Giant multipole resonances in the deformed fissionable nucleus ^{238}U

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The deformed, fissionable nucleus ^{238}U was studied with inelastic scattering of 87.5 MeV electrons between 5 and 40 MeV excitation energy with inelastic momentum transfers ranging from 0.32 fm^{-1} to 0.58 fm^{-1} for an excitation energy of 15 MeV. Resonance cross sections extracted were compared with distorted-wave Born-approximation calculations using the Goldhaber-Teller, Steinwedel-Jensen, and Myers-Swiatecki models of the giant resonance. It is demonstrated that up to the first minimum of the form factor the cross section is nearly completely determined by one parameter, the transition radius R_{tr}. Using the known systematics of various multipole resonances in other, nonfissionable nuclei as a guide, it was found that the assumed ground state radius of ^{238}U had to be enlarged by about 10% for all multipolarities, to bring the strength found into agreement with the systematics and with other experiments in ^{238}U. In particular, while the model-independent values for position and width of the giant dipole resonance agree well with photon experiments, a scaled version of the Myers-Swiatecki model had to be used to produce agreement in strength. Similarly a scaled Goldhaber-Teller model was used for the isoscalar $E2$ resonance at 9.9 MeV. The situation for the isovector states above the giant dipole resonance, $E2$, and $E3$ (or $E0$) is even more complicated. It is argued that with proper caution and consideration of other available data the use of the collective models mentioned above may give valuable insight into the charge distribution of ^{238}U at higher excitation energies.

NUCLEAR REACTIONS ^{238}U(e,e'), $E_0 = 87.5$ MeV. Measured $d^2\sigma/d\Omega dE_x$, bound and continuum states (giant resonances). Deduced multipolarity, reduced matrix element $B(E\lambda)$, radiative width Γ_γ^0, sum rule exhaustion of giant resonances, total width of continuum and clustered states.

I. INTRODUCTION

In recent years, there has been a renewed interest in the structure of the nuclear continuum. This interest has been stimulated by the discovery of numerous electric and magnetic isoscalar and isovector resonances of various multipolarity above particle threshold (giant resonances) which had been predicted a long time ago by Bohr and Mottelson.[1]

Most of the experiments were done with inelastic scattering concentrating on the isoscalar quadrupole state (GQR) at $63A^{-1/3}$ MeV, whereby the analysis was based on macroscopic models.[2-5] The macroscopic models have contributed very much to our understanding of the nucleus because they allow one to describe the dynamics of a complicated many-body system in a simple way.[6,7] The use of the same concept in evaluating data taken with different probes and at different laboratories has made possible comparison of the results, mostly expressed in terms of a model-independent sum rule, which has been very fruitful for the progress of the field.[8] It should be emphasized, however, that although the sum rule depends only on the nuclear ground state charge distribution and is indeed nearly model independent,

the amount exhausted by a certain resonance depends critically on the model used. Another important point is that in the momentum transfer covered by most (e, e') experiments, which rarely went beyond the first minimum in the form factor, the momentum transfer dependence of the cross section is completely determined by just one parameter, the transition radius $R_{tr}^\lambda = [\langle r^{\lambda+2}\rangle / \langle r^2 \rangle]^{1/2}$, as will be shown below, quite similar to the description of the elastic cross section at low momentum transfer by the ground state rms radius.

Since the giant dipole resonance (GDR) fragments in a spectacular way into separate oscillations along long and short axes,[9,10] immediately after the discovery of GQR's as systematic features of heavy nuclei interest focused onto the question whether or not this mode of excitation would fragment correspondingly.

These experiments are complicated because due to the splitting of the GDR its lower branch and the isoscalar $E2$ can no longer be separated by line shape, since they fall on top of each other. Certain assumptions for the form factor of the GDR have, therefore, to be made in order to subtract it from the total cross section measured.

To do so, extensions of the Goldhaber-Teller

model have been used in the past for the GDR by
introducing different spherical transition charge
densities for the dipole oscillations along long
$(R_g, K = 0^-)$ and short $(R_1, K = 1^-)$ axes,[11] based on
scaling the half density radius of the hypothetical
ground state chrage distribution, which enters the
formulas of the hydrodynamic models in the ratio
of long and short axes, respectively, to the
average radius,[12,13,14] This approach has been
criticized by Suzuki and Rowe,[15] who showed that
the scaling assumption in only valid for the $K = 1^-$
branch of the GDR. Since the $K = 0^-$ branch is the
one which is lower in excitation energy, and nearly
completely overlaps the GQR, a quantitative re-
analysis seems to be in order. Fortunately, it
turns out that the quantitative changes are rela-
tively small, at least compared to the changes
due to the use of different models of the giant
dipole resonance.

To conclude the introduction we would like to
quote from the conclusion of an unpublished re-
port,[16] which evaluated the data taken in Monterey
on ^{238}U with a strict, that is not modified in any
way, Goldhaber-Teller model. One of the salient
points in this report was the surprisingly low $E2$
strength, comprising only 40% of the $E2$ sum rule
for both isoscalar and isovector state:

"The low $E2$ strength leads to the question
whether the strict hydrodynamical model which
has been successful in describing the data in
non-fissionable nuclei can also be applied to ^{238}U
for quadrupole excitations, or if the quadrupole
strength is shifted to lower energy or spread out
in a non-resonant way. Although this problem
will need more experimental work, one may rea-
son that for a fissionable nucleus the rms-radius
for the charge distribution in the excited state
may be expected to be greater than the one calcu-
lated from the hydrodynamical model ··· such an
assumption leads to a greater sum rule exhaustion,
but still does not explain why this effect should be
very strong for $E2$ oscillations, but not noticeable
for $E1$ excitations."

In fact, it will become apparent below that what
this problem needs is more theoretical considera-
tion, and that there is a set of parameters which
allows a consistent description of giant resonances
in ^{238}U; consistent concerning results from lighter
nuclei as well as a comparison of different multi-
polarities in ^{238}U itself.

II. PREVIOUS EXPERIMENTS WITH ^{238}U

In recent years, the giant dipole resonance has
been extensively studied with various photon tech-
niques.

Bar-noy and Moreh[17] utilized thermal neutron
capture, Gurevich et al.[18,19] employed bremsstrah-

lung beams, and Veyssière et al.[20] and Caldwell[21]
et al. used quasimonochromatic photons. The four
above research groups located the maximum of the
two branches of the giant dipole resonance in ^{238}U
at excitation energies of about 10.9 and 14.0 MeV.

There is also work on the giant quadrupole reso-
nance. With proton scattering a "bumplike"
resonance was found by Lewis and Horen in the
10-13 MeV excitation energy range which was
interpreted as a quadrupole resonance.[22] Approxi-
mately 85% of the isoscalar sum rule was ex-
hausted. Wolynec, Martins, and Moscati[23] have
used the (e, α) reaction to investigate the giant
quadrupole resonance. 85% of the isoscalar energy
weighted sum rule (EWSR) was exhausted by a
Breit-Wigner shaped resonance centered at 8.9
MeV with width $\Gamma = 3.7$ MeV. Since the α-decay
probability at such low excitation energy is ex-
pected to be very small due to inhibition by the
Coulomb barrier, while there are many open chan-
nels for both fission and neutron decay, the experi-
ment of Ref. 23 has been subject to scrutiny. All
these experiments[24,25] agree in that the reported
(e, α) cross sections in Ref. 23 are in error.

More constructively, several groups have
searched with electrofission methods for $E2$
strength in ^{238}U. Shotter et al.,[26] measuring the
fission products, concluded that the asymetric
component was due to the GDR, and therefore, to
$E1$ absorption, while the symmetric component
possibly could be due to an $E2$ component. Lack
of accurate $E2$ virtual photon calculations for
heavy nuclei prevented these authors from more
definite conclusions. In a latter experiment, how-
ever, Shotter et al. concluded that $E2$ strength
around 10 MeV was needed to give a fit to the
electrofission[27] and electroneutron[28] data, but as-
suming only an $E2$ resonance at 22 MeV explained
the data nearly as well. Since presumably both
are present, their quantitative results may change
somewhat, but it is unclear how much the value of
Γ_n/Γ_f for the $E2$ component, which they deduced
to be 0.3 to 0.6, will change. Kneissl et al.[29] had
to include $E2$ cross section with a height of 40 mb
at 22 MeV to explain a shoulder in their cross
section. If one assumes this $E2$ strength to be
concentrated in a 5 MeV wide resonance, it would
result in approximately 130% of the EWSR (ΔT
$= 1, E2$), a somewhat high but still reasonable
value. Since their spectra go only down to 10 MeV,
no investigation of the isoscalar resonance was
possible. Most recently Arruda Neto et al.[30]
found a GQR at 9.9 MeV with a width of 6.8 ± 0.4
MeV, exhausting 71% of the EWSR. Their data,
in addition, are compatible with an $M1$ resonance
centered at 6.5 MeV with a spreading width of
1.5 MeV.

