PHYSICAL REVIEW C VOLUME 21, NUMBER 2 FEBRUARY 1980

Inelastic pion scattering from ^{18}O

T.-S. H. Lee and R. D. Lawson
Argonne National Laboratory, Argonne, Illinois 60439
(Received 28 September 1979)

The inelastic scattering of 164 MeV pions from ^{18}O is studied using the distorted-wave impulse approximation in momentum space. The sensitivity of the predictions to various shell model descriptions of the yrast 2^+ state is examined in detail. The excitation of the triplet of states (4_1^+, 0_2^+, 2_2^+) near 3.7 MeV and the (3_1^-, 0_3^+, 2_3^+) triplet at about 5.2 MeV are also investigated within the context of the shell model. In order to explain the scattering to natural parity states [those with angular momentum J and parity $(-1)^J$], one must enhance the theoretical results by about the same factor as needed to explain the $B(EJ;\, 0\to J)$ transition rates obtained from electromagnetic excitation. The sensitivity of the results to the isoscalar or isovector character of this enhancement is examined. Agreement between distorted-wave impulse approximation calculations and experiment is found to be satisfactory.

[NUCLEAR REACTIONS, NUCLEAR STRUCTURE (π,π') $E_\pi \approx 164$ MeV; ^{18}O target, theoretical $d\sigma/d\Omega$ based on DWIA in momentum space; effects of different shell model descriptions studied.]

I. INTRODUCTION

In this paper we present theoretical calculations for the inelastic scattering of 164 MeV pions on ^{18}O. The pion-nucleus inelastic scattering cross section is calculated using the distorted-wave impulse approximation (DWIA) in momentum space.[1,2] Compared with the usual Kisslinger[3] or Laplacian models,[4] the main feature of our approach is to include exactly the relativistic nonlocal effects in the distorted-wave integration

$$\left\langle \chi_f^{(-)} \right| \sum_{k=1}^{A} t_k \left| \chi_i^{(+)} \right\rangle ,$$

where $\chi_\alpha^{(\pm)}$ are distorted waves generated from an appropriate optical potential U_0, and t_k is a finite range model[5] of the pion-nucleon t matrix. In Sec. II we shall briefly discuss how this optical potential is generated and review those aspects of the DWIA needed in this paper.

Inelastic scattering data in which the 1.98 MeV yrast 2^+ level in ^{18}O is populated now exist for both π^+ and π^- projectiles.[6-8] In addition to this, data also exist concerning the population of states at about 3.7 and 5 MeV excitation energy in this nucleus.[9] Because of the energy resolution now available, the lower energy excitation corresponds to populating the yrast 4^+ state at 3.55 MeV, the first excited 0^+ level at 3.63 MeV and the second 2^+ at 3.92 MeV. The 5 MeV excitation would involve the yrast 3^- at 5.09 MeV, the third 2^+ at 5.25 MeV, and the third 0^+ at 5.33 MeV. We shall describe all states within the context of the shell model and when this is done, it is well known that in order to obtain agreement with experiment for gamma-ray lifetimes and $B(E\lambda)$ values one must introduce an enhancement factor (effective charge) to take into account neglected configurations. In Sec. III we shall discuss how these enhancement factors are to be incorporated into the calculation of the pion scattering cross section.

We shall use several different models to describe the states in ^{18}O and these will be discussed in Sec. IV. Also in that section we shall explore the sensitivity of pion scattering to the various models, report the results for isoscalar and isovector enhancements, and compare our calculated cross sections with experiment. Finally in Sec. V we shall summarize and discuss our findings.

II. DWIA FORMALISM

Our distorted-wave impulse approximation approach in momentum space has been discussed previously[1,2] and we shall only recall the necessary equations to establish the notation that will be needed in our subsequent presentation. In the partial-wave representation the amplitude for pion-nucleus inelastic scattering is $[\hat{L} = (2L+1)]$

$$T_{fi}(\vec{k}_0'\Lambda',\vec{k}_0\Lambda)=\sum_{LML'M'JT}(\hat{L}\hat{L}'\hat{T}\hat{J}_f)^{1/2}(-1)^{J_f-M_f+M}\begin{pmatrix}J & J_i & J_f\\ M_f-M_i & M_i & -M_f\end{pmatrix}\begin{pmatrix}L & L' & J\\ -M & M' & M_i-M_f\end{pmatrix}$$
$$\times(-1)^{1+\Lambda}\begin{pmatrix}1 & 1 & T\\ \Lambda & -\Lambda' & \Lambda_i-\Lambda_f\end{pmatrix}Y^*_{L'M'}(\hat{k}_0')Y_{LM}(\hat{k}_0)$$
$$\times\int_0^\infty k_1^2dk_1\int_0^\infty k_2^2dk_2\chi^{(-)*}_{L'\Lambda'k_0'}(k_1)U^{fiJT}_{L'L}(k_1,k_2)\chi^{(+)}_{L\Lambda k_0}(k_2)\,. \quad (1)$$

In this equation the $\vec{k}$'s are the pion-nucleus relative momenta in the pion-nucleus center-of-mass frame, $\hat{k}_0'$, $\hat{k}_0$ are unit vectors in the direction of the outgoing and incoming pions respectively, $\Lambda(\Lambda')$ denote the initial and final pion isospins, the nuclear states i and f are specified by the spin and isospin quantum numbers $|J_iM_iT_i\Lambda_i\rangle$ and $|J_fM_fT_f\Lambda_f\rangle$, and the parentheses are $3j$ coefficients.

The distorted waves $\chi^{(\pm)}_{L\Lambda k_0}(k)$ in Eq. (1) are obtained from the solutions $\chi^{(\pm)}_{\Lambda\vec{k}_0}(\vec{k})$ of the relativistic scattering equation[1]

$$\chi^{(\pm)}_{\Lambda\vec{k}_0}(\vec{k})=\sum_{LM}\chi^{(\pm)}_{L\Lambda k_0}(k)Y^*_{LM}(\hat{k})Y_{LM}(\hat{k}_0)$$

with

$$\chi^{(\pm)}_{\Lambda\vec{k}_0}(\vec{k})$$
$$=|\Lambda\vec{k}_0\rangle+\left[\frac{1}{E-E_\pi(\vec{k})-E_A(\vec{k})\pm i\epsilon}\right]U_{00}(E)|\chi_{\Lambda\vec{k}_0}(\vec{k})\rangle\,. \quad (2)$$

The optical potential $U_{00}(E)$ is given by

$$U_{00}(E)=t(E)\rho(r)\,,$$

where $t(E)$ is related to the basic pion-nucleon collision matrix and is evaluated by using the program PIPIT,[10] and the ground state nuclear density is assumed to have a Woods-Saxon form

$$\rho(r)=\frac{\rho_0}{1+\exp[(r-R)/a]}\,.$$

Following the approach of Ref. 11, the parameters R and a are adjusted to fit the elastic scattering cross section, and when $R=2.2$ fm and $a=0.57$ fm the fit to the elastic pion-^{18}O data is shown in Fig. 1. With these values of R and a the rms radius of the density is 2.718 fm, and this is quite close to the electron scattering value of 2.789 fm obtained with an harmonic oscillator form factor.[12] The distorted waves obtained in this way are then used directly in the DWIA calculations of the inelastic scattering cross section.

