# Macroscopic description of isoscalar giant multipole resonances

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On the basis of a simple macroscopic model, we calculate the isoscalar giant-resonance energy as a function of mass number and multipole degree. The restoring force is determined from the distortion of the Fermi surface, and the inertia is determined for the incompressible, irrotational flow of nucleons with unit effective mass. With no adjustable parameters, the resulting closed expression reproduces correctly the available experimental data, namely the magnitude and dependence upon mass number of the giant quadrupole energy and the magnitude of the giant octupole energy for $^{208}Pb$. We also calculate the isoscalar giant-resonance width as a function of mass number and multipole degree for various macroscopic damping mechanisms, including two-body viscosity, one-body dissipation, and modified one-body dissipation. None of these damping mechanisms reproduces correctly all features of the available experimental data, namely the magnitude and dependence upon mass number of the giant quadrupole width and the magnitude of the giant octupole width for $^{208}Pb$.

NUCLEAR STRUCTURE $^{208}Pb$; calculated isoscalar giant-resonance energy and width as functions of mass number and multipole degree and compared with experimental values. Boltzmann equation, distorted Fermi surface, irrotational flow, two-body viscosity, one-body dissipation, wall formula, multiple reflections, modified one-body dissipation, nuclear fission, heavy-ion reactions.

## I. INTRODUCTION

Following the discovery of the isoscalar giant quadrupole resonance in 1971 by Pitthan and Walcher,[1,2] the energy and width of this resonance have been measured for many nuclei throughout the Periodic Table.[3-6] Also, the energy[7-9] and width[9] of the isoscalar giant octupole resonance have been measured for $^{208}Pb$, although with greater uncertainty than for the quadrupole mode. The search is on to find the isoscalar giant octupole resonance in other nuclei, as well as to find evidence for higher multipole resonances.

Most theoretical interpretations of giant multipole resonances are based on microscopic calculations of one kind or another.[10-30] These range all the way from non-self-consistent calculations involving particles in a deformed harmonic-oscillator potential to full self-consistent calculations involving specific nucleon-nucleon interactions treated in the random-phase approximation.

Although a detailed description of giant multipole resonances requires a knowledge of the underlying nucleon-nucleon interaction, Bertsch,[31,32] Blaizot,[33] and Sagawa and Holzwarth[34] have shown that the energies of isoscalar giant quadrupole resonances can be understood on the basis of a simple macroscopic model involving the distortion of the Fermi surface. Attempts have also been made by Auerbach and Yeverechyahu[35,36] and by Hasse and Nerud[37,38] to describe the widths of giant quadrupole resonances in terms of viscous hydrodynamical flow in nuclei with rigid boundaries.

In our work, we extend the distorted-Fermi-surface macroscopic model to calculate the energies for isoscalar giant resonances of arbitrary multipole degree $n$. An isoscalar giant resonance is viewed as a small-amplitude collective oscillation of multipole degree $n$ in which the neutrons and protons undergo in-phase, incompressible, irrotational flow with unit effective mass. The nucleons remain in orbitals characterized by their original nodal structure, which introduces an anisotropy into the Fermi surface of their momentum distribution. Nucleons whose orbitals are compressed by the flow increase their energy, whereas those whose orbitals are expanded by the flow decrease their energy. The net result of this anisotropy is an increase in the total energy of the nucleus, which provides a restoring force against the oscillation. The inertia associated with the oscillation is determined for the incompressible, irrotational flow of nucleons with unit effective mass. We also calculate the widths of isoscalar giant resonances of arbitrary multipole degree $n$ for various macroscopic damping mechanisms in nuclei with deformable boundaries.

## II. GIANT-RESONANCE ENERGY

### A. Moments of the Boltzmann equation

To calculate the anisotropy in the Fermi surface introduced by a small-amplitude oscillation of multipole degree $n$, we start with the Boltzmann equation

$$\frac{D}{Dt}f(\vec{r},\vec{v},t)=\frac{\partial f}{\partial t}+v_i\,\frac{\partial f}{\partial x_i}-\frac{1}{m}\,\frac{\partial U}{\partial x_i}\,\frac{\partial f}{\partial v_i}=I(f)\,. \tag{1}$$

The distribution function $f(\vec{r},\vec{v},t)$ specifies the probability for finding a nucleon moving with velocity $\vec{v}$ at position $\vec{r}$ and time $t$. It is normalized so that the nucleon number density $n(\vec{r},t)$ is given by

$$n(\vec{r},t)=\int f d^3v\,.$$

(Use of the same symbol $n$ to denote multipole degree and nucleon number density should lead to no confusion.) The collision integral $I(f)$ specifies the time rate of change of the distribution function arising from collisions between nucleons, $U(\vec{r})$ is the single-nucleon potential, and $m$ is the mass of the bound nucleon. In Eq. (1) and succeeding equations, until otherwise noted, we use the convention that repeated indices are summed over. We note that it is possible to derive, from the Schrödinger equation, equations of similar form to Eq. (1) for the Wigner function or the $n$-body density matrix.[32,39-41]

