elastic K-nucleus scattering is discussed, with a derivation of an optical potential for the medium-energy region. Scattering near the $\Lambda(1520)$ and in the region of energy near 1670 is discussed. The emphasis is on possibilities with future experiments, but the present data is used where available.

II. ISOBAR-DOORWAY MODEL FOR THE $\bar{K}$-NUCLEUS SYSTEM

The isobar-doorway (I-D) model takes advantage of the fact that resonance formation dominates the meson-nucleon interaction at low and medium energies. The theory makes use of a basic interaction V which mediates meson + baryon → baryon resonance. The resonances are most usefully considered to be as elementary as the stable baryons, as in the quark model, although the theory can be formulated as scattering in a medium. In the present chapter we review the formulation of the model by applying it to kaon-nucleus scattering and reactions.

A. The Y^* resonances

The t matrix for a $\bar{K}N$ reaction

$$\bar{K}+N \to m+B$$

mediated by a Y^* (Λ or Σ) resonance can be written as

$$\langle k' | t^{Y^*}(E) | k\rangle = \frac{\langle mB|V|Y^*\rangle\langle Y^*|V|\bar{K}N\rangle}{E-M_{Y^*}+i\Gamma_{Y^*}(E)/2}\,, \qquad (1)$$

with $|\langle mB|V|Y^*\rangle|^2$ and $|\langle Y^*|V|\bar{K}N\rangle|^2$ the partial width for $Y^* \to mB$ and $Y^* \to \bar{K}N$, respectively, and $\Gamma_{Y^*}(E)$ and M_{Y^*} the mass and width of the Y^* resonance. The interaction V and t matrix are specified in dynamic models. The mass M_{Y^*} and width Γ_{Y^*} can also be calculated[2] from appropriate matrix elements of the interaction V. In the present work we take the matrix elements of V from two-body experiments and make use of them to determine the A-body T matrix and optical potential, as described in Sec. II B.

The well-established Y^* resonances in the low-medium-energy region are given in Table I, with the standard notation for strangeness equal −1 resonances of $\Lambda(M)$ or $\Sigma(M)$ for $I=0$ or 1 resonances of mass M. There are a number of important observations which follow from the parameters of this table. First, note that the widths of the Y^* resonances tend to be smaller than the N^* resonances. The four lowest resonances have widths of from 15 to 50 MeV vs the 110 MeV width of the $\Delta(1232)$ which has been treated in the I-D model for pion-nucleus scattering. Since important dynamic effects of Y^* interactions with nucleons are determined by the effective width and mass of the isobar in nuclear matter, the study of $\bar{K}$ interactions would seem even more promising than pionic interaction by the I-D method.

Note that the $\Lambda(1405)$ is below the $\bar{K}N$ threshold. However, it has been clearly established to have important effects at low energies through the analysis of $\bar{K}$ atoms.[6] From the point of view of the present paper the $\Lambda(1520)$ is certainly *a priori* the most interesting of the Y^*'s, since it is very narrow, is strongly coupled to the $\bar{K}N$ channels, and is isolated. This should make the study of $\bar{K}$-nucleus interactions especially interesting at momenta about 400 MeV/c. At 800–900 MeV/c, where all recent K-nucleus experiments have been done, there are overlapping resonances. However, since the quantum numbers of the resonances vary, it might be possible to distinguish their effects. This will be discussed in greater detail below.

B. The isobar-doorway model for $\bar{K}$-nucleus scattering and reactions

The two characteristic elements of the isobar-doorway theory are the use of the operator V of Eq. (1) as the basic meson-nucleon interaction at resonance, and the introduction of isobar-doorway

TABLE I. Properties of low-lying Y^* resonances. The notation is standard (Ref. 23).

Y^* isobar	L	J^P	Width (MeV)	$K^-_{\rm lab}$ (MeV/c)	% $\bar{K}N$
$\Lambda(1405)$	0	$\frac{1}{2}^-$	40 ± 10	below threshold	
$\Lambda(1520$	2	$\frac{3}{2}^-$	$15\pm\ 2$	392	46%
$\Lambda(1670)$	0	$\frac{1}{2}^-$	40 ± 20	735	15–35
$\Lambda(1690)$	2	$\frac{3}{2}^-$	55 ± 25	777	20–30
$\Lambda(1815)$	3	$\frac{5}{2}^+$	85 ± 15	1044	60
$\Sigma(1670)$	2	$\frac{3}{2}^-$	50 ± 15	735	10–25
$\Sigma(1750)$	0	$\frac{1}{2}^-$	75 ± 25	905	10–40
$\Sigma(1765)$	2	$\frac{5}{2}^-$	130 ± 20	937	41

states. This is most conveniently formulated in terms of projection operators. These concepts have been discussed in detail in Ref. 2 for the π-nucleus problem, so only the results without derivation will be given here.

