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Low-energy absorption of pions on nuclei and the real $\rho^2(r)$ potential

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A Wick-formalism treatment of the $\sigma + \omega$ model dynamics is used to obtain the real and imaginary parts of the π-nucleus optical resulting from the true absorption of "s-wave" pions. Good agreement is obtained with the empirical value of the imaginary potential at threshold. The corresponding (dispersive) real potential is attractive. All parameters are taken from other experiments.

[NUCLEAR REACTIONS $A(\pi^-, NN)B$; imaginary part of π-A optical potential calculated in $\sigma + \omega$ model using Fermi-gas wave functions. Off-shell absorptive amplitude used to calculate corresponding real part from unsubtracted dispersion relation.]

The $\sigma + \omega$ model is a relativistic quantum field theory of interacting pions and nucleons (as well as the σ and ω mesons) which is renormalizable, satisfies PCAC, and gives a good description of low-energy NN, πN, and $\pi\pi$ dynamics. Various aspects of the dynamics of pions interacting with nuclei have recently been explored within the framework of this model, including near-threshold pion production and low-energy elastic scattering of pions.[1,2]

In this paper we investigate the predictions of the $\sigma + \omega$ model for "true" pion absorption. This process is usually represented by a term of the phenomenological pion-nucleus optical potential which is quadratic in the nuclear density (because true absorption is dominated by the emission of *two* nucleons):

$$2k^0 v = -4\pi i\, \mathrm{Im}(B_0)\rho^2(r)\,. \tag{1}$$

The folklore of absorption rate calculations has been that the amplitude is dominated by terms representing off-shell elastic scattering of the incident pion from one active nucleon, followed by its absorption on a second nucleon, as illustrated in Fig. 1.[3] In order to calculate this diagram, one needs the off-shell πN scattering matrix, the πNN absorption vertex, and the initial and final nucleonic wave functions. Several authors have recently calculated true pion absorption in this manner, using phenomenological models of varying sophistication to represent the πN scattering matrix.[4-6] These calculations have been unable to account for more than about 70–75% of the low-energy absorptive cross section. Our calculation of the absorptive amplitude differs from those of other authors primarily in that it is based on evaluation of the leading terms in the $\sigma + \omega$ model through third order in the coupling constant. We find that the missing strength is provided by short-range terms arising from the exchange of the heavy mesons included in this theory. (There are also some differences in technical details of the handling of Fermi-gas wave functions and the avoidance of unitarity-violating unphysical singularities.)

In addition to the imaginary part of the pion-nucleus potential arising from true pion absorption, Eq. (1), we expect a real contribution to B_0 arising from dispersion. The expected analytic properties of the amplitudes imply that $\mathrm{Re}(B_0)$ satisfies a dispersion relation; the expected rapid decrease with energy of the imaginary part, for large energy, implies that the dispersion relation will be unsubtracted. Thus, the algebraic sign of the real part near threshold is determined by the energy dependence of the imaginary part. All microscopic calculations of the absorptive part[5-8] have found it to be peaked at energies well above threshold, so that the resulting dispersive real part is attractive. Our calculation is no exception.

We now describe our calculation. The elastic T matrix for π^--nucleus scattering is given exactly by the Wick formalism in the $\sigma + \omega$ model[2]

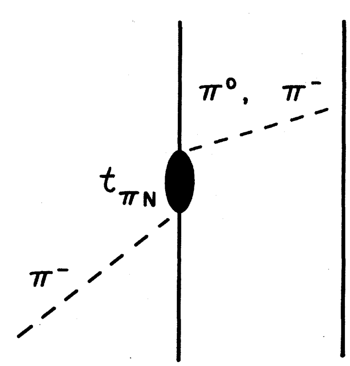

FIG. 1. Rescattering model of $A(\pi, NN)B$ reaction.

$$T_{\vec{k}'\vec{k}}(k^0)=\int\frac{d^3x}{(2\pi)^3 2k^0}e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}\langle \text{g.s.}|\left\{\frac{8M\lambda}{G}\sigma(\vec{x})+4\lambda[\sigma^2(\vec{x})+\vec{\pi}(\vec{x})\cdot\vec{\pi}(\vec{x})]+8\lambda\pi_+(\vec{x})\pi_-(\vec{x})\right\}|\text{g.s.}\rangle$$
$$+\langle \text{g.s.}|J^\dagger_{\vec{k}'}(E_{\text{g.s.}}+k^0+i\eta-H)^{-1}J_{\vec{k}}|\text{g.s.}\rangle+\langle \text{g.s.}|J_{\vec{k}}(E_{\text{g.s.}}-k_0+i\eta-H)^{-1}J^\dagger_{\vec{k}'}|\text{g.s.}\rangle\,, \quad (2)$$

