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3^{-}-2^{+} first-forbidden beta transitions in the decay of $^{72}_{31}Ga_{41}$

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The energy dependence of the beta-gamma angular correlation concerning the outer and inner first-forbidden (3^{-}-2^{+}) beta transitions was measured with a fast-slow scintillation assembly. The present results were combined with the other available observables to determine the beta matrix elements, employing the formalism of Bühring. It is shown that the higher-order matrix elements are important in the analysis. From the sizes of matrix elements, it is concluded that a possible domination of $\int iB_{ij}/\rho$ matrix element and cancellation among the vector matrix elements explain the failure of the Coulomb approximation in describing the two (3^{-}-2^{+}) beta transitions in ^{72}Ga. The conserved vector current ratio ($=D'y/x$) following the present analysis shows a deviation from Fujita's prediction and is consistent with the theory of Damgaard and Winther. A comparison of the features of the two beta transitions reveals similar structure for the levels 2_1^+ and 2_2^+ in the daughter ^{72}Ga nucleus.

[RADIOACTIVITY ^{72}Ga; measured $\beta-\gamma(\theta)$, deduced nuclear matrix elements.]

I. INTRODUCTION

The beta decay modes[1] of ^{72}Ga which are now well established are shown in Fig. 1. The (3^{-}-2^{+}) outer and inner first-forbidden beta transitions are of present interest and are indicated in the decay scheme by bold lines. The outer beta transition with an end-point energy of 3150 keV was experimentally[2-6] well investigated. The nuclear matrix elements (β moments) which govern this transition were, however, extracted employing the beta decay formalism of Morita and Morita[7] who assumed that the electron radial wave function (ERWF) was constant throughout the nuclear volume. The beta decay theory of Bühring,[8] on the other hand, is formulated in a rigorous fashion in that (i) the ERWF is not constant inside the nucleus and expanded in powers of (r/ρ), (ii) the third-forbidden nuclear matrix elements (NME) are included, and (iii) NME due to screening effects also are taken care of.

Simms and Schweitzer[9] have shown that the formalism of Morita and Morita and that of Bühring lead to the same results provided the zeroth order matrix element parameters x and u (in the notation of Kotani[10]) and the higher-order matrix element parameters x' and u' are approximately of the same magnitude as shown in the case of ^{198}Au. On the other hand, it was clearly illustrated that the expressions of Bühring must necessarily be employed[11] for the first-forbidden β decay in ^{140}La for which the ratio $x'/x(=\lambda)$ is about 2.45. *A priori,* it thus becomes necessary to use the general formulas of Bühring. Such an attempt is made in the present work to deduce the beta moments governing the 3150 keV first-forbidden outer beta transition and compare them with those calculated[3,4] on the basis of the formulas of Morita and Morita. The present analysis has yielded information about the higher-order matrix elements which will be of use in nuclear structure studies and in checking the predictions about the conserved vector current (CVC) ratio.[12,13] We have measured the energy dependence of the angular correlation $\epsilon(W)$ of the 3150 keV (β_1)–834 keV (γ_1) cascade (Fig. 1) with our experimental setup and found that the values are in good agreement with the previously reported ones.

The inner beta transition with an end-point energy of 2530 keV which follows the same (3^{-}-2^{+}) spin-parity sequence has a high logft value of 8.5. Its shape is reported to be nonstatistical[5] in nature. There was only one angular correlation measurement earlier by Grenacs *et al.*[2] on the 3^{-}(2530 keV β_2)2^{+}(1464 keV γ_2)0^{+} cascade which indicates a large β-γ anisotropy. These experimental features are seen to be common to both the beta transitions under consideration. The even-even daughter states with 2^{+} character which are populated by the beta decay of the odd-odd parent state were investigated in several nuclei[14-16] earlier, based on a study of the beta decay observables. The odd-odd $^{72}_{31}Ga_{41}$ provides another example to learn about the 2^{+} states in the even-even daughter ^{72}Ge. No attempt seems to have been made for extracting the nuclear matrix elements governing the 2530 keV beta transition. In the present work the energy dependence of the 2530 keV (β_2) and the following 1464 keV (γ_2) cascade was measured employing our fast-slow scintillation system. The angular correlation results of this work were combined with the other