III. PRESENT EXPERIMENT

The present experiments were undertaken to measure the excitation energies, width, and cross sections of discernible resonances of a deformed fissionable nucleus by inelastic electron scattering. Samples of 99.9% enriched ^{238}U were obtained from Ventron Corporation and rolled to 0.004 inches for scattering at 90°, 0.002 inches for the 60° and 75° scattering angles, and 0.001 inches for the 45°. The use of three different target thickness made it possible to optimize count rates as compared to the radiative background while achieving the required statistical accuracy.

87.5 MeV electrons from the NPS 120 MeV electron LINAC were scattered by the self-supporting ^{238}U foils at the scattering angles mentioned above, thus using the variation of the momentum transfer with angle to investigate the multipolarity of the giant resonances. A wider spread of angles was not necessary because the maxima of $E1$ to $E4$ form factors are included in this range (Fig. 1) and would also be very difficult to measure, because at forward angles the radiation tail becomes so dominant that the beam has to be reduced to only several nA; at backward angles the inelastic cross section became very small, a fact which is often overlooked due to the use of relative cross sections σ/σ_{Mott} in the analysis of the data. For example, the cross section for the GDR falls off two orders of magnitude from 45° to 90° at 87.5 MeV.
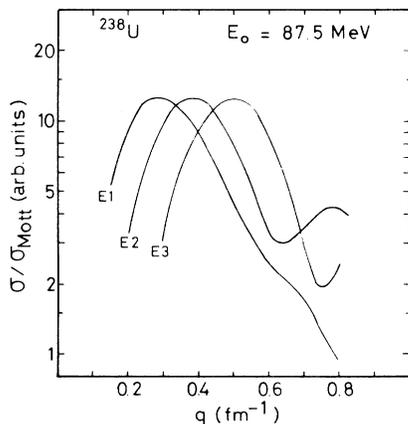

FIG. 1. Comparison of momentum transfer dependence for $E1$ (MS model), $E2$ (GT), and $E3$ (GT) with $c_{tr}/c = 1.0$. The momentum transfer covered is sufficient to distinguish between $E1$, $E2$, and $E3$ excitations. To go to higher and lower momentum transfer would be desirable. To do so is ruled out at the present experimental situation because of the low duty factor (forward angles, low q) and low beam current (backward angle, high q).

The accelerated electrons are momentum analyzed in the symmetry plane of a two 30° sector magnet achromatic deflection system. The electrons scattered from the target are measured by a ten scintillation counter ladder in the focal plane of a 40 cm, 120° double focusing magnetic spectrometer. The momentum bite of the spectrometer if 3%, the stepping width of the magnetic field normally corresponds to 0.1 MeV. The overall resolution of the system is limited by the mechanical dimensions of the scintillators and therefore, maximally 0.3%. It was kept to 0.5%, however, because this value is the optimal compromise between background produced at the energy defining slit system, which rises with better resolution, i.e., narrower slits, and background produced in the beam pipes leading to the spectrometer, which rises with wider slits. The data are sorted into energy bins equal to the stepping width (0.1 MeV). For control purposes the whole excitation range has been measured with a wider stepping width, 2 MeV, before and after each inelastic run. No deviations, indicative of background changes, integrator drifts, etc., were found.

IV. EVALUATION

A. General

The techniques employed were similar to those used in earlier (e, e') experiments with ^{208}Pb, $^{19.}$Au, ^{165}Ho, ^{140}Ce, and ^{89}Y (Refs. 31, 13, 32, 33) so that comparisons between the nuclei could be made without variations from differing methods of evaluation. The points where the evaluation had to differ will be stressed at the end of this section. Special difficulties arise in the case of ^{238}U for extraction of the cross sections because the radiation tail, which has a strength approximately proportional to Z^2, is very large (Fig. 2). Furthermore, because the nucleus is deformed, the resonances are possibly split, as has been observed for the dipole state, or at least broadened and tend to be more spread out than in spherical nuclei,[12-14] thus resulting in a more unfavorable signal to background ratio.

On the other hand, though large, the radiative background in (e, e') is understood, in principle, and although no rigorous treatment is possible yet, due to practical improvements, the calculations account for virtually all the radiative background.[32] It is especially to be noted that in the two regions where one knows the background experimentally (namely between low-lying isolated levels and above 40 to 50 MeV excitation energy), the data are reproduced within a few percent in spherical nuclei.[33,32]
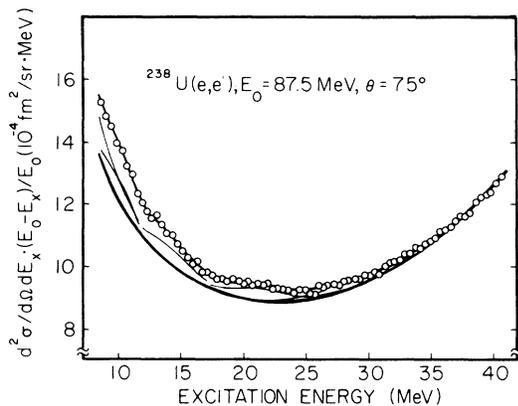
FIG. 2. Spectrum of 87.5 MeV electrons scattered inelastically from ^{238}U at 75°. The total fitted background consisting of radiation tail, general room background, and experimental scattering is represented by the lowest heavy line. The fitted resonances (lines between background and data points) and the composite fitted cross section (top line) are indicated. Note the suppressed scale; the resonant cross section is only a small fraction of the underlying radiation tail. The raising line at the very left is due to the tail of the ghost peak. The cross section has not been corrected for the constant dispersion of the magnetic spectrometer in order to show the data as measured. It is evident that not much can be learned from the data without background subtraction (see Fig. 3).

Our evaluation is sensitive to resonant structure only; more continuous cross sections, e.g., from the tail of the quasi-elastic peak, would not be seen with our method.

The inelastic cross sections were measured relative to the elastic ones. The latter, in turn, were calculated with a phase shift code[34] using the Fermi ground state charge distribution parameters $c = 6.805$ fm and $t = 2.66$ fm. These parameters, which are from elastic electron scattering, were taken from the compilation by de Jager, H. de Vries, and C. de Vries.[35]

The inelastic spectra were evaluated using a least squares line shape fitting program,[33] in which the resonances and the background are fitted simultaneously. Since the elastic cross section σ_{el} can be calculated, and the areas under elastic peak and an inelastic resonance, A_{el} and A_{in}, follow the relation $\sigma_{el}/\sigma_{in} = A_{el}/A_{in}$, σ_{in} is determined by the inelastic area. To determine the latter, the background has to be subtracted. The largest portion of the total background is due to the elastic radiation tail which is caused by photon emission before, during and after the scattering event, plus energy straggling and ionization. The radiation tail was calculated using the Born approximation formulas of Ginsberg and Pratt[36] but

substituting the actual elastic cross section at the energy of the scattered electron, computed with the phase shift code of Fischer and Rawitscher.[34]

In addition to the radiation tail, the experimental background, consisting of general room background and of electrons scattered by the targets and subsequently rescattered by the spectrometer walls, had to be taken into account. The total background was found to be well described with a three parameter function:

$$B(E_f) = P_1 + P_2/E_f + P_3 T_R(E_f)$$

or

$$B(E_f) = P_1 + P_2(E_f - E') + P_3 T_R(E_f),$$

where the P_i are fitting parameters, E_f is the energy of the outgoing electron, E' is the center energy of the fitting range, and T_R is the radiation tail. The parameter P_3 turned out to be close to 1, namely (0.95 to 1.05) ±0.04, which shows that little scaling of the calculated radiation tail was necessary. P_2 was small; otherwise the two functions could not have served equally well in describing the background because this term is the one different in both forms. No difference between the results obtained using the above two background functions was observed. The second function was used in the final analysis of all spectra. In addition, a bump from instrumental scattering (ghost peak) at 6.5 MeV had to be subtracted. The whole procedure has been thoroughly described in Secs. III of Refs. 33 and 32.

