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Proton-carbon bremsstrahlung calculation

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The Feshbach-Yennie approximation is applied to calculate the proton-carbon bremsstrahlung cross sections near the resonance at 1.7 MeV. In all but one case, the predicted cross sections are in very good agreement with the experimental data. The calculation includes not only the principal term of the approximation but also the correction term, which is negligible at an energy far from the resonance but becomes very significant in the region of the resonance.

[NUCLEAR REACTIONS p-^{12}C bremsstrahlung near 1.7 MeV, calculate bremsstrahlung cross section.]

I. INTRODUCTION

Two well-known model independent approximations which have been commonly used in the study of various nuclear bremsstrahlung processes are the soft-photon approximation of Low[1,2] and the Feshbach-Yennie approximation.[3] These two approximations predict about the same cross sections at an incident energy which is far from any resonant state, but they give very different results in the vicinity of a resonance. The soft-photon approximation always gives a bremsstrahlung spectrum with a characteristic $1/k$ dependence (k is the photon energy) whereas the Feshbach-Yennie approximation predicts structure in the region of a resonance.

Recently, the bremsstrahlung cross sections for the scattering of protons by ^{12}C near the 1.7-MeV resonance have been measured by the Bologna group[4] and the Brooklyn group.[5] These groups have observed two types of bremsstrahlung spectra: The spectrum with a simple $1/k$ dependence which can be described by both the soft-photon approximation and the Feshbach-Yennie approximation, and the spectrum with a structure which cannot be described by the soft-photon approximation but may be described by the Feshbach-Yennie approximation.[6] To the best of our knowledge, those spectra which have structure clearly exhibited at a photon energy corresponding to the resonance energy have never before been observed. These spectra with structure are very important not only because they can be used to extract the nuclear time delay, information which allows an unambiguous separation between compound nuclear reactions and direct interaction,[7] but also because they can be used to test the Feshbach-Yennie approximation.

The bremsstrahlung cross section calculated from the Feshbach-Yennie approximation can be written as

$$\sigma = \frac{\sigma_{-1}(k)}{k} + \sigma_0(k)\,, \tag{1}$$

where $\sigma_{-1}(k)/k$ represents the principal term and $\sigma_0(k)$, which depends on the derivatives of the elastic scattering amplitude, represents the correction term. It was always assumed in the past that the contribution from $\sigma_0(k)$ would be small at very low incident energy. If this assumption is correct, then the expression which includes only the principal term

$$\sigma' = \frac{\sigma_{-1}(k)}{k} \tag{2}$$

would be a good approximation for the description of the low energy nuclear bremsstrahlung processes with or without the presence of resonances.

Equation (2) has been used to analyze the Bologna data by the Bologna group and by Perng *et al.*[6] In these analyses, the parameters of the resonances have been used as input for the calculation of the p-^{12}C elastic cross section and p-^{12}C bremsstrahlung cross section (p-^{12}Cγ). The Bologna group was unable to obtain quantitative agreement with Eq. (2) for their spectrum at 1795 keV if the parameters of the resonances in ^{13}N obtained from elastic scattering data[8,9] were used. To get good agreement with their data, the Bologna group had to use a new set of parameters for the resonances. This new set of parameters was studied by Perng *et al.* who found that these parameters give poor agreement with the elastic scattering cross sections near the resonances for some scattering angles. They also found that it was very difficult to find a set of new resonance parameters which would fit both the p-^{12}C elastic data of Refs. 8 and 9 and the p-^{12}Cγ data of the Bologna group. This difficulty led them to conclude that either the bremsstrahlung data at 1795 keV was inconsistent with the elastic data if Eq. (2) was indeed a good approximation, or Eq. (2) was inadequate to describe the spectrum at 1795 keV if these two different sets of data were actually consistent, or Eq. (2) was inadequate and two sets of experimental data were inconsistent.

At Brooklyn College, we have tried to solve the problem by performing a new bremsstrahlung experiment near 1.7 MeV and improving the calculation of Perng *et al.* by taking into account the correction term, i.e., to use the complete expression of the Feshbach-Yennie approximation given by Eq. (1). The experimental result has already been reported elsewhere[5] and the result of the calculation will be presented here.

