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Parametrization of the Paris N-N potential

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In view of practical nuclear structure calculations the Paris N-N potential is parametrized in a simple analytical form. This parametrization consists of a regularized discrete superposition of Yukawa-type terms. Results for phase shifts and deuteron parameters are presented as well as nuclear matter binding energy obtained with this potential.

[NUCLEAR REACTIONS Nucleon-nucleon interaction, parametrization of the Paris N-N potential. 0 to 330 MeV N-N phase shifts.]

Some years ago, we derived[1] a nucleon-nucleon interaction from πN and $\pi\pi$ interactions, which includes the one-pion-exchange (OPE), correlated and uncorrelated two-pion-exchange, and ω-exchange contributions. These theoretical contributions give a fairly realistic description of the long and medium range (LR+MR) N-N forces since

(i) the peripheral ($J>2$) phase shifts calculated from these contributions are in good agreement with the experimental ones,[2]

(ii) an equivalent potential derived from this $(\pi+2\pi+\omega)$ exchange interaction compares very well with the phenomenological potentials of Yale and of Hamada-Johnston down to internucleon distances $r\sim 0.6$ fm for the spin-spin and tensor components and $r\sim 0.8$ fm for the central and spin-orbit components.[1]

Although in some cases (e.g., the isotriplet spin-spin component) the agreement extends to very small values of r, there is no compelling theoretical reason to believe the validity of our potential in the region $r\lesssim 0.8$ fm since the short range (SR) part of the interaction is related to exchange of heavier systems and/or to effects of subhadronic constituents such as quarks, gluons, etc. At present, no reliable calculations of this SR part are available. Thus, we provisionally take the viewpoint that the SR part should be determined phenomenologically, with the hope that our accurately determined LR+MR interaction will provide strong constraints on this SR part, leaving us with only a few degrees of freedom. Along this line, we proposed[3] to describe the core with a very simple phenomenological model; namely, the long and intermediate range $(\pi+2\pi+\omega)$ potential is cut off rather sharply at internucleon distance $r\sim 0.8$ fm and the short range ($r\lesssim 0.8$ fm) is described simply by a constant soft core. This introduces the minimum number (five) of adjustable parameters corresponding to the five components (central, spin-spin, tensor, spin-orbit, and quadratic spin-orbit) of the potential for each isospin state. On the other hand it was found that the central component of the theoretical LR+MR potential has a weak but significant energy dependence and that this energy dependence is, in a very good approximation, linear. As an energy dependence appears already in the LR+MR potential, one expects also an energy dependence in the SR part. Indeed, fitting the data required an energy dependent core for the central potential, the energy dependence being again linear and introducing therefore one additional parameter, the slope of the energy dependence. The proposed SR part is then determined by fitting all the known phase shifts ($J\leq 6$) up to 330 MeV and the deuteron parameters. Although the number of free parameters is small (six in total for each isospin state) the quality of the fit is very good. The χ^2/data are as good as the ones given by the best phenomenological potentials which contain many more free parameters.

The previous model (referred to, in the literature, as the Paris N-N potential) was purposely chosen in its simplest form to demonstrate that, once the LR+MR forces are accurately determined, the SR forces can be described by a model with few parameters that does not affect the LR+MR part. This simple model in which a definite separation between the theoretical and phenomenological parts is made, is designed for providing a clear physical insight into the problem. However, the explicit expression of the resulting potential is not very convenient for practical use in many-body calculations:

(i) the energy dependence of the potential which can be treated naturally in the two-body scattering

TABLE I. The $T=1$ Potential parameters. The notation $a\pm n$ stands for $a\times 10^{\pm n}$.

		Central Singlet Potential V_0^a		
$m_j(\mathrm{fm}^{-1})$	0.684 026 00 + 00	0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
g_j(MeV)	−0.100 774 27 + 02	−0.120 495 64 + 03	−0.212 364 60 + 03	−0.871 741 98 + 04
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	0.543 833 77 + 05	−0.213 421 47 + 06	0.494 583 57 + 06	−0.667 153 34 + 06
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	0.529 575 98 + 06	−0.137 034 12 + 06	−0.346 971 94 + 06	see Eq. (9)
		Central Triplet Potential V_1^a		
$m_j(\mathrm{fm}^{-1})$	0.684 026 00 + 00	0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
g_j(MeV)	0.335 914 22 + 01	−0.864 795 68 + 02	−0.465 931 11 + 03	0.186 730 85 + 04
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	0.385 092 13 + 04	−0.196 743 38 + 05	0.123 231 40 + 06	−0.314 493 61 + 06
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	0.242 424 40 + 06	0.166 904 04 + 06	−0.485 343 64 + 06	see Eq. (9)
		Central Singlet Potential V_0^b		
$m_j(\mathrm{fm}^{-1})$	0.684 026 00 + 00	0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
g_j	0.268 513 93 − 02	0.510 924 55 − 01	−0.842 642 58 + 00	0.147 363 12 + 02
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	−0.145 219 93 + 03	0.841 583 89 + 03	−0.278 611 70 + 04	0.505 645 10 + 04
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	−0.336 742 05 + 04	−0.178 455 29 + 04	0.535 482 66 + 04	see Eq. (9)
		Central Triplet Potential V_1^b		
$m_j(\mathrm{fm}^{-1})$	0.684 026 00 + 00	0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
g_j	−0.895 046 44 − 03	0.374 884 81 − 01	−0.893 730 89 + 00	0.141 234 75 + 02
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	−0.146 601 52 + 03	0.841 914 62 + 03	−0.283 942 73 + 04	0.526 534 27 + 04
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	−0.350 004 30 + 04	−0.248 794 79 + 04	0.730 681 21 + 04	see Eq. (9)
		Spin-Orbit Potential V_{LS}		
$m_j(\mathrm{fm}^{-1})$		0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
g_j(MeV)		−0.426 003 59 + 03	0.262 795 17 + 05	−0.575 570 33 + 06
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	0.600 339 34 + 07	−0.345 194 43 + 08	0.113 554 59 + 09	−0.207 292 09 + 09
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	0.171 315 48 + 09	−0.864 182 22 + 08	see Eq. (9)	see Eq. (9)
		Tensor Potential V_T		
$m_j(\mathrm{fm}^{-1})$	0.684 026 00 + 00	0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
g_j(MeV)	0.335 914 22 + 01	−0.859 458 24 + 00	−0.104 763 40 + 03	0.126 294 65 + 04
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	−0.188 810 61 + 05	0.106 132 46 + 06	−0.332 119 10 + 06	0.555 857 62 + 06
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	−0.349 166 64 + 06	−0.119 450 13 + 06	see Eq. (9)	see Eq. (9)
		Quadratic Spin-Orbit V_{SO2}		
$m_j(\mathrm{fm}^{-1})$		0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
g_j(MeV)		−0.522 186 40 + 00	0.186 445 58 + 03	−0.370 911 15 + 04
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	0.559 131 17 + 05	−0.369 985 60 + 06	0.145 375 43 + 07	−0.313 524 71 + 07
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	0.243 390 81 + 07	see Eq. (9)	see Eq. (9)	see Eq. (9)

TABLE II. The $T=0$ Potential parameters. The notation $a \pm n$ stands for $a \times 10^{\pm n}$.