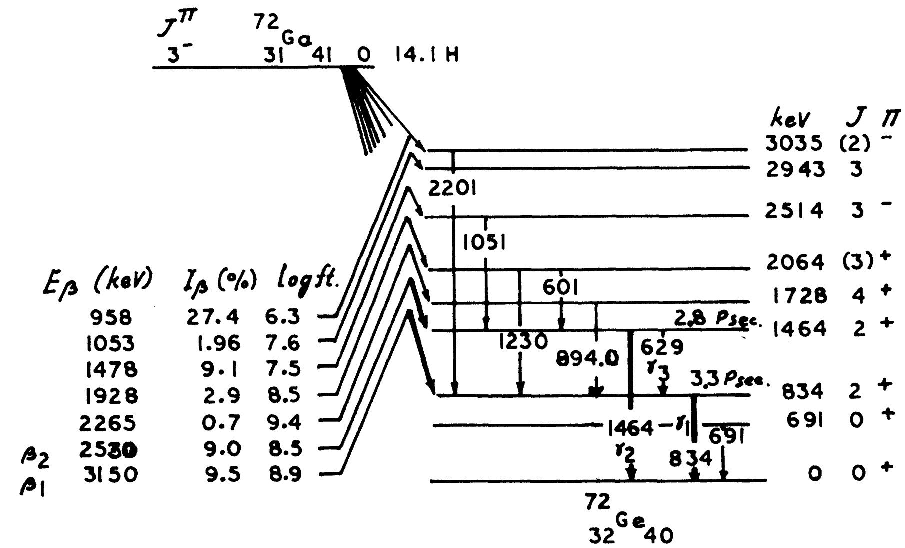

FIG. 1. Partial decay scheme of the ground state decay of ^{72}Ga as taken from Ref. 1. The β-γ cascades of present interest are shown by bold lines.

observables (logft value and spectrum shape reported by earlier workers) to extract the beta matrix elements using the formulas of Bühring.[8] The CVC predictions due to different authors are checked. We have made a comparison of all the available observables including the present ones on the outer and inner (3^--2^+) beta transitions in ^{72}Ga to infer the structure of the 2_1^+ and 2_2^+ levels in the daughter ^{72}Ge nucleus.

II. EXPERIMENT

The fast-slow scintillation assembly used in the present measurements was described in our earlier papers.[17-19] In the same references, one finds details about the standardization of the setup, source preparation, data collection, analysis of the experimental data, and various corrections to be applied to arrive at the final values of the β-γ correlation coefficients at different beta energies.

An enriched sample of ^{71}Ga was obtained from Oak Ridge National Laboratories in oxide form and was irradiated at a flux of 10^{13} neutrons/sec/cm^2 in the CIRUS reactor at Bhabha Atomic Research Centre, Bombay, India. Thin film sources were prepared and correlation experiments were conducted in the usual manner. Figure 2 summarizes the angular correlation results on $\beta_1 - \gamma_1$ (outer) cascade in their final form while Fig. 3 presents those on the $\beta_2 - \gamma_2$ (inner) cascade. The angular correlation function $\epsilon(W)$ shown in Figs. 2 and 3 is the product $A_2(\beta)A_2(\gamma)$.

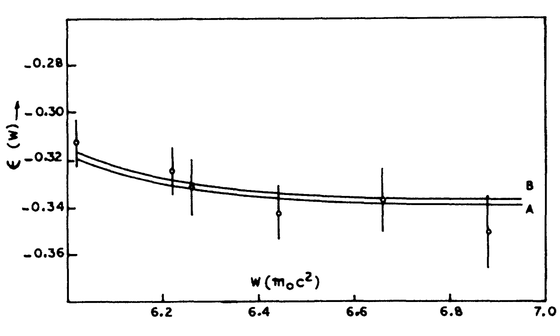

FIG. 2. Energy dependence of the 3150 keV (β_1)-834 keV (γ_1) directional correlation function $\epsilon(W) = A_2(W)/A_0(W)$. The spikes are the experimental points. The smooth curves represent the theoretical predictions of $\epsilon(W)$ for the matrix element parameter sets A and B given in Table II.

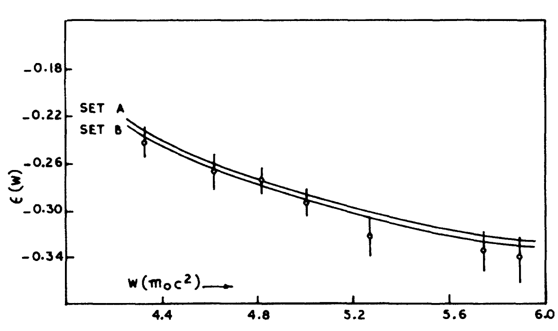

FIG. 3. Energy dependence of the 2530 keV (β_2)-1464 keV (γ_2) directional correlation function from the present investigation. The smooth curves represent the $\epsilon(W)$s corresponding to the typical NME parameter sets (A and B) given in Table V.

III. EXTRACTION OF NUCLEAR MATRIX ELEMENTS

Simms[20] has arranged the formulas of Bühring in a fashion convenient to determine the first-forbidden beta matrix elements from a knowledge of the β observables. The details are given in a series of papers.[11,20-23] We have adopted the same notation regarding the definition of NME parameters and followed the same procedure in the present analysis. Some of the details were given in our earlier papers, also.[24,25]

A. The (3^--2^+) 3150 keV beta transition

Only four zeroth order matrix elements belonging to the tensor ranks 1 and 2 contribute to the beta transition with unit spin change and a change in parity. The NME parameters as defined by Smith and Simms[11] are given as

$$x_0=\frac{x+\frac{4}{5}ax'}{1+\frac{4}{5}a}, \quad y_0=\frac{y+ay'}{1+a}, \qquad u_0=\frac{u+\frac{4}{5}au'}{1+\frac{4}{5}a}, \quad z_0=\frac{z+\frac{4}{5}az'}{1+\frac{4}{5}a}, \tag{1}$$

$$DY=D'y_0=\left[\frac{D(1+d\lambda)(x+u)}{(1+a)}\right], \tag{2}$$

where

$$a=-\tfrac{1}{6}\left[(W\rho+\tfrac{3}{2}\alpha Z)^2-\rho^2\right]. \tag{3}$$

$D(\approx|D'|)$ is used in the same manner as ξ in Ref. 10.