The purpose of this paper is to describe in detail the method of our calculation since the derivation of our bremsstrahlung amplitude is slightly different from the original one first proposed by Feshbach and Yennie, and to demonstrate that the principal term along [i.e., Eq. (2)] cannot be used to describe the bremsstrahlung spectrum near a resonance, since the contribution from the correction term, which is negligible at an energy far from the resonance, becomes very significant in the region of the resonance. Moreover, we show that, except for one puzzling case, all experimental data can be described by the Feshbach-Yennie approximation if both the principal term and the correction term [i.e., Eq. (1)] are properly included in the calculation.

II. BREMSSTRAHLUNG AMPLITUDE AND CROSS SECTION

We consider the proton-carbon bremsstrahlung process:

$$p(q_i^\mu) + {}^{12}\mathrm{C}(p_i^\mu) \rightarrow p(q_f^\mu) + {}^{12}\mathrm{C}(p_f^\mu) + \gamma(k^\mu)\,,$$

where q_i^μ (q_f^μ) and p_i^μ (p_f^μ) are the initial (final) four-momenta of proton and carbon, respectively, and k^μ is the four-momentum of the emitted photon. These five momenta are defined in the laboratory frame as

$$q_i^\mu = (m + E_i, 0, 0, q_i)\,,$$

$$p_i^\mu = (M, 0, 0, 0)\,,$$

$$q_f^\mu = (E_q, q_f \sin\theta_q \cos\phi_q, q_f \sin\theta_q \sin\phi_q, q_f \cos\theta_q)\,,$$

$$p_f^\mu = (E_p, p_f \sin\theta_p \cos\phi_p, p_f \sin\theta_p \sin\phi_p, p_f \cos\theta_p)\,,$$

$$k^\mu = (k, k \sin\theta_\gamma \cos\phi_\gamma, k \sin\theta_\gamma \sin\phi_\gamma, k \cos\theta_\gamma)\,,$$

where $E_q = (m^2 + \vec{q}_f^{\,2})^{1/2}$, $E_p = (M^2 + \vec{p}_f^{\,2})^{1/2}$, m and M are the masses of proton and carbon, respectively, and the θ and ϕ follow the usual conventions in spherical coordinates. They satisfy energy-momentum conservation:

$$q_i^\mu + p_i^\mu = q_f^\mu + p_f^\mu + k^\mu\,. \tag{3}$$

If θ_q, ϕ_q, θ_γ, ϕ_γ, and k are chosen to be independent, then the rest of the four dependent variables, q_f, p_f, θ_p, and ϕ_p, can be calculated from Eq. (3), and the bremsstrahlung cross section in the laboratory system can be written as

$$\frac{d^3\sigma}{d\Omega_q d\Omega_\gamma dk} = \int (2\pi)^4 \delta^{(4)}(q_i + p_i - q_f - p_f - k)\left[\tfrac{1}{2}\sum_{\text{pol,spin}} \langle (M_\mu \epsilon^\mu)^\dagger (M_\mu \epsilon^\mu)\rangle\right] \times \frac{e^2 m^2}{[(p_i\cdot q_i)^2 - m^2M^2]^{1/2}} \frac{q_f^2 dq_f}{(2\pi)^3 2E_q} \frac{d^3p_f}{(2\pi)^3 2E_p} \frac{k^2}{(2\pi)^3 2k}\,, \tag{4}$$

where M_μ is the bremsstrahlung amplitude, ϵ_μ is the proton polarization, e is the proton charge, $d\Omega_q = \sin\theta_q d\theta_q d\phi_q$, $d\Omega_\gamma = \sin\theta_\gamma d\theta_\gamma d\phi_\gamma$, and the summation sign indicates a sum over the photon polarization and a sum over the final and initial proton spins.