		Central Singlet Potential V_0^a		
$m_j(\mathrm{fm}^{-1})$	0.699 536 00 + 00	0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
$g_j(\mathrm{MeV})$	0.322 908 74 + 02	−0.824 656 31 + 02	0.123 293 84 + 04	−0.168 598 79 + 05
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	0.172 926 83 + 06	−0.768 352 77 + 06	0.218 904 75 + 07	−0.384 472 87 + 07
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	0.279 905 59 + 07	0.502 518 28 + 06	−0.260 061 24 + 07	see Eq. (9)
		Central Triplet Potential V_1^a		
$m_j(\mathrm{fm}^{-1})$	0.699 536 00 + 00	0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
$g_j(\mathrm{MeV})$	−0.107 636 25 + 02	−0.429 736 69 + 02	−0.718 568 44 + 03	0.424 691 20 + 04
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	−0.345 740 24 + 05	0.126 711 69 + 06	−0.274 168 41 + 06	0.529 607 24 + 06
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	−0.366 067 13 + 06	−0.223 036 73 + 06	0.406 838 33 + 06	see Eq. (9)
		Central Singlet Potential V_0^b		
$m_j(\mathrm{fm}^{-1})$	0.699 536 00 + 00	0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
g_j	−0.859 800 96 − 02	0.268 143 85 − 01	−0.132 806 93 + 01	0.103 242 89 + 02
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	−0.115 270 67 + 03	0.694 561 75 + 03	−0.238 793 35 + 04	0.423 880 11 + 04
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	−0.245 216 04 + 04	−0.195 128 21 + 04	0.418 011 60 + 04	see Eq. (9)
		Central Triplet Potential V_1^b		
$m_j(\mathrm{fm}^{-1})$	0.699 536 00 + 00	0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
g_j	0.286 600 32 − 02	−0.817 980 46 − 03	−0.533 145 60 + 00	0.831 620 30 + 00
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	−0.311 923 95 + 02	0.300 413 84 + 03	−0.124 150 67 + 04	0.247 622 41 + 04
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	−0.130 430 30 + 04	−0.214 965 77 + 04	0.409 969 17 + 04	see Eq. (9)
		Spin-Orbit Potential V_{LS}		
$m_j(\mathrm{fm}^{-1})$		0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
$g_j(\mathrm{MeV})$		−0.661 764 21 + 02	0.289 036 88 + 04	−0.625 924 00 + 05
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	0.691 461 41 + 06	−0.409 691 46 + 07	0.140 320 93 + 08	−0.268 274 68 + 08
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	0.235 114 42 + 08	−0.146 884 61 + 08	see Eq. (9)	see Eq. (9)
		Tensor Potential V_T		
$m_j(\mathrm{fm}^{-1})$	0.699 536 00 + 00	0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
$g_j(\mathrm{MeV})$	−0.107 636 25 + 02	−0.468 180 29 + 00	0.601 477 39 + 02	0.352 569 41 + 03
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	0.514 321 70 + 03	0.116 373 02 + 05	−0.445 954 15 + 05	0.692 117 38 + 05
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	−0.481 276 68 + 05	0.705 140 08 + 04	see Eq. (9)	see Eq. (9)
		Quadratic Spin Orbit V_{SO2}		
$m_j(\mathrm{fm}^{-1})$		0.160 000 00 + 01	0.230 000 00 + 01	0.300 000 00 + 01
g_j		−0.628 510 20 + 00	−0.762 901 97 + 02	−0.788 275 81 + 03
m_j	0.370 000 00 + 01	0.440 000 00 + 01	0.510 000 00 + 01	0.580 000 00 + 01
g_j	−0.649 047 98 + 04	0.547 343 78 + 04	−0.329 419 12 + 05	0.249 491 32 + 06
m_j	0.650 000 00 + 01	0.820 000 00 + 01	0.990 000 00 + 01	0.113 000 00 + 02
g_j	−0.160 129 56 + 05	see Eq. (9)	see Eq. (9)	see Eq. (9)

case, may be ill-defined in many-body systems;

(ii) the theoretical LR+ MR potential presents itself as a dispersion integral;

(iii) the presence of a sharp cutoff function may be troublesome in numerical works.

This note reports a parametrization of the Paris potential which is convenient enough to facilitate its use in the many-body calculations. We adopted a unique analytical expression for the complete potential, namely a discrete sum of Yukawa-type terms which has the advantage that their forms are simple in both configuration and momentum spaces. Concerning the energy dependence of the central potential, it has been shown[4] that one can transform the linear energy dependence occurring here into a p^2 dependence. In this work we have used this latter form since it can be handled without any amibiguity.

We would like to warn the reader that these forms provide a useful but purely mathematical representation of our potential, valid for all r, without distinction between the theoretical and phenomenological parts. Consequently, no physical interpretation should be attached to the values of the parameters. However, we required the LR + MR theoretical part to be strictly preserved.

PARAMETRIZATION IN CONFIGURATION SPACE

For the two isospin values $T=1$ and $T=0$, the potential is expressed in terms of the usual nonrelativistic invariants:

$$V(\vec{r},p^2)=V_0(r,p^2)\Omega_0+V_1(r,p^2)\Omega_1+V_{LS}(r)\Omega_{LS}+V_T(r)\Omega_T+V_{SO2}(r)\Omega_{SO2} \tag{1}$$

where

$$\begin{aligned}
\Omega_0&=\frac{1-\vec{\sigma}_1\cdot\vec{\sigma}_2}{4}\\
\Omega_1&=\frac{3+\vec{\sigma}_1\cdot\vec{\sigma}_2}{4}\\
\Omega_{LS}&=\vec{L}\cdot\vec{S}\\
\Omega_T&=3\frac{\vec{\sigma}_1\cdot\vec{r}\,\vec{\sigma}_2\cdot\vec{r}}{r^2}-\vec{\sigma}_1\cdot\vec{\sigma}_2\\
\Omega_{SO2}&=\tfrac{1}{2}(\vec{\sigma}_1\cdot\vec{L}\,\vec{\sigma}_2\cdot\vec{L}+\vec{\sigma}_2\cdot\vec{L}\,\vec{\sigma}_1\cdot\vec{L}).
\end{aligned} \tag{2}$$