D and D' are used to distinguish between the nonrelativistic and relativistic matrix elements, respectively,

$$D=\tfrac{1}{2}\alpha Z+W_0\rho, \tag{4}$$

$$d=-\left(\frac{1}{5D}\right)\left[\tfrac{1}{2}\alpha Z-a(3D+2\hat{q})\right], \tag{5}$$

$$\lambda=x'/x, \tag{6}$$

$$\hat{q}=\tfrac{1}{3}(q\rho), \tag{7}$$

where q is the neutrino momentum, W is the beta energy, W_0 is the end-point energy, α is the fine structure constant, and ρ is the nuclear radius of the daughter nucleus. x, u, etc., are the zeroth order NME parameters while x_0, u_0, etc., contain the higher-order parameters (x', u', etc.) also, which arise out of the expansion of the ERWF. The contributions of the third-forbidden matrix element parameters r', s', and t' are reduced by a factor of $1/DD'$ (Ref. 11) (≈ 50 for the present nucleus) relative to the first-forbidden parameters. Thus, in order that r', s', and t' contribute appreciably to the matrix element combinations that occur in the formulas for beta decay observables, they must be approximately a factor of 50 times larger than the largest of the first-forbidden parameters. The maximum value of any matrix element cannot exceed $\sqrt{2}$, the magnitude of the super-allowed matrix element. Hence any forbidden matrix element parameter cannot be larger than $\sqrt{2}/\eta$, where η is the scaling factor determined[11] from a knowledge of the $\log ft$ value and particle parameters $b_{kk'}$. Even if one assumes the maximum value of $\sqrt{2}/\eta_{\rm min}$ for the third-forbidden NME parameters, the ratio of this to the largest of the first-forbidden NME parameters must be as large as 50 in order that the third-forbidden parameters be important in the analysis. To start with, it was assumed that there is no mechanism which would enhance the sizes of the third-forbidden parameters relative to the first-forbidden ones to such an extent. However, once η is determined in the analysis, the validity of this assumption can be checked. The matrix elements that arise out of screening effects are important only at low beta energies and so are not taken into consideration in the present analysis.

There are four unknowns, namely, Y (a linear combination of x_0, u_0, and y_0), x_0, u_0, and z_0 to be determined from the four known experimental observables $\log ft$, $\epsilon(W)$, $C(W)$, and $P_\gamma(\theta)$. This number reduces to three when expressed relative to z_0, which in turn could be determined from a knowledge of the ft value. Thus, it is seen that the situation is quite favorable to determine NME experimentally for the 3150 keV beta transition in ^{72}Ga.

A computer search program called "MATCAL" was developed at the Laboratories for Nuclear Research, Andhra University, to perform the NME computations. First, a coarse search fixed the approximate ranges of NME and then a fine search yielded the exact limits between which NME solutions exist. The parameters of the ERWFs were computed employing Bhalla and Rose[26] tables. The computer was fed with the following data for the extraction of NME parameters:

(1) $\epsilon(W)$ of this work (Fig. 2),

(2) $C(W)$ normalized to the value at $6.44m_0c^2$ units from Ref. 6,

(3) $\beta_1-\gamma_1$ circular polarization results (at $6.26m_0c^2$ units) from Ref. 4.

The computer prints out only those solutions which satisfy all the above experimental data simultaneously, together with the corresponding χ^2 values. The ranges of the values of NME parameters obtained from the present analysis are given below:

TABLE I. NME parameters and the absolute values (in natural units) of the NME together with the single particle estimates for the 3150 keV beta transition in ^{72}Ga corresponding to $\lambda = 2.5$ to 2.8, $\rho = 0.0129$, $D = 0.1477$, $a = -0.0271$, and $d = -0.1756$.