The mean velocity in the $i$th direction is

$$u_i(\vec{r},t)=\frac{1}{n}\int f v_i d^3v\,,$$

and the $ij$ component of the pressure tensor is

$$P_{ij}(\vec{r},t)=m\int f w_i w_j d^3v\,,$$

where

$$w_i=v_i-u_i\,.$$

The equations satisfied by $n$, $u_i$, and $P_{ij}$ are obtained by taking successive velocity moments[32,42-44] of Eq. (1). This leads to the equation of continuity

$$\frac{\partial n}{\partial t}+\frac{\partial}{\partial x_i}(nu_i)=0\,, \tag{2}$$

the conservation-of-momentum equation

$$\frac{\partial}{\partial t}(mnu_i)+\frac{\partial}{\partial x_j}(mnu_iu_j+P_{ij})+n\,\frac{\partial U}{\partial x_i}=0\,, \tag{3}$$

and the pressure-tensor equation

$$\frac{\partial P_{ij}}{\partial t}+\frac{\partial}{\partial x_k}(P_{ij}u_k)+P_{ik}\,\frac{\partial u_j}{\partial x_k}+P_{jk}\,\frac{\partial u_i}{\partial x_k}+m\,\frac{\partial}{\partial x_k}\int f w_i w_j w_k d^3v=m\int I(f)v_iv_jd^3v\,. \tag{4}$$

It is convenient to write

$$P_{ij}=\tfrac{1}{3}mn_0\langle v^2\rangle_0\delta_{ij}+\kappa_{ij}\,, \tag{5}$$

where $\kappa_{ij}$ is the deviation of the pressure tensor from its isotropic, diagonal value for stationary nuclear matter at equilibrium density $n_0$. The mean-square velocity $\langle v^2\rangle_0$ is related to the Fermi velocity $v_F$ at equilibrium density by

$$\langle v^2\rangle_0=\tfrac{3}{5}v_F{}^2\,.$$

Substitution of Eq. (5) into Eq. (4) leads to

$$\frac{\partial\kappa_{ij}}{\partial t}+\frac{\partial}{\partial x_k}(\kappa_{ij}u_k)+\kappa_{ik}\,\frac{\partial u_j}{\partial x_k}+\kappa_{jk}\,\frac{\partial u_i}{\partial x_k}+\tfrac{1}{3}mn_0\langle v^2\rangle_0\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}+\delta_{ij}\,\frac{\partial u_k}{\partial x_k}\right)+m\,\frac{\partial}{\partial x_k}\int f w_iw_jw_kd^3v=m\int I(f)\,v_iv_jd^3v\,, \tag{6}$$

which is still an exact equation.

### B. Specialization to incompressible flow

For our present application to isoscalar giant resonances, we consider small-amplitude collective oscillations in which the neutrons and protons undergo in-phase, incompressible, irrotational flow with unit effective mass. This type of flow pattern is expected to arise when the nucleons remain in orbitals characterized by their original nodal structure.[45]

It follows from Eq. (2) that for incompressible flow,

$$\partial u_i/\partial x_i=0\,. \tag{7}$$

We use this result and neglect the three terms in Eq. (6) that are second order in the amplitude of the excitation, as well as the terms involving the third moment and the collision integral. This leads to the simplified equation[32]

$$\frac{\partial\kappa_{ij}}{\partial t}=-\tfrac{1}{3}mn_0\langle v^2\rangle_0\left(\frac{\partial u_i}{\partial x_j}+\frac{\partial u_j}{\partial x_i}\right), \tag{8}$$

which is valid only for incompressible, small-amplitude flow without collisions between nucleons.

For describing small multipole distortions about a spherical shape, we specify the nuclear surface by means of the equation

$$r=R(\theta)=(R_0/\lambda)[1+\alpha_n P_n(\cos\theta)]\,,$$

where

$$R_0=r_0A^{1/3}$$

is the equivalent-sharp-surface radius of the spherical nucleus. The quantity $\lambda$ is determined in terms of the coordinates $\alpha_n$ by requiring that the volume remain constant; to first order it is unity.

We write the displacement $dx_i$ of matter at a point $\vec{r}$ inside the nucleus as

$$dx_i = a_i^n(\vec{r})d\alpha_n \,, \tag{9}$$

or, alternatively, the mean velocity as

$$u_i = a_i^n(\vec{r})d\alpha_n/dt \,. \tag{10}$$

Upon substituting this result into Eq. (8), we find that for small distortions from an initially spherical shape,[32,39]

$$\kappa_{ij} = -\tfrac{1}{3} m n_0 \langle v^2\rangle_0 \left(\frac{\partial a_i^n}{\partial x_j} + \frac{\partial a_j^n}{\partial x_i}\right)\alpha_n \,. \tag{11}$$

### C. Stiffness coefficient

The increase in energy $E$ of the system associated with small distortions from an initially spherical shape is

$$E = -\int F_i dx_i d^3r \,, \tag{12}$$

where $F_i$ is the force per unit volume in the $i$th direction. According to Eq. (3),

$$F_i = -\frac{\partial P_{ij}}{\partial x_j} - n\frac{\partial U}{\partial x_i} \,. \tag{13}$$

After being substituted into Eq. (12), the second term of this equation leads to the Coulomb, surface, and single-particle energies. These are small compared to the distorted-Fermi-surface energy that results from the first term and are consequently neglected here. This distorted-Fermi-surface restoring force is the same as the nuclear elastic force introduced by Bertsch.[32]