As mentioned earlier, the central components contain a velocity dependent part and V_0 and V_1 are defined as

$$V(r,p^2)=V^a(r)+(p^2/m)V^b(r)+V^b(r)(p^2/m), \tag{3}$$

with

$$p^2=-\hbar^2\left[\frac{1}{r}\frac{d^2}{dr^2}r-\frac{\vec{L}^2}{r^2}\right],$$

$m=938.2592$ MeV for $T=1$, and 938.9055 MeV for

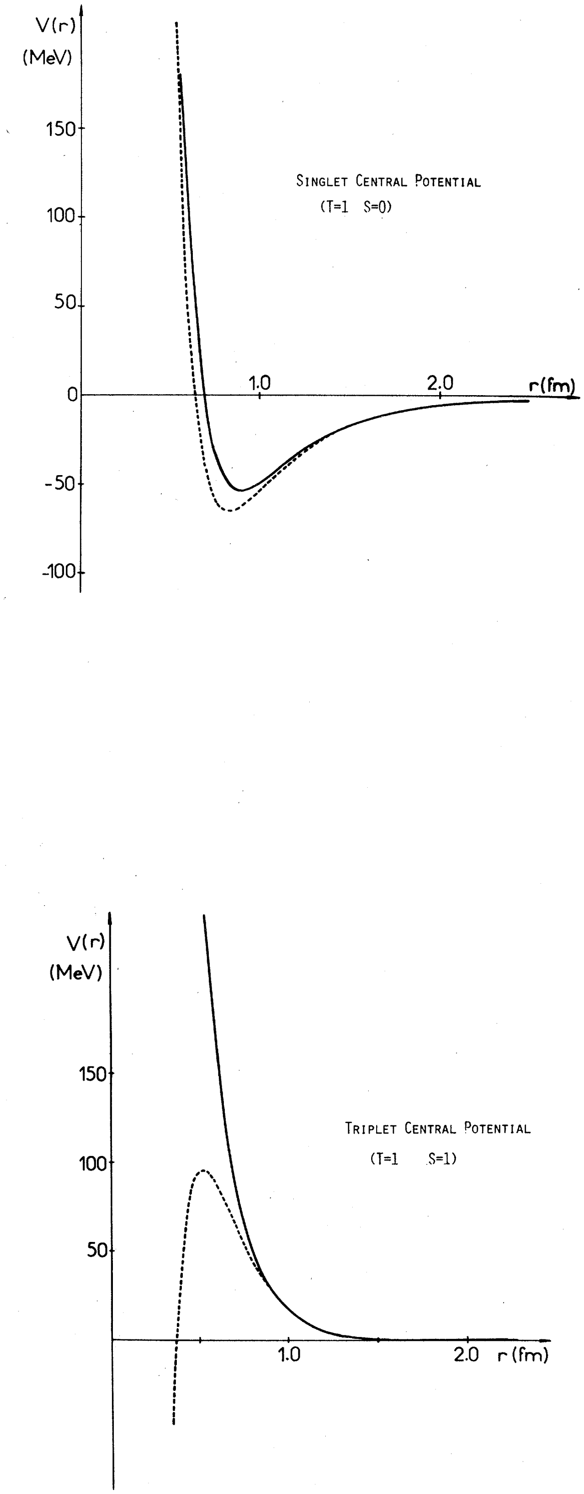

FIG. 1. The solid lines refer to the complete potential, the dashed lines to the theoretical one. In the case of the LS potential ($T=0$ $S=1$) the two curves are indistinguishable.

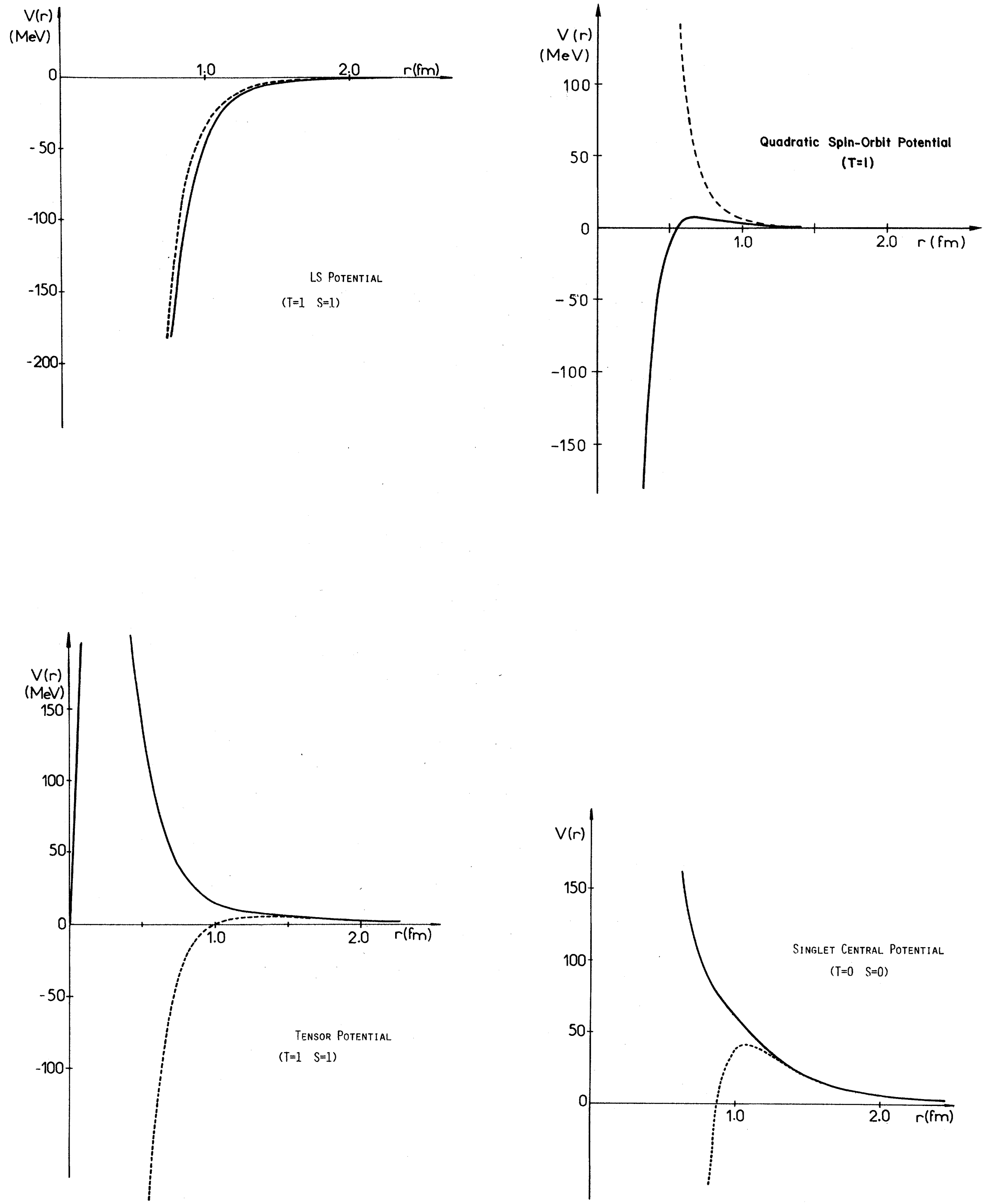

FIG. 1. (*Continued*).

