PHYSICAL REVIEW C VOLUME 21, NUMBER 1 JANUARY 1980

Some convergence tests on medium-energy pion-deuteron elastic scattering amplitudes

A. S. Rinat and Y. Starkand
Weizmann Institute of Science, Rehovot, Israel
(Received 26 February 1979)

With the exact solution of a covariant multiple scattering model for elastic pion-deuteron scattering as a standard, we test (a) the convergence of the multiple scattering series and (b) the sufficiency of the first order and on-shell second order scattering amplitudes.

[NUCLEAR REACTIONS πd elastic scattering; convergence tests of several expansions.]

I. INTRODUCTION

The major part of pion-deuteron elastic amplitudes for energies $T_\pi \lesssim 250$ MeV is adequately described by multiple scattering (MS) theories.[1,2] Beyond that energy range it becomes extremely difficult to exactly compute that MS component of the amplitude, and approximations are called for.

Mainly because of its simplicity the Glauber approach has been a frequently used tool. Regarding elastic πd scattering, Carlson[3] ($T_\pi < 142$ MeV), Gabathuler and Wilkin[4] ($T_\pi = 256$ MeV), and Hoenig and Rinat[5] analyzing the new Virginia-LAMPF data[6] ($515 \gtrsim T_\pi$ (MeV) $\gtrsim 230$) observed agreement in the forward hemisphere and reproduced at least the correct order of magnitude for the large-angle cross sections. In the following we scrutinize the observed agreement.

The literature contains a number of investigations relevant to our topic. Most prominent is the work by Harrington,[7] who has shown how in the eikonal limit ($E \to \infty$, $q \to 0$) a complicated MS amplitude tends to the simple Glauber expression, i.e.,

$$F_d = \sum_n F^{(n)} \xrightarrow[\substack{E\to\infty \\ q\to 0}]{} (F^{(1)} + F^{(2)\text{on}})^{\text{Gl}} \sim (F^{(1)} + F^{(1)} G^{\text{on}} F^{(1)})^{\text{Gl}} . \qquad (1)$$

The only surviving terms are thus the single scattering amplitude $F^{(1)}$ and that part of $F^{(2)}$, where in the intermediate propagator G the system remains on the energy shell (conserves energy). Eikonal limits for both terms in (1) are implied by the superscript Gl. Alternatively one may write

$$F^{(2)\text{off}} + \sum_{n\geq 3} F^{(n)} \xrightarrow[\substack{\text{eikonal} \\ \text{limit}}]{} 0 . \qquad (2)$$

Equation (2) thus implies that in the eikonal limit the off-shell double scattering term either cancels the rest series $\sum_{n\geq 3} F^{(n)}$ or that each part tends to zero individually.

For πd scattering $T_\pi \lesssim 500$ MeV, one is obviously outside the eikonal limit for essentially all angles. The fits obtained by Carlson[3] and Hoenig and Rinat[5] thus imply that the following truncation of the MS series apparently suffices:

$$F^{\text{MS}} \approx F^{\text{tr}} = F^{(1)} + F^{(2)\text{on}} = 2\langle \phi_d | f_{\pi N} | \phi_d \rangle + 2\langle \phi_d | f_{\pi N} G^{\text{on}} f_{\pi N} | \phi_d \rangle . \qquad (3)$$

[Notice that the approximation (3) differs from (1) in that no eikonal limits have been taken in (3). However, Gabathuler and Wilkin[4] did use the standard Glauber amplitude (1). See Refs. 8 and 9 for a discussion of the difference.]

Since the Harrington mechanism is not operative in the energy region under discussion one has to explain the sufficiency of (3). We mention a number of possibilities:

(a) For a dilute system with weak πN forces, $F^{(1)}$ will dominate. It is in that case by no means clear whether the major correction is $F^{(2)\text{on}}$ or the full double scattering amplitude $F^{(2)}$.

(b) A second, far from trivial case for which Eq. (3) apparently holds, is scattering from coinciding scattering centers. Fanchiotti and Osborn[10] and later Agassi and Gal[11] as well as Alexander, Wallace, and Sparrow[12] numerically demonstrated that the truncated amplitude F^{tr}, Eq. (3), closely approximates F^{MS} even for relatively low energies. Separation of the centers rapidly destroys the cancellations (2) unless of course the eikonal conditions are met.