The transition potential $U^{fiJT}_{L'L}(k_1,k_2)$ of Eq. (1) can be written as

$$U^{fiJT}_{L'L}(k_1,k_2)=\sum_{l'lKS}I^{fiJT}_{l'lKS}(k_1,k_2)H^{L'LJT}_{l'lKS}(k_1,k_2)\,, \quad (3)$$

where the function $H^{L'LJT}_{l'lKS}(k_1,k_2)$ is determined by only the pion-nucleon dynamics

$$H^{L'LJT}_{l'lKS}(k_1,k_2)=i^{l'-l}(\hat{K})^{1/2}\hat{l}\hat{l}'\begin{pmatrix}l & l' & K\\ 0 & 0 & 0\end{pmatrix}\sum_\lambda(\hat{\lambda})^{3/2}(-1)^{\lambda-J}\begin{pmatrix}L' & l' & \lambda\\ 0 & 0 & 0\end{pmatrix}\begin{pmatrix}L & l & \lambda\\ 0 & 0 & 0\end{pmatrix}\begin{Bmatrix}L & L' & J\\ \lambda & \lambda & S\\ l & l' & K\end{Bmatrix}$$
$$\times\sum_I\hat{I}(-1)^{I-1/2}\begin{Bmatrix}1 & 1 & T\\ \frac{1}{2} & \frac{1}{2} & I\end{Bmatrix}t^{\lambda I}_S(k_1,k_2,W_0)\,, \quad (4)$$

where the small curly bracket denotes a $6j$ coefficient and the large curly bracket is a $9j$ coefficient. $t^{\lambda I}_S(k_1,k_2,W_0)$ is the λth partial-wave pion-nucleus amplitude for isospin $I=\frac{1}{2}$ or $\frac{3}{2}$ with $S=0$ and 1 denoting the spin independent and spin dependent parts respectively. This amplitude is generated from PIPIT using the finite range model of Londergan, McVoy, and Moniz.[5]

All the nuclear information is included in the function $I^{fiJT}_{l'lKS}(k_1,k_2)$ which is given by

$$I^{fiJT}_{l'lKS}(k_1,k_2)=\int_0^\infty j_{l'}(k_1r)F^{fiJT}_{KS}(r)j_l(k_2r)r^2dr\,, \quad (5)$$

where the $j_l(kr)$ are spherical Bessel functions and

$$F_{KS}^{fiJT}(r)=(-1)^{T_f-\Lambda_f}(\hat{T}_f)^{1/2}\begin{pmatrix} T & T_i & T_f\\ \Lambda_f-\Lambda_i & \Lambda_i & -\Lambda_f\end{pmatrix}\sum_{\alpha\beta}\langle J_fT_f|||[b_\alpha^\dagger\times h_\beta^\dagger]_{JT}|||J_iT_i\rangle(4\pi\hat{j}_\alpha)^{1/2}\langle\alpha||[Y_K(\hat{r})\times\sigma_S]_J||\beta\rangle \times R_{n_\alpha l_\alpha}(r)R_{n_\beta l_\beta}(r)\,. \qquad (6)$$

$\langle\alpha||\ ||\beta\rangle$ is the reduced matrix element as defined by Brink and Satchler.[13] $\langle J_fT_f|||\ \ |||J_iT_i\rangle$ means the reduced matrix element in both space and isospin, $\sigma_S=1$ when $S=0$, $\sigma_S=\vec{\sigma}$ when $S=1$, the operators $b_\alpha^\dagger$ and $h_\alpha^\dagger$ create a particle and hole respectively in the shell model state $(nlj)_\alpha$ and $[\times]_{JT}$ implies coupling to angular momentum J and isospin T.

The reduced matrix element $\langle J_fT_f|||[b_\alpha^\dagger\times h_\beta^\dagger]_{JT}|||J_iT_i\rangle$ contains all the nuclear structure information and in terms of fractional parentage coefficients has the form

$$\langle J_fT_f|||[b_\alpha^\dagger\times h_\beta^\dagger]_{JT}|||J_iT_i\rangle=n\sum_{J_3T_3\gamma}(-1)^{j_\beta+1/2+J_3+J_f-J+T_3+T_f-T}\times[\hat{J}_i\hat{T}_i\hat{J}\hat{T}]^{1/2}\begin{Bmatrix} J_i & J_f & J\\ j_\alpha & j_\beta & J_3\end{Bmatrix}\begin{Bmatrix} T_i & T_f & T\\ \frac{1}{2} & \frac{1}{2} & T_3\end{Bmatrix}\langle(n-1)J_3T_3\gamma,j_\alpha\tfrac{1}{2}|\}nJ_fT_f\rangle \times\langle(n-1)J_3T_3\gamma,j_\beta\tfrac{1}{2}|\}nJ_iT_i\rangle\,, \qquad (7)$$

where $\langle(n-1)J_3T_3\gamma,j_\alpha\frac{1}{2}|\}nJ_fT_f\rangle$ is the fractional parentage coefficient for the n-valence-particle state (J_fT_f) and γ stands for any additional quantum numbers (other than spin and isospin) needed to completely specify the $(n-1)$-particle state (J_3T_3).

From Eqs. (1), (4), (6), and (7) it is clear that the isospin dependence of the pion-nucleus scattering amplitude is

$$Q=[\hat{T}_i\hat{T}_f]^{1/2}\sum_{ITT_3}(-1)^{I+T_3-T-\Lambda'+\Lambda_i}\hat{I}\hat{T}\begin{pmatrix} T & T_i & T_f\\ \Lambda_f-\Lambda_i & \Lambda_i & -\Lambda_f\end{pmatrix}\begin{pmatrix} 1 & 1 & T\\ \Lambda & -\Lambda' & \Lambda_i-\Lambda_f\end{pmatrix}\begin{Bmatrix} 1 & 1 & T\\ \frac{1}{2} & \frac{1}{2} & I\end{Bmatrix}\begin{Bmatrix} T_i & T_f & T\\ \frac{1}{2} & \frac{1}{2} & T_3\end{Bmatrix} \times t_S^{\lambda I}(k_1,k_2,W_0)\,\langle(n-1)J_3T_3\gamma,j_\alpha\tfrac{1}{2}|\}nJ_fT_f\rangle\langle(n-1)J_3T_3\gamma,j_\beta\tfrac{1}{2}|\}nJ_iT_i\rangle\,. \qquad (8)$$

For energies near the pion-nucleon resonance the $I=\frac{3}{2}$ term in the sum is dominant. In this limit one can use Eq. (8) to deduce some interesting selection rules or cross section ratios. In particular if $\Lambda_i=\Lambda_f$ and the n-nucleon states of interest have isospin $T_i=T_f=n/2$ the only value of T_3 that can come into the sum is $T_3=T_i-\frac{1}{2}$ and Eq. (8) becomes

$$Q=-\tfrac{1}{3}\langle(n-1)J_3T_i-\tfrac{1}{2}\gamma,j_\alpha\tfrac{1}{2}|\}nJ_fT_i\rangle\langle(n-1)J_3T_i-\tfrac{1}{2}\gamma,j_\beta\tfrac{1}{2}|\}nJ_iT_i\rangle t_S^{\lambda 3/2}(k_1,k_2,W_0)(2+\Lambda_i\Lambda/T_i)\,,$$

where $\Lambda=+1,\ 0,\ -1$ for π^-,π^0,π^+ respectively.

Thus if one describes ^{18}O as two neutrons outside the doubly closed ^{16}O core, $\Lambda_i=T_i=1$, and, furthermore, if one neglects the difference in distortion between the π^+ and π^- incident waves one arrives at the result

$$\frac{\sigma(\pi^-,{}^{18}\mathrm{O}(J_f))}{\sigma(\pi^+,{}^{18}\mathrm{O}(J_f))}=9/1\,,$$

where J_f is the spin of the final state in ^{18}O. Experimentally for the 2_1^+ state this ratio is closer to 2/1 which provides clear evidence for core excitation in ^{18}O.

