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Static and dynamic deformation effects in the fusion cross section of light heavy ions at sub-barrier energies

M. S. Hussein
Instituto de Física, Universidade de São Paulo, S.P., Brasil

L. F. Canto and R. Donangelo
Instituto de Fisica, Universidade Federal do Rio de Janeiro, RJ, Brasil
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The static and dynamic deformation effects on the sub-barrier fusion cross section of light heavy ions are investigated by performing a coupled channel calculation for the system $^{12}C + ^{16}O$. It is found that dynamic effects are negligible whereas static effects could be important, and they appear to show up partly through absorption under the barrier.

[NUCLEAR REACTIONS Heavy-ion reactions; low-energy fusion cross section calculated.]

It is a common practice in sub-barrier light-heavy ion data analysis, particularly in the case of fusion, to utilize spherically symmetric potentials.[1] Although the quality of the fits obtained is often quite good, there are, however, several deformed systems, e.g., $^{12}C + ^{12}C$, $^{16}O + ^{12}C$, where this type of analysis has always presented difficulties.

It would seem natural to resort to a coupled channel calculation in order to fit the fusion data of deformed nuclei even at sub-barrier energies. Recently a measurement of the sub-barrier fusion cross section of systems with varying degrees of deformation was carried out.[2] It was found[2] that an increase of the deformation of the fusing system results in an increase in σ_{fusion}. This raises the question of how to mock up the effect of deformation in an effective one channel description of the fusion process. In the present work we discuss the static and dynamic effects that the deformed shape of nuclei produces in the fusion cross section at sub-barrier energies. As an example, we consider the light system $^{16}O + ^{12}C$.

The fusion process will depend on the relative orientation of the two nuclear surfaces as the corresponding densities start overlapping. In the case of the fusion of a spherical nucleus 1 with a deformed nucleus 2, the sum of the nuclear radii is given by

$$R(\theta)=r_0\{A_1^{1/3}+A_2^{1/3}[1+(5/4\pi)^{1/2}\beta_2P_2(\cos\theta)]\}, \tag{1}$$

where θ is the angle subtended between the symmetry axis of 2 and the line connecting the centers of 1 and 2, A_1 and A_2 are the mass numbers of the two nuclei, β_2 the quadrupole deformation parameter of 2, and P_2 the second order Legendre polynomial.

Considering the specific case of ^{16}O impinging on ^{12}C ($\beta_2=-0.47$), the largest and smallest spatial extentions of the fusing system are given by

$$R_>=R(\pi/2), \quad R_<=R(0), \tag{2}$$

leading to a relative change in the barrier height, as compared to the spherical case

$$\frac{\Delta E_b}{\bar{E}_b}=\frac{1/R_<-1/R_>}{1/\bar{R}}\sim 25\%. \tag{3}$$

The above result is a clear indication of possible static effects due to the deformation of ^{12}C effecting the sub-barrier fusion cross section.

Besides this static effect there might be dynamical effects due to coupling to inelastic channels. Consideration of the coupling to the 2^+ excited state in ^{12}C has been made by Imanishi[3] several years ago. The importance of this coupling in explaining several features of the fusion cross section of $^{12}C + ^{12}C$ and $^{16}O + ^{12}C$ was emphasized by this author.

In order to study both static and dynamic effects on the fusion cross section, we have performed coupled-channel calculations on the system $^{16}O + ^{12}C$ involving both the ground and the first excited ($E_{2^+}=4.43$ MeV) states of ^{12}C, using the code CHUCK.[4] The optical potential used was a deformed, surface-transparent, Woods-Saxon potential. The diffuseness, a_W, of the imaginary part of this potential was restricted to values satisfying the condition[5]

$$a_W\leq\frac{\hbar}{(8\mu E_b)^{1/2}}. \tag{4}$$

In Eq. (4) μ is the reduced mass of the fusing system and E_b the Coulomb barrier height.

These calculations were performed for the experimental value of the deformation parameter of ^{12}C, $\beta_2 = -0.47$,[6] as well as for $\beta_2 = 0.0$ and $\beta_2 = -1.27$, for the purpose of comparison and later discussion. Other parameters entering the calculation are given in the caption of Fig. 1, where the results are shown. As can be seen, increasing deformation implies an increase in the slope of the rising ratio $\sigma_F(\beta \neq 0)/\sigma_F(\beta = 0) \equiv F(\beta)$ as the center of mass energy is decreased.