(c) Remler finally studied scattering of a pair of free particles from an infinitely heavy target.[13] Case (b) and case (c) do not apply to πd scattering and we thus turn to convergence tests of the multiple scattering series.

II. MS CONVERGENCE TESTS FOR πd SCATTERING

We are aware of only a few studies of the convergence of the MS in general[14] and actually none focuses on the truncated amplitude (3). (The Padé approximant techniques used by Giraud *et al.*[2] produce the required expansion in principle, but these authors, aiming at a calculation of exact amplitudes, did not publish convergence results, nor did they investigate the part (3) above.) Yet numerous calculations of πd elastic scattering have been based on the single scattering term $F^{(1)}$ in (3).[15] It is evidently of interest to study the convergence of the MS series in particular for πd scattering, since available exact solutions can serve as a standard. Indeed, scattering from a deuteron is particularly instructive, because its MS (Faddeev) expansion exactly accounts for binding effects, Pauli blocking, etc., all of which can only approximately be assessed for heavier targets.

We chose a covariant version of the MS theory with a dominant P_{33} πN amplitude and applied it around the highest energies permitting accurate computation.[1] We shall use standard separable interactions, which is in particular justified for the resonating P_{33} channel and solve first the coupled integral equations (for a [12,32] grid; cf. Eq. (3.7) Ref. 1):

$$\begin{aligned} T_{\pi d,\pi d} &= 2B_{\pi d,N\Delta}G_\Delta T_{N\Delta,\pi d}\,, \\ T_{N\Delta,\pi d} &= B_{N\Delta,\pi d}+B_{N\Delta,\pi d}G_d T_{\pi d,\pi d} \\ &\quad + B_{N\Delta,N\Delta}G_\Delta T_{N\Delta,\pi d}\,. \end{aligned} \tag{4}$$

In (4), $B_{\alpha\beta}$ is a single particle exchange amplitude between channels α,β (Ref. 16),

$$B_{\alpha\beta}=g_\alpha G_0 g_\beta\,, \tag{5}$$

where g_α is the vertex function (form factor) for the dissociation of the pair α and where G_0 is the unperturbed propagator. If, as shown in Fig. 1, ϵ_p, $\epsilon_{p'}$, and $\epsilon_{|\vec p+\vec p'|}$ are energies of respectively the initial and final spectator (i.e., a particle not in an interacting pair) and of the particle exchanged then[16] (s is the squared total energy)

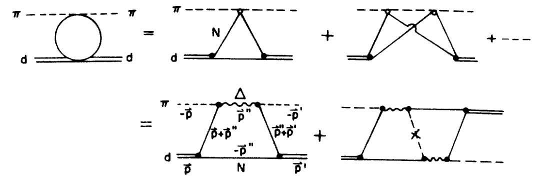

FIG. 1. Graphical representation of the elastic amplitude $T_{\pi d}$. (a) MS series which, if $t_{\pi N}$ is dominated by Δ, coincides with the solution of Eq. (4). (b) Lowest order and on-shell second order term of truncated series (3). Cross indicates that pion is on its mass-shell.

$$G_0 \propto [s-(\epsilon_p+\epsilon_{p'}+\epsilon_{|\vec p+\vec p'|})^2+i\eta]^{-1}\,. \tag{6}$$

Finally, G_α in (4) represents the propagator of the interacting pair α in the presence of a spectator.

The exact solution $T_{\pi d,\pi d}$ can now be compared with the iterative expansion. (Notice [Eqs. (3), (7), and Fig. 1] that in the expansion of $T_{\pi d,\pi d}$ first order in $t_{\pi N}$ means second order in B, etc.) In particular we are interested in the following truncated series [cf. Eq. (3)]:

$$\begin{aligned} T^{\rm MS}_{\pi d,\pi d} &\approx T^{\rm tr}_{\pi d,\pi d} \\ &\equiv 2B_{\pi d,N\Delta}G_\Delta B_{N\Delta,\pi d} \\ &\quad + 2B_{\pi d,N\Delta}G_\Delta B^{\rm on}_{N\Delta,N\Delta}G_\Delta B_{N\Delta,\pi d}\,, \end{aligned} \tag{7}$$

where the superscript "on" reminds one to compute B with (Fig. 1) the exchanged pion on its mass shell. Using Eq. (5), the form $t_\alpha=g_\alpha G_\alpha g_\alpha$ for separable t matrices as well as $g_d G_0=\phi_d$, one retraces the form (3):

$$T^{\rm tr}=2\langle\phi_d|t_{\pi N}|\phi_d\rangle+2\langle\phi_d|t_{\pi N}G_D^{\rm on}t_{\pi N}|\phi_d\rangle\,. \tag{8}$$