III. ENHANCEMENT FACTORS

In any shell model calculation one is forced to make a severe truncation of the basis states in order to make the calculation tractable. Because of this one must deal with effective operators when calculating nuclear properties. For example, in order to understand electric multipole gamma-ray transitions one has to endow the nucleon with an effective charge. Thus instead of dealing with the bare operator $(\delta_p=\delta_n=0)$

$$T_{K\mu}=\sum_i\left\{\frac{e(1+\delta_p)}{2}[1-\tau_z(i)]+\frac{e\delta_n}{2}[1+\tau_z(i)]\right\}r_i^KY_{K\mu}(\hat{r}_i)$$
$$=\sum_i\left\{\frac{e(1+\delta_p+\delta_n)}{2}-\frac{e(1+\delta_p-\delta_n)}{2}\tau_z(i)\right\}r_i^KY_{K\mu}(\hat{r}_i)\,, \qquad (9)$$

where e is the proton charge, one must take δ_p and δ_n both different from zero. It is clear by comparing

Eq. (6) to Eq. (9) that when $\sigma_S = 1$ pion scattering depends on the same kind of operator as that which governs electric multipole gamma decay. Consequently if, for a given model space, an effective charge is introduced to explain electromagnetic decays then a similar enhancement of the pion-nucleon operator must be expected.

To be specific, let us consider the simplest model for the yrast 0^+ and 2^+ states in ^{18}O, namely that ^{16}O is an inert core and the two valence neutrons are confined to the $1d_{5/2}$ level. This model has obvious deficiencies—in addition to neglecting the four particle-two hole component in these states[14] it neglects the effects of single particle excitation out of the ^{16}O core and these are precisely the effects that give rise to the effective charge. Thus a more realistic wave function to be used in calculating matrix elements of single particle operators would be

$$\Psi_{00} = (\nu d_{5/2}^2)_{00} + \sum_{j_1 j_2} \Big\{ \alpha_0(j_1 j_2)[(\nu d_{5/2}^2)_2 \times [\nu j_1 \times (\nu j_2)^{-1}]_2]_{00} + \beta_0(j_1 j_2)[(\nu d_{5/2}^2)_2 \times [\pi j_1 \times (\pi j_2)^{-1}]_2]_{00} \Big\} + \sum_j \gamma_0(j)[\nu d_{5/2} \times \nu j]_{00}\,, \tag{10a}$$

$$\Psi_{2M} = (\nu d_{5/2}^2)_{2M} + \sum_{j_1 j_2} \Big\{ \alpha_2(j_1 j_2)[(\nu d_{5/2}^2)_0 \times [\nu j_1 \times (\nu j_2)^{-1}]_2]_{2M} + \beta_2(j_1 j_2)[(\nu d_{5/2}^2)_0 \times [\pi j_1 \times (\pi j_2)^{-1}]_2]_{2M} \Big\} + \sum_j \gamma_2(j)[\nu d_{5/2} \times \nu j]_{2M}\,, \tag{10b}$$

where $[\nu j_1 \times (\nu j_2)^{-1}]_{2\mu}$ ($[\pi j_1 \times (\pi j_2)^{-1}]_{2\mu}$) denotes a neutron (proton) particle-hole state with the hole occurring in the ^{16}O core. If one uses the bare operator given by Eq. (9) with $\delta_p = \delta_n = 0$ to describe the gamma decay and, furthermore, assumes that only terms linear in the admixture coefficients are important, the $E2$ matrix element is proportional to β_0 and β_2 and the effective neutron charge to be used in the $d_{5/2}$-model space is defined by the equation

$$\Big\langle \Psi_{00} \Big| e \sum_i \frac{[1-\tau_z(i)]}{2} r_i^2 Y_{2\mu}(\hat{r}_i) \Big| \Psi_{2M} \Big\rangle = e \sum_{j_1 j_2} \Big\{ \beta_2(j_1 j_2) \Big\langle (\nu d_{5/2}^2)_{00} \Big| \sum_i r_i^2 Y_{2\mu}(\hat{r}_i) \Big| [(\nu d_{5/2}^2)_0 \times [\pi_{j_1} \times (\pi_{j_2})^{-1}]_2]_{2M} \Big\rangle + \beta_0(j_1 j_2) \Big\langle [\nu d^2{}_{5/2})_2 \times [\pi_{j_1} \times (\pi_{j_2})^{-1}]_2]_{00} \Big| \sum_i r_i^2 Y_{2\mu}(\hat{r}_i) \Big| (\nu d_{5/2}^2)_{2M} \Big\rangle \Big\}$$

$$\equiv e\delta_n \Big\langle (\nu d_{5/2}^2)_{00} \Big| \sum_i r_i^2 Y_{2\mu}(\hat{r}_i) \Big| (\nu d_{5/2}^2)_{2M} \Big\rangle\,.$$

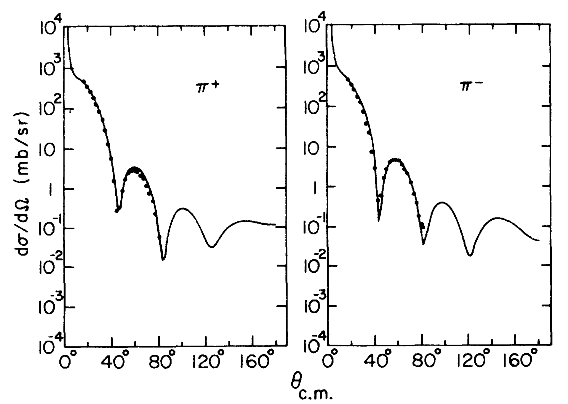

FIG. 1. Elastic scattering of 164 MeV pions on ^{18}O. The experimental data are those of Iversen *et al.*, Refs. 6 and 9. The solid curve is the DWIA prediction when a Woods-Saxon form with rms radius 2.718 fm and diffuseness parameter a= 0.57 fm is used for the ground state nuclear density.

When the same model space is used for ^{18}Ne Eqs. (10a) and (10b) describe the 0^+ and 2^+ states provided one makes the replacement proton $\rightleftarrows$ neutron. Thus the gamma decay of the 2^+ state in ^{18}Ne has, in addition to the zeroth order $(\pi d_{5/2})_2^2 \to (\pi d_{5/2})_0^2$ contribution, terms proportional to $\alpha_J(j_1 j_2)$ and $\gamma_J(j)$, and to take these into account a polarization charge $e\delta_p$ must be added to the bare proton charge.

One would expect the same sort of enhancement factors should be taken into account when one discusses the pion scattering that excites the yrast 2^+ level in ^{18}O if one uses the $(\nu d_{5/2})^2$ model space. In the neutron-proton representation we can write

$$[b_\alpha^\dagger \times h_\beta^\dagger]_{JM,T0} = \frac{1}{\sqrt{2}}\{[b_{\alpha\nu}^\dagger \times h_{\beta\nu}^\dagger]_{JM} + (-1)^T [b_{\alpha\pi}^\dagger \times h_{\beta\pi}^\dagger]_{JM}\}\,,$$

where the subscripts ν and π on the operators indicate neutron and proton operators respectively. It follows that when we deal with the spin independent contribution to the pion cross section the reduced matrix element should be

$$\langle J_f=2^+, T_f=1|||[b^\dagger_{5/2}\times h^\dagger_{5/2}]_{2;T}|||J_i=0, T_i=1\rangle = \left\{\frac{T+1}{3}\right\}^{1/2}(1+\delta_p+(-1)^T\delta_n). \quad (11)$$

The only other operator that can contribute to the excitation of the 2^+ state is $[Y_2\times\sigma]_{2\mu}$ and this has vanishing diagonal matrix elements.