In view of the fact that the inelastic cross section resulting from our coupled channel calculation was found to be quite small (10^{-3}–10^{-8} mb), such an increase in σ_F should be attributed exclusively to the static deformation effects. Such an increase in σ_{fusion} can be mocked up by changing the parameters of the absorptive potential or in a lowering and/or narrowing of the Coulomb barrier. The latter possibilities are connected to the choice of the real part of the optical potential. We argue below that of all the parameters of the optical potential which have to be changed to mock up the effects of static deformation, the diffuseness a_W seems to play a major role in giving rise to the behavior of $F(\beta)$ as a function of the center of mass energy depicted in Fig. 1.

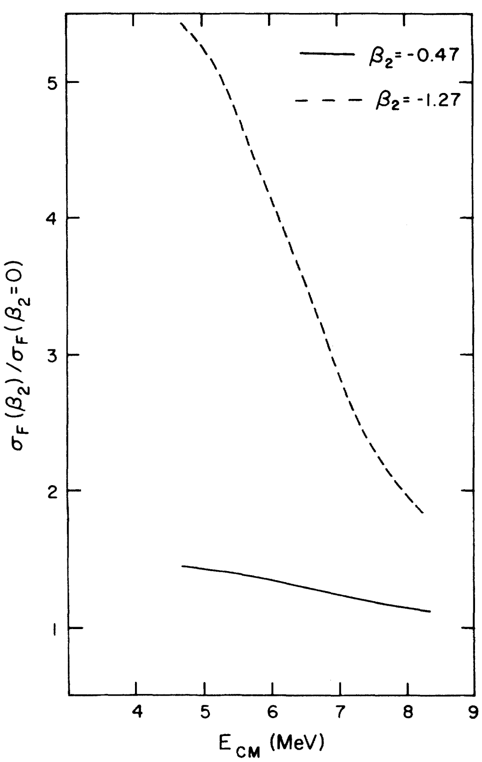

FIG. 1. The ratio $F(\beta) \equiv \sigma_F(\beta \neq 0)/\sigma_F(\beta = 0)$ plotted vs $E_{\text{c.m.}}$ for two values of β: -0.47 (full line) and -1.27 (dashed lines). The calculation was performed using the coupled channel (cc) code CHUCK. A deformed Woods-Saxon optical potential was used with the following values of its parameters: $V_0 = 50$ MeV, $a_R = 0.4$ fm, $r_{0,R} = 1.4$ fm, $W_0 = 8$ MeV, $a_W = 0.1$ fm, and $r_{0,W} = 1.4$ fm (see text for further details). Included in the cc calculation was the 2^+ excited state of ^{12}C ($E_{2^+} = 4.43$ MeV).

A natural way to take the effects of static deformation into account is to average the fusion cross section, calculated assuming a particular orientation of the symmetry axis of the deformed system, over all orientations, namely

$$\sigma_F(E) = \frac{1}{4\pi} \int d\Omega\, \sigma_F(E, \Omega), \tag{5}$$

where Ω corresponds to the Euler angles that specify the orientation of the symmetry axis of the deformed target relative to the line connecting the centers of the two ions. At the low energies we are considering, the dominant contribution to σ_F seems to come from the $l = 0$ transmission coefficient, $T_{l=0}$, and one may therefore calculate the above average using a WKB estimate of the assumed barrier penetration probability form for $T_{l=0}$:

$$\sigma_F(E, \Omega) \simeq \frac{\pi}{k^2} \exp\left[-\int_{a_i(\Omega)}^{a_0(\Omega)} 2\,\mathrm{Re}\,k(r, \Omega)\, dr\right], \tag{6}$$

where $k(r, \Omega)$ is the local complex Ω-dependent wave number given by

$$k(r, \Omega) = \left(\frac{2\mu}{\hbar^2}[V_{\text{opt}}(r, \Omega) - E]\right)^{1/2}, \tag{7}$$

with $V_{\text{opt}}(r, \Omega) = V(r, \Omega) + iW(r, \Omega)$ being the deformed optical potential and E the center of mass energy $= \hbar^2 k^2 / 2\mu$. The limits $a_i(\Omega)$ and $a_0(\Omega)$ are the inner and outer classical turning points respectively, given by the two solutions of the equation

$$k(r, \Omega) = 0\,. \tag{8}$$

Short of being able to actually perform the above average exactly, we could obtain a gross estimate of the amount by which the diffusivities a_R and a_W of V_{opt} are affected by static deformation by merely calculating the average of $V_{\text{opt}}(r, \Omega)$, namely,

$$\overline{V}_{\text{opt}}(r) = \frac{1}{4\pi} \int d\Omega\, V_{\text{opt}}(r, \Omega)\,. \tag{9}$$