We emphasize again that the off-shell scattering matrix $t_{\pi N}$ above (containing binding effects) is not replaced by its on-shell, eikonal limit.

We return to $T^{\rm tr}$, Eq. (7) which is evaluated by a standard partial wave analysis.[1] In particular the on-shell part of the partial wave representation of $T^{(2)}$ in (3) reads ($B_{d\Delta}\equiv B_{\pi d,N\Delta}$, etc.)

$$\begin{aligned} [T^J_{Ld,L'd}(pp,s)]^{(2)\,\rm on} &= 2\sum_{L''S''}\int\frac{p'^2dp'}{16\pi^3E_{p'}}B^J_{Ld,L''S''\Delta}(pp's)G_\Delta(s,p') \\ &\quad\times\sum_{L''S''}\int\frac{p''^2dp''}{16\pi^3E_{p''}}Y^J_{L'S'\Delta,L''S''\Delta}(p'p'',s)G_\Delta(s,p'')B^J_{L''S''\Delta,Ld}(p''p,s)\,, \end{aligned} \tag{9}$$

where LSJ are channel angular momentum, channel spin, and total angular momentum. Y in (9) is that part of $B_{\Delta\Delta}$ which describes an exchanged pion on its mass shell and from Eqs. (5) and (6) its construction is seen to amount to the replacement in (6) of

TABLE I. Some partial wave amplitudes (in 10^{-4} fm) to first, second, and third order [see remark after Eq. (6)] their exact values and the truncated amplitude (3).

n_{max} \ $J^{\pi}L$	$0^+, 1$	$1^+, 1$	$1^-, 0$	$2^+, 1$	$4^+, 3$
			$T_{\pi}=180$ MeV, $P_D=6.7\%$, Hulthén		
1	$487+901i$	$568+1699i$	$576+2086i$	$1316+4548i$	$170+585i$
2	$499+904i$	$531+1658i$	$465+2041i$	$933+4372i$	$173+593i$
3	$476+894i$	$498+1629i$	$455+2028i$	$840+3944i$	$173+593i$
exact	$477+894i$	$500+1630i$	$456+2027i$	$934+3968i$	$173+592i$
trunc.	$592+994i$	$621+1636i$	$661+1869i$	$1092+4061i$	$226+631i$
			$T_{\pi}=265$ MeV, $P_D=6.7\%$, Hulthén		
1	$-152+807i$	$-421+1091i$	$-684+1460i$	$-1147+2581i$	$-228+486i$
2	$-155+815i$	$-401+1064i$	$-661+1383i$	$-1074+2358i$	$-235+489i$
3	$-146+796i$	$-396+1042i$	$-654+1376i$	$-803+2338i$	$-235+489i$
exact	$-146+797i$	$-397+1043i$	$-654+1377i$	$-907+2356i$	$-235+489i$
trunc.	$-232+887i$	$-378+1122i$	$-524+1481i$	$-912+2378i$	$-256+531i$
			$T_{\pi}=256$ MeV, $P_D=5.76\%$, McGee		
1	$-59+886i$	$-383+1108i$	$-689+1554i$	$-1220+2732i$	$-217+480i$
2	$-63+895i$	$-353+1079i$	$-675+1474i$	$-1123+2478i$	$-224+484i$
3	$-86+872i$	$-356+1065i$	$-674+1460i$	$-945+2523i$	$-224+484i$
trunc.	$-156+974i$	$-334+1135i$	$-510+1583i$	$-960+2506i$	$-224+526i$

$$[s-(\epsilon_p+\epsilon_{p'}+\epsilon_{|\vec{p}+\vec{p}'|})^2]^{-1} \to -i\pi\delta(s-(\epsilon_p+\epsilon_{p'}+\epsilon_{|\vec{p}+\vec{p}'|})^2) .$$