Substitution of Eq. (9) and the first term of Eq. (13) into Eq. (12) yields

$$E = \int \frac{\partial P_{ij}}{\partial x_j} a_i^n d^3r\, \alpha_n \,,$$

which becomes, after an integration by parts,

$$E = -\int P_{ij}\frac{\partial a_i^n}{\partial x_j} d^3r\, \alpha_n \,.$$

For incompressible flow, substitution of Eq. (5) and use of Eqs. (7) and (10) leads to

$$E = -\int \kappa_{ij}\frac{\partial a_i^n}{\partial x_j} d^3r\, \alpha_n \,.$$

After substitution of Eq. (11), this becomes finally

$$E = \tfrac{1}{3} m n_0\langle v^2\rangle_0 \int \frac{\partial a_i^n}{\partial x_j}\left(\frac{\partial a_i^n}{\partial x_j} + \frac{\partial a_j^n}{\partial x_i}\right) d^3r\, \alpha_n^{\,2} \,.$$

The stiffness coefficient associated with an oscillation of degree $n$ is

$$C_n = \frac{d^2E}{d\alpha_n^{\,2}} = \tfrac{2}{3} m n_0\langle v^2\rangle_0 \int \frac{\partial a_i^n}{\partial x_j}\left(\frac{\partial a_i^n}{\partial x_j} + \frac{\partial a_j^n}{\partial x_i}\right) d^3r \,. \tag{14}$$

In this equation and succeeding equations, there is no summation over $n$.

For incompressible, irrotational, small oscillations about a spherical shape,[46]

$$a_i^n = \frac{1}{nR_0^{n-2}}\frac{\partial}{\partial x_i}[r^n P_n(\cos\theta)] \,.$$

For this type of flow, Eq. (14) can be readily evaluated because the integral appearing in it is identical to the one evaluated in calculating the Rayleigh dissipation function for ordinary two-body viscosity.[47-50] The result is

$$C_n = \frac{8\pi(n-1)}{3n} m n_0 \langle v^2\rangle_0 R_0^{\,3} = \frac{12}{5}\frac{(n-1)}{n} A E_F$$

where

$$E_F = \tfrac{1}{2} m v_F^{\,2} = \frac{(9\pi)^{2/3}\hbar^2}{8 m r_0^{\,2}}$$

is the Fermi energy, with $\hbar$ denoting Planck's constant divided by $2\pi$.

### D. Inertia and energy

For incompressible, irrotational, small oscillations about a spherical shape, the inertia associated with an oscillation of degree $n$ is[46]

$$M_n = \frac{3}{n(2n+1)} M_0 R_0^{\,2} \,, \tag{15}$$

where

$$M_0 = Am$$

is the total mass of the nucleus.

The energy of the isoscalar giant resonance of multipole degree $n$ is therefore

$$E_n = \hbar\left(\frac{C_n}{M_n}\right)^{1/2} = \frac{2\hbar}{r_0}\left(\frac{E_F}{5m}\right)^{1/2}\frac{[(n-1)(2n+1)]^{1/2}}{A^{1/3}} = \frac{(9\pi)^{1/3}}{\sqrt{10}m}\left(\frac{\hbar}{r_0}\right)^2\frac{[(n-1)(2n+1)]^{1/2}}{A^{1/3}} \,. \tag{16}$$

For the values of the three constants that appear, we use[51,52]

$$r_0 = 1.18\ \text{fm} \,,$$

$$\hbar = 197.32858\ \text{MeV fm}/c \,,$$

and

$$m = 931.5016\ \text{MeV}/c^2 \,,$$

where $c$ is the speed of light. Insertion of these values into Eq. (16) leads to

$$E_n = 28.92\ [(n-1)(2n+1)]^{1/2} A^{-1/3}\ \text{MeV}\,.$$

For the isoscalar giant quadrupole, octupole, and hexadecapole resonances, this gives

$$E_2 = 64.7\ A^{-1/3}\ \text{MeV}\,, \qquad (17)$$

$$E_3 = 108.2\ A^{-1/3}\ \text{MeV}\,,$$

and

$$E_4 = 150.3\ A^{-1/3}\ \text{MeV}\,.$$

Our result for the giant quadrupole energy has been obtained previously by Bertsch.[31,32] Although the general result derived in Ref. 32 gives the correct value for $n=2$, the dependence upon multipole degree calculated there is incorrect. In an earlier paper,[53] the correct dependence upon multipole degree was obtained, but an overall factor of $\sqrt{2}$ was missing from the energy expression, which led to the erroneous conclusion that calculated giant quadrupole energies are 30% lower than experimentally observed values.

### E. Comparison with experiment

The above predictions for the energies of the isoscalar giant quadrupole, octupole, and hexadecapole resonances are shown as functions of mass number in Fig. 1. The solid points give experimental values for the giant quadrupole energy[4,5] and are to be compared with the solid curve. The calculations reproduce, with no adjustable parameters, both the magnitude and dependence upon mass number of the giant quadrupole energy. However, for light nuclei, the theoretical curve lies slightly above the experimental points, and the calculated proportionality factor of 64.7 MeV in Eq. (17) is 3% larger than the experimental value[4,5] of 63 MeV.

The three open points in Fig. 1 give experimental values for the giant octupole energy[7-9] for $^{208}$Pb and are to be compared with the dashed curve. The calculated value reproduces, with no adjustable parameters, the giant octupole energy for $^{208}$Pb to within the experimental uncertainty.