The Y^* isobar-doorway states are the complete set of states $|D_i\rangle$ in the D subspace of one Y^* and $A-1$ nucleons for a target of mass number A. The operator V connects the doorway space D to P space, consisting of the $\bar{K}$-nucleus ground state and other states to be explicitly treated, and Q space, the subspace of all other states. We also refer to the Y^*-doorway states as Y^* hypernuclei. This is a useful name in that it emphasizes that these states are interesting nuclear states with one negative unit of strangeness, like the Λ hypernuclei. The T matrix for the reaction $|p_i\rangle \to |p_f\rangle$ is given by

$$T = T^{\rm NR} + \sum_{jk} \frac{\langle P_f|V|D_j\rangle\langle D_k|V|P_i\rangle}{(E-E_j)\delta_{jk} - (\Delta^{\rm el}_{jk}+\Delta^{\rm in}_{jk}) + i(\Gamma^{\rm el}_{jk}+\Gamma^{\rm in}_{jk})/2}, \quad (2)$$

where the initial and final states satisfy the correct boundary conditions, and E_j are the energy eigenvalues of the states $|D_j\rangle$ with the interaction limited to D space. $T^{\rm NR}$ is the nonresonant T matrix obtained in the absence of the Y^* resonances. The elastic and inelastic energy shifts and widths in Eq. (2) are defined by (and can be calculated in a model from)

$$\Delta^{\rm el}_{jk} + i\Gamma^{\rm el}_{jk}/2 = \left\langle D_j \left| V\frac{1}{E-H_p+i\epsilon} V \right| D_k\right\rangle,$$
$$\Delta^{\rm in}_{jk} + i\Gamma^{\rm in}_{jk}/2 = \left\langle D_j \left| V\frac{1}{E-H_Q} V \right| D_k\right\rangle, \quad (3)$$

where H_p and H_Q are the Hamiltonian projected onto P space and Q space, respectively. In other words $\Delta^{\rm el}$ and $\Gamma^{\rm el}$ are the energy shift and the width arising from the decay of the Δ-nuclear states into the elastic and other P channels, while $\Delta^{\rm in}$ and $\Gamma^{\rm in}$ are the corresponding quantities arising from coupling of the doorway states to the Q space. For elastic scattering (the P space limited to the nuclear ground state) $\Gamma^{\rm in}$ is approximately Γ_{Y^*}, the free width, since the Y^* doorway most likely decays into an inelastic nuclear hole state.

The isobar-doorway model is achieved by introducing doorway states which are eigenstates of the effective interaction in D space, so that

$$(\Delta^{\rm el}_{jk}+\Delta^{\rm in}_{jk}) + i(\Gamma^{\rm el}_{jk}+\Gamma^{\rm in}_{jk})/2 \to (\Delta_j + i\Gamma_j)\delta_{jk}. \quad (4)$$

The resulting form for the t matrix is

$$T = T^{\rm NR} + \sum_j \frac{\langle p_f|V|D_j\rangle\langle D_j|V|p_i\rangle}{E-E_{D_j}-\Delta_j+i\Gamma_j/2}. \quad (5)$$

For elastic scattering one has from Eqs. (1) and (5) the $\bar{K}$-nucleus T matrix

$$\langle k'|T(E)|k\rangle = \langle k'|T^{\rm NR}|k\rangle + \sum_j \frac{\langle k'|t|k\rangle(E-M_{Y^*}+i\Gamma_{Y^*}/2)}{E-E_{D_j}-\Delta_j+i\Gamma_j/2}\rho_j(k,k), \quad (6)$$

where $\rho_j(k',k)$ is the nuclear form factor—a type of mixed density function[2] involving the nucleon and Y^* wave functions. The optical potential which corresponds to this T matrix is of similar form with only the inelastic energy shift and width in the energy denominators. For a single doorway or a narrow collection of doorways the optical potential can be written as

$$\langle \vec{k}'|V|\vec{k}\rangle = \langle\vec{k}'|V^{\rm NR}|\vec{k}\rangle + \frac{(E-M_{Y^*}+i\Gamma_{Y^*}/2)}{(E-M_{Y^*}-\Delta E+i\beta\Gamma_{Y^*}/2)} \times\langle\vec{k}'|t^{Y^*}|\vec{k}\rangle\rho_{Y^*}(\vec{k}',\vec{k},\lambda). \quad (7)$$

The parameter ΔE defined by Eq. (7) has the physical significance of being the average binding energy difference between a nucleon and the Y^*. The parameter β measures the average modification of the width of the resonance in nuclear matter. If $\beta > 1$, there is "collision broadening" of the resonance, while $\beta < 1$ would indicate the predominance of Pauli damping and phase space effects of binding. The function $\rho_{Y^*}(\vec{k}',\vec{k},\lambda)$ is a mixed density function defined in Ref. 2 for the Δ, with λ giving the effect of nonlocality due to many-body effects on the propagator of the resonance.

The present results for the I-D model for pion scattering in the region of the $\Delta(1232)$ ($100 \lesssim E \lesssim 300$ MeV) show[7] that $\beta \approx 1.1$ (there is a 10% broadening of the Δ). There also seems to be evidence that $\Delta E \approx 10$ MeV, so that the Δ seems to be less bound than the nucleon. The nonlocality in this region is small, with $0 \lesssim \lambda \lesssim 0.5$ fm being obtained. With such a small nonlocality, it seems that a closure approximation is justified.