The bremsstrahlung amplitude M_μ consists of the external scattering amplitude $M_\mu^{(E)}$ and the internal scattering amplitude $M_\mu^{(I)}$:

$$M_\mu = M_\mu^{(E)} + M_\mu^{(I)}\,. \tag{5}$$

The external scattering amplitude $M_\mu^{(E)}$ is the sum of those terms which describe photon emission from the external protons and carbons. If we treat carbon as a single elementary particle of mass M and charge Ze, where Z is the atomic number of carbon, then the external scattering amplitude $M_\mu^{(E)}$ can be expressed in the form

$$M_\mu^{(E)} = \bar{u}(q_f, \nu_f)\left(\Gamma_\mu \frac{1}{\not{q}_f + \not{k} - m} T_a + T_b \frac{1}{\not{q}_i - \not{k} - m}\Gamma_\mu + \frac{Z}{p_f\cdot k} p_{f\mu} T_c - \frac{Z}{p_i \cdot k} p_{i\mu} T_d\right) u(q_i, \nu_i)\,. \tag{6}$$

Here, T_a, T_b, T_c, and T_d are the half-off-mass-shell T matrices for p-^{12}C scattering (corresponding to Figs. 1(a), 1(b), 1(c), and 1(d), respectively) and Γ_μ is given by

$$\Gamma_\mu = \gamma_\mu - \frac{i}{2m}\lambda\sigma_{\mu\nu}k^\nu\,,$$

where $\sigma_{\mu\nu} = i/2[\gamma_\mu, \gamma_\nu]$, and the proton anomalous magnetic moment λ is 1.79. Since the incident proton energy is about 1.7 MeV, the contribution from those terms which involve λ will be negligible. Therefore, they will be ignored in the low energy

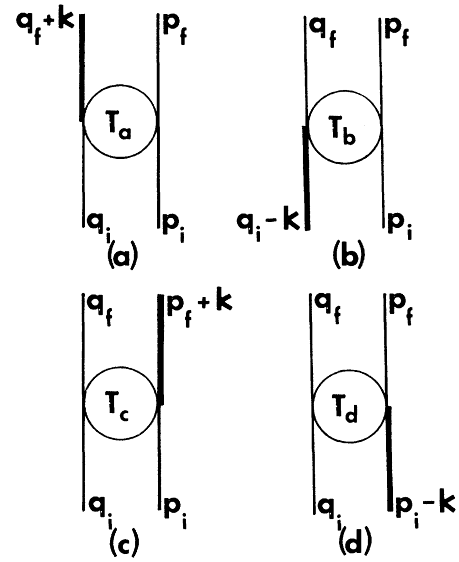

FIG. 1. Four half-off-mass-shell T matrices.

scattering case. Moreover, since $k \leqslant 250$ keV/c, $k/q_i \ll 1$, and $k/q_f \ll 1$, we may also make the following approximations:

$$\bar{u}(q_f,\nu_f)\gamma_\mu \frac{1}{\not{q}_f+\not{k}-m} = \bar{u}(q_f,\nu_f)\frac{q_{f\mu}+\gamma_\mu \not{k}/2}{q_f\cdot k} \approx \bar{u}(q_f,\nu_f)\frac{q_{f\mu}}{q_f\cdot k},$$
$$\frac{1}{\not{q}_i-\not{k}-m}\gamma_\mu u(q_i,\nu_i) = \frac{-q_{i\mu}+\not{k}\gamma_\mu/2}{q_i\cdot k}u(q_i,\nu_i) \approx -\frac{q_{i\mu}}{q_i\cdot k}u(q_i,\nu_i). \tag{7}$$

With all these approximations, Eq. (6) can be simplified as

$$M_\mu^{(E)} = \bar{u}(q_f,\nu_f)\mathfrak{M}_\mu^{(E)}u(q_i,\nu_i), \tag{8}$$

where

$$\mathfrak{M}_\mu^{(E)} = \frac{1}{q_f\cdot k}q_{f\mu}T_a - T_b q_{i\mu}\frac{1}{q_i\cdot k} + \frac{Z}{p_f\cdot k}p_{f\mu}T_c - T_d p_{i\mu}\frac{Z}{p_i\cdot k}.$$