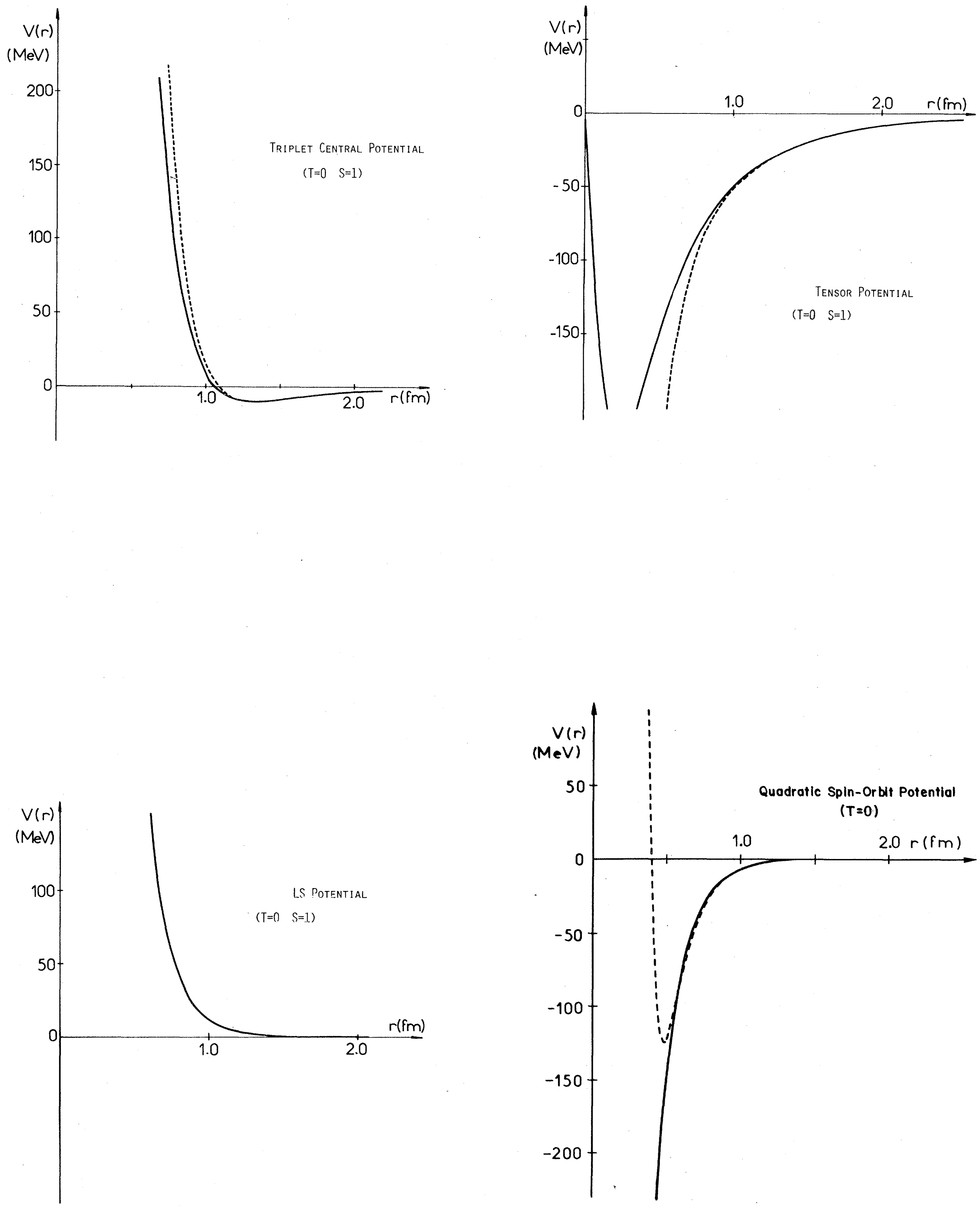

FIG. 1. (*Continued*).

$T=0$. For each component $V(r)$ the following parametrization is used:

$$V(r)=\sum_{j=1}^{n} g_j F(m_j r)\frac{e^{-m_j r}}{m_j r}, \tag{4}$$

where

$$\begin{aligned} F(m_j r)&=1 \text{ for } V_0^a,\ V_0^b,\ V_1^a, \text{ and } V_1^b \\ F(m_j r)&=\frac{1}{m_j r}+\frac{1}{(m_j r)^2} \text{ for } V_{LS} \\ F(m_j r)&=1+\frac{3}{m_j r}+\frac{3}{(m_j r)^2} \text{ for } V_T \\ F(m_j r)&=\frac{1}{(m_j r)^2}\left[1+\frac{3}{m_j r}+\frac{3}{(m_j r)^2}\right] \text{ for } V_{SO2}. \end{aligned} \tag{5}$$

The masses m_j are the same for all components, the first term ($j=1$) corresponds to the OPE and appears only in V_0, V_1, and V_T.

The potential is regularized at the origin $r=0$. Consequently, the parameters g_j are imposed to satisfy the following relationships:

$$\sum_{j=1}^{n}\frac{g_j}{m_j}=0 \text{ for } V_{0,1}^a \text{ and } V_{0,1}^b \tag{6}$$

$$\sum_{j=1}^{n}\frac{g_j}{m_j}=0 \text{ and } \sum_{j=1}^{n}\frac{g_j}{m_j^3}=0 \text{ for } V_T \text{ and } V_{LS} \tag{7}$$

$$\sum_{j=1}^{n}\frac{g_j}{m_j}=0,\quad \sum_{j=1}^{n}\frac{g_j}{m_j^3}=0, \text{ and } \sum_{j=1}^{n}\frac{g_j}{m_j^5}=0 \text{ for } V_{SO2}. \tag{8}$$

It should be noted that these constraints have to be satisfied with an accuracy related to the precision required for the value of V at the origin $r=0$, so that in Tables I and II the last values of the g_j's (the last one for $V_{0,1}^a$ and $V_{0,1}^b$, the last two for V_{LS} and V_T, and the last three for V_{SO2}) are not listed. They are to be computed by the users, according to the accuracy they will need, with the following formulas:

$$g_n=-m_n\sum_{j=1}^{n-1}\frac{g_j}{m_j} \text{ for } V_{0,1}^a \text{ and } V_{0,1}^b,$$

$$\left.\begin{aligned} g_{n-1}&=\frac{m_{n-1}^3}{m_{n-1}^2-m_n^2}\left(m_n^2\sum_{j=1}^{n-2}\frac{g_j}{m_j^3}-\sum_{j=1}^{n-2}\frac{g_j}{m_j}\right)\\ g_n&=\frac{m_n^3}{m_n^2-m_{n-1}^2}\left(m_{n-1}^2\sum_{j=1}^{n-2}\frac{g_j}{m_j^3}-\sum_{j=1}^{n-2}\frac{g_j}{m_j}\right)\end{aligned}\right\}\text{for } V_{LS} \text{ and } V_T, \tag{9}$$

$$g_{n-2}=\frac{m_{n-2}^5}{(m_n^2-m_{n-2}^2)(m_{n-1}^2-m_{n-2}^2)}\left[-m_{n-1}^2m_n^2\sum_{j=1}^{n-3}\frac{g_j}{m_j^5}+(m_{n-1}^2+m_n^2)\sum_{j=1}^{n-3}\frac{g_j}{m_j^3}-\sum_{j=1}^{n-3}\frac{g_j}{m_j}\right] \text{ for } V_{SO2}$$

and two other relations deduced by circular permutation of $n-2$, $n-1$, and n.