In the closure approximation, in which the energy denominators in Eq. (6) are replaced by average values, the sum over doorway states leads to the replacement of the mixed density function of the I-D model [Eq. (7)] by the nuclear density

$$\rho_{Y^*}(\vec{k}',\vec{k},\lambda) \xrightarrow[\rm closure]{} \rho(\vec{k}',\vec{k}) = \frac{1}{(2\pi)^{3/2}}\int e^{i(\vec{k}-\vec{k}')\cdot\vec{r}}\sum_\sigma|\phi_\sigma(r)|^2 d^3r \quad (8)$$

where $\sum|\phi_\sigma(r)|^2$ is the one-particle nuclear density. This gives for the optical model

$$\langle \vec{k}' | V | \vec{k} \rangle^{\text{closure}} = \langle \vec{k}' | V^{\text{NR}} | \vec{k} \rangle + \sigma(E) \langle \vec{k}' | V^{(1)} | \vec{k} \rangle , \tag{9a}$$

where

$$\sigma(E) = (E - M_{Y^*} + i\,\Gamma_{Y^*}/2)(E - M_{Y^*} - \Delta E + i\beta \Gamma_{Y^*}/2) , \tag{9b}$$

$$\langle k' | V^{(1)} | k \rangle = \langle \vec{k}' | t^{Y^*} | \vec{k} \rangle \, \rho(\vec{k}', \vec{k}) . \tag{9c}$$

The potential $V^{(1)}$ in Eqs. (9) is simply the first-order optical potential in the impulse approximation. The parameter $\sigma(E)$ is the ratio of the propagator of the Y^* in the medium to the free propagator of the Y^*. This factor gives all of the effects of isobar formation on a bound nucleon, which are extraordinarily difficult to calculate reliably in a microscopic theory, within a closure approximation.

Some "true" absorptive processes have been neglected here. However, a large part of the true absorption absent in conventional multiple scattering theory is included in the I-D model.[7] For more accurate treatments in the future one should include the remainder of the true absorption and use the general form of the I-D model (without closure).

III. K^- ATOMS AND THE I-D MODEL: THE $\Lambda(1405)$

During the past decade a great deal of data has been obtained[6] from x-ray studies of kaonic atoms. Measurements of the deviation of the energies and widths of the atomic states from those expected with a purely electromagnetic interaction have been made for a number of states. The analysis is done using a potential of the form

$$V = V^{\text{em}} + V^{\text{opt}}$$

for the kaons, where V^{opt} is the effective strong interaction which causes the deviation from ordinary atomic properties. In a number of analyses the optical potential has been taken as the $E = M_N + M_K$ limit of the impulse approximation

$$V^{\text{opt}} \Rightarrow V^{(1)}(E = M_N + M_K) = \frac{2\pi}{\mu} \bar{a} \rho(r) , \tag{10}$$

where $\bar{a}$ is the s-wave scattering length averaged over isospin. This quantity has been used as a fitting parameter in a number of calculations,[6,8] and it turns out that one can find an average value which gives an approximate fit to the data. Here we use the result of Batty *et al.*,[8,20]

$$a_{\text{fit}} = (0.34 + 0.81 i) \text{ fm}, \tag{11}$$

which is a typical value. This should be contrasted with the free scattering length[9]

$$\bar{a}_{\text{free}} = (-0.15 + 0.68\, i) \text{ fm} . \tag{12}$$

From Eqs. (11) and (12) it seems, superficially, as if the free interaction is repulsive (giving $\text{Re}[\bar{a}_{\text{free}}] < 0$), while the interaction with the bound nucleon is attractive ($\text{Re}[\bar{a}_{\text{fit}}] > 0$).

It has been convincingly demonstrated in a number of calculations that one can explain this reversal in sign of the real part of the scattering length by the below threshold $\Lambda(1405)$ resonance and binding effects.[10] Recall that at resonance the real part of the amplitude due to the resonance vanishes

$$\text{Re}[t^{Y^*}(E = M_{Y^*})] = 0 .$$

Therefore,

$$\text{Re}[t^{Y^*}(E > M_{Y^*})] > 0 ,$$

or

$$\text{Re}\, \bar{a}^{\Lambda(1405)}(E = M_N + M_k) < 0$$

as is seen in Eq. (12). On the other hand, with scattering by a bound nucleon the Fermi motion and other effects can lead to a change in the sign of real part of the t matrix. Of course the optical potential cannot be represented by $V_{\text{opt}} = t_{\text{free}}\,\rho$ in this situation.