The half-off-mass-shell T matrices, T_i($i = a,b,c,d$), are functions of three Lorentz invariants. We can choose these invariants to be the total energy squared s, the momentum transfer squared t, and the square of the invariant mass of the off-mass-shell leg on which the photon emission occurs Δ. In terms of these invariants, T_i can be written in the form

$$T_a = T(s_a,t_a,\Delta_a),$$
$$T_b = T(s_b,t_b,\Delta_b),$$
$$T_c = T(s_c,t_c,\Delta_c),$$
$$T_d = T(s_d,t_d,\Delta_d), \tag{9}$$

where

$$s_a = s_c = (q_i+p_i)^2 = (m+M)^2 + 2ME_i,$$
$$s_b = s_d = (q_f+p_f)^2 = (m+M)^2 + 2M(E_i-k) + 2\vec{q}_i\cdot\vec{k} - 2E_i k - 2mk \approx (m+M)^2 + 2M(E_i-k),$$
$$t_a = t_b = (p_f-p_i)^2 \approx t - 2(\bar{p}_f - p_i)\cdot(R+k),$$
$$t_c = t_d = (q_f-q_i)^2 \approx t + 2R\cdot(\bar{q}_f - q_i), \tag{10}$$
$$\Delta_a = (q_f+k)^2 \approx m^2 + 2\bar{q}_f\cdot k,$$
$$\Delta_b = (q_i-k)^2 = m^2 - 2q_i\cdot k,$$
$$\Delta_c = (p_f+k)^2 \approx M^2 + 2\bar{p}_f\cdot k,$$
$$\Delta_d = (p_i-k)^2 = M^2 - 2p_i\cdot k,$$
$$t = (\bar{p}_f - p_i)^2 = (\bar{q}_f - q_i)^2.$$

In deriving Eq. (10), the following expressions

$$q_f^\mu = \bar{q}_f^\mu + R^\mu,$$
$$p_f^\mu = \bar{p}_f^\mu - R^\mu - k^\mu,$$
$$\bar{q}_f^\mu = \lim_{k\to 0} q_f^\mu, \tag{11}$$
$$\bar{p}_f^\mu = \lim_{k\to 0} p_f^\mu,$$

where

$$R^\mu = N_R^\mu(\bar{p}_f\cdot k),$$
$$N_R^\mu = [m^2 p_i^\mu - (p_i\cdot\bar{q}_f)\bar{q}_f^\mu]/[(p_i\cdot\bar{q}_f)(\bar{p}_f\cdot\bar{q}_f) - m^2(p_i\cdot\bar{p}_f)],$$

have been used to expand t_i and Δ_i ($i = a,b,c,d$). The expressions given by Eq. (10) can be used to expand T_i. We obtain, keeping only terms to order k^0,

$$\mathfrak{M}_\mu^{(E)} = \left(\frac{q_{f\mu}}{q_f\cdot k} + \frac{Zp_{f\mu}}{p_f\cdot k}\right)T(s_i,t) - \left(\frac{q_{i\mu}}{q_i\cdot k} + \frac{Zp_{i\mu}}{p_i\cdot k}\right)T(s_f,t)$$
$$- 2(\bar{p}_f - p_i)\cdot(R+k)\left[\frac{q_{f\mu}}{q_f\cdot k}\frac{\partial T(s_i,t)}{\partial t} - \frac{q_{i\mu}}{q_i\cdot k}\frac{\partial T(s_f,t)}{\partial t}\right]$$
$$+ 2(\bar{q}_f - q_i)\cdot R\left[\frac{Zp_{f\mu}}{p_f\cdot k}\frac{\partial T(s_i,t)}{\partial t} - \frac{Zp_{i\mu}}{p_i\cdot k}\frac{\partial T(s_f,t)}{\partial t}\right]$$
$$+ 2\bar{q}_{f\mu}\frac{\partial T(s_i,t,\Delta_a)}{\partial\Delta_a} + 2q_{i\mu}\frac{\partial T(s_f,t,\Delta_b)}{\partial\Delta_b}$$
$$+ 2Z\bar{p}_{f\mu}\frac{\partial T(s_i,t,\Delta_c)}{\partial\Delta_c} + 2p_{i\mu}Z\frac{\partial T(s_f,t,\Delta_d)}{\partial\Delta_d}, \tag{12}$$

where

$$s_i=(m+M)^2+2ME_i\,,$$
$$s_f\approx(m+M)^2+2ME_f\,,$$
$$E_f=E_i-k\,.$$

It should be pointed out that although $T(s_i,t)$ is independent of k, $T(s_f,t)$ is still a function of k. That T has been evaluated at two different energies is a distinguishing characteristic of the Feshbach-Yennie theory.