It is quite evident from Fig. 2, that a fit which attempts to let the χ^2 method work its way to a minimum all by itself is not possible. Reasonable good starting values for background and resonances have first to be found, and constraints have to be put on some of the three resonance parameters position, height and width. As a matter of principle, the height always was left variable. Position and width of some resonances were fixed in one computer run, but were fitted in other runs.

Three alternative criteria have been generally used for assuming the presence of a resonance in the spectrum in the first place: (1) the observation of the resonance peaking above the flat expanse of the radiation tail and background, (2) the knowledge of resonances found by photonuclear and photofission experiments, (3) the necessity to add a resonance to achieve a consistent overall fit. In the case of uranium, it is difficult to use the first criterion for reasonable placement. As can be seen in the inelastic spectrum for 75° (Fig. 2), very few of the collective states are visible to the naked eye. It is only after the subtraction of the radiation tail and continuous spectrum due to

bremsstrahlung that the spectrum begins to exhibit the structure of the giant multipole resonances (Fig. 3).

The line shape used throughout was Breit-Wigner. This choice is based on the observation that the strength function, but not the cross section, for the GDR is best described by this form.[37] However, use of Lorentz form gives only slightly different resonance parameters, except for 0.2–0.5 MeV shift in excitation energy, which depends on excitation energy and multipolarity. We want to emphasize that all parameters given are those of the strength function, which is the only invari-
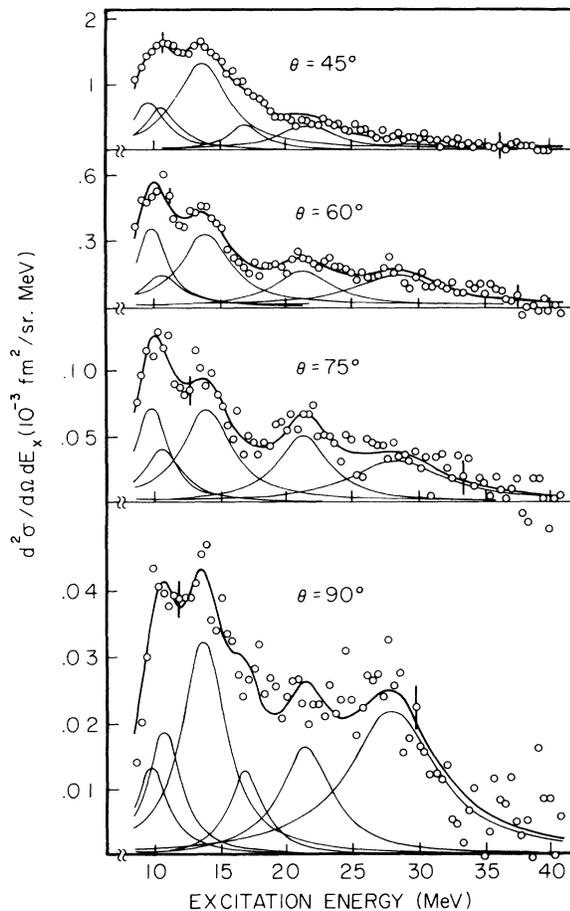

FIG. 3. 87.5 MeV electrons scattered inelastically from ^{238}U at 45°, 60°, 75°, and 90°. The fitted background, consisting of radiation tail, general room background, and instrumental scattering, has been subtracted. Comparison of part c with Fig. 2 shows that after subtraction of background many more details show up. The data have been corrected for the constant dispersion of the magnetic spectrometer in order to show the cross sections of the various resonances in their true relation. The relative differences in peak height for different resonances at different angles show that several multipolarities contribute.

able, and not those of the cross section, which may be different for different experiments and probes.[37]

As pointed out in the beginning, the reduced transition strength for giant resonances is often expressed as fractions of the electromagnetic sum rules. This is particularly appropriate for electron scattering because here the sum rules depend only on the nuclear charge distribution of the ground state (for a discussion of inherent difficulties in the case of hadronic probes, see Ref. 38). In this paper the sum rule for $\lambda > 1$

$$S(E\lambda) = \frac{Z\lambda(2\lambda+1)^2\hbar^2}{8\pi M_p} \langle R^{2\lambda-2} \rangle$$

was used, where M_p is the mass of the proton and $\langle R^{2\lambda-2} \rangle$ the $(2\lambda-2)$ moment of the ground state charge distribution of the nucleus.[39,40] As to the distribution of strength between isoscalar and isovector parts, a fraction of Z/A was thought to be isoscalar, and the remainder, N/A, to be isovector.[40] This sum rule does not account for interference terms between isoscalar and isovector excitations, an assumption which is not true for $N > Z$ nuclei,[41] and may even by invalid for $N = Z$ nuclei as light as ^{28}Si when due to the Coulomb force isospin is no longer a good quantum number. One should, therefore, regard the use of sum rules only as a generally accepted convenient measure for strength. Isoscalar and isovector sums in this simplified picture are thus related by

$$S(E\lambda, \Delta T = 1) = S(E\lambda, \Delta T = 0)(N/Z).$$

The energy-weighted isovector sum rule for the electric dipole resonance has to be modified because the center of mass motion has to be zero[42]:

$$S(E1) = \frac{9\hbar^2}{8\pi M_p} (NZ/A).$$

In the monopole case (breathing mode) the requirement of volume conservation leads to the equation[43]

$$S(E0) = \frac{\hbar^2}{M_p} A \langle r^2 \rangle.$$

The energy-weighted sum rules for ^{238}U, calculated with $\langle R^2 \rangle^{1/2} = 5.730$ fm and $\langle R^4 \rangle^{1/4} = 6.124$ fm, which in turn were calculated by numerical integration of the ground state charge distribution, are

$$S(E0) = 3.24 \times 10^5 \text{ MeV fm}^4,$$

$$S(E2) = 2.49 \times 10^5 \text{ MeV fm}^4,$$

$$S(E3) = 3.15 \times 10^7 \text{ MeV fm}^6,$$

$$S(E1, \Delta T = 1) = 839 \text{ MeV fm}^2.$$

The inelastic cross section is presented in units of the Mott cross section

$$(d\sigma/d\Omega)_{Mott} = (\alpha Z/2E_i)^2 \cos^2(\theta/2)/\sin^4(\theta/2),$$

which describes the elastic scattering of an electron from an infinitely heavy, spinless, pointlike nucleus with charge Ze. Expressing the cross section in these units takes out most of the purely kinematical contributions, makes the nuclear contribution to the cross section more evident, and allows a convenient comparison with theoretical predictions. Through this comparison the determination of multipolarities and strength is made.

B. Models

In the present work the determination of multipolarity and strength is based on calculations using the distorted wave computer code of Tuan, Wright, and Onley,[44] which requires a model for the nuclear transition charge, current, and magnetization densities. For strongly collective transitions it has been found that it is sufficient to take into account the charge contribution only.[45] The problem then rests with the choice of model and the question whether or not the ground state charge distribution, $\rho_0(r)$, which enters all collective models, has to be modified. No such changes have been found necessary up to now for giant resonances, which may be not too surprising, because the continuum excitations of nuclear matter are the hydrodynamical modes of the nucleus.[46] The attempt to fit the transition charge densities to the experimental data at higher momentum transfer $(q \gtrsim 0.8~\mathrm{fm^{-1}})$ in the case of ^{181}Ta leads to transition charge densities which are more concentrated in the nuclear interior than those of the hydrodynamical model.[47] The transition strengths found with these densities are a factor of two to three smaller than those from either other (e, e') experiments[48] in ^{181}Ta or from comparable nuclei.[8] The deviation between DWBA calculations and experiment in ^{181}Ta is probably due to not accounting for higher multipole strength instead of a failure of the hydrodynamical model. Further support for the application of the strict hydrodynamical model to giant resonances comes from the generally good agreement in strength extracted from (e, e'), (γ, n) and inelastic hadron scattering. For ^{238}U the case seems to be different as evidenced by the results of Ref. 16. Figures 4 and 5 demonstrate for the quadrupole resonances how much the results for ^{238}U deviate from what one would expect from measurements at lower A.[8]

For our purpose a convenient parametrization for calculating the transition charge density has been introduced by Ziegler and Peterson[45] by defining a half-density thickness c_{tr} and skin thick-
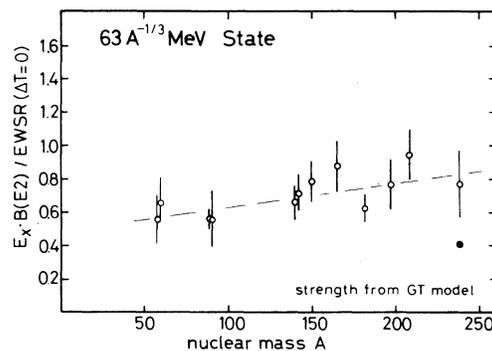

FIG. 4. Strength of giant isoscalar quadrupole resonance as measured by (e, e') using a 'strict' (for definition see text) Goldhaber-Teller model. The result for ^{238}U (full circle) is much lower than the value one would expect from any extrapolation of the data at lower A. The dashed line is for the guidance of the eye only and does not imply any theoretically or otherwise motivated fit to the data.

ness t_{tr}, which replace the parameters c and t in the two-parameter Fermi distribution

$$\rho_0 = C^0 \{ 1 + \exp[(r - c)/z] \}^{-1},$$

where $z = 4t\ln3$. A parametrization $c_{tr}/c = 1.1$, $t_{tr}/t = 1$, e.g., would mean that the "hypothetical" ground state charge distribution half-density radius has been enlarged by 10% while keeping the surface thickness constant.