Different authors	$z_0(=z)$	x	u	$D'y_0(D'y)$	Y	η	$\Lambda_{CVC}^{expt} = (D'y_0/x)$
Present authors (Bühring formalism)	1.0	Region I					
		-0.23 ± 0.07	-0.18 ± 0.13	-0.03 ± 0.005	0.16 ± 0.02	0.07 ± 0.006	0.09 ± 0.02
		Region II					
		0.83 ± 0.10	-0.21 ± 0.26	0.08 ± 0.05	0.18 ± 0.04	0.05 ± 0.01	0.11 ± 0.04
Newsome and Fischbeck (formalism of Morita and Morita) Ref. 3	1.0	(i) -0.1	0.0	...	0.85	...	...
		(ii) 0.9	0.3	...	2.0	...	...
Camp *et al.* (formalism of Morita and Morita) Ref. 4	1.0	(i) -0.126	-0.174	...	1.8	...	...
		(ii) 0.55	-0.235	...	3.16	...	...
		$\left\lvert \int \frac{iB_{ij}}{\rho} \right\rvert$	$\left\lvert \int \frac{i\vec{r}}{\rho} \right\rvert$	$\left\lvert \int \frac{i\vec{\sigma} \times \vec{r}}{\rho} \right\rvert$	$\left\lvert \int \vec{\alpha} \right\rvert$		
Absolute values of NME of present authors		Region I					
		0.064 ± 0.003	0.017 ± 0.006	0.016 ± 0.008	0.002 ± 0.001		
		Region II					
		0.043 ± 0.007	0.042 ± 0.003	0.012 ± 0.012	0.004 ± 0.001		
Single particle estimates, Ref. 28		1.0	1.0	1.0	0.1		

Region I	Region II
$-0.42 \leq x_0 \leq -0.05$,	$0.7 \leq x_0 \leq 0.95$,
$-0.5 \leq u_0 \leq -0.06$,	$-0.45 \leq u_0 \leq 0.10$,
$0.05 \leq Y \leq 0.2$,	$0.1 \leq Y \leq 0.4$.
and $z_0 = 1.0$,	

1. The CVC ratio and higher-order matrix elements

The CVC prediction of Fujita[12] is given by

$$\Lambda_{\text{CVC}}^0 = 2.4\xi\rho + (W_0 - 2.5)\rho \,, \tag{8}$$

where $\xi (= \alpha Z/2\rho)$ is the Coulomb energy and W_0 is the end-point energy. The CVC prediction as modified by Damgaard and Winther[13] (DW) is given by

$$\Lambda_{\text{CVC}}^{\text{DW}} = \int \vec{\alpha} \Big/ \int \frac{i\vec{r}}{\rho} = \Lambda_{\text{CVC}}^0 + \tfrac{1}{2}\alpha Z (0.6 - \lambda) \,, \tag{9}$$

where

$$\lambda = x'/x = \int \frac{i\vec{r}}{\rho}\left(\frac{r}{\rho}\right)^2 \Big/ \int \frac{i\vec{r}}{\rho} \,,$$

and if $\lambda = 0.6$, $\Lambda_{\text{CVC}}^{\text{DW}} = \Lambda_{\text{CVC}}^0$ and

$$\int \vec{\alpha} \Big/ \int \frac{i\vec{r}}{\rho} = D'y/x \tag{10}$$

(in the present notation).

The connection between $\Lambda_{\text{CVC}}^{\text{expt}}$ and x_0, u_0, and Y can be deduced as

$$\Lambda_{\text{CVC}}^{\text{expt}} = \left[\frac{DY}{x_0(1+0.8a)} + \frac{D(x_0+u_0)}{x_0(1+a)}\right] + \left[\frac{DY(0.8a)}{x_0(1+0.8a)} + \frac{D(x_0+u_0)d}{x_0(1+a)}\right]\lambda \tag{11}$$

from Eqs. (1) and (2). By equating $\Lambda_{\text{CVC}}^{\text{DW}}$ [Eq. (9)] and $\Lambda_{\text{CVC}}^{\text{expt}}$ [Eq. (11)] one gets the λ value for each set of solutions x_0, u_0, z_0, and Y. This was a part of the computer program and the range of λ values thus obtained corresponding to the experimentally obtained sets of NME parameters is given in Table I. A knowledge of λ helps the breakup of x and x' in x_0 for each solution. Since x and u have the same radial dependence one can reasonably take $x'/x \approx u'/u = \lambda$. Hence the separation of u and u' in u_0 is also possible. $D'y_0$ can be determined using the relationship (Eq. 2). The ranges of x, u, Y, and $D'y_0$ thus obtained are summarized in Table I. In the same table the numerical values of the different constants ρ, a, d, and D are given. For a comparison, the values of NME parameters of the earlier authors are also included in the same table, who used the theory of Morita and Morita.

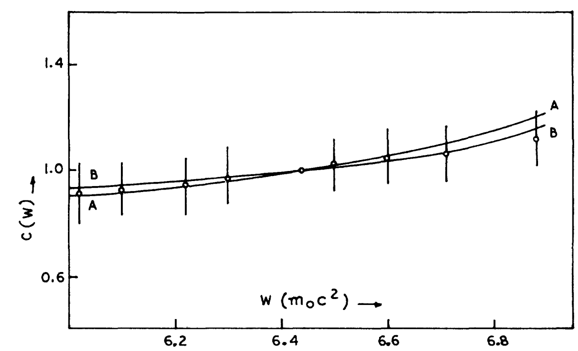

FIG. 4. Shape correction factor $C(W)$ vs W (in m_0c^2 units) as taken from Ref. 6 for the 3150 keV beta (β_1) transition along with the theoretical predictions due to the sets (A and B) given in Table II.