## III. GIANT-RESONANCE WIDTH

We now calculate the isoscalar giant-resonance width as a function of mass number and multipole degree for various macroscopic damping mechanisms, including two-body viscosity, one-body dissipation, and modified one-body dissipation. While our goal is to discriminate between these possible mechanisms, we must bear in mind from the outset that the small-amplitude oscillations involved in isoscalar giant resonances are different from the large-scale motion involved in fission and heavy-ion reactions, as well as from ordinary

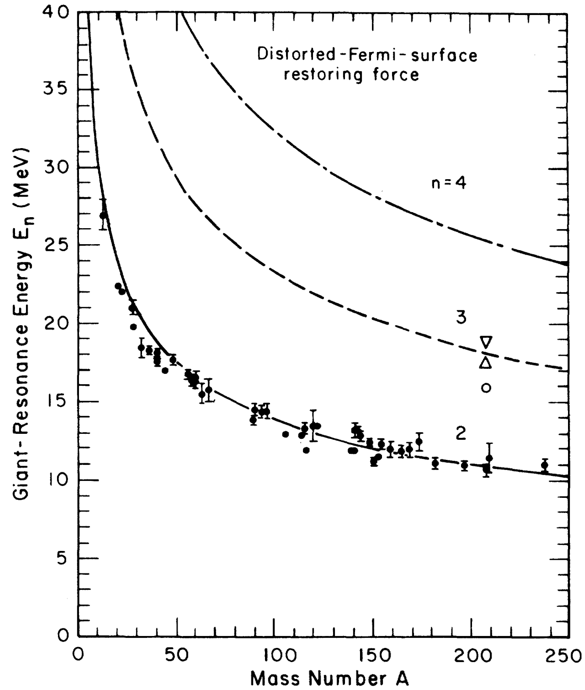


FIG. 1. Comparison of experimental isoscalar giant-resonance energies with values calculated in a distorted-Fermi-surface macroscopic model. The solid circles give experimental values of giant quadrupole energies (Refs. 4 and 5), and the open points give experimental values of the giant octupole energy for $^{208}$Pb. The downward-pointing triangle is taken from Ref. 7, the upward-pointing triangle is taken from Ref. 8, and the open circle is taken from Ref. 9.

quadrupole and octupole vibrations.

In the harmonic approximation, isoscalar giant multipole resonances satisfy the equations of motion

$$M_n \ddot{\alpha}_n + \eta_n \dot{\alpha}_n + C_n \alpha_n = 0\,,$$

where $\eta_n$ is the damping coefficient. Since the width for a giant resonance of multipole degree $n$ is given by

$$\Gamma_n = \hbar \eta_n / M_n\,, \qquad (18)$$

our task is to evaluate $\eta_n$ for the various damping mechanisms.

### A. Two-body viscosity

For ordinary two-body viscosity, Auerbach and Yeverechyahu[35,36] as well as Hasse and Nerud[37,38] have calculated the widths of both isoscalar and isovector giant quadrupole resonances under the assumption that the boundary of the nucleus is rigid. Although their approach takes into account the deviation of the hydrodynamical flow from irrotational flow that is caused by viscosity, the assumption of a rigid boundary is inappropriate for our macroscopic model.

We therefore calculate the widths for nuclei with deformable boundaries, under the assumption of incompressible, irrotational, small-amplitude nuclear flow. The adequacy of this approximation is discussed in the appendix of Ref. 49. For this type of flow, the damping coefficient is given by[47-50]

$$\eta_n^{2\text{-body}} = \frac{8\pi(n-1)}{n} R_0^{\,3}\mu ,$$

where $\mu$ is the ordinary two-body viscosity coefficient. Upon substituting this result and Eq. (15) into Eq. (18), we find that

$$\Gamma_n^{2\text{-body}} = \frac{8\pi\hbar r_0 \mu(n-1)(2n+1)}{3mA^{2/3}} . \tag{19}$$

The value of the two-body viscosity coefficient that is obtained from comparisons of calculated and experimental fission-fragment kinetic energies, when account is taken of the rupture of the neck at a finite radius, is[54]

$$\mu = 0.03 \pm 0.01 \text{ TP},$$

where

$$1 \text{ TP} = 10^{12} \text{ dyn s/cm}^2$$
$$= 6.24\times 10^{-22} \text{ MeV s/fm}^3 .$$

Insertion of this value, along with the values of the other constants given in Sec. II, into Eq. (19) yields

$$\Gamma_n^{2\text{-body}} = 11.76(n-1)(2n+1)A^{-2/3} \text{ MeV} .$$

For ordinary two-body viscosity the integrals that appear in the stiffness coefficient and the damping coefficient are identical. Therefore, for this particular damping mechanism, the width of a resonance is related to its energy according to

$$\Gamma_n^{2\text{-body}} = \frac{80(9\pi)^{1/3} m r_0^{\,5}\mu E_n^{\,2}}{27\hbar^3}$$
$$= 0.01405 E_n^{\,2}/\text{MeV} .$$

The above predictions for the isoscalar giant quadrupole, octupole, and hexadecapole widths are shown as functions of mass number in Fig. 2. The solid points give experimental values for the giant quadrupole width[4,5] and are to be compared with the solid curve. The calculated values are somewhat smaller than the experimental values, except for a few light nuclei, and the calculated dependence upon mass number is more rapid than is observed experimentally.