The cross sections calculated with the DWBA code are normalized to B $(E\lambda, E_x = 0) = 1.0~\mathrm{fm}^{2\lambda}$, so that the experimental transition probability is simply calculated by fitting the calculated curve to the measured points.

Figures 6, 7, and 8 show the importance of the correct choice of model as well as parametrization for $E1$, $E2$, and $E3$ transitions, respectively, because a 10% change in radial dependence may produce more than 50% change in height at the maximum of the calculated relative cross section.
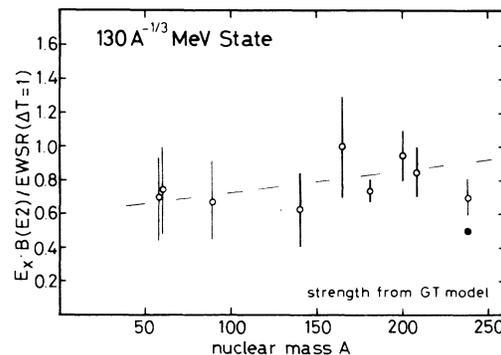

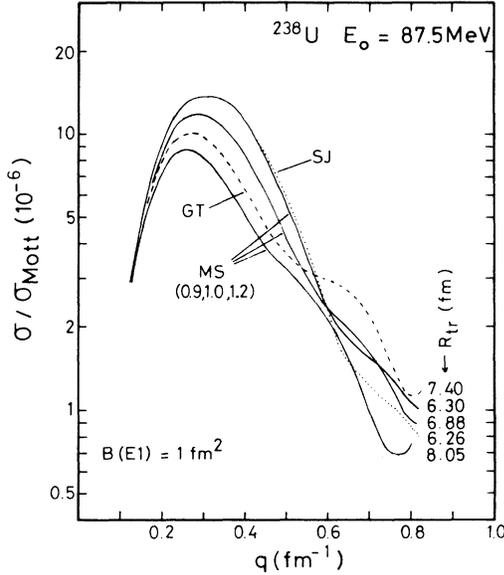FIG. 5. Similar to Fig. 4, but for the isovector GQR.

FIG. 6. Model dependency study of the giant dipole resonance. DWBA calculations (Ref. 44) for the Gold-haber-Teller (Refs. 48, 49), Steinwedel-Jensen (Refs. 42, 50), and Myers-Swiatecki (Ref. 52) models are shown. The meaning of the parametrization (Ref. 45) of the MS model is explained in the text. For GT and SJ models, an unmodified ground state charge distribution was assumed. From comparison of the transition radii (R_{tr}) connected with certain models and parametrizations it is clear that the behavior of the relative cross section σ/σ_{Mott} (form factor) is determined by the value of R_{tr} alone up to approximately the first minimum or what is left of it in a heavy nucleus. The emphasis of the calculations was put on the MS model because recent experiments (Ref. 54) show it to describe the experimental data very well. The trend of the cross section to become smaller with larger R_{tr}'s is clearly visible, thus introducing a model dependence. The curves are normalized to unity in the reduced transition probability.

Before we discuss in more detail Figs. 6, 7, and 8, we will elaborate somewhat on the models used. The Tassie model[48] is generally regarded as the classical hydrodynamical model. Its transition charge distribution is identical with the Goldhaber-Teller (GT) model,[49]

$$\rho_{tr}(r) = C^{GT} r^{\lambda-1} d\rho_0(r)/dr ,$$

corresponding to the oscillation of a rigid proton volume versus a rigid neutron volume. The Steinwedel-Jensen (SJ) model,[50]

$$\rho_{tr}(r) = C^{SJ} j_\lambda(rk_\lambda/c)\rho_0(r) ,$$

corresponds to the assumption of two interpenetrating proton and neutron liquids within one surface.
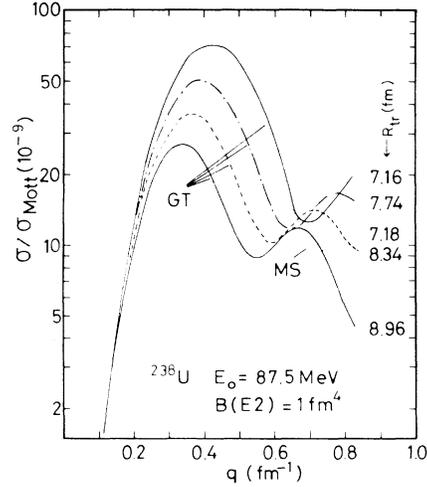
It has been difficult to determine experimentally

FIG. 7. Similar to Fig. 6 but for an $E2$ transition. The emphasis here is on the GT model because it is expected to describe the isoscalar excitation correctly (Ref. 46); a calculation with the MS model using α (238, 2) $= 1.0$ shows that similar to the dipole case R_{tr} determines the height of the curves, with the difference that, using the same parametrization as for Fig. 6, the differences for different parameters are even more pronounced. The GT calculations correspond to parameter c_{tr}/c (Ref. 45, see text) $= 0.9$, 1.0, 1.1, and 1.2, respectively, the MS model to $c_{tr}/c = 1.1$.
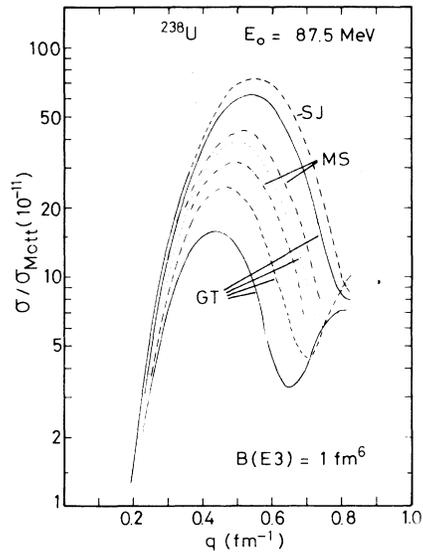

FIG. 8. Similar to Figs. 6 and 7, but for an $E3$ transition. The trend to larger model dependences with higher multipolarities, as already indicated by comparing Figs. 6 and 7, is continued. The four GT curves correspond to the same values of c_{tr}/c as in Fig. 7, the SJ curve to 1.1. The MS curves similarly use a ground state charge distribution expanded by 10%, but vary in the additional parameter of the MS model, the mixture ratio α of SJ and GT mode. The higher MS curve thus corresponds to $\alpha = 1.0$, the lower to $\alpha = 0.5$.

which of these models is correct, since photon absorption measurements, the most exact method, is practically model independent.

Only recently has experimental evidence in the form of the Myers-Swiatecki (MS) droplet model fits[51] been applied to this problem.[52] This approach resulted in a description of the GDR as a mixture of both GT and SJ modes:

$$\rho_{tr}^{MS}(r) = C^{MS}[\rho_{tr}^{GT}(r) + \alpha(A,\lambda)\rho_{tr}^{SJ}(r)],$$

with $\alpha(A,1)$ rising from approximately 0.5 for Ni to 0.9 for ^{238}U. This concept has been also applied by Kodama[53] to higher isovector multipoles resulting in $\alpha = 0.5$, while $\alpha = 0$ is expected for isoscalar states. Recent experiments in ^{140}Ce (Refs. 32, 54) have shown that the data are well described by the predictions $\alpha(140,1) = 0.65$ (Ref. 52) and are compatible with $\alpha(140,2) = 0.5$ (Ref. 53).