PARAMETRIZATION IN MOMENTUM SPACE

By a Fourier transform of Eq. (1), one gets[5] in the center-of-mass system

$$V(\vec{p}_i,\vec{p}_f)=\tilde{V}_0(\vec{p}_i,\vec{p}_f)\tilde{\Omega}_0+\tilde{V}_1(\vec{p}_i,\vec{p}_f)\tilde{\Omega}_1+\tilde{V}_{LS}(\Delta^2)\tilde{\Omega}_{LS}+\tilde{V}_T(\Delta^2)\tilde{\Omega}_T+\tilde{V}_{SO2}(\Delta^2)\tilde{\Omega}_{SO2} \tag{10}$$

where

$$\begin{aligned} \tilde{\Omega}_0&=\frac{1-\vec{\sigma}_1\cdot\vec{\sigma}_2}{4},\\ \tilde{\Omega}_1&=\frac{3+\vec{\sigma}_1\cdot\vec{\sigma}_2}{4},\\ \tilde{\Omega}_{LS}&=\frac{i}{2}(\vec{\sigma}_1+\vec{\sigma}_2)\cdot\vec{p}_i\times\vec{p}_f,\\ \tilde{\Omega}_T&=\Delta^2\vec{\sigma}_1\cdot\vec{\sigma}_2-3\vec{\sigma}_1\cdot\vec{\Delta}\,\vec{\sigma}_2\cdot\vec{\Delta},\\ \tilde{\Omega}_{SO2}&=\vec{\sigma}_1\cdot(\vec{p}_i\times\vec{p}_f)\vec{\sigma}_2\cdot(\vec{p}_i\times\vec{p}_f), \end{aligned} \tag{11}$$

with $\vec{\Delta}=\vec{p}_f-\vec{p}_i$.

Here, the velocity dependent central components $\tilde{V}_0$ and $\tilde{V}_1$ are defined as

$$\tilde{V}(\vec{p}_i,\vec{p}_f)=\tilde{V}^a(\Delta^2)+\frac{\hbar^2}{m}(p_i^2+p_f^2)\tilde{V}^b(\Delta^2). \tag{12}$$

For each component $\tilde{V}(\Delta^2)$

$$\tilde{V}(\Delta^2)=\frac{1}{2\pi^2}\sum_{j=1}^{n}\frac{g_j}{m_j}\frac{1}{m_j^2+\Delta^2}f(m_j), \tag{13}$$

where

$$\begin{aligned} f(m_j)&=1 \text{ for } V_{0,1}^a \text{ and } V_{0,1}^b\\ f(m_j)&=\frac{1}{m_j^2} \text{ for } V_{LS} \text{ and } V_T\\ f(m_j)&=\frac{1}{m_j^4} \text{ for } V_{SO2} \text{ (disregarding additional logarithmic terms).} \end{aligned} \tag{14}$$

TABLE III. The calculated *pp* phase shifts (in degrees).

E_{lab} (MeV)	25	50	95	142	210	330
$^1S_0$	48.51	38.74	25.85	15.68	4.03	−11.80
$^1D_2$	0.75	1.81	3.72	5.56	7.66	9.37
$^1G_4$	0.04	0.17	0.43	0.71	1.14	1.93
$^3P_0$	8.87	11.82	10.35	6.31	−0.16	−10.79
$^3P_1$	−5.04	−8.41	−12.76	−16.47	−21.27	−28.90
$^3P_2$	2.44	5.73	10.66	14.06	16.49	16.44
ϵ_2	−0.85	−1.78	−2.67	−2.91	−2.71	−2.00
$^3F_2$	0.11	0.35	0.77	1.04	1.08	0.14
$^3F_3$	−0.24	−0.73	−1.53	−2.19	−2.93	−3.95
$^3F_4$	0.03	0.14	0.48	0.95	1.75	3.31
ϵ_4	−0.05	−0.21	−0.54	−0.86	−1.23	−1.64
$^3H_4$	0.01	0.03	0.11	0.21	0.37	0.59
$^3H_5$	−0.02	−0.09	−0.30	−0.55	−0.87	−1.31
$^3H_6$	0.00	0.01	0.05	0.13	0.27	0.59
ϵ_6	−0.00	−0.03	−0.12	−0.23	−0.40	−0.66
$^3K_6$	0.00	0.00	0.02	0.04	0.09	0.19

TABLE IV. The calculated $T=0$ phase shifts (in degrees).

E (MeV)	25	50	95	142	210	330
$^1P_1$	−7.11	−10.95	−15.33	−18.60	−22.21	−27.00
$^1F_3$	−0.44	−1.20	−2.31	−3.18	−4.19	−5.77
$^1H_5$	−0.03	−0.17	−0.52	−0.86	−1.27	−1.77
$^3S_1$	80.35	62.28	43.83	31.06	17.49	0.06
ϵ_1	1.69	1.89	2.10	2.50	3.35	5.27
$^3D_1$	−2.95	−6.77	−12.33	−16.61	−20.95	−25.32
$^3D_2$	3.96	9.60	17.91	23.51	27.52	28.25
$^3D_3$	0.04	0.29	1.17	2.30	3.70	4.75
ϵ_3	0.59	1.72	3.56	5.00	6.40	7.76
$^3G_3$	−0.06	−0.28	−0.95	−1.81	−3.10	−5.05
$^3G_4$	0.18	0.77	2.18	3.70	5.75	8.76
$^3G_5$	−0.01	−0.05	−0.16	−0.25	−0.29	−0.07
ϵ_5	0.04	0.22	0.72	1.28	2.04	3.14
$^3I_5$	−0.00	−0.02	−0.12	−0.28	−0.56	−1.11
$^3I_6$	0.01	0.10	0.41	0.86	1.56	2.80

FIT OF THE N-N PHASE SHIFTS AND DATA

The values of the parameters m_j and g_j are listed in Tables I and II, the missing ones (g_n for $V^a_{0,1}$ and $V^b_{0,1}$; g_n, g_{n-1} for V_T and V_{LS}; and g_n, g_{n-1}, g_{n-2} for V_{SO2}) were computed from Eqs. (9). The values shown in the tables were determined by a balanced fitting of the shapes of the potentials, the phase shifts, and the data themselves. Of course, departures from the simple "rounded steplike" model of Ref. (3) are unavoidable with a superposition of Yukawa-type terms even though they are regularized at $r=0$. Another origin of the departures is the use of a p^2 dependence for the central potential instead of an energy dependence as in Ref. (3). However, our requirement that the complete potential should reproduce the LR+MR theoretical part is fulfilled, as can be seen from Fig. 1. The agreement is up to a few percent beyond 1 fm or so, except for the isospin one spin-orbit and tensor potentials.