In Table I we show a few dominant amplitudes for $T_{\pi}=180$ and 256 MeV, calculated by exact solution of (4), by finite order iterations, and finally by evaluation of (7). Generally we employ a Hulthén deuteron wave function with 67% D state probability but results have also been obtained for a more realistic McGee d-wave function.[7] In Table II we test Eq. (2) and give on- and off-shell parts of $F^{(2)}$, $F^{\text{rest}}=\sum_{n\geq 3}F^{(n)}$ and F^{exact}. In Table III we entered some tensor polarizations[1] computed for corresponding approximations. Finally we display in Fig. 2 differential crosssections and can then make the following conclusions:

(1) Although the partial wave MS series converges rapidly for peripheral L values, the convergence is slower for low L in particular for the relatively small real parts ReF. Both $|\text{Re}F|$ and $|\text{Im}F|$ always decrease to the exact values with increasing order n.

(2) For the dominant $J^{\pi}, L=2^+, 1$ amplitudes there is no doubt that F^{tr}, Eq. (3), is an excellent approximation to F and of better quality than $F^{(1)}+F^{(2)}$ ($F^{(2)}=F^{\text{on}}+F^{\text{off}}$). We thus infer (Table II) that effectively a Harrington cancellation (2) takes place, while neither $F^{(2)\text{off}}$ nor $\sum_{n\geq 3}F^{(n)}$ is particularly small compared to $F^{(2)\text{on}}$. Although less obvious the same is the case for the other large partial wave amplitudes. The fact that the sufficiency of F^{tr} is better for 256 MeV than for 180 MeV is presumably not related to closer proximity to the eikonal region. We rather blame the sensitivity of ReF around the resonance region.

(3) Deviations of F^{tr} from F often affect real and imaginary parts in different directions but

$$\begin{aligned} &|\text{Re}(F-F^{\text{tr}})| \ll |\text{Re}F| \\ &\qquad\qquad\qquad\qquad \text{or } |F| \sim |(\text{Re}F+i\,\text{Im}F)^{\text{tr}}| . \\ &|\text{Im}(F-F^{\text{tr}})| \ll |\text{Im}F| \end{aligned} \tag{10}$$

TABLE II. On- and off-shell parts of some partial wave amplitudes in 10^{-4} fm ($T_{\pi}=265$ MeV), the rest amplitude $F^{\text{rest}}=\sum_{n\geq 3}F^{(n)}$ and F^{exact}.

J^{π}, L	$F^{(2)}$	$F^{(2)\text{on}}$	$F^{(2)\text{off}}$	F^{rest}	F^{exact}
1^-0	$24-77i$	$161+22i$	$-137-99i$	$7-6i$	$-654+1377i$
1^+1	$20-27i$	$43+31i$	$-23-59i$	$4-21i$	$-397+1043i$
2^+1	$73-223i$	$235-202i$	$-162-21i$	$167-2i$	$-907+2356i$

TABLE III. Some values for the analyzing power computed in various approximations with $(\phi_d)_{\text{Hulthén}}$ ($P_D=6.7\%$).

	$T_{\pi}=180$ MeV		$T_{\pi}=256$ MeV	
	$t_{20}(90°)$	$t_{20}(180°)$	$t_{20}(90°)$	$t_{20}(180°)$
Third order	−0.325	−1.125	−0.225	−1.164
exact	−0.373	−1.073	−0.241	−1.138
trunc.	−0.211	−0.838	−0.190	−1.316

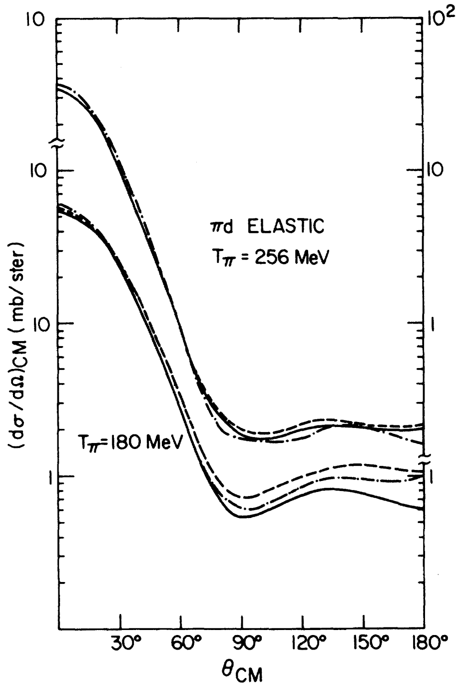

FIG. 2. Differential cross section for πd elastic scattering (T_π=180, 256 MeV, Hulthén wave function, P_D=6.7%). (–) exact solution of (4); (---) same up to and including second order in E; (–•–) same with only first order and on-shell second order included.