Figure 6 shows several DWBA calculations for the GDR in ^{238}U in comparison. Based on the experiments in ^{140}Ce we believe that the model of Myers *et al.*[52] is the best available description for the dipole case and calculations for three parameters, $c_{tr}/c = 0.9$, 1.0, and 1.2, are shown. The skin thickness was not changed. There are several important points to make. In PWBA the variable displayed, σ/σ_{Mott}, is a true form factor and can be written as

$$F^2(q) = \sigma/\sigma_{Mott}$$

$$= \frac{4\pi}{Z^2}(2\lambda + 1)\left|\int \rho_{tr}(r)j_\lambda(qr)r^2\,dr\right|^2,$$

$$q = \frac{1}{\hbar c}(E_i^2 + E_f^2 - 2E_f E_f\cos\theta)^{1/2}.$$

E_i initial, E_f final electron energy. F^2 is a function of q only and displays the typical pattern of a spherical Bessel function. In heavy nuclei, where the plane wave approximation breaks down, two things happen. First, the form factor is a function of two variables, E_i and θ. Consequently, curves for the same q, but different E_i, will be different.

Secondly, the diffraction minima are filled in. For $E1$ excitations the first minimum is barely visible, and occurs at lower q compared to PWBA. Another important point is that, despite all these changes, up to the first minimum, or what is left of it, the curves are nearly identical for calculations with the same initial energies E_i, but different transition charge densities, if the transition radii

$$R_{tr} = \langle r^{\lambda+2}\rangle_{tr}/\langle r^2\rangle_{tr}$$

are equal. But it is also apparent that differences beyond the first minimum can be quite large. This can, for example, be seen from comparison between the MS model ($c_{tr}/c = 0.9$), $R_{tr} = 6.30$ fm, and the SJ model ($c_{tr}/c = 1$), $R_{tr} = 6.26$ fm. As one would expect from the general trend, the calculation with the lower R_{tr} results in the higher curve. This is understandable because, as mentioned above, the cross sections in the DWBA code are normalized to

$$B(E\lambda) = (2\lambda + 1)\left|\int r^\lambda\rho_{tr}(r)r^2\,dr\right|^2 = 1\,\text{fm}^{2\lambda}.$$

Since the integral will be larger when R_{tr} is larger, the curves have to be lower. A compilation of various models and parametrizations and the R_{tr} which go with them is given in Table I. Figure 7 shows similar calculations for the quadrupole case, but with the emphasis on the GT model ($c_{tr}/c = 0.9$, 1.0, 1.1, and 1.2). Again, a calculation with the MS model ($c_{tr}/c = 1$) which yields a R_{tr} close to the GT case ($c_{tr}/c = 0.9$) is nearly identical with the latter.

Two features emerge comparing $E2$ to the $E1$ calculations (Fig. 6): the differences in height between $E2$ curves with the same parameters as in the $E1$ case are larger (and will be even larger for higher multipolarities, see Fig. 8) and the first minima are less washed out. Figure 8 finally shows a set of calculations for an octupole transition.

TABLE I. Transition radii $R_{tr} = \langle r^{\lambda+2}\rangle_{tr}/\langle r^2\rangle_{tr}$ for various combinations of models, parametrizations, and multipolarities. This table is useful for a quantitative analysis of the model dependence as indicated by Figs. 4, 5, and 6.

c_{tr}/c	E1 R_{tr}^{GT} (fm)	E1 R_{tr}^{MS} (fm)[a]	E1 R_{tr}^{SJ} (fm)	E2 R_{tr}^{GT} (fm)	E2 R_{tr}^{MS} (fm)[b]	E2 R_{tr}^{SJ} (fm)	E3 R_{tr}^{GT} (fm)	E3 R_{tr}^{MS} (fm)[b]	E3 R_{tr}^{SJ} (fm)
0.9	6.78	6.30	5.74	7.16	6.61	6.02	7.59		6.16
1.0	7.34	6.88	6.26	7.64	7.18	6.57	8.13		6.76
1.1	8.03	7.23	6.78	8.35	7.76	7.12	8.69	8.04	7.34
1.2	8.67	8.05	7.31	8.96	8.34	7.67	9.27	8.62	7.92
1.24	8.923								
1.36		8.99							

[a] $\alpha = 0.9$.
[b] $\alpha = 1.0$.

The conclusion from the actual variation in the models, displayed in Figs. 6 to 8 is that the strength extracted from (e, e') may be quite model dependent, but that the multipolarity determined from the position of the first maximum of the form factor is much less sensitive.

Nevertheless, for all practical purposes in the momentum transfer covered by this experiment, a GT calculation for an $E2$ with $c_{tr}/c = 0.9$ is indistinguishable from calculation for $E3$ with $c_{tr}/c = 1.2$. This has to be kept in mind below where we try to get a unified and consistent picture and description of giant resonances in ^{238}U.

C. Errors

The error assignment to giant resonance cross sections has been found difficult by most authors (see, e.g., Ref. 55). Since many uncertainties enter in the background determination, the purely statistical error from solving the error matrix in the χ^2 fit is mostly too small. The errors shown for the cross sections in this paper are estimated total errors, which came out to be approximately twice the statistical errors. The estimate was based on how the areas under the curves, positions, and width could be changed during the fits due to different choice of resonance parameters, neighboring lines, background, ghost peak subtraction, etc., while still maintaining $\chi^2 < 1.1$ per degree of freedom. As may be seen from Figs. 9–15, the typical estimated uncertainty for a given peak area for one angle is 20 to 30%. All other experimental errors are very minor in comparison (elastic phase shift $<\frac{1}{2}\%$, charge accumulation $<0.1\%$, inelastic DWBA $<1\%$, etc.) and have been, therefore, neglected.
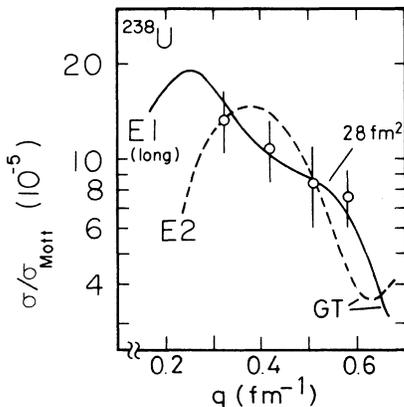

FIG. 9. Comparison between experimental and DWBA (GT, $c_{tr}/c = 1$) form factor for the GDR branch at 11 MeV (oscillation along long axis). The data are extremely well described by the calculation.
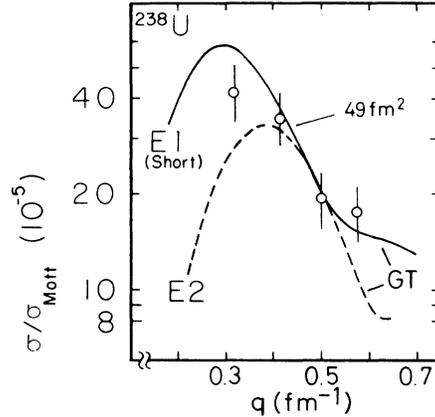

FIG. 10. Similar to Fig. 9, but for the GDR resonance at 14 MeV.

Since we have measured four angles the estimated error per resonance given in Table II is 10 to 15% (average of four angles).

The assignment of the error in giant resonance measurements is subject to criticism. The subjective impression seems to be that the assigned errors are too small.