To obtain the internal scattering amplitude, $M_\mu^{(I)}\equiv\bar{u}(q_f,\nu_f)\mathfrak{M}_\mu^{(I)}u(q_i,\nu_i)$, we now impose the gauge-invariance (current conservation) condition:

$$k^\mu\mathfrak{M}_\mu^{(I)}=-k^\mu\mathfrak{M}_\mu^{(E)} \tag{13a}$$

$$\begin{aligned}&=(1+Z)[T(s_f,t)-T(s_i,t)]-2\bar{q}_f\cdot k\frac{\partial T(s_i,t,\Delta_a)}{\partial\Delta_a}\\&\quad-2q_i\cdot k\frac{\partial T(s_f,t,\Delta_b)}{\partial\Delta_b}-2\bar{p}_f\cdot kZ\frac{\partial T(s_i,t,\Delta_c)}{\partial\Delta_c}-2p_i\cdot kZ\frac{\partial T(s_f,t,\Delta_d)}{\partial\Delta_d}\\&\quad+[2(\bar{p}_f-p_i-Z\bar{q}_f+Zq_i)\cdot R+2(\bar{p}_f-p_i)\cdot k]\left[\frac{\partial T(s_i,t)}{\partial t}-\frac{\partial T(s_f,t)}{\partial t}\right].\end{aligned} \tag{13b}$$

From Eq. (13b), we obtain

$$\begin{aligned}\mathfrak{M}_\mu^{(I)}&=\frac{(q_i+p_i)_\mu}{(q_i+p_i)\cdot k}(1+Z)T(s_f,t)-\frac{(q_f+p_f)_\mu}{(q_f+p_f)\cdot k}(1+Z)T(s_i,t)\\&\quad+[2(\bar{p}_f-p_i-Z\bar{q}_f+Zq_i)\cdot N_R\bar{p}_{f\mu}+2(\bar{p}_f-p_i)_\mu]\left[\frac{\partial T(s_i,t)}{\partial t}-\frac{\partial T(s_f,t)}{\partial t}\right]-2\bar{q}_{f\mu}\frac{\partial T(s_i,t,\Delta_a)}{\partial\Delta_a}\\&\quad-2q_{i\mu}\frac{\partial T(s_f,t,\Delta_b)}{\partial\Delta_b}-2\bar{p}_{f\mu}Z\frac{\partial T(s_i,t,\Delta_c)}{\partial\Delta_c}-2p_{i\mu}Z\frac{\partial T(s_f,t,\Delta_d)}{\partial\Delta_d}.\end{aligned} \tag{14}$$

The total bremsstrahlung amplitude is then obtained by Eq. (5), i.e., to combine the expression of $\mathfrak{M}_\mu^{(I)}$ given by Eq. (14) with the expression of $\mathfrak{M}_\mu^{(E)}$ given by Eq. (12):

$$\begin{aligned}M_\mu&=\bar{u}(q_f,\nu_f)\{\mathfrak{M}_\mu^{(E)}+\mathfrak{M}_\mu^{(I)}\}u(q_i,\nu_i)\\&=\bar{u}(q_f,\nu_f)\left\{\left[\frac{q_{f\mu}}{q_f\cdot k}+\frac{Zp_{f\mu}}{p_f\cdot k}-\frac{(1+Z)(q_f+p_f)_\mu}{(q_f+p_f)\cdot k}\right]T(s_i,t)-\left[\frac{q_{i\mu}}{q_i\cdot k}+\frac{Zp_{i\mu}}{p_i\cdot k}-\frac{(1+Z)(q_i+p_i)_\mu}{(q_i+p_i)\cdot k}\right]T(s_f,t)\right.\\&\quad+\frac{\partial T(s_i,t)}{\partial t}\left[\frac{2(\bar{q}_f-q_i)\cdot R}{p_f\cdot k}Zp_{f\mu}-\frac{2(\bar{p}_f-p_i)\cdot(R+k)}{q_f\cdot k}q_{f\mu}\right.\\&\qquad\left.+2(\bar{p}_f-p_i-Z\bar{q}_f+Zq_i)\cdot N_R\bar{p}_{f\mu}+2(\bar{p}_f-p_i)_\mu\right]\\&\quad-\frac{\partial T(s_f,t)}{\partial t}\left[\frac{2(\bar{q}_f-q_i)\cdot R}{p_i\cdot k}Zp_{i\mu}-\frac{2(\bar{p}_f-p_i)\cdot(R+k)}{q_i\cdot k}q_{i\mu}\right.\\&\qquad\left.\left.+2(\bar{p}_f-p_i-Z\bar{q}_f+Zq_i)\cdot N_R\bar{p}_{f\mu}+2(p_f-p_i)_\mu\right]\right\}u(q_i,\nu_i),\end{aligned} \tag{15}$$