Preliminary results were reported at the Vancouver Conference on the N-N Interaction. Since then, several improvements have been achieved: (i) New results[6] for the S and P wave $N\bar{N}\rightarrow 2\pi$ amplitudes have been included in the LR+MR theoretical potential. The main change is the weakening of the tensor component and an increase of the spin-orbit and central ones; (ii) the effective ω vector coupling constant $g_\omega^2/4\pi$ has been changed from 9.5 to 11.75, the tensor/vector ratio being kept at -0.12. Another part of the 3π exchange represented by the A_1 is included with a coupling constant value $g_{A_1}^2/4\pi=14$. (iii) the determination of the core parameters is now performed by fitting not only the phase shifts but also the scattering data themselves. The fit was carried out via a two-step procedure, first the best fit of the MAW

TABLE V. The calculated deuteron and low energy parameters.

E_D = 2.2249	$Q_D = 0.279$
P_D = 0.0577	$\mu_D = 0.853$
D/S = 0.026 08	$r(-E_D, 0) = 1.765$
a_{np} = 5.427	$r(0,0) = 1.766$
a_{pp} = −7.810	$r_{pp} = 2.797$
a_{nn} = −17.612	$r_{nn} = 2.881$

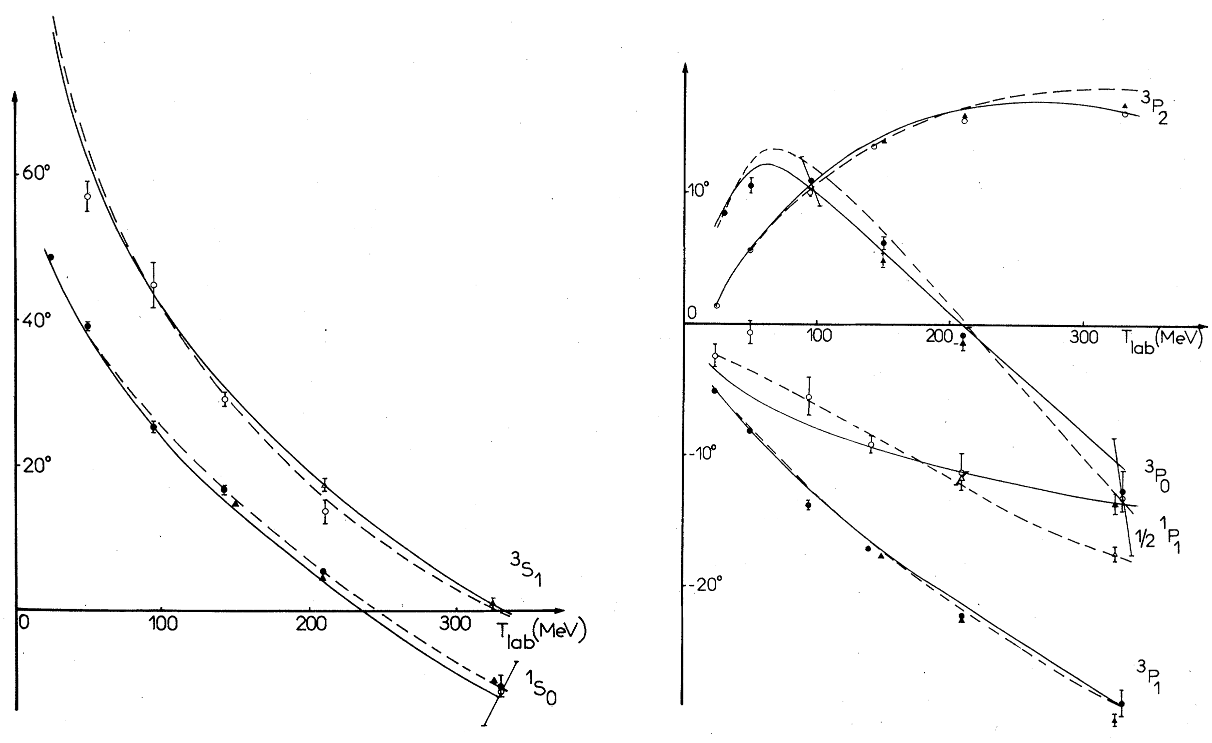

FIG. 2. The solid lines refer to the results of the present work, the dashed lines to the energy dependent phase shift analysis of Ref. (11), the circles and triangles to the energy independent phase shift analyses of Ref. (7) and Ref. (12), respectively.