We compared recently, in the case of ^{140}Ce (Ref. 32), the total estimated error with the standard deviation of the sum rule exhaustion (Sec. III B and Table X of Ref. 32). Since the latter is a measure for the fit to the model, i.e., the determination of the multipolarity, it should be smaller, but of the same order of magnitude as the total

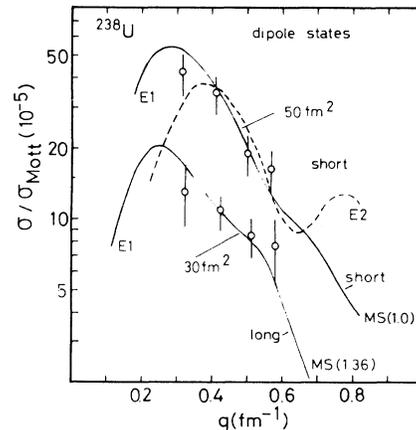FIG. 11. Comparison between MS model DWBA calculations, assuming a ground state charge distribution enlarged by 10% and the experimental data for both branches of the GDR. The resulting strength is close to the one extracted with the GT model in Figs. 9 and 10. On this basis no decision about the validity of either model and other underlying assumptions could be made (see text for more details).
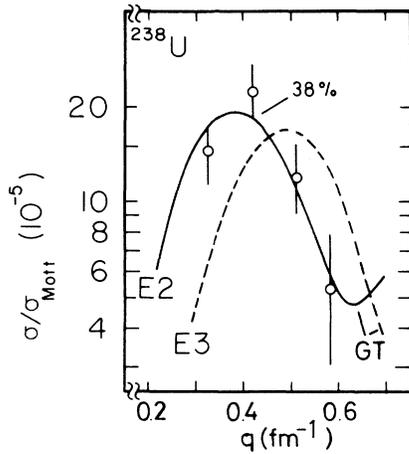
FIG. 12. Comparison between experimental data for the 9.9 MeV resonance and $E2$ and $E3$ GT DWBA calculations. An $E2$ assignment is favored but results in the relatively low sum rule value indicated in Fig. 6.

estimated uncertainty. This was found to be the case in Ref. 32. In the present paper the result of such a comparison is similar: The standard deviation is smaller then, but of the same order of magnitude as, the total error. If we would arbitrarily enlarge the estimated error of the areas under the resonances, say be a factor of 2, the χ^2 for the fits of the DWBA curves in Figs. 9–15 to the data points would become ridiculously small.

In our attempts to fit the four spectra shown in Fig. 3 we have tried to artificially change the areas under the resonances by keeping parameters of resonances at larger or smaller values than those determined by the fits as described above and in Refs. 33 and 32. No fit with a reasonable χ^2 per
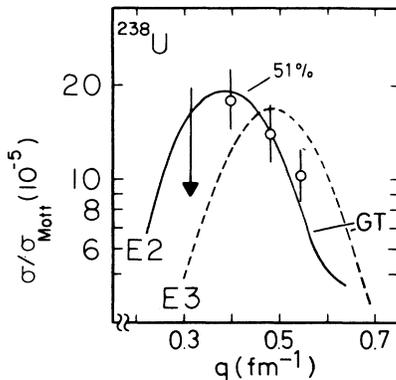

FIG. 13. Similar to Fig. 12 but for the resonance at 21.6 MeV, which is assumed to be isovector in nature. The datum at 0.32 fm^{-1} is an upper limit which was estimated from the statistical error of the 45° measurement and the width as determined from other angles.
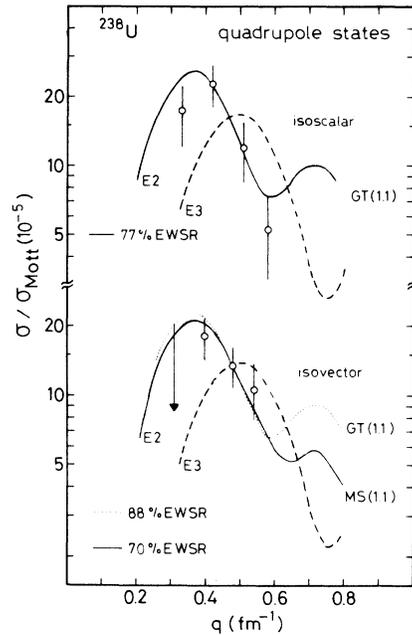

FIG. 14. Comparison between the experimental data of Figs. 12 and 13 and DWBA calculations based on modified GT and MS models. If we follow the underlying hypothesis of this paper, namely the assumption of a spatially larger ^{238}U in the excited state as compared to the ground state, the data are well described by the DWBA calculations and are in agreement with other experiments and what we would expect from Figs. 4 and 5,
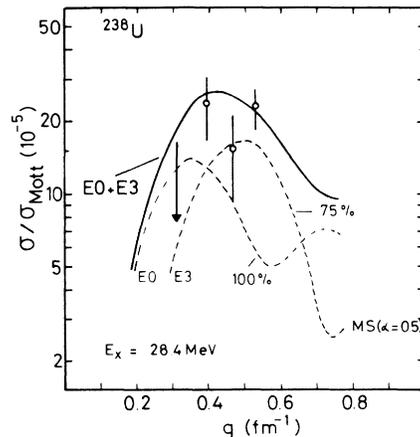
FIG. 15. Comparison between the cross section of the 28.4 MeV resonance and DWBA calculations. Choosing $c_{tr}/c = 1.1$ in agreement with the parametrization for the other resonances is compatible with 100% of the isovector monopole plus 75% of the isovector octupole EWSR in this region. It should be emphasized, however, that assumption of 90% of the $E3$ EWSR (on the basis of the GT model) to be present fits the data nearly as well without any monopole contribution. Similar to Fig. 13 the upper limit at 0.30 fm^{-1} was estimated from the statistical error of the 45° data at 28.4 MeV and the width as determined from other angles. We think that the χ^2 fit would be sensitive to any larger cross section.

TABLE II. Summary of the quantitative results of this paper. While the excitation energy and the width of the resonant structures found is relatively insensitive to multipolarity and models used, the strength is not. For each resonance, two values are given. The upper value corresponds to a straight application of the GT model to the data. The lower value corresponds to the assumption of an ^{238}U nucleus which is spatially enlarged by approximately 10% as compared to the ground state. In addition, the MS model (Refs. 52, 53) was used for the isovector excitations. These assumptions lead to a greater consistency of the strength with other available data in lighter nuclei (Ref. 8) and for ^{238}U.

E_x (MeV)	$E\lambda$	Model (c_{tr}/c)	$E_x(A^{-1/3}$ MeV)	ΔT	Γ (MeV)	$B(E\lambda)$ (fm$^{2\lambda}$)	Γ_γ^0 (eV)	SPU	$R(\%)$ [a]
9.9 ±0.2	E2	GT(1.0)	62	0	$2.9^{+0.8}_{-0.4}$	3700	56	17	38 ± 10
		GT(1.1)				7500	114	35	77 ± 20
10.8 ±0.3	E1	GT(1.24)	67	1	3.2 ± 0.4	28	1.3×10^4	5	36 ± 4
		MS(1.35) [b]				30	1.3×10^4	5	39 ± 4
13.9 ±0.3	E1	GT(0.9)	86	1	$4.5^{+0.3}_{-0.4}$	49	4.6×10^4	10	81 ± 8
		MS(1.0) [b]				50	4.7×10^4	10	83 ± 8
21.6 ±0.6	E2	GT(1.0)	133	1	5.0 ± 0.6	3600	2.7×10^3	17	51 ± 8
		MS(1.1) [c]				5000	3.7×10^3	23	70 ± 11
28.4 ±1.2	E3	GT(1.0)	176	1	8.1 ± 1.1	6.2×10^5	5.4×10^2	78	91 ± 15
		MS(1.1) [d]				5.1×10^5	6.6×10^2	64	75 ± 12

[a] $R = E_x \times B(E\lambda)/\text{EWSR}(E\lambda, \Delta T) \times 100$.
[b] $\alpha(238,1) = 0.9$.
[c] $\alpha(238,2) = 1.0$.
[d] $\alpha(238,3) = 0.5$.

degree of freedom was possible. The fit to a single resonance or even a single spectrum may be arbitrary. It is the simultaneous fit to all the resonances in all the spectra which leads to our error assignment.

The only other large uncertainty comes from the model dependence and concerns only the extracted strength. The values quoted can easily be changed by a factor of two by changing the model or its parametrization. The question as to the model dependence has no easy answer, but it does not even have to be answered, because the thrust of this paper, as may be deduced from the quote from Ref. 16 in the beginning, rather is whether or not the charge distribution for the excited states is more extended than that of the ground state, and whether or not the sum rule values can be brought in to agreement with the systematic expectations (Figs. 4 and 5) and other experiments, with reasonable changes of the models used.

V. RESULTS

A. General

Figure 3 shows the spectra taken and evaluated for the present work after subtraction of total background including the ghost peak, which in our spectrometer occurs at 92% of the elastic energy, i.e., approximately 7 MeV excitation energy. The resonances required for the simultaneous fit of spectra and background are indicated.