The open point in Fig. 2 gives the experimental value for the giant octupole width[9] for $^{208}$Pb and

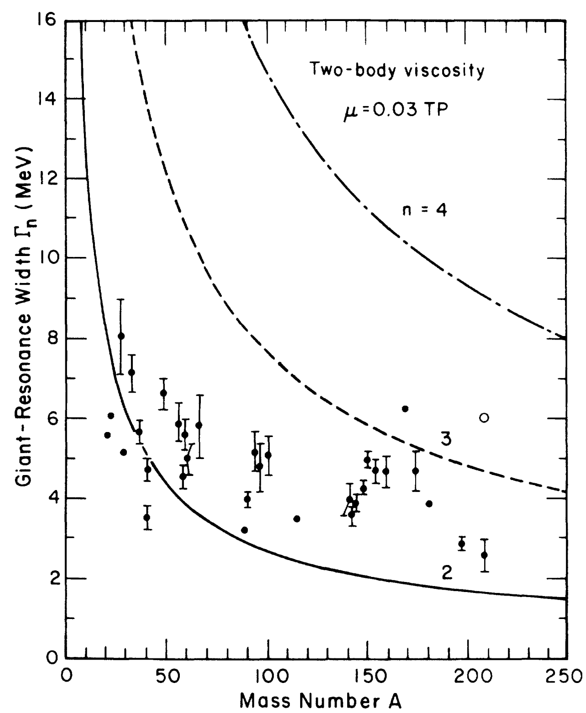


FIG. 2. Comparison of experimental isoscalar giant-resonance widths with values calculated for an ordinary two-body viscosity coefficient $\mu = 0.03$ TP. The solid circles give experimental values of giant quadrupole widths (Refs. 4 and 5), and the open circle gives the experimental value of the giant octupole width for $^{208}$Pb (Ref. 9).

is to be compared with the dashed curve. The calculated value is somewhat smaller than the experimental value.

### B. One-body dissipation

Because of the Pauli exclusion principle, the mean free path of nucleons inside a nucleus at low excitation energy is long compared to the nuclear radius, which invalidates the conditions that are necessary for the applicability of ordinary two-body viscosity. An alternative damping mechanism is one-body dissipation, which arises from collisions of nucleons with the moving nuclear surface.[55,56]

Under the assumption that the velocity distribution of nucleons striking the moving nuclear surface is completely randomized, Swiatecki and his colleagues have derived a wall formula, in which the rate of energy dissipation is proportional to the integral over the nuclear surface of the square of the normal velocity of the surface.[55,56] The corresponding damping coefficient is[50,55]

$$\eta_n^{\mathrm{wall}} = \frac{3\pi}{(2n+1)} m n_0 v_F R_0^4$$
$$= \frac{9(9\pi)^{1/3}\hbar A^{4/3}}{8(2n+1)}\,.$$

Upon substituting this result and Eq. (15) into Eq. (18), we obtain

$$\Gamma_n^{\mathrm{wall}} = \frac{3(9\pi)^{1/2}\hbar^2 n}{8 m r_0^2 A^{1/3}}\,.$$

This becomes, after insertion of the values of the constants given in Sec. II,

$$\Gamma_n^{\mathrm{wall}} = 34.3 n A^{-1/3}\ \mathrm{MeV}.$$

The resulting predictions for the giant quadrupole and octupole widths are shown in Fig. 3, along with the experimental values.[4,5,9] By comparing the solid curve with the solid circles (giant quadrupole) and the dashed curve with the open circle (giant octupole), we see that for both resonances the wall-formula predictions are about 3.0 times as large as the experimental values.

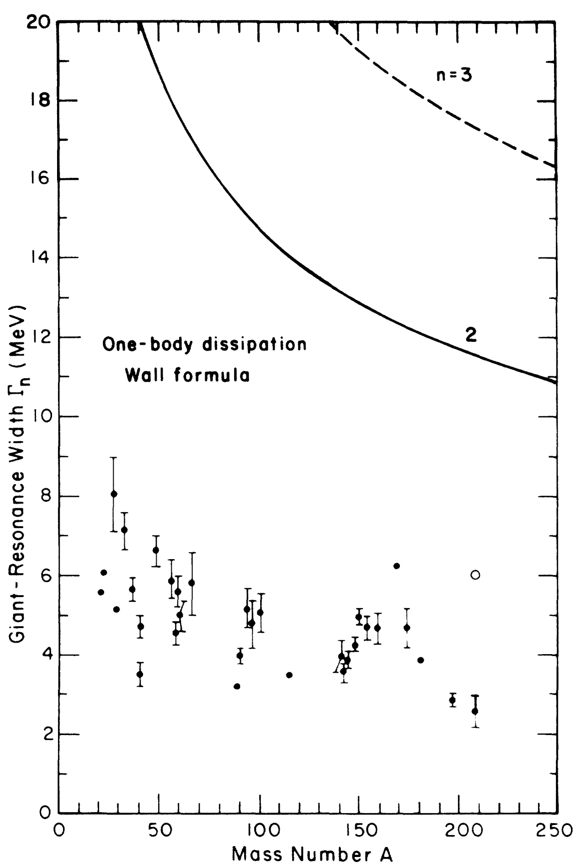


FIG. 3. Comparison of experimental isoscalar giant-resonance widths with values calculated for one-body dissipation by use of the wall formula. The experimental points are the same as those in Fig. 2.