This leads us to ask the question: What is happening to the $\Lambda(1405)$ in nuclear matter? One can most readily answer the question by considering the $\Lambda(1405)$ hypernuclei via the isobar-doorway model of the previous section. Let us assume that the entire $E = M_N + M_K$ interaction is given by the coupling of the $\bar{K}N$ to the $\Lambda(1405)$. Then the isobar-doorway model would give

$$V = V^{\text{res}} = V^{\text{fit}} = \frac{2\pi}{\mu} \bar{a}_{\text{fit}} \rho .$$

Recalling that

$$\frac{V^{\text{res}}}{V^{(1)}} = \sigma(E) = \frac{E - M_{1405} + i\,\Gamma_{1405}/2}{E - M_{1405} - \Delta E + i\beta \Gamma_{1405}/2}$$

and that

$$V^{(1)} = \frac{2\pi}{\mu} \bar{a}_{\text{free}} \rho,$$

one finds that

$$\left. \frac{E - M_{1405} + i\,\Gamma_{1405}/2}{E - M_{1405} - \Delta E + i\beta \Gamma_{1405}/2} \right|_{E \cong M_N + M_K} = \left. \frac{\bar{a}_{\text{fit}}}{\bar{a}_{\text{free}}} \right|_{E \cong M_N + M_K} = 1.03 - 0.73\, i . \tag{13}$$

This gives the result shown in Table II, that

$$\Delta E(\Lambda(1405)) \cong 19 \text{ MeV},$$
$$\beta(\Lambda(1405)) \approx 1.29 . \tag{14}$$

Therefore, the closure estimate assuming complete $\Lambda(1405)$ dominance at $E = M_N + M_K$ is that there is a 29% collision broadening and that the $\Lambda(1405)$ is effectively only about 9 MeV below threshold when produced by a $\bar{K}$. This is a satisfactory result, but it should not be taken too seriously before more careful calculations are carried out. Still, the general conclusions are consistent with pion-nucleus scattering at the energies of the $\Delta(1232)$ as seen in Table II and discussed in Sec. II. It is not inconsistent with the results of other theoretical calculations using conventional potentials.[6,10]

As has been mentioned in the Introduction, there has been a great deal of work on the effect of the $\Lambda(1405)$ on K-mesic atoms. The work closest in spirit to the present work makes use of $\Lambda(1405)$-holes states,[11] similar to the Δ-hole states used for pion-nucleus scattering.[3-5] This is a specific model for the I-D theory, and application was made for ^{12}C and ^{32}S kaonic atoms. In a local approximation changes in the mass and the width of the $\Lambda(1405)$ considerably smaller than those found in the present work were obtained. Corrections to the closure approximation and other effects can produce significant changes in such a parametrization.[2,7,11] A phenomenological inclusion of nonlocality shows that there are ambiguities in the result.[11]

The model of the present paper averages over all partial waves and many-body states, and the resulting parameters ΔE and β are average values. They include some compensating effects, such as nucleon binding, Fermi motion, Pauli blocking, and collision broadening, all of which must be calculated in detail in a microscopic model. It is of special interest to note that in the present model *all of the data* which has been accumulated on kaonic atoms has been approximately included, as one can see from Eq. (13) and the discussion following Eq. (10). It should also be noted that the present I-D theory provides a partial justification for the approach of Rook[12] in fitting the K-mesic data with a potential that included a term with the form of a complex parameter $\times$ nuclear density.

TABLE II. Binding energy and width modification of isobars (see text).

Resonance	ΔE	β
$\Delta(1232)$	10	1.1
$\Delta(1405)$	19	1.29
$\Delta(1520)$	10.5	1.3

There has been an explicit calculation of the mass shift and width of the $\Lambda(1405)$ in nuclear matter,[13] using a $\Sigma\pi$ model for the resonance. The resulting mass and width modifications are small compared to our phenomenological result.

IV. $\bar{K}$-NUCLEUS ELASTIC SCATTERING. THE OPTICAL POTENTIAL

A. Optical potential

We shall make use of the closure approximation for the optical potential given in Eq. (9). Let us first consider the first-order potential $V^{(1)}$ in that expression. From Table I it is clear that for a potential to be useful through 1 GeV/c it must include $L = 3$ $\bar{K}N$ partial waves, and even at 100 MeV the $L = 2$ partial wave plays an essential role. This will lead to an optical potential with strong surface effects. Recall that the strong $L = 1$ resonance in the π-N system leads to a potential involving gradients of the nuclear density. The $\bar{K}N$ systems should have even stronger surface effects. Physically, the system must produce effective centrifugal barriers of particle dimension distributed over the nuclear surface.

The form of the optical potential which is used here is[14]

$$2E_{k^-} V_{k^-} = A[\, c_0 \rho_L{}^2 \rho(r) + c_1 \nabla \cdot \rho \nabla + c_2 \nabla^2 \rho + c_3 \nabla^4 \rho] . \tag{15}$$

The derivation of this potential is given in the Appendix. The parameters $c_i(E)$ are obtained from the $\bar{K}N$ amplitude as given in the Appendix.[15]

As in the pion-nucleus case, the form of the optical potential is not unique since it depends upon the off-shell behavior of the $\bar{K}N$ amplitude, which is not known. The particular form used in Eq. (15) is an (almost) local form which should give an adequate representation of elastic scattering, which is not very sensitive to off-shell behavior. As with the pion optical potential, the kaon optical potential is actually a truly nonlocal operator. The part of the "true" absorption not included in the I-D model, and off-shell form factors,[16] will introduce nonlocalities best treated in momentum space. However, the form of Eq. (15) is adequate for present purposes and the use of more elaborate forms is not consistent with the closure approximation for binding corrections, which will now be discussed.