Several features are apparent in Fig. 3 without a detailed quantitative analysis. We know from

photon work the position of the two branches of the dipole state, 11 and 14 MeV. If we take these resonances as reference, it is apparent from Fig. 1 that the resonances at 10 and 22 MeV are of higher multipolarity because they rise with the momentum transfer compared to E1. The states at 10 and 22 MeV are presumably E2, because these energy positions correspond to the well-known 63 and $135A^{-1/3}$ MeV dependence for the isoscalar and isovector E2.[8] The state at 28 MeV rises faster with angle or momentum transfer than any other resonance and thus has to be of higher multipolarity. The excitation energy, compared in $A^{-1/3}$ MeV units, is lower than the resonancelike structure found in ^{208}Pb and ^{197}Au (Ref. 31), but agrees with the data from ^{181}Ta (Ref. 47) and ^{165}Ho (Ref. 12). All these arguments assume that only *one* multipolarity contributes to each resonant structure.

The one feature of Fig. 3 which does not fit into the simple picture as inferred from Fig. 1 is the resonant cross section at 17 MeV because it appears only at 45° and 90°. However, a resonance shape was required at this excitation energy to achieve a satisfactory χ^2 per degree of freedom, i.e., $\chi^2 < 1.1$, at these angles.

B. The giant dipole resonance

The splitting of the GDR in ^{238}U into two branches has been measured by $(\gamma, n) + (\gamma, f)$[18-21] and (γ, γ') measurements.[17] The values for position and width agree well with our results (Table III), which can

TABLE III. Comparison of giant dipole resonance parameters from several experiments. The γ cross sections (Refs. 18-21) have been converted into reduced transition probabilities using Eq. 9 of Ref. 37. The other parameters, excitation energy and width, were not converted to those of the strength function, because for the $E1$ the resulting changes are relatively small (Ref. 37), in any case smaller than the errors given. The strength given from this work is based on the "strict" GT model; more detailed information can be found in the text and Table II.

Ref.	Reaction	E_x (MeV)	Long axis Γ (MeV)	B (fm^2)	E_x (MeV)	Short axis Γ (MeV)	B (fm^2)
20	(γ,n)	10.96 ± 0.09	2.90 ± 0.14	31	14.04 ± 0.13	4.53 ± 0.13	46
18	(γ,γ')	10.95 ± 0.06	2.62 ± 0.19	28	14.00 ± 0.68	4.53 ± 0.20	48
21	(γ,n)	10.80	2.44	28	13.85	5.12	65
19	(γ,n)	10.97 ± 0.13	2.99 ± 0.48	30	14.25 ± 0.18	5.10 ± 0.63	49
This work	(e,e')	10.75 ± 0.25	3.2 ± 0.4	28	13.95 ± 0.25	$4.5^{+0.2}_{-0.3}$	49

be taken from the fit to the spectra without much model dependence. Getting the strength [reduced transition probability $B(E\lambda)$, sometimes called B-value] is more involved.

In the Danos-Okamoto model of the GDR the splitting in deformed nuclei is interpreted as being due to difference in the resonant energies of oscillations directed along the longer and shorter radii, R_z and R_1, of the nuclear spheroid.[9,10] As outlined in the Introduction, these oscillations are treated separately. The model parameters, R_z and R_1, are determined in the framework of the Danos-Okamoto model from the giant dipole energies, E_z and E_1. If we assume a uniform density within the spheroid we get a volume constraint

$$R_{eq}^3 = \tfrac{5}{3}\langle r^2 \rangle^{3/2} = R_z \times R_1^2 \,,$$

where R_{eq} is the equivalent radius of a sphere. The Danos-Okamoto equation

$$R_z/R_1 = (E_1/E_z - 0.089)/0.911$$

then enables the calculation of c_{tr}, yielding $c_{tr}^z = 1.24c$ and $c_{tr}^1 = 0.90c$. Constant skin thickness has been assumed for these calculations, because variations in the skin thickness influence the resulting R_{tr} only weakly. If one uses an unmodified, or "strict," Goldhaber-Teller model for all resonances, the GDR is the only one for which reasonable values, or values in agreement with other experiments, concerning the sum rule are achievable. Figures 9 and 10 show a fit to DWBA calculations which were performed with the parameters of the strict GT model yielding $B(E1, \text{long}) = 28$ fm^2 and $B(E1, \text{short}) = 49$ fm^2, which agrees rather well with the γ results (Table II). In contrast, the MS model with $c_{tr}/c = 1.24$ and 0.9 does not do that well and would yield 23 and 37 fm^2, respectively. Since for the MS model values for both axes are smaller than the photon results (Table II), we have done the obvious and enlarged c_{tr} by approximately 10%,

which brings c_{tr}/c to 1.0 and 1.36, respectively. Calculations using this value with the Myers-Swiatecki model are compared in Fig. 11 to the experimental data, resulting in $B_{MS}(E1, c_{tr}/c = 1.36, E_x = 11 \text{ MeV}) = 50$ fm^2 and $B_{MS}(E1, c_{tr}/c = 0.99, E_x = 14 \text{ MeV}) = 30$ fm^2. The strengths calculated with both models and both parameterizations are shown in Table II together with the model independent γ results.

This is a rather confusing situation which seemingly does not lend itself to easy interpretation. However, the physical aspects simplify, if we look at Table I, and compare the transition radii for the different model and parametrization combinations which give similar strength values. One sees that $R_{tr}(\text{GT}, 0.9) \approx R_{tr}(\text{MS}, 1.0)$ and $R_{tr}(\text{GT}, 1.24) \approx R_{tr}(\text{MS}, 1.36)$ and consequently, due to the connection between R_{tr} and $B(E\lambda)$, the above result is no longer surprising.

It has been shown, that a large fraction of the $E1$ cross section[20,21] as well as the $E2$ cross section[30] is due to fission. Consequently, the enlargement of the ^{238}U nucleus in the giant resonance region should be less a function of multipolarity but of excitation energy. The above analysis will only appear reasonably sound if an analysis along these lines ("blow-up" of the excited state charge distribution by approximately 10%) improves the agreement between results from other experiments and (e,e') in ^{238}U for all multipolarities. In Sec. VC we will, therefore, investigate the quadrupole states.

In summary, if one assumes the MS model to be the better choice (for arguments see, e.g., Ref. 54), one has to conclude that the ^{238}U nucleus in the excited state is 10% larger than in the ground state.

C. Isoscalar and isovector quadrupole giant resonance

Besides the problem of the strength of the $E2$

states and its dependence on the model and param-
etrization used, as outlined earlier, the question
of their widths is also of importance. Since this
parameter is only weakly model dependent, it may
serve as a measure for the reliablility of an ex-
periment. The width measured, or assumed, also
influences the cross section under a resonance,
especially in inelastic scattering where the back-
ground is fitted simultaneously with the resonances.
Since assumption of a wrong width will influence
the results at all angles it will influence the deter-
mination of multipolarities only weakly. The phy-
sical motivation for measuring the width of de-
formed nuclei has been given in the Introduction.

There have been several calculations, both with
microscopic and macroscopic theories, as to the
width of the isoscalar quadrupole states in de-
formed nuclei. Comparison of ^{144}Sm and ^{154}Sm by
Kishimoto et al.[56] showed a total theoretical split-
ting of 6 MeV for the $K = 0, 1, 2$ components, while
the experimental result from α scattering was
only 0.8 ± 0.3 MeV, namely from 3.9 MeV to 4.7
MeV for the broadening of the overall line shape.[56]
Requiring self-consistency for the quadrupole-
quadrupole interaction brought the total splitting
down to 2 MeV, which results in a broadening in
agreement with the experimental value. An (e, e')
experiment[12] in the same A region of the nuclear
system, ^{142}Nd compared to ^{152}Nd, showed a broad-
ening of 2.2 MeV, namely from 2.8 ± 0.2 MeV to
5.0 ± 0.2 MeV, while (e, e') on ^{165}Ho showed $\Gamma = 3.9$
± 0.4, which was compared to the GQR in ^{140}Ce
and ^{208}Pb, both 2.8 MeV wide.[13] Disparate results
given in Refs. 13 and 56 seem now to be recon-
ciled.[57]

Microscopic calculations using a quasiparticle
RPA[58] predicted a broadening which varied ap-
preciably in the three rare earth nuclei investi-
gated (from 1.0 to 2.9 MeV, depending on the nu-
cleus and assumption of the unperturbed linewidth),
but no measurements have been done in these

nuclei. Macroscopic calculations based on a
viscosity model by Auerbach and Yeverechyahu,[59]
by definition applicable to all A, give rough agree-
ment with the general trend,[8] but do not account for
shell or deformation effects. An application of the
theory of Ref. 59 on deformed nuclei[60] shows qual-
itative agreement, as do more recent macroscopic
calculations by Suzuki and Rowe.[61] The most de-
tailed description of deformed nuclei has been pub-
lished by Soloviev and co-workers[41,62] on the basis
of Soloviev's semimicroscopic model (for a more
complete list of references to this work see Ref.
63). Table IV shows a comparison between this
theory and experiment. In general, the agreement
is good and shows the tendency of the width to
decrease with increasing mass, expected from
simple macroscopic considerations,[59] but also
shows individual variations from nucleus to nu-
cleus as expected from a microscopic theory.[58]

The next figures (Figs. 12–14) show that the
resonances at 9.9 ± 0.2 MeV with a width of $2.9^{+0.8}_{-0.4}$
MeV, and at 21.6 ± 0.7 MeV with a width of 5.0
± 0.6 MeV, conform to an $E2$ momentum transfer
predicted by the DWBA calculations described in
the captions to Figs. 12–14. Figures 4 and 5 had
shown that the application of the "strict" Gold-
haber-Teller model lead to sum rule fractions
which were lower than expected from other nuclei.
Figures 12 and 13 show that despite that low
strength the momentum transfer dependence is
correctly described.