It has been recognized from the outset that for shapes with high symmetry, such as quadrupole shapes, the randomization assumption breaks down and the wall formula predicts too much dissipation.[55,56] Koonin and Randrup have taken into account the symmetry of the nuclear shape, in the limit of zero-frequency oscillations, by following the multiple reflections of nucleons inside the nucleus.[57] According to their calculations, the effect of multiple reflections is to simply multiply the wall formula prediction for a given multipole degree $n$ by a factor $f_n$. For quadrupole, octupole, and hexadecapole oscillations, these factors are[57]

$$f_2 = 0.00\,,$$

$$f_3 = 0.85\,,$$

and

$$f_4 = 0.45\,.$$

The corresponding predictions are shown in Fig. 4, along with the experimental values for the two lowest modes.[4,5,9] By comparing the solid line at zero with the solid circles (giant quadrupole) and the dashed curve with the open circle (giant octupole), we see that the inclusion of multi-

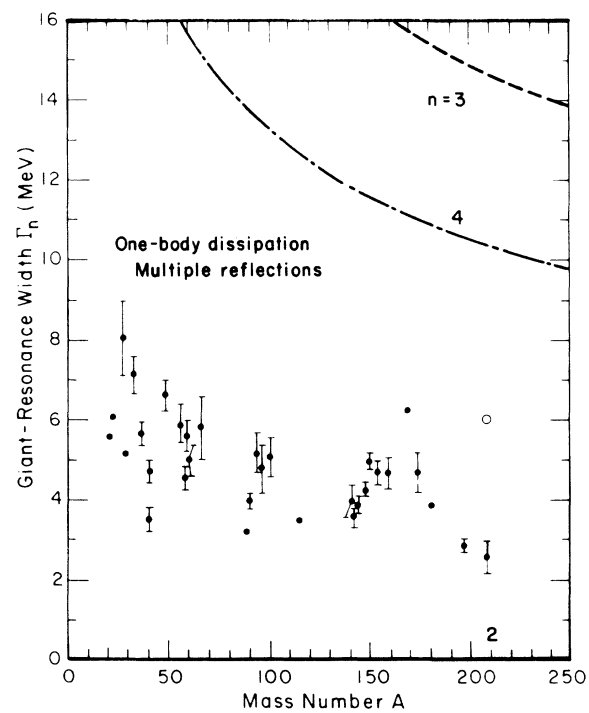


FIG. 4. Comparison of experimental isoscalar giant-resonance widths with values calculated for one-body dissipation by taking into account the multiple reflections of nucleons inside the nucleus, in the limit of zero-frequency oscillations. The experimental points are the same as those in Fig. 2.

ple reflections in the wall formula, in the limit of zero-frequency oscillations, does not reconcile the discrepancies. The calculated giant quadrupole widths are reduced to zero, and the calculated giant octupole width for $^{208}$Pb is now 2.5 times the experimental value.

### C. Modified one-body dissipation

We calculate finally, for completeness, the widths corresponding to a modified one-body dissipation that attempted to incorporate self-consistency[50] into the wall formula. For modified one-body dissipation and incompressible, irrotational flow, the damping coefficient is[50]

$$\eta_n^{\text{modified}} = \frac{3\pi(n-1)^2}{(2n+1)} m n_0 v_F \lambda^2 R_0^{\,2} = \frac{9(9\pi)^{1/3}(n-1)^2\hbar\lambda^2 A^{2/3}}{8(2n+1)r_0^{\,2}},$$

where the effective distance $\lambda$ specifies the magnitude of the dissipation. Upon substituting this result and Eq. (15) into Eq. (18), we obtain

$$\Gamma_n^{\text{modified}} = \frac{3(9\pi)^{1/3}(n-1)^2 n\hbar^2\lambda^2}{8m r_0^{\,4} A}$$

The value of $\lambda^2$ that is obtained from comparisons of calculated and experimental fission-fragment kinetic energies, when account is taken of the rupture of the neck at a finite radius, is[50]

$$\lambda^2 = 3 \pm 1 \text{ fm}^2 .$$

Insertion of this value, along with the values of the other constants given in Sec. II, leads to

$$\Gamma_n^{\text{modified}} = 73.9(n-1)^2 n A^{-1} \text{ MeV} .$$

The resulting predictions for the widths of the isoscalar giant quadrupole, octupole, and hexadecapole resonances are shown in Fig. 5, along with the experimental values for the two lowest modes.[4,5,9] By comparing the solid curve with the solid circles, we see that the calculated quadrupole widths are smaller than the experimental values and that the calculated dependence upon mass number is significantly more rapid than is observed experimentally. By comparing the dashed curve with the open circle, we see that the calculated octupole width is smaller than the experimental value.