The dynamic effects of isobar and nucleon interactions are introduced via the isobar-doorway model described in the previous section. This is done by using a parametrization of the $\bar{K}N$ amplitude in each partial wave of the form of a resonance and a background amplitude

$$t^{\alpha\beta}=t_B^{\alpha\beta}+t_{\text{res}}^{\alpha\beta}\,,$$

with

$$t_{\text{res}}^{\alpha\beta}=\frac{[\Gamma_\alpha(E)\Gamma_\beta(E)]^{1/2}e^{i\phi}}{M-E-i\Gamma(E)}\,, \tag{16}$$

the resonant form of the T-matrix element for the α,β channels. The parameters M, Γ_α, Γ, the background amplitude t_B, and phase ϕ are given by various fits to the data.[10] For the isobar-doorway model the resonant amplitude is replaced by

$$(t_{\text{res}}^{\alpha\beta})_{\text{bound}}=\frac{M-E-i\Gamma(E)}{M+\Delta E-E+i\beta\Gamma(E)}\,t_{\text{res}}^{\alpha\beta}\,, \tag{17}$$

thus the parameters $c_i(E)$ of Eq. (13) are obtained by using the equations in the Appendix with the modification of Eq. (17).

B. Elastic scattering at the Λ(1520)

The only published $\bar{K}$-nucleus elastic scattering to data at low energy are emulsion studies of two decades ago.[17] Fortunately, these experiments were done in the region of the Λ(1520), which is at an energy of 87 MeV in the $\bar{K}$-nucleon center of mass system. The main difficulty in using this data is that one must average over energy as well as target nuclei. Although the errors in the data are large by current standards we were able to do an analysis in the isobar-doorway model.[18]

The data of Melkanoff *et al.*[17] is at an average energy of 100 MeV, some 13 MeV above the resonance. Using the closure form (9a) and assuming resonance dominance, we find[18] that the ratio of the I-D potential to the first-order potential is

$$\alpha^{\Lambda(1520)}_{(E\cong 100\,\text{MeV})}\approx 1-i\,. \tag{18}$$

Using the expression (9b) it follows that

$$\Delta E(\Lambda(1520))=10,\quad \beta(\Lambda(1520))=1.3\,.$$

This is an interesting, although tentative, result. Again referring to Table II, the pattern seems to be developing that, on the average, baryon resonances are less bound in nuclei than nucleons, or even unbound in their lowest states, and that they undergo collision broadening.

For more definitive conclusions about the properties of the Λ(1520) in nuclei accurate angular distributions must be measured for a variety of energies in the region of 100 MeV. These must be fit with more accurate theoretical forms, especially without the use of the closure approximation. In order to show the possibilities, some theoretical estimates of the differential cross sections in the region of the Λ(1520) are given in Fig. 1. These are obtained by using the optical potential of Eq. (15) with the modification of Eq. (17). The three curves on the left are the impulse approximation results, showing the rapid change in angular distribution as a function of energy in the region of the isobar. The three curves to the right show the strong dependence on the ΔE parameter of the I-D model as seen by comparing curves on the left and right at equal energy. These results are somewhat exaggerated, since the Fermi averaging will smooth out some of the energy dependence, but clearly this is the most promising region for learning details of a Y^* interaction. A systematic study of $\bar{K}$-nucleus scattering in the region of the Λ(1520) is one of the most interesting problems in nuclear physics.

C. Other regions, elastic scattering near $E=1670$

From Table I it can be seen that only the Λ(1405) and Λ(1520) are isolated Y^* resonances. However, there are other regions of interest. Consider the

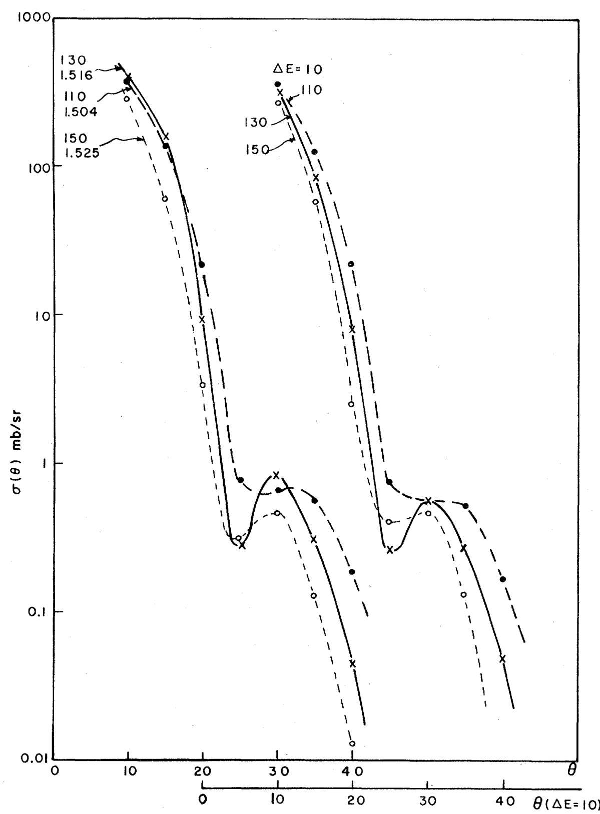

FIG. 1. The three curves on the left give the K^--^{12}C elastic differential cross sections at three energies near the Λ(1520) resonance. The labels are $\bar{K}N$ laboratory energy (MeV) and center of mass total energy (GeV). The curves on the right are the same with the N-Λ(1520) binding energy $\Delta E=10$ MeV.