To test our hypothesis that ^{238}U at higher exci-
tation energy is more extended than in the ground
state, we have performed calculations in analog
to the dipole case. Figure 14 shows a fit of DWBA
calculations based on the GT model for the iso-
scalar $E2$ at 9.9 MeV (upper part) and both GT
and MS model for the isovector $E2$ at 21.6 MeV for
the isovector $E2$ (lower part) with a ground state
change distribution scaled by 10% to the experi-
mental data. It is obvious that the fits are just as

TABLE IV. Comparison of the experimental spreading width of the isoscalar giant quadru-
pole resonance in deformed nuclei resonance as measured with (e, e'), (e, f), and (α, α'), with
theoretical predictions based on Soloviev's semimicroscopic model (Ref. 41).

Nucleus	Γ_{exp} (MeV)	Γ_{theory} (MeV)	Ref. (exp)	Ref. (theory)
^{150}Nd	5.0 ± 0.2	5.0	12	39
^{154}Sm	4.7 ± 0.3	4.5	5	39
^{165}Ho	3.9 ± 0.4	3.9	13	a
^{181}Ta	3.13 ± 0.55		45	
^{238}U	5.1 ± 0.3^b	2.9	28	39
	$2.9^{+0.8}_{-0.4}$		present work	39

^{a} S. V. Akulinichev and L. A. Malov, DUBNA Report No. E4-9758, quoted in Ref. 41.
^{b} Width of the strength function, that of the cross section as given in Ref. 30 is 6.8 ± 0.4
MeV.

good as the ones in Figs. 12 and 13. Table V shows that, for the isoscalar state, the sum rule fraction of 77% is in reasonable agreement with other measurements though still lower than expected from Fig. 6. But this difference is meaningless in view of the strong model dependence.

There are no other direct measurements of the isovector $E2$ at 21.6 MeV. The only quantiative inference made from the work of Kneissl et al.[29] would lead to 130% of the EWSR (see Sec. II), for which we estimate an error of 50% on the basis of the data of Ref. 29. Our result, 70 to 88%, depending on the model used, has, as the other strengths given, a strong model dependence. As Fig. 14 shows this is mainly due to the fact that the data points are on the falling region of the form factor. To overcome this difficulty data would have to be taken in the region of the second maximum, a difficult enterprise in itself due to the above mentioned rapid falloff of the cross sections, and the increasingly strong excitation of higher multipoles as evident from Fig. 1.

The MS model calculation shown in Fig. 12 was based on $\alpha(238,2) = 1.0$. Using $\alpha = 0.5$ from Kodama's work would raise the EWSR fraction from 70% to approximately 80%, but, as one may infer from Fig. 14, not change the assignment of $E2$ or any other conclusions.

It is somewhat dangerous to rely on the extrapolations from lower A in judging what might or might not be a reasonable value. But still, the strength has to be somewhere. So it either might be spread out through direct or semidirect processes or shifted to very low excitation energy. On the other hand, support for the hypothesis of assuming a "blown-up" ^{238}U nucleus comes from the simultaneous improvement for both $E1$ and $E2$ states in comparison with results from other methods.[18-21,29,30]

D. The state at 28 MeV

From Fig. 3 it is evident that the resonancelike structure, called resonance in the following, at 28.4 $(176A^{1/3})$ MeV rises faster with momentum transfer than the $E2$ states at 10 and 22 MeV. In

TABLE V. Isoscalar $E2$ resonance.

Reference	E_x (MeV)	Γ (MeV)	R (%)[a]	Reaction
22	10 − 13		85	(p,p')
28	9.9 ± 0.2	5.1 ± 0.3	71	(e,f)
This work	9.9 ± 0.2	$2.9^{+0.8}_{-0.4}$	77[b]	(e,e')

[a] $R = E_x \times B(E2)/\mathrm{EWSR}(E2, \Delta T = 0)$.

[b] Value based on a modified GT model with $c_{tr}/c = 1.1$.

measurements in ^{140}Ce (Ref. 32), ^{165}Ho (Ref. 13), ^{181}Ta (Ref. 47), and ^{197}Au and ^{208}Pb (Ref. 31), a resonance at about the same excitation energy has been found, whereby a definite assignment proved difficult due to the high excitation energy which results in a large width and small peak cross section. Interpretation has been oscillating between $E0$ and $E3$ (Ref. 8). Macroscopic predictions project the isovector $E3$ state[64] at around $190A^{-1/3}$ (30.5) MeV, and the isovector monopole[65] at $178A^{-1/3}$ (28.5) MeV. Microscopic calculations give compatible results (see, e.g., Ref. 66).

From experiment it was noted[8,32] that a resonance appeared at $195A^{-1/3}$ MeV in spherical nuclei but at $\sim 180A^{-1/3}$ MeV in deformed nuclei. It has, therefore, been speculated[32] that the higher energy resonance (if they are indeed of different multipolarity) might be $E3$, which fragments in a deformed potential and disappears in the background. The $E0$, on the other hand, is in spherical nuclei between isovector $E2$ and $E3$ and becomes visible only in deformed nuclei. While decisive experiments still have to be performed, Fig. 15 shows that the data in ^{238}U are compatible with this explanation, because the cross section is large enough to accommodate 100% of the isovector monopole and 75% of the isovector octupole EWSR. For the $E0$ DWBA calculations of Fig. 15 the model by Schucan[67]

$$\rho_{tr}(r) = -3\rho_0(r) + d\rho_0(r)/dr$$

was used, which is identical with the one proposed by Satchler.[68] No model dependence was investigated for the monopole.

E. The cross section at 17 MeV

Figure 3 shows for the 45° and 90° spectra, but not the 60° and 75° spectra, a resonance at 17 MeV. Since this behavior is somewhat at odds with the regular behavior predicted by Fig. 1 no definite answer as to the origin is possible. But this "resonance" was necessary to fit the spectra as outlined earlier. If taken as a Breit-Wigner shaped state its excitation energy is $E_x = 17 \pm 2$ MeV and its width 3.9 ± 1.5 MeV. Although the resonance energy of $106A^{-1/3}$ MeV closely follows the isoscalar $E3$ resonance predicted by the self-consistent shell model,[7,64] it does not follow the angular distribution for an $E2$ cross section. However, the momentum transfer found could be explained by a mixture of $M1$ and $E3$, because at 45° an $M1$ would be at the maximum of its form factor. Since the measurements were not extended to sufficiently large or small angles to isolate transverse contributions, no more definite statement can be made.

VI. CONCLUSIONS

We have investigated the giant resonance region of the fissionable, deformed nucleus ^{238}U with inelastic electron scattering. While other deformed nuclei have been investigated in the past, this experiment constitutes to our knowledge the first (e, e') measurement of a fissionable nucleus in the continuum. The difference to these other measurements is that the hydrodynamic models (GT for $E2$ and $E3$, and MS for the $E1$) apparently fail to describe the strength correctly.

Does that mean they break down and cannot and should not be used for the evaluation? It is clear from our paper that we concluded otherwise. As in other cases the collective models give an insight into the phenomena of a complicated many body system that is difficult to achieve otherwise. In the present case it tells us that ^{238}U expands as it is excited to higher excitation energies. While it is tempting to refine the present analysis by investigating if the scaling factor c_{tr}/c is a function of the excitation energy, we think that such an attempt would overtax our present data base.

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