where $8\lambda M^2/G^2=m_\sigma{}^2-m_\pi{}^2$, M is the nucleon mass, $\sigma(\vec{x})$ and $\vec{\pi}(\vec{x})$ are the nuclear σ-meson and pion fields, and $J_{\vec{k}}$ is the Fourier transform of the source term in the Klein-Gordon equation

$$\left(\nabla^2-m^2-\frac{\partial^2}{\partial t^2}\right)\pi_-(\vec{x})=J(\vec{x})\,,$$
$$J_{\vec{k}}=\int\frac{d^3x}{(2\pi)^{3/2}\sqrt{2k^0}}e^{i\vec{k}\cdot\vec{x}}J(\vec{x})$$
$$\equiv\int\frac{d^3x}{(2\pi)^{3/2}\sqrt{2k^0}}e^{i\vec{k}\cdot\vec{x}}\times\left\{iGN^\dagger(\vec{x})\gamma^0\gamma^5\tau_-N(\vec{x})+\frac{8M\lambda}{G}\sigma(\vec{x})\pi_-(\vec{x})+4\lambda\pi_-(\vec{x})[\sigma^2(\vec{x})+\vec{\pi}(\vec{x})\cdot\vec{\pi}(\vec{x})]\right\}. \quad (3)$$

As noted in Ref. 2, the field operators in (3) include interactions, hence include terms of all orders in G. Here we shall concentrate on terms through $O(G^3)$, which arise in the expansion of the $N^\dagger N$ and $\sigma\pi$ terms. The $\lambda\pi(\sigma^2+\pi^3)$ term is $O(G^5)$ (at least), so is dropped.

The first term of Eq. (2) contains the effects of pion interactions with the fluctuating meson fields of the target nucleus. The next two terms are handled by using those states containing zero or one asymptotic mesons to spectrally represent the energy denominators in which H appears. The zero meson states of low energy give a contribution which is the sum over nucleons of the driving terms of the (free) π-nucleon Low equation, and which leads to the impulse contribution to the optical potential.[9] The zero-meson states of higher energies (2p-2h states not reached by matrix elements of $J_{\vec{k}}$ involving single nucleons only) lead to an imaginary part of T (true pion absorption) which is physically distinct from that arising from elastic and inelastic unitarity. This absorptive part is given by the imaginary parts of the terms of Eq. (2) involving J's, in which the intermediate states have no free pions. In fact, we know from experiment that the dominant process in low-energy pion absorption is the emission of two high-energy nucleons, so we shall write

$$\text{Im}[T^{0\pi}_{\vec{k}'\vec{k}}(k^0)]\simeq-\pi\sum_{2p\text{-}2h}\langle 2\text{p-2h}|J_{\vec{k}'}|\text{g.s.}\rangle^*\times\langle 2\text{p-2h}|J_{\vec{k}}|\text{g.s.}\rangle\times[\delta(E_x-k^0)+\delta(E_x+k^0)]\,, \quad (4)$$

where E_x is the energy difference between the 2p-2h and ground states, and where we have assumed $N=Z$. Note that Eq. (4) is strictly correct only for $k^0\approx m_\pi$. At such low energies, neither pion-nucleus inelastic scattering nor meson production can occur. At higher energies both inelastic scattering and meson production represent an important part of Im(T) which is omitted deliberately from Eq. (4). Note that the spectral function [negative imaginary part of $T_{\vec{k}'\vec{k}}$ from Eq. (2)] is a positive operator in the sense that, for arbitrary functions ψ,

$$\int d\vec{k}'\int d\vec{k}\,\psi^*(\vec{k}')[-\text{Im}(T_{\vec{k}'\vec{k}}(k^0)]\psi(\vec{k})\geq 0\,.$$

(It is this property which guarantees that the imaginary part of the optical potential is absorptive, rather than emissive, even when it is nonlocal.)