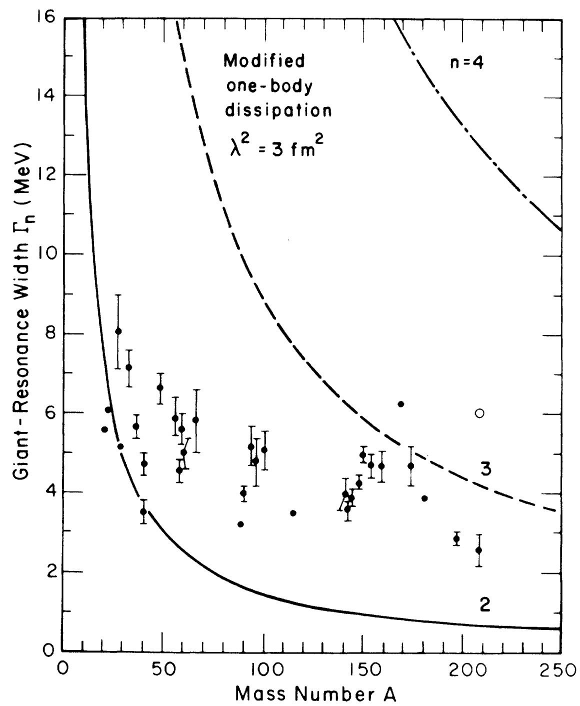


FIG. 5. Comparison of isoscalar giant-resonance widths with values calculated for a modified one-body dissipation coefficient $\lambda^2 = 3$ fm$^2$. The experimental points are the same as those in Fig. 2.

## IV. SUMMARY AND CONCLUSION

We have calculated the isoscalar giant-resonance energy as a function of mass number and multipole degree by use of a simple macroscopic model that takes into account the distortion of the Fermi surface. With no adjustable parameters, the resulting closed expression reproduces correctly the available experimental data, but these data are limited to giant quadrupole energies for nuclei throughout the Periodic Table and to the giant octupole energy for $^{208}$Pb. Clearly, additional experimental measurements that include the isoscalar giant octupole resonance in other nuclei, as well as higher multipole resonances, are needed to further test the predictions of the model.

We have also calculated the isoscalar giant-resonance width for various macroscopic damping mechanisms that are currently being studied in nuclear fission and heavy-ion reactions. The calculated dependence of the width upon mass number and multipole degree is substantially different for each of the mechanisms that we considered. The giant quadrupole and octupole widths calculated for ordinary two-body viscosity are somewhat smaller than the experimental values, and the calculated dependence upon mass number of the giant quadrupole width is more rapid than is observed experimentally.

The giant quadrupole and octupole widths calculated for one-body dissipation by use of the wall formula are about 3.0 times the experimental values. The inclusion of multiple reflections, in the limit of zero-frequency oscillations, does not reconcile the discrepancies, since the calculated giant quadrupole widths are reduced to zero and the calculated giant octupole width for $^{208}$Pb is now 2.5 times the experimental value. Finally, the giant quadrupole and octupole widths calculated for a modified one-body dissipation are smaller than the experimental values, and the calculated dependence upon mass number of the giant quadrupole width is significantly more rapid than is observed experimentally.

While none of the macroscopic damping mechanisms that we considered reproduce all features of the experimental widths, we must bear in mind once again that the small-amplitude oscillations involved in isoscalar giant resonances are different from the large-scale motion involved in fission and heavy-ion reactions. Clearly, further theoretical work is required before we can claim to understand the mechanism of nuclear damping.