$\Lambda_{\text{CVC}}^{\text{expt}} (= D'y/x)$ was obtained by equating $D'y_0$ to $D'y$. The error introduced in this approximation is only a few percent (~3%) as Y_0 and y differ by an additional term $\simeq ay$. ("a" for the present nucleus is -0.02.) The values of $\Lambda_{\text{CVC}}^{\text{expt}}$ are also included in Table I. The limits on the errors of NME parameters were set based on the χ^2 values. Three typical sets A, B, and C were chosen from the ranges of NME parameters given in Table I and are furnished in Table II. Figures 2, 4, and 5 show the theoretically predicted functions of $\epsilon(W)$, $C(W)$, and $P_\gamma(\theta)$ corresponding to these sets along with the experimental values. It may be seen from these figures that the experimental observables are not sensitive to small variations in NME parameters.

$\Lambda_{\text{CVC}}^{\text{expt}}$ [Eq. (11)] is shown as a function of λ in Fig. 6 for the limiting values of $\Lambda_{\text{CVC}}^{\text{expt}}$ given by set C and set A (Table II). The variation of $\Lambda_{\text{CVC}}^{\text{th}}$ due

TABLE II. Typical matrix element parameters sets for the 3150 keV beta (β_1) transition in ^{72}Ga obtained from regions I and II given in Table I.

Set	z_0	x	u	Y	$D'y_0$	λ	$\Lambda_{\text{CVC}}^{\text{expt}}$	η
A (region I)	1.0	0.782	−0.365	0.2	0.061	2.848	0.078	0.055
B (region II)	1.0	−0.295	−0.415	0.18	−0.032	2.603	0.108	0.076
C (region I)	1.0	0.932	0.0	0.18	0.102	2.558	0.109	0.046

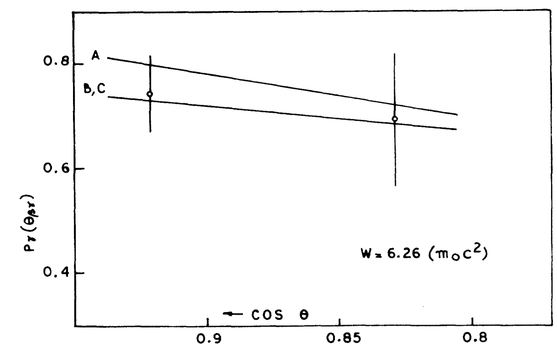

FIG. 5. Angular dependence of the β_1-γ_1 circular polarization as taken from Ref. 4. The straight line represents the theoretical prediction corresponding to the sets given in Table II.

to Damgaard and Winther [Eq. (9)] is also shown as a function of λ in the same figure. The points of intersection of Λ_{CVC}^{th} and Λ_{CVC}^{expt} plots give the limiting values of λ as $\lambda_{min} = 2.54$ and $\lambda_{max} = 2.84$. These values are consistent with the theoretical limits given in Table II.

2. *Scaling factor η and the evaluation of absolute values of NME*

The procedure suggested in Ref. 11 was adopted to obtain η from a knowledge of the ft value, x_0, u_0, and Y. The range of η values thus obtained is given in Table I. The ranges of the absolute values of the matrix elements corresponding to the NME parameters given in Table I are also included in the same table. The absolute values of matrix elements for the sets given in Table II are reported in Table III. For the purpose of evaluation of $\int iB_{ij}/\rho$ (the rank-2 matrix element), z_0 is taken as z just as $D'y_0$ is taken as Dy since there is no way of separating y and y' in y_0 and z

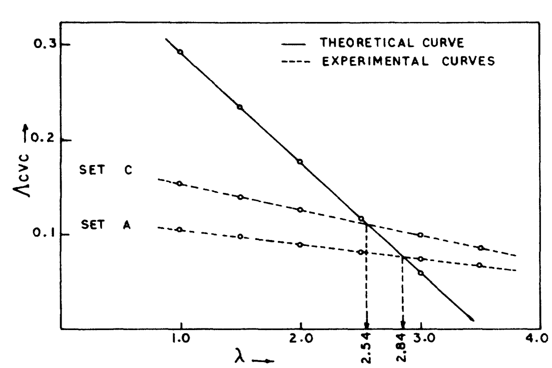

FIG. 6. CVC ratio (Λ_{CVC}) vs $\lambda (= x'/x)$ for the β_1 transition. Λ_{CVC}^{expt} are due to the two limiting sets of solutions A and B given in Table II for NME parameters. Λ_{CVC}^{th} are from Eq. (9) and Λ_{CVC}^{expt} are from Eq. (11).

TABLE III. Absolute values of nuclear matrix elements for the typical sets given in Table II.