If these results are inserted into Eq. (1) it follows that on resonance

$$\frac{\sigma(\pi^-;{}^{18}\mathrm{O}(2^+))}{\sigma(\pi^+;{}^{18}\mathrm{O}(2^+))}=\left|\frac{2(1+\delta_p+\delta_n)+(1+\delta_p-\delta_n)}{2(1+\delta_p+\delta_n)-(1+\delta_p-\delta_n)}\right|^2, \quad (12)$$

provided the π^+ and π^- distorted waves are the same. Values of δ_p and δ_n can be obtained from the measured $B(E2;2^+\to0^+)$ in ^{18}O and ^{18}Ne. For the former we take[15] $B(E2)=7.42\ e^2\,\mathrm{fm}^4$ and for the latter[16] $52\ e^2\,\mathrm{fm}^4$. This leads to the values

$$\delta_n=1.089\,,\qquad \delta_p=1.883 \quad (13)$$

and consequently to a π^-/π^+ ratio of 2.5. In an exact DWIA calculation for 164 MeV pions, discussed in the next section, this ratio becomes 2.1 which is in excellent agreement with the experimental results of Iversen *et al.*[6] and the most recent values obtained by the SIN group.[8]

If a larger model space is used to describe the states involved in the scattering, the matrix element $\langle\alpha||[Y_2\times\sigma]_2||\beta\rangle$ will, in general, not vanish and must be taken into account. There is no experimental evidence for any enhancement associated with this operator, and as a consequence none will be introduced. In the following section we present theoretical calculations for the inelastic pion scattering to several states in ^{18}O using various shell model descriptions for the states. In all cases we shall assume that

$$\begin{aligned}\langle J_f, T_f=1|||[b^\dagger_\alpha\times h^\dagger_\beta]_{J_f;T}|||J_i=0,T_i=1\rangle\langle\alpha||[Y_K\times\sigma_S]_{J_f}||\beta\rangle &= \langle J_f, T_f=1|||[b^\dagger_\alpha\times h^\dagger_\beta]_{J_f;T}|||J_i=0,T_i=1\rangle\\ &\quad\times(1+\delta_p(J_f)+(-1)^T\delta_n(J_f))\langle\alpha||Y_{K=J_f}||\beta\rangle \text{ when } \sigma_S=1\\ &=\langle J_f, T_f=1|||[b^\dagger_\alpha\times h^\dagger_\beta]_{J_f;T}|||J_i=0,T_i=1\rangle\\ &\quad\times\langle\alpha||[Y_K\times\sigma]_{J_f}||\beta\rangle \text{ when } \sigma_S=\sigma\,,\end{aligned} \quad (14)$$

with the enhancement factors chosen to fit the observed $B(EJ;J\to0)$ for the model space under consideration.

IV. RESULTS

In this section we compare our calculated cross sections for inelastic scattering of 164 MeV π^+ and π^- mesons to experiment. The 1.98 MeV yrast 2^+ state in ^{18}O is well separated in energy from other states so that one can observe the inelastic scattering to this state alone. On the other hand, a triplet of states exist around 3.7 MeV excitation energy (a 4^+ at 3.55 MeV, a 0^+ at 3.63 MeV, and a 2^+ at 3.92 MeV) and these states will not be resolved. In addition, the scattering to the 3^- state at 5.09 MeV undoubtedly contains an appreciable contribution from the 2^+ state at 5.25 MeV.

A. 1.98 MeV 2^+ state

In this paper three different models have been considered to describe the ground and yrast 2^+ states in ^{18}O:

(i) *The $(d_{5/2})^2$ model.* In this case ^{16}O is taken to be an inert core and the two valence neutrons are restricted to the $1d_{5/2}$ orbit. In Sec. III we discussed the enhancement factors needed to explain the ^{18}O and ^{18}Ne gamma-ray lifetimes and these are given in Eq. (13) and listed in the first line of Table I.

(ii) *The $(d_{5/2}, s_{1/2})$ model.* Again ^{16}O is taken to be an inert core, but now the two valence nucleons are allowed to occupy the $d_{5/2}$ and $s_{1/2}$ single-particle levels. The residual two-body interaction between the valence particles is taken to be the one deduced by Cohen *et al.*[17] If one assumes j-independent enhancement factors the ^{18}O and ^{18}Ne lifetimes are fitted by the values of δ_p and δ_n in the second line of Table I.

(iii) *LSF model.* In this case[14] the model space consists of two neutrons outside a closed shell and these nucleons are restricted to the $1d_{5/2}$, $2s_{1/2}$, and $1d_{3/2}$ single-particle states with at most one particle in the $1d_{3/2}$ level. In addition one collective state (four-particle two-hole) of spin 0^+, 2^+, and 4^+ was allowed. In this model, the wave functions were adjusted so as to give a best fit to all the one and two nucleon transfer data, the $M1$ and $E2$ gamma-ray lifetimes and the static multipole moments of the three lowest 0^+ and 2^+ states and the two lowest 4^+ levels. In the present paper the wave functions labeled "constrained II" given in Tables III, IV, and V of Ref. 14 were

TABLE I. Enhancement factors used in conjunction with Eq. (14). These factors were calculated under the assumption that the nucleon eigenfunctions are harmonic oscillator wave functions $[\sim \exp - \frac{1}{2}(r/b)^2]$ with $b=1.63$ fm.

Model		Isovector 2^+ δ_p	Isovector 2^+ δ_n	Isoscalar, $\delta_p=\delta_n=\delta$ 4^+	Isoscalar, $\delta_p=\delta_n=\delta$ 3^-
$(d_{5/2})^2$		1.883	1.089		
$(d_{5/2}, s_{1/2})$		1.369	0.895		
	δ_{dd}	1.047	0.547		
LSF	δ_{ds}	1.113	0.613	0.630	
	δ_c	0.753	0.753		
$(d_{5/2})^3(p_{1/2})^{-1}$					0.992

used. The enhancement factors δ_p and δ_n were obtained as follows: δ_{dd}, the enhancement when we deal with an $E2$ matrix element involving $1d$ nucleons was obtained from the quadrupole moment of ^{17}O and δ_{ds}, the enhancement factor for a $1d$ to $2s$ transition, came from the lifetime of the $\frac{1}{2}^+$ state in ^{17}O. These values are listed in column 3 of Table I. In the LSF calculation the value of δ_c, the enhancement of the four-particle two-hole part of the wave function, was never explicitly required; only the matrix element $\langle\Psi_0 \| E2 \| \Psi_2\rangle = -19.64\, e^2\,\text{fm}^4$ was used. However, in this calculation δ_c is needed and its value, given in Table I, was obtained by assuming this value for the collective matrix element, using single-particle harmonic oscillator wave functions, $\varphi \sim \exp[-\frac{1}{2}(r/b)^2]$ with $b=1.63$ fm and taking the SU(3) limit for the four-particle two-hole intruder states [i.e., Eqs. (14) and (15) of the LSF paper]. To find δ_p from the ^{18}Ne lifetime we have assumed that the value of δ_c is the same as in ^{18}O and that

$$(\delta_p)_{dd} = (\delta_n)_{dd} + \delta\,,$$

$$(\delta_p)_{ds} = (\delta_n)_{ds} + \delta$$

with δ adjusted to fit experiment.

The calculated results that emerge from each of these models are shown by the dotted curves in Figs. 2–4. In order to test the sensitivity of the results to changes in δ_p and δ_n we have also plotted, as a solid curve, the values one would obtain if δ_p were taken equal to δ_n with δ_n obtained from the ^{18}O lifetime data. At forward angles all three models give results that fall below the experimental results. On the other hand, it is difficult to obtain very precise values for the inelastic cross sections at small angles and as a consequence this is not necessarily a defect of the DWIA theory. Consequently, we shall only concern ourselves with the data at angles $\gtrsim 30°$ in our subsequent discussions.