We are interested in calculating the optical potential v, but Eqs. (2) and (4) involve $T(k^0)$. Liu and Shakin[10] have shown that in order to obtain formal expressions for v from those involving $T(k^0)$, one need merely remove from $T(k^0)$ all terms involving intermediate π-nuclear eigenstates, in which the nucleus remains in its ground state. Clearly, no terms involving the pion plus nuclear ground state appear in the right-hand side of Eq. (4). Thus we may write, ignoring nonlocality which is clearly present in Eq. (4),

$$\text{Im}[T^{0\pi}_{\vec{k}'\vec{k}}(k^0)]\simeq\text{Im}\int\frac{d^3x}{(2\pi)^3}e^{i(\vec{k}-\vec{k}')\cdot\vec{x}}v(\vec{x},k^0)\,. \quad (5)$$

[It will be reasonable to ignore nonlocal effects since they have a range smaller than $\sim(2m_\pi)^{-1}$, and for low-energy pions, $T_\pi\leq 50$ MeV, both k and k' $< m_\pi$.]

To simplify the calculation we treat the nucleus as a Fermi gas and ignore surface effects. Then for small momenta $\vec{k}$ and $\vec{k}'$,

$$\text{Im}[V(k^0)]\simeq-\frac{(2\pi)^3\pi}{\Omega}\sum_{2p\text{-}2h}|\langle 2\text{p-2h}|J_0|\text{g.s.}\rangle|^2\times[\delta(E_x-k^0)+\delta(E_x+k^0)] \quad (6)$$

and

$$\text{Re}[V(k^0)]=-\frac{P}{\pi}\int_{-\infty}^{\infty}d\nu\frac{\text{Im}[V(\nu)]}{k^0-\nu}\,, \quad (7a)$$

or

$$\text{Re}[V(k^0)]=\frac{2}{\pi}P\int_0^\infty\frac{d\nu\,\nu}{(k^0)^2-\nu^2}(-\text{Im}[V(\nu)])\,, \quad (7b)$$

where $V(\nu) = v(x=0,\nu)$ and Ω is the nuclear volume. In obtaining Eq. (6) we have replaced the left-hand side of Eq. (5) by Eq. (4) and then taken an inverse Fourier transform. The result, (7a), is essentially the dispersive part of Eq. (2)—Eq. (7b) is obtained from (7a) via Eq. (6) (crossing symmetry for a $T=0$ target). Thus, $-\mathrm{Im}V(\nu)$ defined in (6) approximates the spectral function of the true absorption contribution to the pion-nucleus scattering amplitude.

The off-shell dynamics specified by the Low equation are exhibited in Eqs. (6) and (7). To calculate $V(k^0)$ one must first obtain the absorptive transition matrix for incident pions of four-momentum $k \approx (k^0, \vec{0})$. Contributions to $\mathrm{Re}V(k^0)$ occur for values of ν ranging from 0 to ∞, so the incident pions are, in general, far off the mass-shell $[k^2 = (k^0)^2 \neq m_\pi^2]$. This specification has not been used by previous authors.

An important difference between this calculation and that of Hachenberg and Pirner[5] is that our spectral function, Eq. (6), is positive definite, whereas theirs is of indefinite sign. [This would remain true even had we kept the nonlocal part of the optical potential (arising from the nuclear surface), in the sense that the spectral function would be a positive-definite integral kernel.]

The matrix elements $\langle 2\text{p-}2\text{h}|J_0|\text{g.s.}\rangle$ are obtained from covariant perturbation theory, and the diagrams of lowest order (G^3) in the $\sigma+\omega$ model are displayed in Fig. 2. To this order no terms involving the pion plus nuclear ground state appear. The single nucleon term, proportional to $\vec{\sigma}_1 \cdot \vec{k}$, is expected to provide negligible contributions here, so is not included.

The approximate evaluation of the diagrams of Fig. 2 leads to the following two-nucleon absorption operator

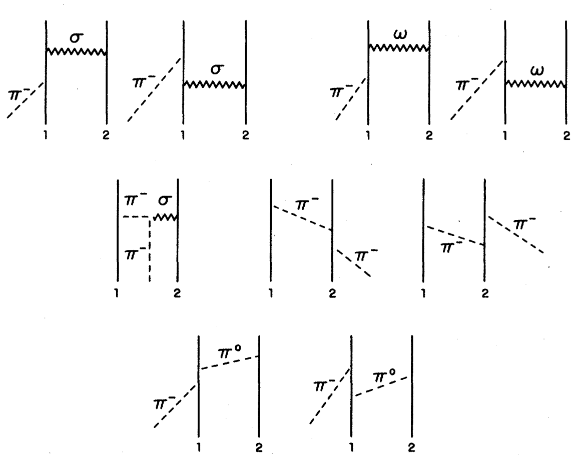

FIG. 2. The leading contributions $[O(G^3)]$ to $A(\pi^-, NN)B$ in the $\sigma+\omega$ model.