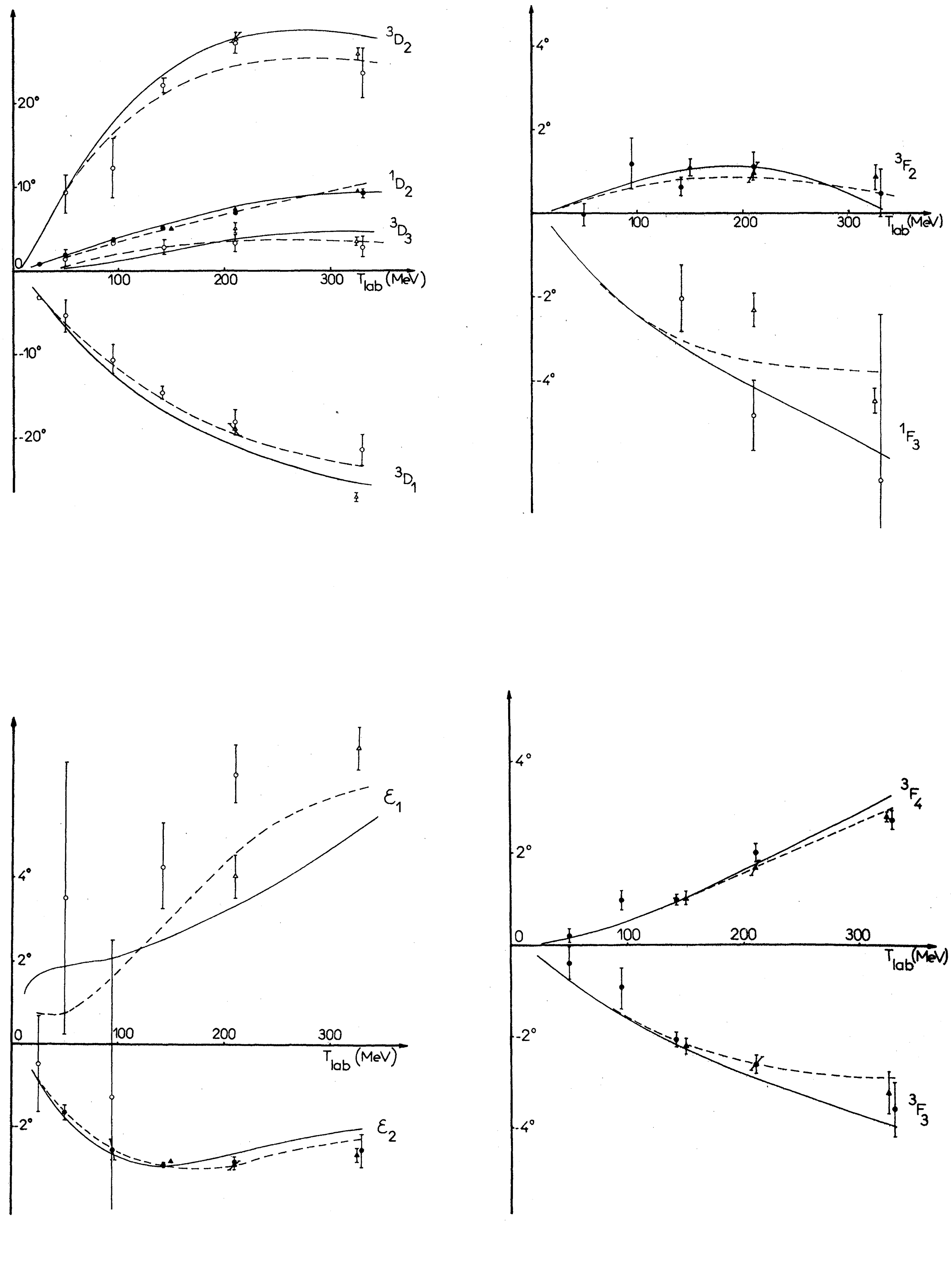

FIG. 2. (*Continued*).

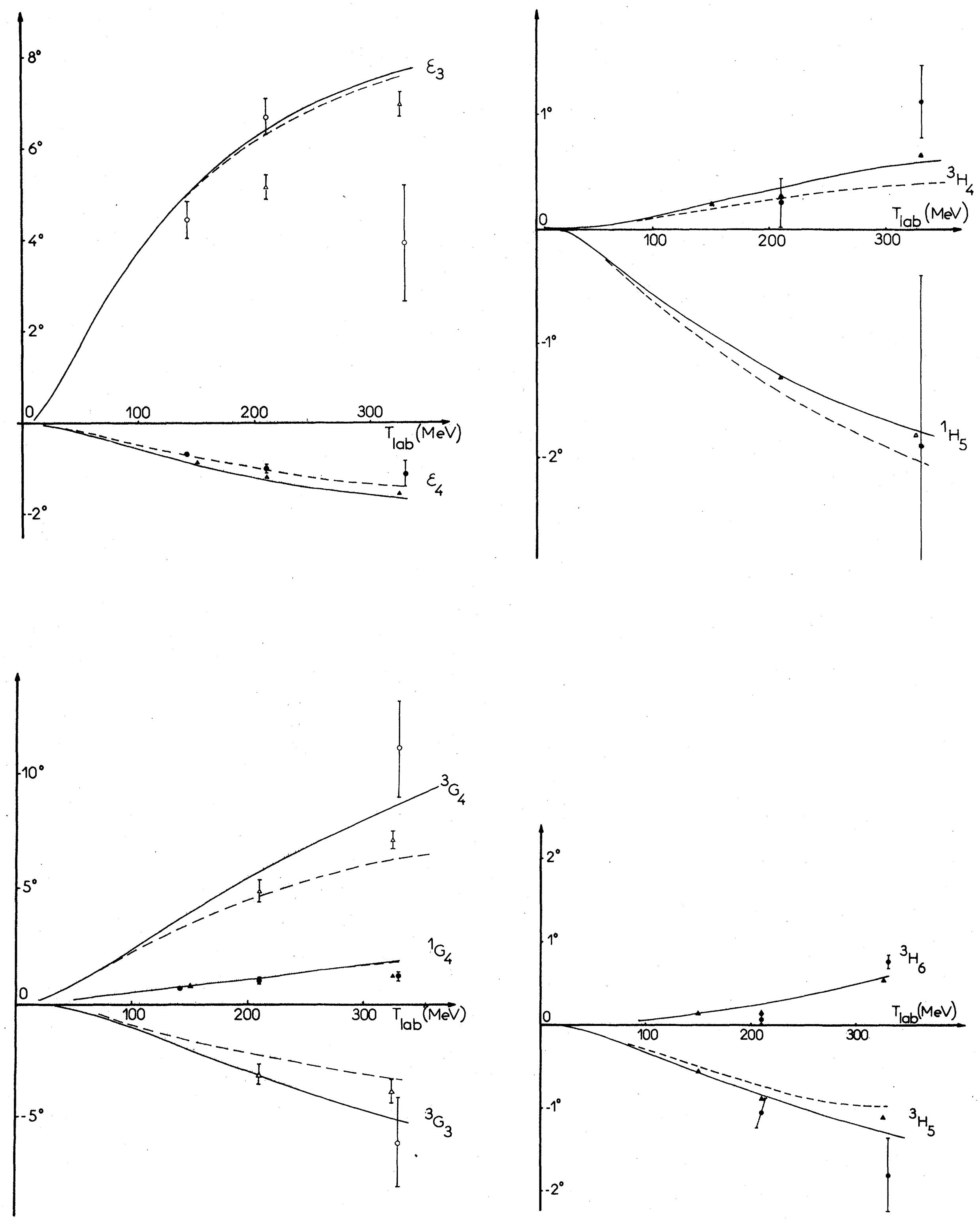

FIG. 2. (*Continued*).