We first compare the results obtained for the three different models when an isoscalar enhancement ($\delta_p=\delta_n$) is assumed. The LSF and $(d_{5/2}, s_{1/2})$ models give essentially identical magnitudes and angular distributions for the cross sections whereas the pure $(d_{5/2})^2$ model gives a slightly smaller cross section. When one compares with experiment one sees that the $(d_{5/2})^2$ model is better for the second maximum whereas the LSF and $(d_{5/2}, s_{1/2})$ models are superior for the primary

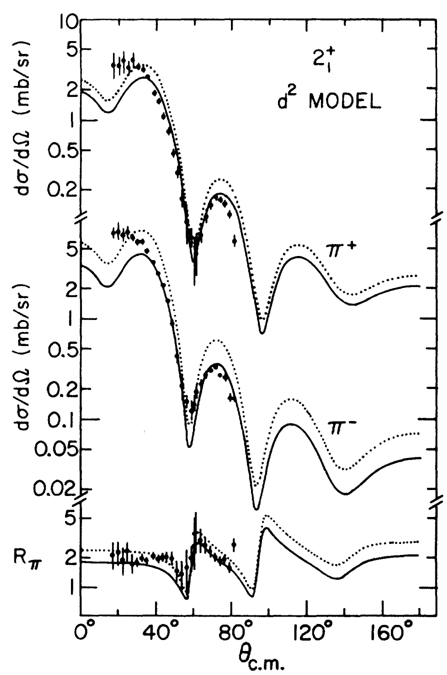

FIG. 2. Comparison of theory and experiment for the excitation of the yrast 2^+ state in ^{18}O. The experimental data are those of Iversen *et al.*, Refs. 6 and 9. The bottom curve is the ratio of the π^- to π^+ cross section as a function of angle. The theoretical curves are for the $(d_{5/2})^2$ model described in the text. The dotted curve is for an isovector enhancement with δ_p and δ_n, used in Eq. (14), given in the first line of Table I. The solid curve is for an isoscalar enhancement, $\delta_p=\delta_n$, with δ_n given in the first line of Table I.

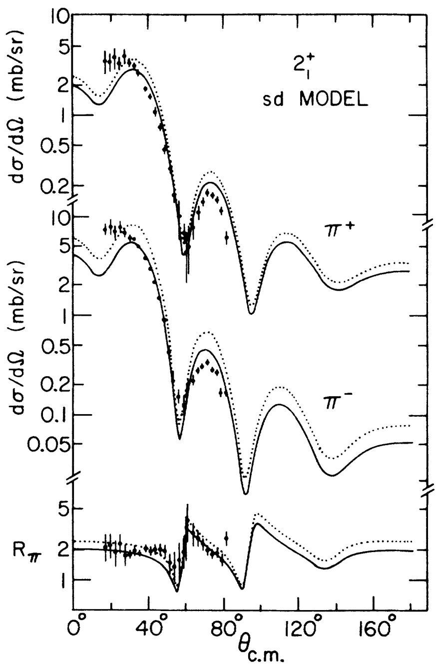

FIG. 3. Comparison of theory and experiment for the excitation of the yrast 2^+ state in ^{18}O. The theoretical curves are for the $(d_{5/2}, s_{1/2})$ model described in the text with enhancement factors δ_p and δ_n, given in the second line of Table I. See the caption to Fig. 2 for further details.

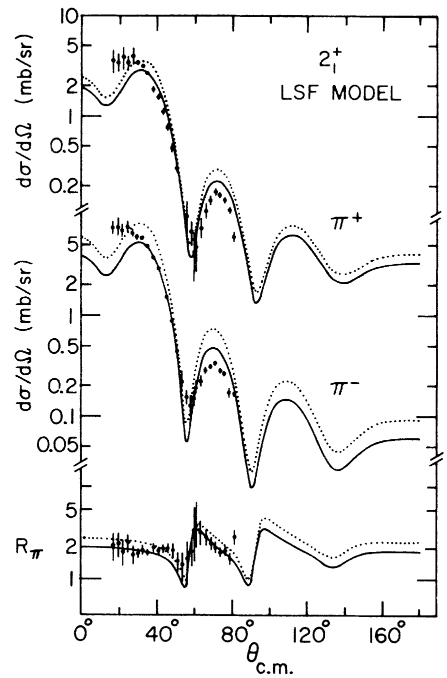

FIG. 4. Comparison of theory and experiment for the excitation of the yrast 2^+ state in ^{18}O. The theoretical curves are for the LSF model described in the text with enhancement factor given by the lines labeled LSF in Table I. See the caption to Fig. 2 for further details.

maximum. One would expect higher order processes to be less important when discussing the first maximum. Consequently one would conclude that the LSF and $(d_{5/2}, s_{1/2})$ model both give a better description of the data. However, an isoscalar enhancement which gives the same $B(E2; 2_1^+ \rightarrow 0_1^+)$ for these two different models gives indistinguishable π^+ and π^- inelastic scattering cross sections.

When an isovector enhancement is taken ($\delta_p \neq \delta_n$) with the values of δ_p and δ_n given in Table I, all three models give essentially the same results. In this case both the π^- and π^+ cross sections are increased over the isoscalar result because the enhancement factors in Eq. (14) $[1+\delta_p+(-1)^T\delta_n]$ are >1 for both $T=0$ and $T=1$. Moreover, because δ_p, which measures the neutron excitation in ^{18}O, is larger than δ_n, the π^- cross section is increased more than the π^+. A comparison of the solid and dashed curves in Figs. 2–4 does not lead to a definitive conclusion as to whether an isoscalar or isovector enhancement is called for. One might argue that using the ^{18}Ne data to determine the neutron core excitation in ^{18}O gives an overestimate of the value of δ_p. The reason is that the enhancement factor δ_p is not measured directly by the electromagnetic decay but instead the product

$$me = \delta_p \int R_{n'l'}(r) r^2 R_{nl}(r) r^2 dr$$

is determined. Since ^{18}Ne is more loosely bound than ^{18}O the value of the radial integral is larger for the former nucleus which would mean the value of δ_p determined in this way is larger than should be used in ^{18}O. Thus a curve somewhere between the solid and dashed ones in Figs. 2–4 would probably be more appropriate and this would certainly equally well fit the experimental data.

The curves labeled R_π in Figs. 2, 3, and 4 show a comparison of theory and experiment for the π^- to π^+ ratio for excitation of the yrast 2^+ state as a function of angle. All three models reproduce the observed angular variation of this ratio extremely well and once more the data are equally well fitted by either an isoscalar (solid line) or isovector (dotted line) enhancement.

B. 3.55 MeV 4^+, 3.63 MeV 0^+, 3.92 MeV 2^+ states

These three states, which lie fairly close in energy, are not resolved in the inelastic pion scattering and as a consequence must be discussed together. It is well known that the $(d_{5/2}, s_{1/2})$ model does not do well in predicting the properties of the 0^+ and 2^+ members of this triplet—

particularly for the 0^+ which is dominated by the four-particle two-hole configuration. Consequently we shall discuss the scattering to all three states only within the context of the LSF model and use the wave functions labeled "constrained II" (see Tables III, IV, and V of Ref. 14).

In the LSF model a state independent effective charge was used so that the appropriate values of the enhancement factors for excitation of the 3.92 MeV level are those given in Table I. There is no Coulomb excitation or gamma-decay data concerning the 4^+ state in ^{18}Ne so that we have no way of obtaining from experiment a value of δ_p to be used in Eq. (14). Therefore, we shall assume an isoscalar enhancement for the 3.55 MeV level ($\delta_p = \delta_n$) with $\delta_n(4^+)$ determined so as to give $B(E4; 4^+ \to 0^+) = 155\, e^2\ \mathrm{fm}^8$, the value obtained from the polarized proton scattering data of Escudié *et al.*[18] If we assume $\delta_{dd} = \delta_{ds} = \delta_c = \delta_n(4^+)$ and use the SU(3) limit for the core contribution [Eqs. (14) and (15) of LSF] one needs $\delta_n(4^+) = 0.630$ to fit experiment. Finally the cross section to the excited 0^+ was evaluated with all enhancement factors set equal to zero.