Set	$\lvert\eta\rvert$	$\left\lvert\int \frac{iB_{ij}}{\rho}\right\rvert$	$\left\lvert\int \frac{i\vec{r}}{\rho}\right\rvert$	$\left\lvert\int \frac{i\vec{\sigma}\times\vec{r}}{\rho}\right\rvert$	$\left\lvert\int \vec{\alpha}\right\rvert$
A	0.055	0.046	0.043	0.017	0.003
B	0.076	0.063	0.022	0.026	0.002
C	0.046	0.038	0.043	0.0	0.005

and z' in z_0. The error thus introduced is only about 3%. The smallest value of η from the present analysis is 0.049. The maximum value of any third-forbidden matrix element parameter could be $\sqrt{2}/0.049$ (≈ 28) which is only about 28 times as large as the largest of the first-forbidden NME parameters given in Table I. The third-forbidden NME must be at least 50 times larger than the largest of the first-forbidden one in order to be of consequence in the analysis as noted earlier. Thus, the neglect of the third-forbidden matrix elements in the present analysis could be justified.

3. *Discussion*

From Table I, it is seen that there is a significant difference between the present values of Y and those obtained by the earlier authors because of the different theoretical approaches in the analysis. Y is important in that it helps in the estimation of CVC ratio.

The values of Λ_{CVC} from this analysis vary between 0.074 and 0.148 as against the theoretical prediction of 0.34 of Fujita. The present values agree with the theory proposed by Damgaard and Winther for λ lying between 2.54 and 2.84. From the values of $\lambda (= x'/x)$ one recognizes the necessity of using the theory of Bühring in the analysis of 3150 keV first-forbidden beta transition in ^{72}Ga. In the present situation, the reason for the deviation of Λ_{CVC} from Fujita's estimate can be ascribed to the small value of $\int i\vec{r}/\rho (= \eta x)$ as seen from its values given in Table III. Such a situation has been encountered in nuclei ^{140}La,[11] ^{124}Sb,[23] and ^{122}Sb.[27]

In $^{72}_{31}$Ga$_{41}$, both of the transforming nucleons lie within the same major shell ($28 \leq Z \leq 50$). If no configuration mixing is involved, the odd spin and odd parity of the beta decaying parent state is probably due to the coupling of the 41st neutron in $1g_{9/2}$ orbital and the 31st proton in $2p_{3/2}$ orbital. The final beta decaying state may be a configuration mixture of three states $2p_{3/2}$, $1f_{5/2}$, and $2p_{1/2}$. The rank-2 matrix element will play a major role in governing this transition if the final proton

TABLE IV. NME parameters and the absolute values of NME for the 2530 keV beta transition in ^{72}Ga corresponding to $\lambda = 2.8$ to 3.2.

NME parameters	Absolute values of NME
$z_0 = 1.0$	
$x = 0.35 \pm 0.16$	$\left\lvert \int \frac{iB_{ij}}{\rho} \right\rvert = 0.132 \pm 0.011$
$u = 0.21 \pm 0.19$	$\left\lvert \int \frac{i\vec{r}}{\rho} \right\rvert = 0.055 \pm 0.025$
$D'y_0 = 0.02 \pm 0.01$	$\left\lvert \int \frac{i\vec{\sigma}\times\vec{r}}{\rho} \right\rvert = 0.029 \pm 0.016$
$Y = 0.0 \pm 0.03$	$\left\lvert \int \vec{\alpha} \right\rvert = 0.002 \pm 0.001$
$\eta = 0.16 \pm 0.01$	
$\Lambda_{CVC}^{expt} = \left(\frac{D'y_0}{x}\right) = 0.05 \pm 0.03$	

state is $1f_{5/2}$. Single particle estimates[28] for the matrix elements $\int i\vec{r}/\rho$, $\int i\vec{\sigma}\times\vec{r}/\rho$, and $\int iB_{ij}/\rho$ will be of the order of unity while relativistic matrix element $\int \vec{\alpha}$ is about 0.1 as given in Table I. In the extreme case of modified B_{ij} approximation, $\int iB_{ij}/\rho$ is $\simeq 0.2$ while the other matrix elements vanish. A comparison of the theoretical and experimental values in Table I shows that $\int i\vec{r}/\rho$, $\int i\vec{\sigma}\times\vec{r}/\rho$, and $\int iB_{ij}/\rho$ are, on the average, suppressed by a factor of 50 while $\int \vec{\alpha}$ by a factor of about 30. If one considers the "modified B_{ij} approximation," the experimental value is reduced only by a factor of 4. This suggests a domination of B_{ij} matrix element. It can be further clarified only when the limits set on the NME are narrowed down. It will, perhaps, be possible when additional experimental observables, namely, longitudinal polarization of electrons, angular distribution of gamma rays, as well as electrons from oriented nuclei are available. The small value of Y indicates a cancellation of rank-1 ma-

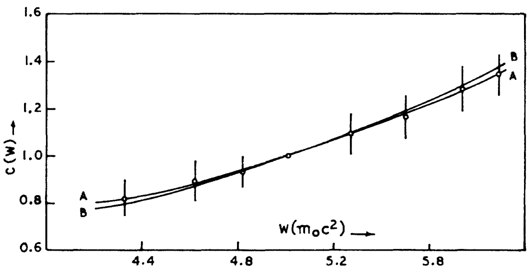

FIG. 7. Experimental shape correction factor $C(W)$ vs W for β_2 transition as taken from Ref. 5. The theoretical predictions for the typical NME sets (A and B) given in Table V are shown by the smooth curves.

trix elements which could be a reason for the large $\log ft$, large $\beta-\gamma$ anisotropy and large spectrum shape correction factor, all of which suggest the failure of the Coulomb approximation in describing the present 3150 keV beta transition in ^{72}Ga.