energy region of 1670 MeV. Note that three Y^*'s occur at this energy, so that one must deal with strongly overlapping isobar-doorway states. However, the quantum numbers are quite different, so that one can hope to sort out the effects of the individual isobars. The strongest overlap occurs between the $\Lambda(1670)$ and $\Sigma(1670)$. However, since these are $L = 0$ and 2 resonances, respectively, their momentum dependence (on- and off-shell) is quite different (see Appendix). Therefore, the parameters of the optical potential are affected very differently by the two resonances. The $\Lambda(1670)$ will contribute only to the c_0 parameter, while the $\Sigma(1670)$ can contribute to all of the c_i [see Eq. (15)]. Therefore, if one can determine the individual parameters from detailed experimental and theoretical studies, the energy dependence will give information about the individual resonances. Also, one can use the isospin properties to try to distinguish between the Λ and Σ resonances by comparing experiments with different isotopes as targets.

In order to give a general impression about such possibilities, the energy and ΔE dependence of the differential cross sections are shown in Fig. 2.

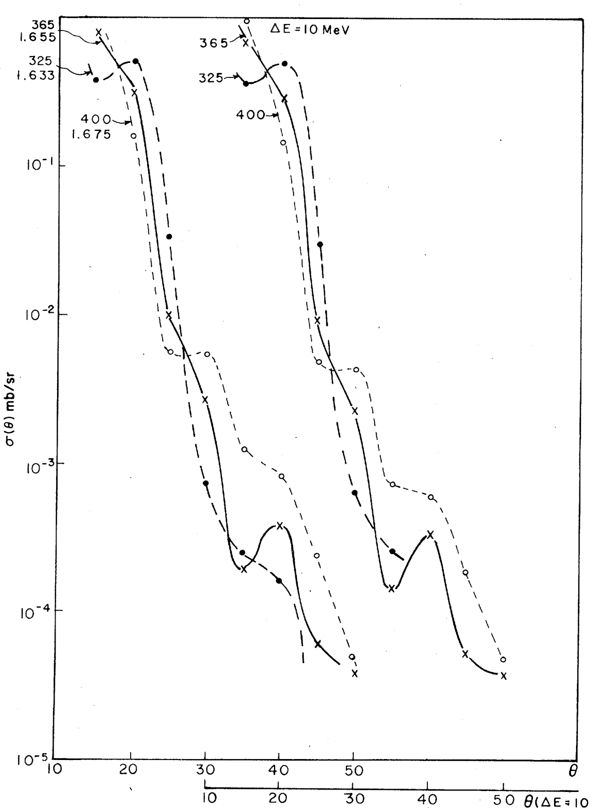

FIG. 2. $\bar{K}$-^{12}C elastic scattering near 1670 MeV. See caption of Fig. 1.

Note the strong energy dependence and considerable sensitivity to the binding energy parameter. This suggests that the program just described might be possible, although difficult. Once more, Fermi averaging would reduce some of the sensitivity.

The attempt of the present paper to strongly motivate research in particular energy regions should not distract the reader from more general considerations. An extensive program of accurate experiments in the entire low medium-energy region of zero to several hundred MeV is needed to explore the physics and complete the development of an accurate optical potential for $\bar{K}$-nucleus scattering and reactions. Some experiments have now been carried out.[19] Knowing that $\bar{K}$ beam time is currently limited, I have concentrated my discussion on aspects which I feel will be particularly rewarding. For the forward medium-energy kaon scattering needed for inelastic studies the impulse and eikonal approximations should be satisfactory,[20,21] and optical models have been proposed[20] for this purpose.

V. CONCLUSIONS

The medium-energy kaon-nucleus system has been seen to be rich in possibilities of learning about Y^* interactions in nuclei. Using the closure approximation in the I-D model, it has been shown that kaonic atom data suggests that the $\Lambda(1405)$ is only weakly bound (vs the naive expectation of being 35 MeV below threshold) and that its width is broadened by some 29% in nuclei. Very sketchy experimental information and the closure approximation indicate that the $\Lambda(1520)$ also is unbound and experiences a 30% broadening of its width. These properties are consistent with those observed for the Δ in pion-nucleus scattering.