In Fig. 5 the predicted excitations of the individual states are shown as dotted lines and the solid curve gives the sum of the cross sections which is to be compared to the experimental results of Iverson *et al.*[9] The contribution of the 0^+ state is negligible except at forward angles and even then it is about a factor of 10 smaller than that for the 2^+ state. The primary maximum of both the π^+ and π^- scattering is dominated by excitation of the 3.92 MeV 2^+ and for the results shown in this figure an isoscalar enhancement in which δ_p was set equal to δ_n was used. On the other hand, the shoulder on the curves occurring in the 50°–60° range is primarily due to excitation of the 3.55 MeV 4^+ level.

The theoretical predictions for both the π^+ and π^- cross sections at the primary maximum are somewhat lower than experiment. For the first 2^+ state the introduction of the isovector enhancement given in Table I substantially increased the calculated value. However, for the second 2^+ there is virtually no difference in the π^+ and only about a 10% increase in the π^- scattering when this effect is considered. This underestimate of the scattering cross section is consistent with the fact that the LSF wave functions give too small a branching ratio for decay of the 2_2^+ level to the ground state. Experimentally this branching ratio is $(13 \pm 3)\%$ whereas the "constrained II" eigenfunctions give only 7.75%. Consequently we have recalculated the 2_2^+ cross section using an isoscalar enhancement in which each of the values of δ_n given in column 3 of Table I was increased

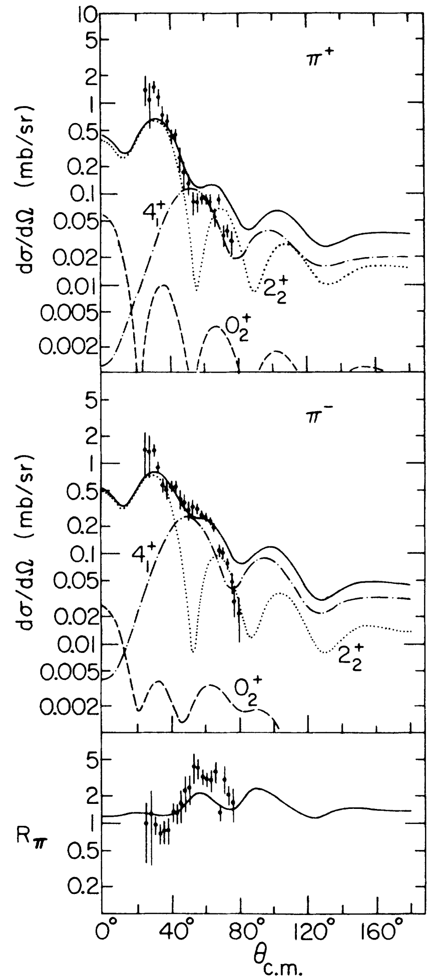

FIG. 5. Comparison of theory and experiment for excitation of the 4_1^+(3.55 MeV), 0_2^+(3.63 MeV) and 2_2^+(3.92 MeV) states in ^{18}O. The experimental data are those of Iversen *et al.*, Ref. 9. The 4_1^+ excitation (—— · ——) was calculated using the isoscalar 4^+ enhancement factor given in Table I. The theoretical predictions for the 0_2^+ state (---) were calculated with $\delta_p = \delta_n = 0$ and for the 2_2^+ excitation (· · ·) an isoscalar enhancement, $\delta_p = \delta_n$, was assumed with δ_n given in the lines labeled LSF (column 3) of Table I. The solid curve corresponds to the incoherent sum of the three predicted cross sections. R_π is the ratio of the π^- to π^+ cross section as a function of angle.

by 40% (this would make the branching ratio approximately 13%). The results of these calculations combined with the original 0^+ and 4^+ predictions are compared with experiment in Fig. 6. It is clear that one now has a very satisfactory fit to the π^- data for angles up to 60°. For π^+ scattering the fit to the primary maximum is excellent, however, the predicted 50° results are almost a factor of 2 larger than experiment. The origin of this difficulty could be due to the fact

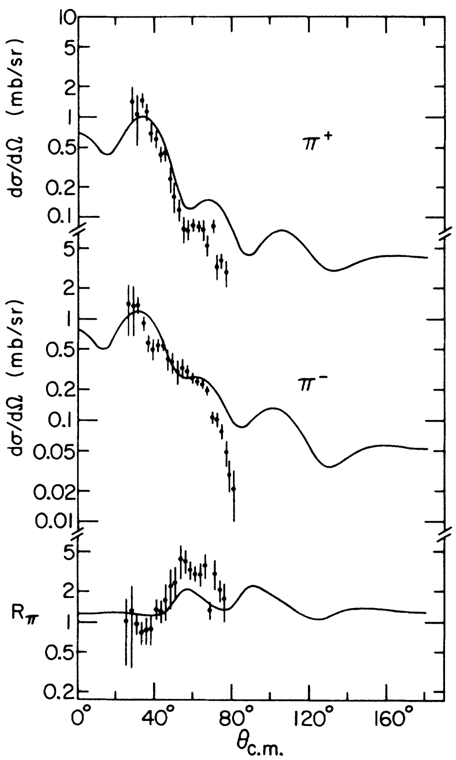

FIG. 6. Comparison of theory and experiment for excitation of the 4_1^+, 0_2^+, and 2_2^+ states in ^{18}O. The solid curve corresponds to the incoherent sum of the three predicted excitations. For this curve the contribution of the 2_2^+ state was calculated using the LSF model and an isoscalar enhancement with each of the δ_n's given in column 3 of Table I increased by 40%. The 4_1^+ and 0_2^+ contributions were taken to be the same as those shown in Fig. 5.

that in this angular range the cross section is an order of magnitude smaller than the primary maximum and consequently higher order processes may be more important. Alternatively, it follows from Eq. (12) that the π^+ cross section increases more rapidly with $\delta_n(4^+)$ than does the π^- and these results may imply that a somewhat smaller enhancement factor should be used.

The ratio R_π of the π^- to π^+ cross sections is shown at the bottom of Figs. 5 and 6. For angles less than 50° the theoretical and experimental values of R_π are in good agreement. However, for larger angles the experimental ratio is larger than the theoretical one, and this comes about because the predicted π^+ cross section is too large at these angles.

C. 5.09 MeV 3^-, 5.25 MeV 2^+, 5.33 MeV 0^+ states

A second strongly excited triplet of states occurs at about 5 MeV excitation energy in ^{18}O. In making theoretical predictions for these we assume that the LSF model with "constrained II" wave functions is appropriate to describe the 0^+ and 2^+ states.

In our calculations for the yrast 2^+ state we found that the simple $(d_{5/2})^2$ model gave almost as good a fit to the data as did the more complicated $(d_{5/2}, s_{1/2})$ and LSF models. Consequently in order to simplify the calculations for the 3^- state we shall assume that the ^{18}O ground state is $(d_{5/2})^2_0$ and that the 3^- is described by

$$\psi_{3M;11} = \alpha[(d_{5/2})^3_{5/2;1/2} \times (p_{1/2})^3_{1/2;1/2}]_{3M;11} + \beta[(d_{5/2})^3_{5/2;3/2} \times (p_{1/2})^3_{1/2;1/2}]_{3M;11} \tag{15}$$

where the notation $(d_{5/2})^3_{5/2;1/2}$ denotes the configuration in which three $d_{5/2}$ particles couple to spin $\frac{5}{2}$ with seniority one and isospin $\frac{1}{2}$ and $[\times]_{3M;11}$ implies angular momentum coupling in both spin and isospin with quantum numbers $(3, M)$ and $(1, T_z = 1)$ respectively.