It may be concluded from the present work on the 3150 keV beta transition in ^{72}Ga that (i) the formulas of Bühring have yielded different results from those of Morita and Morita as seen in the values of x' and u', (ii) the small magnitudes of $\int i\vec{r}/\rho (=\eta x)$ explain the deviation of CVC ratio (Λ_{CVC}) from Fujita's prediction, and (iii) a cancellation in the rank-1 matrix elements and a possible domination of rank-2 matrix element $\int iB_{ij}/\rho$ are the causes for the failure of the Coulomb approximation in describing this transition.

B. Inner 2530 keV ($3^- - 2^+$) beta transition

1. Analysis and results

The analysis was performed in an identical manner as for the outer beta transition. The following data were used to extract the matrix element parameters:

(i) the beta-gamma angular correlation $\epsilon(W)$ from the present work and

(ii) the beta spectrum shape of Langer from Ref. 5.

The circular polarization data were not available for the inner $\beta_2-\gamma_2$ cascade. The ranges of NME parameters obtained from the fine search are given

TABLE V. Typical sets of matrix element parameters for the 2530 keV inner beta (β_2) transition in ^{72}Ga.

Set	Y	x	u	z_0	λ	η	$D'y_0$	Λ_{CVC}^{expt}
A	−0.010	0.23	−0.104	1.0	3.141	0.170	0.007	0.028
B	0.020	0.44	−0.314	1.0	3.159	0.141	0.011	0.025
C	0.001	0.48	−0.050	1.0	2.839	0.149	0.029	0.061
D	0.001	0.32	−0.217	1.0	3.207	0.159	0.004	0.021

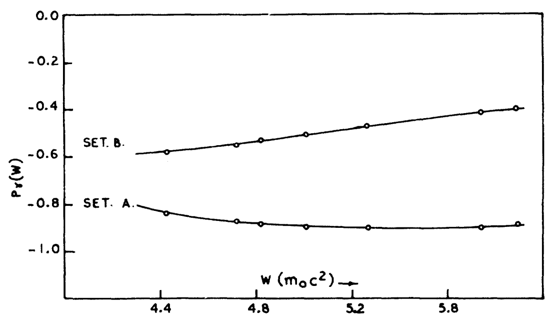

FIG. 8. Theoretical prediction for the energy dependence of the circular polarization of γ_2 following β_2 $[P_\gamma(W)]$ at $\theta = 180°$ corresponding to the sets (A and B) given in Table V.

as

$$x_0 = 0 \text{ to } 0.5, \quad u_0 = -0.2 \text{ to } 0.02,$$

$$Y = -0.02 \text{ to } +0.02, \quad z_0 = 1.0.$$

The values of $\lambda(=x'/x \approx u'/u)$, x, u, and $\Lambda_{\mathrm{CVC}}^{\mathrm{expt}}$ were determined as described earlier and are given in Table IV along with the other parameters. Four typical sets labeled as A, B, C, and D are given in Table V. The theoretical functions corresponding to sets A and B for $\epsilon(W)$ and $C(W)$ are shown in Figs. 3 and 7, respectively. These figures show a good fit to the experimental data. The energy dependence of beta-gamma circular polarization was predicted at $\theta = 180°$ as a part of the program and is shown in Fig. 8. This parameter will be useful when experiments on circular polarization of the gamma rays following the inner beta-decay are attempted.

The variation of $\Lambda_{\mathrm{CVC}}^{\mathrm{expt}}$ and $\Lambda_{\mathrm{CVC}}^{\mathrm{th}}$ with λ are shown in Fig. 9. $\Lambda_{\mathrm{CVC}}^{\mathrm{expt}}$ in this figure corresponds to the limiting sets selected from those given in Table V. The values of λ emphasize the need to use

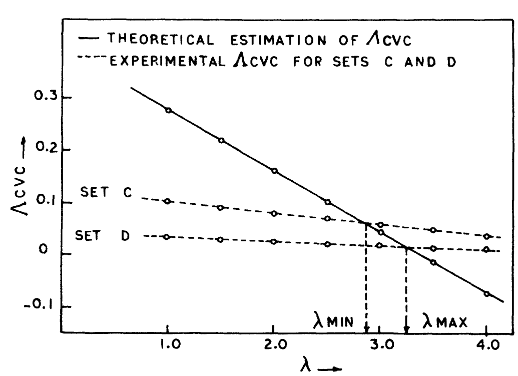

FIG. 9. Λ_{CVC} vs $\lambda(=x'/x)$ plot. $\Lambda_{\mathrm{CVC}}^{\mathrm{expt}}$ are for the sets C and D given in Table V and from Eq. (11). $\Lambda_{\mathrm{CVC}}^{\mathrm{th}}$ are from Eq. (9).