which is used in Eq. (4) for our calculation of bremsstrahlung cross section. Practically, however, we have used the parameters of the resonances as input to determine the amplitude $f_{\mu'\mu}(E)$ which is then used in our calculation. [See Refs. 6 and 8 for the definition of the amplitude $f_{\mu'\mu}(E)$]. This can be done by using a method already described in Ref. 6. It is obvious that if the amplitude given by Eq. (15) is used, then the bremsstrahlung cross section calculated from Eq. (4) must contain both the principal term and the correction term, and it can be written in the form given by Eq. (1).

III. RESULTS AND COMPARISON WITH EXPERIMENT

We have applied Eq. (4) to calculate the bremsstrahlung cross section in the laboratory system as a function of k. The resonance parameters used in this calculation are given in Table I. These parameters are the best-fit parameters obtained from the elastic data by Armstrong *et al.* The calculated cross sections are compared with the Bologna data.

The Brooklyn group did not measure the absolute bremsstrahlung cross section $d^3\sigma/d\Omega_q\,d\Omega_\gamma dk$. They have measured only the ratio of the bremsstrahlung cross section to the elastic cross section in the laboratory system:

TABLE I. Parameters of the resonances in ^{13}N. These parameters are from Ref. 8.

J^π	$\frac{1}{2}^+$	$\frac{3}{2}^-$	$\frac{5}{2}^+$
Reduced width γ^2 (MeV/cm $\times 10^{-13}$)	7.58	0.515	3.55
Characteristic energy E_λ (keV)	951	3510	3609

$$\sigma_{\rm rel} = (d^3\sigma/d\Omega_q d\Omega_\gamma dk)_{\rm lab}/(d\sigma/d\Omega_q)_{\rm lab} .$$

In order to compare our predictions with this measurement, we have also calculated the elastic cross section in the laboratory system $(d\sigma/d\Omega_q)_{\rm lab}$ and have used it to calculate $\sigma_{\rm rel}$.

In Figs. 2, 3, and 4, the bremsstrahlung cross sections in the laboratory system are plotted as a function of k at three bombarding energies, 1765, 1795, and 1895 keV, and they are compared with the measurements of the Bologna group. (See Ref. 4 for the detailed experimental arrangement). The bremsstrahlung spectrum at 1765 keV is shown in Fig. 2. In this figure, we present three different calculations. The solid curve represents the result of our most complete calculation which includes both the principal term and the correction term, i.e., the result obtained from Eq. (4) or Eq. (1). The dahsed curve represents the result of calculation without the correction term, i.e., the result obtained in Ref. 6 [or the result calculated from Eq. (2)]. Finally, the dotted curve represents the result of the soft-photon approximation calculated in Ref. 6. From this figure, it is clear that the observed spectrum, which is quite different from the soft-photon prediction, can be described by the Feshbach-Yennie approximation. The agreement between experiment and the Feshbach-Yennie prediction is better if both the principal and correction terms are included. In Fig. 3, we present similar calculations at 1795 keV. This is a puzzling case since the agreement between experiment and all three different calculations is very poor. The inclusion of the correction term (the solid curve) does not seem to improve the agreement between theory and experiment. Since this happens to be the only case which gives poor agreement between theory and experiment, we would support the conclusion reached by Perng *et al.* that this set of bremsstrahlung data is inconsistent with the elastic data of Refs. 8 and 9. In Fig. 4, we present three different calculations at 1895 keV. There is no structure due to resonance exhibited in the observed spectrum since the maximum photon energy measured was 100 keV and the expected structure would appear around 160 keV. As we can see from this figure, all three calculations give very similar results which are in very good agreement with the data.