Although it will turn out that β is small, even a small admixture gives a significant contribution to π^- scattering. That this is true follows from Eq. (8) which tells us that in the absence of any enhancement factors

$$\sigma(\pi^\pm) \sim |\sqrt{14}\,\alpha(2\pm 1) - 2\beta(4\mp 1)|^2 . \tag{16}$$

Thus when $\alpha = 1$ the π^+ cross section is nine times the π^-. With no enhancement this would be the predicted ratio for any single-particle excitations out of the p shell to the (sd) shell if the three (sd) nucleons have $T = \frac{1}{2}$ since for this (sd) isospin only proton excitation can contribute. However, when $\beta = -\sqrt{0.1}$ this ratio drops to 3.49 even when no enhancement is allowed for.

In order to calculate α and β one must know the residual two-body force acting between the valence nucleons. For the $(p_{1/2} - d_{5/2})$ interaction we have used the Millener-Kurath[19] matrix elements and for $(d_{5/2} - d_{5/2})$ we have assumed that it is such that the $(d_{5/2})^3_{J=5/2;T=3/2}$ state lies 7.34 MeV above the $(d_{5/2})^3_{J=5/2;T=1/2}$ which is the observed splitting[20] between the yrast $J = \frac{5}{2}$, $T = \frac{3}{2}$ and $\frac{1}{2}$ states in ^{19}F. With these assumptions it follows that

$$\alpha = 0.952, \quad \beta = -0.306 . \tag{17}$$

The 3^- state does not gamma decay directly to the ground state and so the enhancement factors to be used in the calculation must be determined in some other way. From the electron scattering results of Groh *et al.*[21] one obtains $B(E3; 3^- \to 0^+) = 160\, e^2$ fm^6 or $90\, e^2$ fm^6 depending on whether the Helm of liquid drop model is used to extrapolate to zero momentum transfer. Alternatively, the polarized proton scattering data of Escudié *et al.*[18] give a deformation of 0.2 which translates to $B(E3; 3^- \to 0^+) = 77\, e^2$ fm^6. Thus there is some ambiguity as to what value should be used. There is, however, a lifetime measurement[22] for the $3^- \to 0^+$ decay in the neighboring nucleus ^{16}O and this yields $B(E3; 3^- \to 0^+) = 224\,e^2\,fm^6$. Since one would not expect the $B(E3)$ in ^{18}O to be much different from that in ^{16}O we have assumed that in ^{18}O the value extracted from electron scattering using the Helm model is the correct one. If one assumes an isoscalar enhancement and uses Eqs. (15) and (17) to describe the 3^- state, $B(E3; 3^- \to 0^+) = 160\ e^2\,fm^6$ for decay to the $(d_{5/2})_0^2(p_{1/2})_0^4$ ground state is fitted when $\delta = 0.992$.

In Fig. 7 the predicted cross sections for these states are compared with the experimental results of Iverson *et al.*[9] The 0^+ cross section, which was calculated with $\delta_p = \delta_n = 0$, is predicted to be small for both the π^+ and π^- scattering and gives a negligible contribution to the total cross section shown by the solid line in this figure.

The 3^- excitation was calculated using $\delta_p = \delta_n = 0.992$ in Eq. (14). The predicted peak π^+ (π^-) cross section is about $\frac{2}{3}$ ($\frac{1}{2}$) the observed value. From our experience with the yrast 2^+ level we would expect that the primary maximum would not be very sensitive to the details of the wave function provided the enhancement factor used in Eq. (14) gives the correct $B(E3)$. Consequently we would expect that if the experimental data correspond only to population of the 3^- we would have come much closer to it. Therefore, these results strongly suggest that the scattering to states in the neighborhood of 5 MeV excitation energy in ^{18}O must have a substantial contribution corresponding to population of the 5.25 MeV 2^+ state.

The predictions for excitation of the 5.25 MeV state are also shown in Fig. 7. For the results presented in this figure the LSF "constrained II" wave functions were used and the isovector enhancement given in columns 2 and 3 of Table I was employed. The predicted cross sections for both π^+ and π^- scattering are an order of magnitude smaller than those for the 3^-. Moreover, in contrast to the other two 2^+ states, the π^+ cross section has a peak value that is more than $1\frac{1}{2}$ times the π^-. This comes about because in calculating $B(E2)$ for the 2_3^+ state the neutron contribution to the matrix element almost cancels out, and in fact if one uses an isoscalar enhancement in which δ_p is set equal to δ_n the predicted π^- cross section drops by a factor of 10.

Because of the simplified model we have used for the 3_1^- state, one cannot say definitely that the entire discrepancy between theory and experiment is to be attributed to a shortcoming of the LSF 2_3^+ wave function. However, the results are certainly very suggestive and are consistent with the fact that the LSF wave function gives a smaller value for $B(E2; 2_3 \to 0_1^+)$ than obtained from

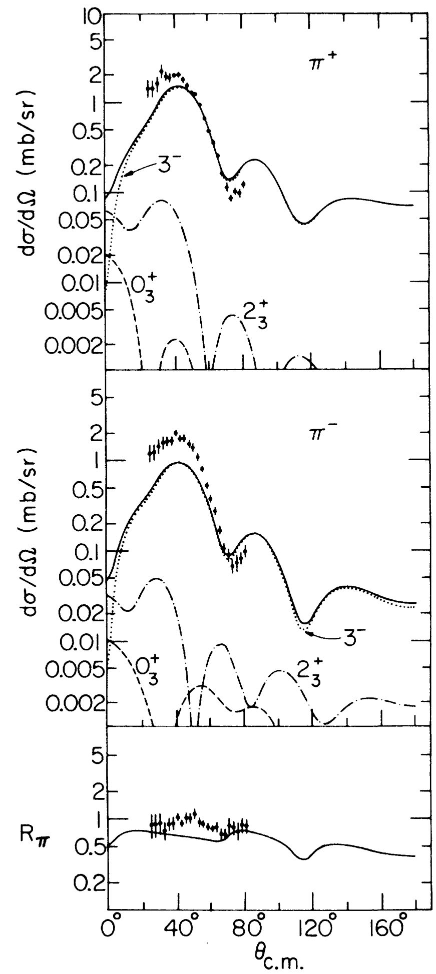

FIG. 7. Comparison of theory and experiment for excitation of the 3_1^-(5.09 MeV), 2_3^+(5.25 MeV), and 0_3^+(5.33 MeV) states in ^{18}O. The experimental data are those of Iversen *et al.*, Ref. 9. The 0_3^+ and 2_3^+ cross sections were calculated using the LSF wave functions with $\delta_p = \delta_n = 0$ for the former and the isovector enhancement given in Table I for the latter. For the 3^- excitation the isoscalar 3^- enhancement given in Table I was used together with Eqs. (15) and (17) for the 3^- level and $(d_{5/2})_0^2(p_{1/2})_0^4$ for the ground state. The solid curve corresponds to the incoherent sum of the three predicted cross sections. R_π is the ratio of the π^- to π^+ cross section as a function of angle.

electron scattering. The theoretical prediction for this quantity is $1.39\,e^2\,fm^4$ whereas experiment[21] gives $4.8\ e^2\,fm^4$. Because the neutron and proton contributions interfere destructively, one would have to multiply the enhancement factors in Table I by four to bring theory and experiment into agreement and this is, of course, unreasonable. On the other hand, a crude way to take this into account is to merely scale the predicted 2_3^+ cross sections by 4.8/1.93. If this is done the sum of the $0_3^+ + 2_3^+ + 3_1^-$ cross sections follows the experimental shapes of the primary maxima for both π^+ and π^- scattering. However, theory is about 20% below experiment for the π^+ and a factor of 2 below experiment for the π^-.