TABLE VI. Absolute values (in natural units) of the NME for the typical (Table V) sets governing the 2530 keV inner beta (β_2) transition in ^{72}Ga.

Set	η	$\left\lvert\int \frac{iB_{ij}}{\rho}\right\rvert$	$\left\lvert\int \frac{i\vec{r}}{\rho}\right\rvert$	$\left\lvert\int \frac{i\vec{\sigma}\times\vec{r}}{\rho}\right\rvert$	$\left\lvert\int \vec{\alpha}\right\rvert$
A	0.170	0.142	0.039	0.015	0.001
B	0.142	0.118	0.062	0.037	0.002
C	0.149	0.124	0.071	0.006	0.004
D	0.159	0.132	0.051	0.029	0.001

the formalism of Bühring in obtaining matrix elements that govern the 2530 keV beta transition.

The scaling factor η was estimated to be 0.159 $\pm$0.013. The largest value of the third-forbidden NME parameters that contributes to the beta transition could be $\sqrt{2}/\eta_{\min}$ ($\approx$10). It is seen from Table IV that the ratio of the third-forbidden to the largest of the first-forbidden matrix element parameters is only 10, even in the extreme case, whereas it should be at least 50 in order that the third-forbidden matrix elements are to be important. Thus, the neglect of third-forbidden nuclear matrix elements is justifiable. The ranges for the absolute values of NME (η times the NME parameters) were evaluated and are given in Table IV. For the four typical sets A, B, C, and D the absolute values are given in Table VI.

2. *Discussion*

The $\Lambda_{\mathrm{CVC}}^{\mathrm{expt}}$ values (Table IV) show disagreement with the prediction of Fujita. This is attributed to the large contribution from the higher-order matrix elements x' (x' ranging from 2.8 to 3.25 times of x) and the small values of $\int i\vec{r}/\rho$. This is in agreement with the observations of Damgaard and Winther.

It is seen from Table IV that the magnitudes of the rank-1 matrix elements, namely, $\int i\vec{r}/\rho$, $\int i\vec{\sigma}\times\vec{r}/\rho$, and $\int\vec{\alpha}$ are much smaller than $\int iB_{ij}/\rho$ (rank-2 matrix element). This again suggests a domination of $\int iB_{ij}/\rho$ matrix element in describing the 2530 keV beta transition in the decay of ^{72}Ga. In this situation, the failure of the Coulomb approximation in describing the 2530 keV beta transition is obvious. The small value of Y indicates a cancellation of rank-1 matrix elements and is consistent with the deviation from Coulomb approximation. The B_{ij} domination can be understood when the transforming nucleon is in the $1g_{9/2}$ orbital and a predominant contribution to the second 2^+ state is from a proton in the $1f_{5/2}$ orbital which results in $\Delta J = 2$ with a change in parity.

For a ready comparison of the inner and outer first-forbidden beta transitions from the ground

TABLE VII. Comparison of the outer and the inner beta transitions in ^{72}Ga.

Ser. no.	Feature	β_1 decay from ^{72}Ga to first 2^+ state in ^{72}Ge	β_2 decay from ^{72}Ga to second 2^+ state in ^{72}Ge
1.	Log ft	8.9	8.5
2.	Beta spectrum shape	Large deviation from statistical shape (mod. B_{ij})	Nearly B_{ij} shape
3.	Beta-gamma anisotropy	Large (≈35%)	Large (≈40%)
4.	Beta-gamma polarization	Varies with angle	...
5.	Beta-gamma reduced correlation coefficient	Energy independent within experimental errors (equal to about −0.052)	Energy independent within experimental errors (equal to about −0.061)
6.	Domination of B_{ij}	Present	Present
7.	Cancellation of vector matrix elements	Possible	Possible
8.	$\lambda\,(=x'/x \approx u'/u)$	2.54 to 2.84	2.87 to 3.25
9.	$\Lambda_{\mathrm{CVC}}^{\mathrm{expt}}$	0.090 ± 0.02	0.05 ± 0.03

state decay of ^{72}Ga feeding the first and second 2^+ states in the daughter ^{72}Ge nucleus, the various features are summarized in Table VII. An examination of this table shows that the outer and inner beta transitions are nearly identical in nature, thus suggesting a nearly identical structure for the 2_1^+ and 2_2^+ states in the ^{72}Ge nucleus. The energy levels of second $0^+(0_2^+)$, second $2^+(2_2^+)$, and 4^+ states in ^{72}Ge (Fig. 1) are not located at the right places either for a vibrational or for a rotational description of the nucleus. In this mass region of even-even Ge and Se nuclei, the low-lying levels were described by assuming the coexistence[29,30] of spherical and nearly deformed shapes corresponding to a vibrational band built on the 0_1^+ level and a rotational band built on the 0_2^+ level, respectively. A mixing of these levels results in an energy level pattern of the type of ^{72}Ge nucleus. It will be interesting to make a theoretical attempt in this direction to obtain the first-forbidden beta matrix elements also.

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