In Figs. 5, 6, and 7, we show the result of $\sigma_{\rm rel}$ calculation as a function of k. Again, we have used three different approximations to calculate $\sigma_{\rm rel}$. The solid and the dashed curves represent, respectively, the prediction of the Feshbach-Yennie

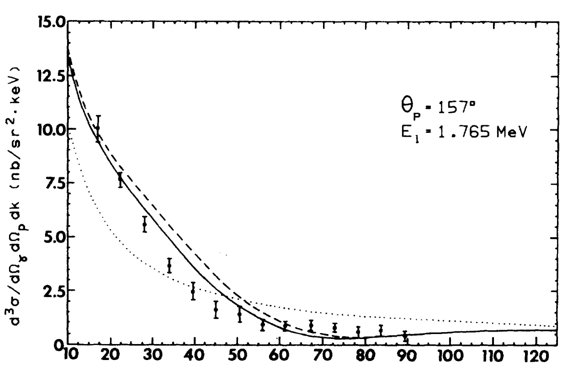

FIG. 2. The proton-carbon bremsstrahlung cross sections in the laboratory system as a function of photon energy at an incident proton energy of 1765 keV. The solid curve represents our calculation using the Feshbach-Yennie approximation which includes both the principal term and the correction term. The dashed curve represents the calculation using the principal term of the Feshbach-Yennie approximation. The dotted curve represents the calculation using the leading term of the soft-photon approximation. The experimental data are from Ref. 4.

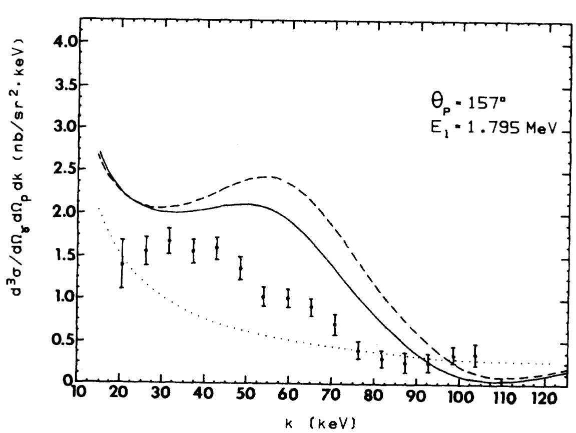

FIG. 3. Same as Fig. 2 but at an incident proton energy of 1795 keV.

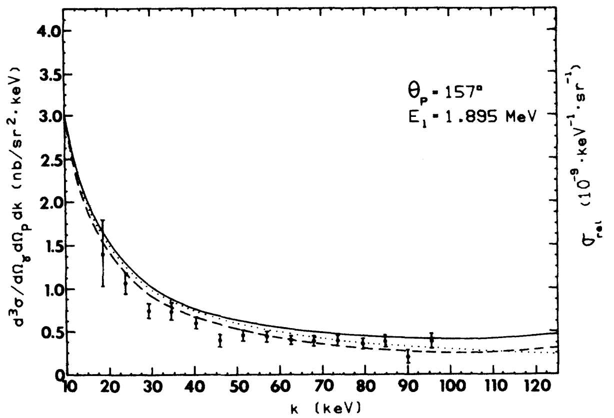

FIG. 4. Same as Fig. 2 but at an incident proton energy of 1895 keV.

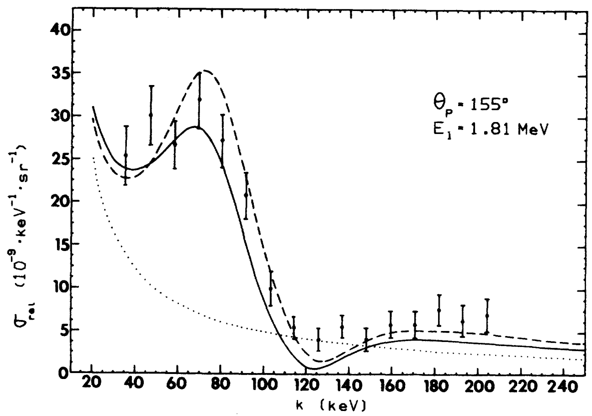

FIG. 6. Same as Fig. 5 but at an incident proton energy of 1810 keV. No dash-dotted curve is shown in this figure.