$$J_0 = \frac{G\sqrt{2}}{(2\pi)^{3/2}} \frac{1}{\sqrt{2k^0}} \frac{1}{\Omega}\, \delta_{\vec{p}_f, \vec{p}_i} u^2(q)$$
$$\times \{-\vec{\sigma}_1 \cdot \vec{q}A - \vec{\sigma}_1 \cdot [\vec{p}_2 + \vec{p}_2{}' + i\vec{\sigma}_2 \times (\vec{p}_2{}' - \vec{p}_2)]B$$
$$+ \tau_3(2)(\vec{\sigma}_1 - \vec{\sigma}_2) \cdot \vec{q}C\} \tag{8}$$

with

$$A = \frac{G^2}{q^2 + m_\sigma^2} - \frac{G_\omega^2}{q^2 + m_\omega^2} - C\,,$$
$$B = \frac{G_\omega^2}{q^2 + m_\omega^2} \frac{k^0}{2M + k^0}\,, \tag{9}$$
$$C = \frac{1}{2} \frac{G^2}{q^2 + m_\pi^2} \frac{k^0}{2M + k^0}\,,$$

and where $\vec{q} = \vec{p}_2{}' - \vec{p}_2$. Our convention is always that the proton absorbing the negative charge is labeled "1", and the other nucleon is labeled "2". (See Fig. 2.) The meson-nucleon vertex function $u(\vec{q}^2)$ is taken to be

$$u(q^2) = (1 + q^2/M^2)^{-1}\,. \tag{10}$$

For simplicity we have assumed that the vertex function is the same for all meson-nucleon couplings. (Although the dispersion integral would converge with point vertex functions, as in other calculations extended vertices are required to cut off the integral at a physically reasonable energy, since what happens above $\nu \sim 1$ GeV clearly cannot influence the threshold pion-nucleus scattering significantly.) In our evaluations of Eqs. (8) and (9) we use empirically determined values $G^2/4\pi = 14.2$, $G_\omega^2/4\pi = 12.9$, and $m_\sigma = 700$ MeV.[11]

To obtain $\mathrm{Im}[V(k^0)]$, we square the matrix element of J_0 (between the ground and 2p-2h states) and perform the sum over all such intermediate states. We assume in common with other authors[4-8] that the sum is dominated by those terms in which each final nucleon has energy $k^0/2$. For $k^0 \gtrsim m_\pi$ the momentum of each final nucleon is much greater than the momenta of the initial nucleons in the ground state. Hence, we neglect in (8) the momenta of the initial nucleons. This approximation, made also in Refs. 4-8, gives the simple result $q^2 = Mk^0$. In order to obtain $\mathrm{Re}V(k^0)$ we need $\mathrm{Im}[V(\nu)]$ for small values of ν for which the preceding approximation fails. However, $-\mathrm{Im}[V(\nu)]$ peaks at (as we shall show, Fig. 3) $\nu \approx 250$ MeV, so that our approximation is quite good for the energy interval which dominates the integral representation of $\mathrm{Re}V(k^0)$.

With the use of the above approximations, the evaluation of $\langle 2\text{p-}2\text{h}|J_0|\text{g.s.}\rangle$ is straightforward except for the consequences of the antisymmetry of the ground and 2p-2h wave functions. If the

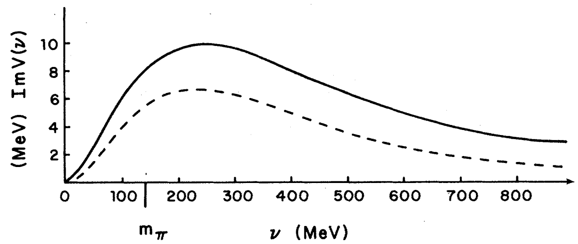

FIG. 3. The imaginary s-wave part of the pion-nucleus optical potential arising from true pion absorption, Fig. 2. (See text for details.)

intermediate state contains two neutrons, this state must be antisymmetrized. If the intermediate state contains a neutron-proton pair, the ground state contains a proton-proton pair which must be antisymmetrized. Using Eqs. (8)–(10) to evaluate Eq. (6), we find

$$\mathrm{Im}[V(k^0)] = -\frac{G^2\rho^2}{256\pi}(Mk^0)^{1/2}(\hbar c)^6 u^2(Mk^0) F(Mk^0/k_F{}^2) \times [(A+B)^2+(A+3B)^2+4C^2]\,. \qquad (11)$$