This interpretation not only is to be regarded as only semiquantitative due to the simplified theoretical treatment, but also it has been made by averaging over all of the l-wave projections of the doorway state without an attempt to extract more detailed information. With accurate experimental data over a range of energies, one could attempt to learn much more about the doorway states, and thus the Y^* properties. The region of the $\Lambda(1520)$ would be most interesting in this regard and is certainly the most promising. From the experimental point of view it is difficult to work at this low an energy with present $\bar{K}$ beams. However, if it could be done it should be most rewarding. Note, however, that only systematic studies would be conclusive.

In addition to the possibility of gaining some information about Y^* interactions and propagation, the determination of the optical potential in itself is sufficient motivation for an intensive and extensive study of $\bar{K}$ elastic scattering. Even though the form of the optical potential is not unique (see Sec. III), the result given in Eq. (5) for the $\bar{K}$-nucleus potential is significant. Due to the formation of $\bar{K}N$ resonances with relatively large relative orbital angular momenta in the nuclear surface, there must be rapidly varying spatial and energy dependence of the optical potential. The replacement of the I-D closure prescription for binding corrections [Eq. (15)] can produce important qualitative modifications[2-5] However, just as the physics of Δ formation with pions led to the gradient potential, suitably modified, the formation of Y^*'s in the nuclear surface will lead to a most interesting optical potential, with unusual surface properties.

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APPENDIX

The impulse approximation for the optical potential in momentum space is

$$\langle \vec{k}' | V | \vec{k} \rangle = \langle \vec{k}' | t | \vec{k} \rangle \rho(|\vec{k} - \vec{k}'|) . \tag{A1}$$

The procedure of the present paper is to use the closure approximation for the I-D model to include binding corrections. As a consequence, the potential is of the form (A1) with the bound state t matrix having the same $\vec{k}$, $\vec{k}'$ dependence as the free t matrix, but with modified energy dependence. Consider the partial wave expansion of the off-shell t matrix:

$$\langle \vec{k}' | t | \vec{k} \rangle \cong \frac{1}{p} \sum_{l,I} [(l+1) t^I_{lj^+}(E,k,k') + l\, t^I_{lj^-}(E,k,k')] \times P_l(\cos\theta), \tag{A2}$$

where $E = (p^2 + m^2)^{1/2}$, $\cos\theta = \vec{k} \cdot \vec{k}'/kk'$, I labels isospin, and spin flip has been neglected. To complete the model one must assume a model for the partial wave amplitudes $t_{lj}(E,k,k')$ which reduce to the on-shell value when $E = (k^2 + m^2)^{1/2} = (k'^2 + m^2)^{1/2}$.

For the off-shell behavior we make the following *Ansätze*

$$t_{l=0}(E,k,k') = a_0(E) , \tag{A3}$$

$$t_{l=1,j}(E,k,k') = a_{1j}(E) , \tag{A4}$$

$$t_{l=2,j}(E,k,k') p_2(\cos\theta) = a_{2j}(E) \left(1 - \frac{3}{2} \frac{|\vec{k} - \vec{k}'|^2}{p^2} + \frac{3}{8} \frac{|\vec{k} - \vec{k}'|^4}{p^4}\right) , \tag{A5}$$

$$t_{l=3,j} p_3(\cos\theta) \cong a_{3j}(E) \left(1 - \frac{5}{2} \frac{|\vec{k} - \vec{k}'|^2}{p^2} + \frac{5}{8} \frac{|k - k'|^4}{p^4}\right) , \tag{A6}$$

where the $a_{lj}(E)$ are the on-shell partial wave amplitudes modified by the I-D considerations discussed in the text. The first two forms result in the gradient potential.[22] Equation (A5) is one of the possible off-shell extrapolations and has been chosen so as not to introduce additional second-order derivative of the kaon wave function. The form (A6) for the F-wave term is not only an arbitrary *Ansatz*, but is only accurate for $\hat{k} \cdot \hat{k} \approx 1$. Since large-angle scattering proceeds mainly through a series of small-angle steps at medium energy, this is not a serious limitation.

Taking the Fourier transform of (A1) with the use of (A2)–(A6), one obtains for the potential in coordinate space, $\langle \vec{r}' | V | \vec{r} \rangle = V(r) \delta(\vec{r} - \vec{r}')$,

$$2EV(r) = A[c_0 p_L{}^2 \rho(r) + c_1 \nabla \cdot \rho(r) \nabla + c_2 \nabla^2 \rho(r) + c_3 \nabla^4 \rho(r)] \tag{A7}$$

with

$$c_0 = B(\langle a_0 \rangle - \langle a_2 \rangle - \langle a_3 \rangle), \tag{A8}$$

$$c_1 = B\langle a_1 \rangle ,$$

$$c_2 = -(B/2)(3\langle a_2 \rangle + 5\langle a_3 \rangle) , \tag{A9}$$

$$c_3 = (2/4 p_L{}^2) c_2 , \tag{A10}$$

and

$$B = 4\pi(m_k{}^2 + M^2 + 2ME_k{}^2)/p_L{}^3 M^2 A , \tag{A11}$$

$$\langle a_0 \rangle = N f^1_{0,1/2} + Z(f^1_{0,1/2} + f^0_{0,1/2})/2 , \tag{A12}$$