approximation with and without the correction term, and the dotted curve represents the soft-photon prediction. These curves are compared with the Brooklyn data (see Ref. 5 for the experimental details). Here, we should point out that because of the finite size of the photon detector used in the experiment, all three predictions are actually averaged over the solid angle of the photon detector. The result of our σ_{rel} calculation at 1594 keV is shown in Fig. 5. There is not any resonance in the energy region 20 keV < k < 250 keV, and all calculations give very similar results which are in very good agreement with data. Two most important comparisons between the Feshbach-Yennie prediction and the Brooklyn data are shown in Figs. 6 and 7. In these figures, the structures due to resonances are clearly observed experimentally and they can be successfully described by the Feshbach-Yennie approximation if both the principal term and the correction term are included. The contribution from the correction term is not negligible in the region of the resonance as it was usually assumed in the past. The effects of this term are twofold: It changes substantially the magnitude of the bremsstrahlung cross section near the resonance and shifts the energy of the peak thus giving good agreement with the experimental data. The agreement between the prediction calculated from a complete expression of the Feshbach-Yennie approximation and

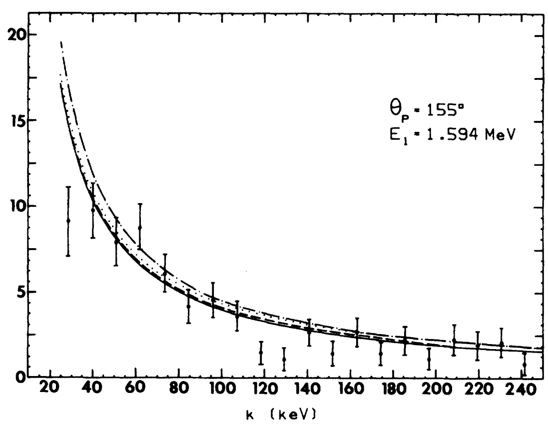

FIG. 5. The bremsstrahlung cross section relative to the elastic scattering cross section as a function of photon energy at an incident proton energy of 1594 keV The solid curve represents our result calculated from the Feshbach-Yennie approximation, which includes both the principal term and the correction term, and averaged over the solid angle of the photon detector. The dash-dotted curve represents our result calculated from the same approximation but without averaging over the solid angle of the photon detector. The dashed curve represents the result, averaged over the solid angle of the photon detector, of the calculation using the principal term of the Feshbach-Yennie approximation. The dotted curve represents the result, averaged over the solid angle of the photon detector, of the calculation using the leading term of the soft photon approximation. The experimental data are from Ref. 5.

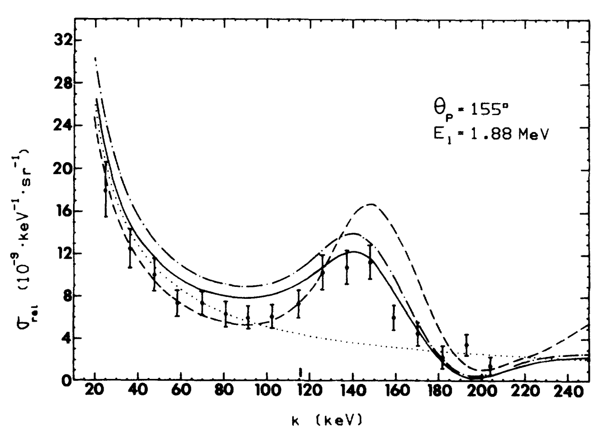

FIG. 7. Same as Fig. 5 but at an incident proton energy of 1880 keV.

the Brooklyn data is very good. As we have already mentioned, all three calculations presented in these figures are the average σ_{rel} over the solid angle of the photon detector. In order to see the effects of the finite size of the photon detector on our results, we have included in Figs. 5 and 7 a dash-dotted curve which represents the results of our complete (including both the principal and correction terms) Feshbach-Yennie prediction without averaging over the solid angle of the photon detector. The effects can be easily seen from the comparison of this curve with the solid curve in these figures.

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