In Eq. (11) ρ is the density of nuclear matter (0.166 fm^{-3}), k_F is the Fermi momentum, and F is a factor which results from the requirements that the two initial momenta lie within, and the two intermediate momenta lie outside, the Fermi sphere. The function F is displayed in Fig. 4 and given by

$$F(Mk^0/k_F{}^2) = \left\{\frac{M}{2}\left(\frac{4\pi}{3}k_F{}^3\right)^2 (Mk^0)^{1/2}\right\}^{-1} \times \left\{\int d\vec{p}_1{}' \int d\vec{p}_2{}' \int d\vec{p}_1 \int d\vec{p}_2\, \theta(p_1' - k_F)\theta(p_2' - k_F)\theta(k_F - p_1)\theta(k_F - p_2)\delta\left(\frac{p_1'^2 + p_2'^2 - p_1{}^2 - p_2{}^2}{2M} - k^0\right)\right\}. \qquad (12)$$

The numerical evaluation of Eq. (11) is displayed in Fig. 3. The low-energy cutoff is provided by the function F, since the phase space for producing a 2p-2h excitation by absorbing energy k^0 decreases rapidly as k^0 approaches zero. At threshold, $\mathrm{Im}[V] = -8.0$ MeV, in excellent agreement with the experimental value.[12] (This value of $\mathrm{Im}[V]$ corresponds to $\mathrm{Im}[B_0] = 0.042 m_\pi{}^{-4}$.) In the energy region of current interest for π-nuclear scattering ($m_\pi < k^0 < 3m_\pi$) $\mathrm{Im}[V(k^0)]$ is relatively constant in magnitude, never deviating by more than 10% from its mean value of about 9 MeV. About one-third of $\mathrm{Im}[V(k^0)]$ arises from the inclusion of heavy meson exchange.

The function $-\mathrm{Im}[V(\nu)]$ peaks at about 250 MeV and falls off slowly at high energies. This falloff results both from the large-$\vec{q}^{\,2}$ behavior of the mesonic propagators appearing in Eq. (9), as well as from the meson-nucleon vertex functions.

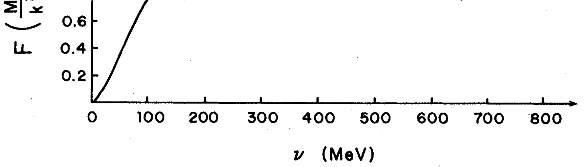

FIG. 4. The phase-space factor for 2p-2h excitations of energy k^0, using Fermi-gas wave functions, relative to its asymptotic (large k^0) form.

Given $\mathrm{Im}[V(\nu)]$ from Fig. 3 and Eq. (7), we expect $\mathrm{Re}V(k^0)$ to be negative (attractive) near threshold and to decrease with increasing energy. This behavior is shown in Fig. 5. The real potential has the value −12 MeV at threshold and decreases monotonically to −4 MeV at 310 MeV.

Our attractive potential (at threshold) is about 60% smaller than that obtained in Ref. 5. Pionic atom data have required a real *repulsive* term about equal in magnitude to the imaginary potential.[12,13] The present work yields an attractive dispersive potential in qualitative agreement with all

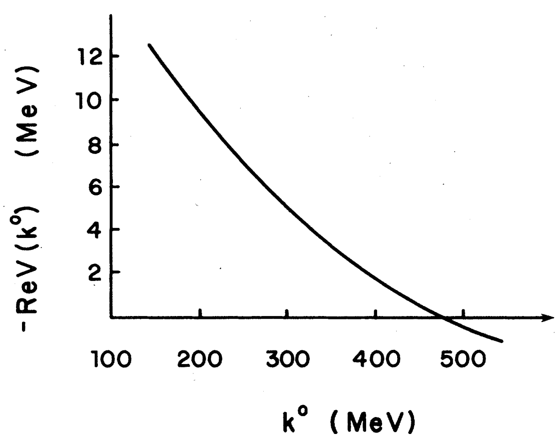

FIG. 5. The dispersive real part of the s-wave pion nucleus optical, arising from the substitution of Im $V(\nu)$ (from Fig. 3) into Eq. (7b).

other microscopic calculations of the dispersive real part, and in disagreement both with the early work of Brueckner[14] and of Thouless,[15] as well as with the more recent discussion of Hüfner.[13] In other words, the dispersive real part of B_0 cannot account for the definitely repulsive effect required by experiment.

Finally, we note that Mizutani and Koltun,[16] and Rinat[17] have proposed formalisms alternative to Eq. (2)ff, based on amalgamating multiple scattering theory and field theory. We feel that the application of the $\sigma + \omega$ model is more transparently expressed in terms of Eq. (2), however.

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