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[1]R. Pitthan and T. Walcher, Phys. Lett. 36B, 563 (1971).
[2]R. Pitthan and T. Walcher, Z. Naturforsch. 27A, 1683 (1972).
[3]G. R. Satchler, Phys. Rep. 14, 97 (1974).
[4]G. R. Satchler, in *Proceedings of the International School of Physics "Enrico Fermi," Varenna, Italy, 1976*, Course LXIX, *Elementary Modes of Excitation in Nuclei* (Societa Italiana di Fisica, Bologna, 1977), p. 271.
[5]F. E. Bertrand, Annu. Rev. Nucl. Sci. 26, 457 (1976).
[6]R. Pitthan, H. Hass, D. H. Meyer, F. R. Buskirk, and J. N. Dyer, Phys. Rev. C 19, 1251 (1979).
[7]M. Nagao and Y. Torizuka, Phys. Rev. Lett. 30, 1068 (1973).
[8]R. Pitthan, F. R. Buskirk, E. B. Dally, J. N. Dyer, and X. K. Maruyama, Phys. Rev. Lett. 33, 849 (1974).
[9]M. Sasao and Y. Torizuka, Phys. Rev C 15, 217 (1977).
[10]B. R. Mottelson, in *Proceedings of the International Conference on Nuclear Structure, Kingston, Canada, 1960*, edited by D. A. Bromley and E. W. Vogt (University of Toronto Press, Toronto, Canada, 1960), p. 525.
[11]A. Bohr and B. R. Mottelson, Phys. Scr. 10A, 13 (1974).
[12]A. Bohr and B. R. Mottelson, *Nuclear Structure*, (Benjamin, Reading, 1975), Vol. II, pp. 507–512.
[13]B. R. Mottelson, Rev. Mod. Phys. 48, 375 (1976).
[14]T. Suzuki, Nucl. Phys. A217, 182 (1973).
[15]T. Suzuki and D. J. Rowe, Nucl. Phys. A292, 93 (1977).
[16]G. F. Bertsch, Phys. Rev. Lett. 31, 121 (1973).
[17]G. F. Bertsch and S. F. Tsai, Phys. Lett. 50B, 319 (1974).
[18]G. F. Bertsch and S. F. Tsai, Phys. Rep. 18, 125 (1975).
[19]S. Shlomo and G. F. Bertsch, Nucl. Phys. A243, 507 (1975).
[20]G. R. Hammerstein, H. McManus, A. Moalem, and T. T. S. Kuo, Phys.Lett. 49B, 235 (1974).
[21]D. H. Jakubassa, Z. Phys. 268, 409 (1974).
[22]P. Ring and J. Speth, Nucl. Phys. A235, 315 (1974).
[23]S. Krewald and J. Speth, Phys. Lett. 52B, 295 (1974).
[24]S. Krewald, J. Birkholz, A. Faessler, and J. Speth, Phys. Rev. Lett. 33, 1386 (1974).
[25]D. Zawischa and J. Speth, Phys. Rev. Lett. 36, 843 (1976).
[26]A. Faessler, in *Proceedings of the International Conference on Selected Topics in Nuclear Structure, Dubna, USSR, 1976*, Joint Institute for Nuclear Research Report No. JINR-D-9920, 1976 (unpublished), Vol. II, p. 242.
[27]H. Flocard and D. Vautherin, Phys. Lett. 55B, 259 (1975).
[28]J. Martorell, O. Bohigas, S. Fallieros, and A. M. Lane, Phys. Lett. 60B, 313 (1976).
[29]M. Golin and L. Zamick, Nucl. Phys. A249, 320 (1975).
[30]L. Zamick, in Proceedings of the Sendai Conference on Electro- and Photo-Excitations, Sendai, Japan, 1977, Tohoku University Laboratory of Nuclear Science Research report, Vol. 10, Supplement, 1977 (unpublished), p. 85.
[31]G. F. Bertsch, Nucl. Phys. A249, 253 (1975).
[32]G. F. Bertsch, in *Nuclear Physics with Heavy Ions and Mesons*, 1977 Les Houches Lectures, edited by R. Balian, M. Rho, and G. Ripka (North-Holland, Amsterdam, 1978), Vol. 1, p. 175.
[33]J. P. Blaizot, Phys. Lett. 78B, 367 (1978).
[34]H. Sagawa and G. Holzwarth, Prog. Theor. Phys. 59, 1213 (1978).
[35]N. Auerbach and A. Yeverechyahu, Ann. Phys. (N. Y.) 95, 35 (1975).
[36]N. Auerbach and A. Yeverechyahu, Phys. Lett. 62B, 143 (1976).
[37]R. W. Hasse and P. Nerud, J. Phys. G 2, L101 (1976).
[38]R. W. Hasse, Rep. Prog. Phys. 41, 1027 (1978).
[39]C. Y. Wong and J. A. McDonald, Phys. Rev. C 16, 1196 (1977).
[40]G. Holzwarth and G. Eckart, Z. Phys. A284, 291 (1978)
[41]E. A. Remler, Ann. Phys. (N. Y.) 119, 326 (1979).
[42]K. Huang, *Statistical Mechanics* (Wiley, New York, 1963), pp. 95–99.
[43]F. H. Harlow and A. A. Amsden, Los Alamos Scientific Laboratory Report No. LA-4700, 1971 (unpublished).
[44]S. E. Koonin, Ph. D. thesis, Massachusetts Institute of Technology, 1975 (unpublished).
[45]P. Möller and J. R. Nix, Nucl. Phys. A296, 289 (1978).
[46]J. R. Nix, Ann. Phys. (N. Y.) 41, 52 (1967).
[47]J. Schirmer, S. Knaak, and G. Süssmann, Nucl. Phys.

A199, 31 (1973).
[48]R. W. Hasse, Ann Phys. (N. Y.) 93, 68 (1975).
[49]K. T. R. Davies, A. J. Sierk, and J. R. Nix, Phys. Rev. C 13, 2385 (1976).
[50]A. J. Sierk, S. E. Koonin, and J. R. Nix, Phys. Rev. C 17, 646 (1978).
[51]W. D. Myers, At. Data Nucl. Data Tables 17, 411 (1976).
[52]C. Bricman, C. Dionisi, R. J. Hemingway, M. Mazzucato, L. Montanet, N. Barash-Schmidt, R. C. Crawford, M. Roos, A. Barbaro-Galtieri, C. P. Horne, R. L. Kelly, M. J. Losty, A. Rittenberg, T. G. Trippe, G. P. Yost, and B. Armstrong, Phys. Lett. 75B, i (1978).
[53]G. F. Bertsch, Ann. Phys. (N. Y.) 86, 138 (1974).
[54]K. T. R. Davies, R. A. Managan, J. R. Nix, and A. J. Sierk, Phys. Rev. C 16, 1890 (1977).
[55]J. Błocki, Y. Boneh, J. R. Nix, J. Randrup, M. Robel, A. J. Sierk, and W. J. Swiatecki, Ann. Phys. (N. Y.) 113, 330 (1978).
[56]J. Randrup and W. J. Swiatecki, Nordisk Institut for Teoretisk Atomfysik Report No. NORDITA-78/38, 1978 (unpublished).
[57]S. E. Koonin and J. Randrup, Nucl. Phys. A289, 475 (1977).