TABLE VI. The deuteron wave functions. Values for smaller meshes can be obtained on request.

r (fm)	U	W	r	U	W
0.200 00 − 01	0.151 44 − 02	0.103 75 − 05	0.300 00 + 01	0.432 21 + 00	0.962 30 − 01
0.300 00 − 01	0.239 06 − 02	0.362 51 − 05	0.320 00 + 01	0.414 65 + 00	0.877 44 − 01
0.400 00 − 01	0.334 82 − 02	0.894 55 − 05	0.340 00 + 01	0.397 37 + 00	0.800 13 − 01
0.500 00 − 01	0.438 76 − 02	0.181 94 − 04	0.360 00 + 01	0.380 51 + 00	0.729 94 − 01
0.600 00 − 01	0.550 89 − 02	0.327 02 − 04	0.380 00 + 01	0.364 13 + 00	0.666 36 − 01
0.700 00 − 01	0.671 17 − 02	0.539 41 − 04	0.400 00 + 01	0.348 29 + 00	0.608 83 − 01
0.800 00 − 01	0.799 56 − 02	0.835 66 − 04	0.420 00 + 01	0.333 02 + 00	0.556 83 − 01
0.900 00 − 01	0.935 85 − 02	0.123 19 − 03	0.440 00 + 01	0.318 32 + 00	0.509 80 − 01
0.100 00 + 00	0.107 99 − 01	0.174 59 − 03	0.460 00 + 01	0.304 21 + 00	0.467 27 − 01
0.200 00 + 00	0.290 20 − 01	0.174 39 − 02	0.480 00 + 01	0.290 67 + 00	0.428 78 − 01
0.300 00 + 00	0.536 70 − 01	0.645 30 − 02	0.500 00 + 01	0.277 69 + 00	0.393 92 − 01
0.400 00 + 00	0.862 97 − 01	0.158 20 − 01	0.520 00 + 01	0.265 26 + 00	0.362 32 − 01
0.500 00 + 00	0.129 43 + 00	0.310 76 − 01	0.540 00 + 01	0.253 36 + 00	0.333 65 − 01
0.600 00 + 00	0.183 88 + 00	0.525 75 − 01	0.560 00 + 01	0.241 97 + 00	0.307 60 − 01
0.700 00 + 00	0.246 95 + 00	0.788 28 − 01	0.580 00 + 01	0.231 09 + 00	0.283 91 − 01
0.800 00 + 00	0.312 23 + 00	0.106 32 + 00	0.600 00 + 01	0.220 68 + 00	0.262 33 − 01
0.900 00 + 00	0.372 13 + 00	0.130 91 + 00	0.650 00 + 01	0.196 64 + 00	0.216 30 − 01
0.100 00 + 01	0.421 46 + 00	0.149 90 + 00	0.700 00 + 01	0.175 18 + 00	0.179 47 − 01
0.110 00 + 01	0.458 79 + 00	0.162 68 + 00	0.750 00 + 01	0.156 05 + 00	0.149 78 − 01
0.120 00 + 01	0.485 35 + 00	0.170 08 + 00	0.800 00 + 01	0.139 00 + 00	0.125 66 − 01
0.130 00 + 01	0.503 30 + 00	0.173 35 + 00	0.850 00 + 01	0.123 81 + 00	0.105 93 − 01
0.140 00 + 01	0.514 69 + 00	0.173 66 + 00	0.900 00 + 01	0.110 28 + 00	0.896 86 − 02
0.150 00 + 01	0.521 17 + 00	0.171 89 + 00	0.950 00 + 01	0.982 20 − 01	0.762 29 − 02
0.160 00 + 01	0.523 94 + 00	0.168 68 + 00	0.100 00 + 02	0.874 82 − 01	0.650 20 − 02
0.170 00 + 01	0.523 89 + 00	0.164 48 + 00	0.105 00 + 02	0.779 17 − 01	0.556 38 − 02
0.180 00 + 01	0.521 69 + 00	0.159 63 + 00	0.110 00 + 02	0.693 97 − 01	0.477 44 − 02
0.190 00 + 01	0.517 82 + 00	0.154 34 + 00	0.115 00 + 02	0.618 08 − 01	0.410 75 − 02
0.200 00 + 01	0.512 67 + 00	0.148 81 + 00	0.120 00 + 02	0.550 48 − 01	0.354 20 − 02
0.220 00 + 01	0.499 59 + 00	0.137 45 + 00	0.125 00 + 02	0.490 29 − 01	0.306 11 − 02
0.240 00 + 01	0.484 12 + 00	0.126 23 + 00	0.130 00 + 02	0.436 68 − 01	0.265 05 − 02
0.260 00 + 01	0.467 32 + 00	0.115 52 + 00	0.135 00 + 02	0.388 93 − 01	0.229 89 − 02
0.280 00 + 01	0.449 87 + 00	0.105 49 + 00	0.140 00 + 02	0.346 39 − 01	0.199 72 − 02

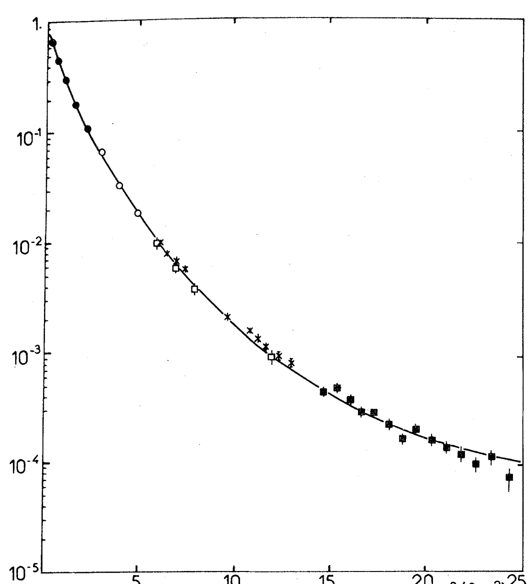

FIG. 3. The deuteron form factor $A(q^2)$. Experimental results are from Ref. (13).

phase shifts[7] was searched for and then the results were tuned up by fitting the data themselves. The set of data used consists of 913 data points with 60 normalizations for pp scattering in the energy range $3 < E_{\text{lab}} < 330$ MeV,[8] and by 2239 data points with 156 normalizations for np scattering in the energy range $13 < E_{\text{lab}} < 350$ MeV.[9] The χ^2 per degrees of freedom (degrees of freedom=number of data points minus number of normalizations) are, respectively, 1.99 for pp scattering and 2.17 for np scattering. The details of this analysis will be published elsewhere.[10] To our knowledge, this is one of the best, if not the best, χ^2 presently available. The results for the phase shifts and the deuteron parameters obtained by this fit are displayed in Tables III–V and Fig. 2. The deuteron wave function and form factor are shown in Table VI and Fig. 3.

NUCLEAR MATTER PROPERTIES

Since our fit of the two nucleon system parameters appears to be very satisfactory, it is interesting to test our potential for the other extreme situation presented by nuclear matter. The purpose is not, for the moment, to predict definite nuclear matter parameters in performing the most sophisticated calculations but rather to check whether the result obtained in the lowest order is sensible. We therefore calculated the binding energy in the lowest order Brueckner theory. This gives a minimum $E/A \sim -11.22$ MeV at the density $k_F \simeq 1.51\ \mathrm{fm}^{-1}$ with an average wound integral $K=0.11$, this latter value indicates the softness of our potential, implying a good convergence in many-body calculations.

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