V. DISCUSSION

The DWIA provides a satisfactory description of the primary maximum for the inelastic scattering of pions to the yrast 2^+ state in ^{18}O. Furthermore, when one takes into account the deficiencies of the LSF wave function for the 2_2^+ state it also gives a satisfactory description of the primary maximum for scattering to the $(4_1^+, 0_2^+, 2_2^+)$ states at ~3.7 MeV. However, the secondary maximum at 50°–60° is overestimated in these cases for both π^+ and π^- scattering. One should not be too surprised by this result considering the simplifications that go into the DWIA. In particular, the phenomenological procedure of adjusting R and a to obtain the optical potential may not be adequate to describe the effects of "true pion absorption" and other higher order nucleon-nucleon correlation effects[23] that are expected to be important at large angles.

To fit the cross section at the primary maximum, it is necessary to enhance the $S=0$ contribution to the cross section by the same factor as is needed to fit the $B(EL; 0^+ \to L=J)$ value obtained from the electromagnetic properties of the state [see Eq. (14)]. In this respect our enhancement differs from that proposed by Bernstein *et al.*,[24] who suggest that the entire contribution to the scattering scale as do the $B(EL)$'s [i.e., both the $(L,S=0)$ and $(L,S=1)$ be enhanced]. In principle the pion data could be used to decide whether an isoscalar $(\delta_p = \delta_n)$ or an isovector $(\delta_p \neq \delta_n)$ enhancement is needed. However, the accuracy of the present data does not allow one to draw any conclusions about this point.

Brown and Fortune[25] have suggested that one should enhance the contribution of the deformed components in the ^{18}O wave function differently than we have done. Because pion scattering is a surface phenomenon, they argue that the deformed state, which possesses more surface, should have a larger enhancement than that given by the electromagnetic $B(EL)$ value. Since the deformed states in ^{18}O should be similar to the ground state rotational band in ^{20}Ne a test of this conjecture would be provided by inelastic pion scattering off a ^{20}Ne target. Using the enhancement factor δ_c given in Table I we predict the peak cross section for π^+ excitation of the 2^+ state in ^{20}Ne to occur at about 30° and have the value 11.2 mb/sr. A significantly larger value than this would bear out the Brown-Fortune conjecture.

Pion scattering is fairly insensitive to the structure of the yrast 2_1^+ state. All three models discussed give essentially the same cross section provided the enhancement factors of Eq. (14) are chosen to fit the observed $B(E2; 0_1^+ \to 2_1^+)$. On the other hand, the calculated cross section for both the 2_2^+ and 2_3^+ states is smaller than needed to fit experiment. This is consistent with the fact that the LSF wave functions give a branching ratio for the gamma decay $2_2^+ \to 0_1^+$ that is smaller than experiment by about 40% and that the $B(E2; 0_1^+ \to 2_3^+)$ obtained from electron scattering is about 2.5 times the LSF prediction. Thus in these cases pion scattering confirms the results found with other nuclear probes.

The most important property of the pion is its ability to differentiate between the neutron and proton contribution to a given excitation. This comes about because on resonance the π^+ (π^-) scattering amplitude off a proton (neutron) is three times that off a neutron (proton). By virtue of this property one could pick out nuclear states corresponding, for example, to almost pure neutron excitation and such a case seems to have been observed[26,27] in ^{13}C. In ^{18}O the LSF wave functions predict that in $B(E2; 0_1^+ \to 2_3^+)$ the proton contribution is larger than the neutron part. If the isoscalar enhancement of Table I is used the predicted π^- cross section to the 2_3^+ state is an order of magnitude smaller than the π^+. On the other hand, the isovector enhancement of Table I leads to the conclusion that the π^+ cross section is about twice the π^-. Both these results are in disagreement with experiment, so it would appear that the LSF wave function for the 2_3^+ not only underestimates the magnitude of the $B(E2)$ but also gives the wrong neutron-proton structure for the state.

ACKNOWLEDGMENTS

We would like to thank Dr. D. Kurath for discussions and reading the manuscript. This work was performed under the auspices of the U. S. Department of Energy.

[1] T.-S. H. Lee and F. Tabakin, Nucl. Phys. A226, 253 (1974).
[2] T.-S. H. Lee and D. Kurath, Phys. Rev. C 21, 293 (1980).
[3] L. S. Kisslinger, Phys. Rev. 98, 761 (1955).
[4] G. E. Edwards and E. Rost, Phys. Rev. Lett. 26, 785 (1971); D. A. Sparrow, Nucl. Phys. A276, 365 (1977), and references therein.

[5]T. J. Londergan, K. W. McVoy, and E. J. Moniz, Ann. Phys. (N.Y.) 86, 147 (1974).
[6]S. Iversen, H. Nann, A. Obst, Kamal K. Seth, N. Tanaka, C. L. Morris, H. A. Thiessen, K. Boyer, W. Cottingame, C. Fred Moore, R. L. Boudrie, and D. Dehnhard, Phys. Lett. 82B, 51 (1979).
[7]J. Jansen, J. Zichy, J. P. Albanèse, J. Arvieux, J. Bolger, E. Boschitz, C. H. Q. Ingram, and L. Pflug, Phys. Lett. 77B, 359 (1978).
[8]E. Boschitz, private communication.
[9]S. Iversen, Ph.D. thesis, Northwestern Univ., Los Alamos Scientific Laboratory Report No. LA-7828-T (1979).
[10]R. A. Eisenstein and F. Tabakin, Comp. Phys. Comm. 12, 237 (1976).
[11]C. Olmer, D. F. Geesaman, B. Zeidman, S. Chakravarti, T.-S. H. Lee, R. L. Boudrie, R. H. Siemssen, J. F. Amann, C. L. Morris, H. A. Thiessen, G. R. Burleson, M. J. Devereux, R. E. Segel, and L. W. Swenson, Phys. Rev. C 21, 254 (1980).
[12]C. W. DeJager, H. DeVries, and C. DeVries, At. Data Nucl. Data Tables 14, 479 (1974).
[13]D. M. Brink and G. R. Satchler, Angular Momentum (Oxford University Press, London, 1968).
[14]R. D. Lawson, F. J. D. Serduke, and H. T. Fortune, Phys. Rev. C 14, 1245 (1976).
[15]Z. Berant, C. Broude, G. Engler, and D. F. H. Start, Nucl. Phys. A225, 55 (1975).
[16]A. B. McDonald, T. K. Alexander, C. Broude, J. S. Forster, O. Häusser, F. C. Khanna, and I. V. Mitchell, Nucl. Phys. A258, 152 (1976).
[17]S. Cohen, R. D. Lawson, M. H. Macfarlane, and M. Soga, Phys. Lett. 9, 180 (1964).
[18]J. L. Escudié, R. Lombard, M. Pignanelli, F. Resmini, and A. Tarrats, Phys. Rev. C 11, 639 (1975).
[19]D. J. Millener and D. Kurath, Nucl. Phys. A255, 315 (1975).
[20]F. Ajzenberg-Selove, Nucl. Phys. A300, 1 (1978).
[21]J. L. Groh, R. P. Singhal, H. S. Caplan, and B. S. Dolbilkin, Can. J. Phys. 49, 2743 (1971).
[22]F. Ajzenberg-Selove, Nucl. Phys. A281, 1 (1977).
[23]T.-S. H. Lee and S. Chakravarti, Phys. Rev. C 16, 273 (1977).
[24]A. M. Bernstein, V. R. Brown, and V. A. Madsen, Phys. Rev. Lett. 42, 425 (1979).
[25]G. E. Brown and H. T. Fortune, private communication.
[26]D. Dehnhard, S. J. Tripp, M. A. Franey, G. S. Kyle, C. L. Morris, R. L. Boudrie, J. Piffaretti, and H. A. Thiessen, Phys. Rev. Lett. 43, 1091 (1979).
[27]T.-S. H. Lee and D. Kurath (unpublished).