Since the differential cross section is dominated by a few partial waves, (10) readily explains why the approximation (3) suffices for $d\sigma/d\Omega$ and also indicates the lesser quality (still surprisingly good) predictions for polarizations. (The latter are strongly affected by genuine absorption corrections[18] which we have disregarded altogether. The comparison made holds only for the pure MS contributions to $F_{\pi d}$.) To a lesser degree the same results when the eikonal limit, i.e., the Glauber amplitude

$$F^{tr} \underset{\substack{E\to\infty\\ q\to 0}}{\longrightarrow} F^{Gl}$$

is used.

The reported results are numerical observations and do not explain the mechanism which brings about the approximate Harrington cancellation. In particular, the relatively large pn separation in the d, although definitely working in the desired direction, cannot be the complete answer because agreement of comparable quality has been obtained for pion scattering on heavier targets. Successful Glauber fits seem hardly related to eikonal conditions as expected, but bear on the convergence of *selected parts* of lowest order terms. An analysis as performed above for $A \geq 3$ targets would be much more complicated and has until now not been performed. We are thus still ignorant of whether a full lowest order expansion or one with on-shell parts only is actually preferable above a standard first order optical potential fit.[19]

[1]A. S. Rinat and A. W. Thomas, Nucl. Phys. A282, 365 (1977).

[2]A. W. Thomas, Nucl. Phys. A258, 417 (1976); V. B. Mandelzweig, H. Garcilazo, and J. M. Eisenberg, *ibid.* A256, 461 (1976); A264, 531 (1976); N. Giraud, G. H. Lamot, and C. Fayard, Phys. Rev. Lett. 40, 438 (1978); Phys. Lett. 77B, 141 (1978); Phys. Rev. C 19, 465 (1979).

[3]C. Carlson, Phys. Rev. C 2, 1224 (1970).

[4]K. Gabathuler and C. Wilkin, Nucl. Phys. B70, 215 (1974).

[5]M. M. Hoenig and A. S. Rinat, Phys. Rev. C 10, 2102 (1974); 18, 2683 (1978).

[6]R. H. Cole, J. C. McCarthy, R. C. Minehardt, and E. A. Wadlinger, Phys. Rev. C 17, 681 (1978).

[7]D. R. Harrington, Phys. Rev. 184, 1745 (1969).

[8]S. A. Gurvitz, Y. Alexander, and A. S. Rinat, Ann. Phys. (N.Y.) 93, 152 (1975); 98, 346 (1976).

[9]S. A. Gurvitz, J. P. Dedonder, and R. D. Amado, Phys. Rev. C 19, 142 (1979).

[10]H. Fanchiotti and T. A. Osborn, Nucl. Phys. A197, 273 (1971).

[11]D. Agassi and A. Gal, Phys. Rev. Lett. 31, 961 (1973).

[12]Y. Alexander, S. J. Wallace, and D. Sparrow, Nucl. Phys. A280, 285 (1977).

[13]E. Remler, Ann. Phys. (N.Y.) 67, 114 (1971).

[14]I. Sloan, Phys. Rev. 185, 1361 (1969).

[15]E.g., H. Garcilazo, Phys. Rev. C 15, 1940 (1977); M. van der Velde, Nuovo Cimento 40A, 97 (1977).

[16]R. Aaron, R. D. Amado, and J. E. Young, Phys. Rev. 174, 2022 (1968).

[17]I. G. McGee, Phys. Rev. 151, 722 (1966).

[18]A. S. Rinat, Y. Starkand, E. Hammel, and A. W. Thomas, Phys. Lett. 80B, 166 (1979) and to be published.

[19]M. Thiess, Phys. Lett. 63B, 39, 43 (1976).