In the present paper $V_{\text{opt}}(r, \Omega)$ is given by the deformed Woods-Saxon form

$$V_{\text{opt}}(r, \Omega) = -\frac{V_0}{1 + \exp\{[r - R(\theta)]/a_R\}} - \frac{iW_0}{1 + \exp\{[r - R(\theta)]/a_W\}}, \tag{10}$$

where $R(\theta)$ is given by Eq. (1).

In the limiting case where the diffuseness parameter is zero, the average potential $\overline{V}_{\text{opt}}(r)$

acquires an effective diffuseness:

$$a_{\rm eff} \simeq 1.2 A_2^{1/3} |\beta_2| \ (\text{fm}) . \tag{11}$$

If the diffuseness parameter is not zero, the averaging procedure indicated in Eq. (9) will also result in an increase in the diffusivity of the potential, although $a_{\rm eff}$ will not be given by such a simple expression as that of Eq. (11). To see this, we have evaluated $\bar{V}_{\rm opt}(r)$ of Eq. (9) for different values of the deformation parameter, β_2, and extracted an effective diffuseness $a_{\rm eff}(\beta_2)$. In Fig. 2 we present the result for $(a_W)_{\rm eff}(\beta_2)$ and $(a_R)_{\rm eff}(\beta_2)$. Specifically, we obtain $(a_W)_{\rm eff}(\beta_2 = -0.47) = 0.27$ fm and $(a_W)_{\rm eff}(\beta_2 = -1.27) = 0.665$ fm. These values are much smaller than those predicted from Eq. (11). It seems, therefore, that Eq. (11) is valid only for small values of β_2. As is clear from Fig. 2, $(a_R)_{\rm eff}$ exhibits a slower increase with β_2 than does $(a_W)_{\rm eff}$ owing to the larger value of $a_R (= 0.4$ fm) than that of $a_W (= 0.1$ fm) considered for $\beta_2 = 0$.

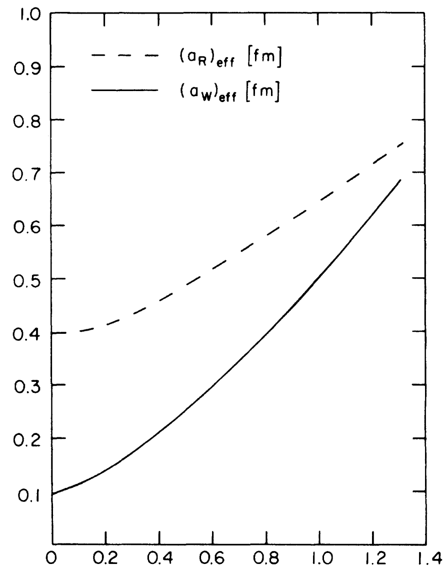

FIG. 2. The effective disfussivities $(a_R)_{\rm eff}$ and $(a_W)_{\rm eff}$ plotted vs β_2. The parameters of $V_{\rm opt}(r,\theta)$ used in the calculation of Eq. (9) are the same as those used in the cc calculation (see the caption of Fig. 1).

Turning now to a detailed discussion of the consequences of the increase in a_R and a_W due to static deformation, we first argue that the change in slope of the increasing $F(\beta)$ with decreasing center of mass energy is a clear indication of absorption under the barrier.[5,7] Insofar as the diffuseness of the imaginary part of the optical potential is chosen to be that of Eq. (4), it is guaranteed that no absorption under the barrier would result in $\sigma_F(\beta = 0)$. One also knows from Ref. 5 that for $a_W > \hbar/\sqrt{8\mu E_b}$ absorption under the barrier becomes important at energies lower than a certain critical energy, E_c, given by

$$E_c \cong E_b - \frac{\hbar^2}{2\mu}\left(\frac{1}{2a_W}\right)^2 . \tag{12}$$

The above observations suggest that the form of $F(\beta)$ at sub-barrier energies may be calculated following the procedure of Ref. 7:

$$F(\beta) = \left[1 + 2\frac{2\mu\,|W(r_0^+)|}{\hbar^2}\,\frac{1}{|k(r_0^+)|}\left(\frac{\pi}{[(d/dr)\,{\rm Re}\,k(r)]_{r_0^+}}\right)^{1/2} \exp\left(2\int_{a_i}^{r_0^+} {\rm Re}\,k(r')\,dr'\right)\right] \frac{T_{l=0}^{\rm BP}(\beta)}{T_{l=0}^{\rm BP}(\beta=0)} , \tag{13}$$

where the optical parameters entering in the above expression are the effective ones which contain, on the average, the effect of deformation. The quantity $T_{l=0}^{\rm BP}(\beta)$ is the $l = 0$ barrier penetration (BP) probability calculated for strong volume absorption (no absorption under the barrier) and with a real potential determined by the effective parameters. For $\beta = 0$, the quantity $F(0)$ is 1 since in this case no absorption under the barrier is present by assumption.

The radius r_0^+ is the position at which the integrand of the transmission coefficient that determines σ_F, has a maximum in the barrier region.[7] The equation that determines r_0^+ is

$$\frac{1}{a_W} = 2\,{\rm Re}\,k(r_0^+) \equiv 2\left(\frac{\mu}{\hbar^2}\right)^{1/2} [V(r_0^+) - E]^{1/2} \left\{1 + \left[1 + \left(\frac{W(r_0^{(+)})}{V(r_0^{(+)}) - E}\right)^2\right]^{1/2}\right\}^{1/2} . \tag{14}$$

Equation (12) is an approximation to Eq. (14) obtained by dropping the factor $W(r_0)/[V(r_0^+) - E]$ and by approximating $V(r_0^+) \simeq E_b$. The most important factor which determines the energy dependence of $F(\beta)$ is the exponential which tends to increase as the center of mass energy is lowered. This is clear from the fact that decreasing E amounts to an increase in $a_i - r_0^+$. If absorption under the barrier were not present, then the ratio $F(\beta)$ would change basically because of the change in the barrier penetration resulting from the effect of deformation on the real part of the optical potential. As a matter of fact, at energies near the barrier height one might expect $F(\beta)$ to be larger than unity primarily because of the reduction in the height, as well as the shift in the position of the maximum, of the barrier. It is this effective reduction in the height of the barrier which seems to be responsible for the increase in $F(\beta=-1.27)$ down to an energy $E\sim7.4$ MeV with an almost constant slope. This same mechanism seems to hold in the case $\beta=-0.47$ throughout the energy range 8.5–5.2 MeV within which we were able to calculate σ_{fusion} with the coupled channel code. The rather sudden change in slope of $F(\beta=-1.27)$ at $E\sim7.4$ MeV is an indication of the onset of absorption under the barrier. A similar effect should also occur in $F(\beta=-0.47)$, but at much lower energy. Unfortunately, the code CHUCK does not function well once the coupled 2^+ channel becomes a closed one and therefore we could not trust our results at $E<5$ MeV.

We now use Eq. (12) to obtain an estimate of $(a_W)_{\text{eff}}$. In the case $\beta=-1.27$, we take for E_c the value 7.4 MeV. This gives $(a_W)_{\text{eff}}=0.83$ fm which is close to the value 0.665 fm obtained from Eq. (9). As was discussed previously, the change in slope of $F(\beta=-0.47)$ occurs at much smaller energies, owing to the small value of $(a_W)_{\text{eff}}$ for this case.

It should be clear that our discussion above was based on the $l=0$ transmission coefficient which gives the dominant contribution to σ_{fus}. Considering the contributions from the few $l\neq0$ transmission coefficients would not change the picture given above.

We hope to have demonstrated in this work how static deformation affects sub-barrier heavy ion fusion cross section. Our conclusion is that static deformation results mainly in two effects; the initial rise in $F(\beta)$ due to the overall lowering of the barrier height and then, at a lower energy, the rather abrupt change in the slope of $F(\beta)$ due to absorption under the barrier. We are presently testing our findings on the data of Ref. 2, in which case the dynamic effects are expected to be important owing to the larger probability for exciting the 2^+ state in the deformed Sm nuclei. However, the Coulomb polarization potential derived recently in Refs. 8 and 9 and extended to include the reorientation of the 2^+ exactly in Ref. 10 should account rather well for the dynamic effect in a one-channel optical model description of the sub-barrier fusion process.

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