$$\langle a_1\rangle = N(2f^1_{1,3/2}+f^1_{1,1/2}) + Z(f^1_{1,3/2}+f^1_{1,1/2}/2+f^0_{1,3/2}+f^0_{1,1/2}/2)\,, \tag{A13}$$

$$\langle a_2\rangle = N(3f^1_{2,5/2}+2f^1_{2,3/2}) + (Z/2)[3(f^1_{2,5/2}+f^0_{2,5/2}) + 2(f^1_{2,3/2}+f^0_{2,3/2})]\,, \tag{A14}$$

$$\langle a_3\rangle = N(4f^1_{3,7/2}+3f^1_{3,5/2}) + (Z/2)[4(f^1_{3,7/2}+f^0_{3,7/2}) + 3(f^1_{3,5/2}+f^0_{3,5/2})]\,, \tag{A15}$$

with $f_\alpha = e^{i\delta}\sin\delta$ and the notation f^I_{lj}. For the bound state modification these partial wave amplitudes are multiplied by the Δ propagator ratio as given in Eq. (15).

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[1]L. S. Kisslinger and W. L. Wang, Phys. Rev. Lett. 30, 1071 (1973).

[2]L. S. Kisslinger and W. L. Wang, Ann. Phys. (N.Y.) 99, 374 (1976).

[3]M. Hirata, J. Koch, F. Lenz, and E. Moniz, Phys. Lett. 70B, 281 (1978); MIT report (unpublished).

[4]K. Klingenbeck, M. Dillig, and M. G. Huber, Phys. Rev. Lett. 41, 387 (1978); Erlanger report (unpublished).

[5]E. Oset and W. Weise, Regensburg report (unpublished).

[6]R. Seki and C. E. Wiegand, Annu. Rev. Nucl. Sci. 25, 241 (1975). This review contains references to experiment and theory.

[7]L. S. Kisslinger and A. N. Saharia, in *Meson-Nuclear Physics—1976*, Proceedings of the International Topical Conference on Meson-Nuclear Physics, edited by P. D. Barnes, R. A. Eisenstein, and L. S. Kisslinger (AIP, New York, 1976), p. 184; and TRIUMF report (unpublished).

[8]C. J. Batty *et al.*, Nucl. Phys. A329, 407 (1979).

[9]A. D. Martin, Phys. Lett. 65B, 346 (1976).

[10]M. Alberg, E. M. Henley, and L. Wilets, Ann. Phys. (N.Y.) 96, 43 (1976). This article contains references to the large body of theoretical work which has been done on the effects of the $\Lambda(1405)$ on $\bar{K}$ atoms. For recent references see S. Wycech, in *Meson-Nuclear Physics—1979*, Proceedings of the 2nd International Topical Conference on Meson-Nuclear Physics, Houston, edited by E. V. Hungerford (AIP, New York, 1979), p. 390.

[11]W. Weise and L. Tauscher, Phys. Lett. 64B, 424 (1976); R. Brockmann, W. Weise, and L. Tauscher, Nucl. Phys. A308, 365 (1978); M. Thies, Phys. Lett. 70B, 401 (1977); Nucl. Phys. A298, 344 (1978).

[12]J. R. Rook, Nucl. Phys. A249, 466 (1975).

[13]J. M. Eisenberg, Phys. Rev. C 14, 2343 (1976).

[14]A preliminary version of the I-D model for the K^- optical potential is given by L. S. Kisslinger, *Meson-Nuclear Physics—1976* (see Ref. 7), p. 159.

[15]P. Baillon and P. J. Lichfield, Nucl. Phys. B94, 39 (1975). This article contains other references on $\bar{K}N$ analyses.

[16]R. H. Landau, S. C. Phatak, and F. Tabakin, Ann. Phys. (N.Y.) 78, 299 (1973).

[17]M. Melkanoff, D. J. Prowse, and D. H. Stork, Phys. Rev. Lett. 4, 183 (1960).

[18]L. S. Kisslinger and Nguyen Van Giai, Phys. Lett. 72B, 19 (1977).

[19]D. Marlow, P. Barnes, S. Dytman, S. Bart, D. Hancock, R. Hackenberg, E. Hungerford, L. Pinsky, T. Williams, R. Chrien, N. Palevsky, and R. Sutter, in *Meson-Nuclear Physics—1979*, Proceedings of 2nd International Conference on Meson-Nuclear Physics, Houston, 1979, edited by E. V. Hungerford (AIP, New York, 1979).

[20]D. B. Dover and G. E. Walker, Phys. Rev. C 19, 1393 (1979); S. R. Cotanch and F. Tabakin, *ibid.* 15, 1379 (1977).

[21]G. N. Epstein, F. Tabakin, L. M. Vincent, A. Gal, and L. S. Kisslinger, Phys. Rev. C 17, 1501 (1978).

[22]L. S. Kisslinger, Phys. Rev. 98, 761 (1955).

[23]Particle Data Group, Rev. Mod. Phys. 48, 51 (1976).