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Finite range distorted-wave Born approximation analysis of (p,t) reactions with a realistic triton wave function

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Exact finite range distorted-wave Born approximation analysis of the ground state reactions ^{208}Pb$(p,t)^{206}$Pb and ^{18}O$(p,t)^{16}$O are presented. The calculations are carried out using a realistic triton wave function comprising a spatially symmetric S and mixed symmetric S' and D states. The transfer interaction is treated consistently with the interaction used in obtaining the triton wave function. The use of a realistic wave function and transfer potential yields improved agreement between experimental and theoretical angular distributions. Calculations using the wave function of the transferred neutron pair suggest it is possible to explain both the absolute magnitude and shape of the angular distribution for these transitions.

[NUCLEAR REACTIONS (p,t), distorted-wave Born approximation analyses.]

I. INTRODUCTION

The two nucleon transfer reactions (p,t) and (t,p) are enormously rich and useful processes for the understanding of nuclei as is demonstrated by the vast amount of experimental work in the area.[1] Despite the great amount of experimental work, the theoretical development is far from complete. The zero-range distorted-wave Born approximation (DWBA) is usually employed[2,3] in analyzing data of angular distributions. Unfortunately zero-range analyses can only predict shapes of angular distributions with arbitrary normalization introduced to fit absolute magnitudes of data. Although these analyses can often suggest the orbital angular momentum being transferred, they are not a useful tool in studying nuclear wave functions which differ by two neutrons. Such wave functions are seldom made up of pure configurations but are composed of linear combinations of elementary configurations whose amplitudes add coherently in yielding angular distributions. The same superposition of elementary configurations produces the strong correlations responsible for determining the collective properties of the nuclear wave functions. For most cases studied[4] the shapes of angular distributions are not sensitive to the individual configurations employed. Thus in order to properly assess nuclear collectivity from (p,t) and (t,p) processes one must use a theory which can predict absolute magnitudes of angular distributions.

We present exact finite-range DWBA calculations[5,6] which predict both absolute magnitudes and shapes of angular distributions. Our work differs from earlier finite-range (p,t) studies[7-10] in several important ways. A fully realistic triton wave function is employed comprising a state of symmetric orbital permutation symmetry and total orbital angular momentum zero (S state) and two states of mixed orbital permutation symmetry having total orbital angular momentum zero and two respectively (S' and D states). A type of consistency is maintained wherein the realistic two-body potential used in calculating the triton ground state is also the interaction causing the transfer. This feature is essential for reliable predictions[11] of absolute magnitudes of angular distributions. Our method employs a modification of the half-separation energy method for obtaining neutron pair wave functions, but can easily be extended to include wave functions derived from any method. Preliminary results[12,13] of our theory demonstrate a systematic agreement between theory and experiment for the ground state transitions ^{18}O$(p,t)^{16}$O, ^{208}Pb$(p,t)^{206}$Pb, and ^{92}Zr$(t,p)^{90}$Zr which is a significant improvement on the previous finite-range calculations.

It should be emphasized that the spatial part of the two-neutron wave function employed throughout is a modification of the standard half-separation energy method. Our endeavor has been to focus on the development and improvement of the transition interactions and the three-body wave functions. However, our formalism, as described below, is sufficiently general to accommodate any neutron pair wave function.

In Sec. II we briefly outline the conventions followed in obtaining the exact finite-range DWBA amplitude, the form factor, and the three nucleon

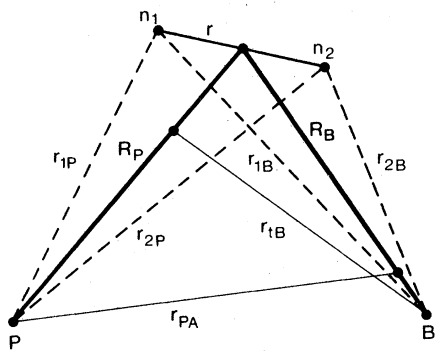

FIG. 1. The coordinate system for the reaction $A(p,t)B$. The three linearly independent vector coordinates appearing in the DWBA amplitude are $\vec{r}_{pA}$, $\vec{r}_{tB}$, and $\vec{r}$. The form factor is a function of the two vector coordinates $\vec{R}_p$ and $\vec{R}_B$.

wave functions. General selection rules for both natural and unnatural parity transitions are given in Sec. III. Results are reported for the reactions $^{18}O(p,t)^{16}O(0^+\text{g.s.})$ and $^{208}Pb(p,t)^{206}Pb(0^+\text{g.s.})$ in Sec. IV. Section V summarizes the major conclusions of this work.

II. EXACT FINITE-RANGE DWBA AMPLITUDE

Consider a reaction $A(p,t)B$ with coordinates as shown in Fig. 1. Following the usual convention in finite-range DWBA,[14] we chose the vectors $\vec{r}_{pA}$, $\vec{r}_{tB}$, and $\vec{r}$ as the three linearly independent vectors. $\vec{r}_{pA}$ and $\vec{r}_{tB}$ are the initial and final channel vectors designating, respectively, the position of the proton relative to the target and the position of the triton relative to the residual nucleus. $\vec{r}$ is the relative separation of the two neutrons transferred in the reaction. The finite-range amplitude is obtained by evaluating a nine-dimensional integral over the above three vectors. Usually, this task is separated into two parts. A form factor is first evaluated by integrating over the vector $\vec{r}$ yielding a function of the vectors $\vec{R}_p$ and $\vec{R}_B$, and is characterized by several angular momenta some of which contribute coherently and the others incoherently to the cross section. The method of Austern *et al.*[15] is used to transform the form factor to a function of the channel vectors $\vec{r}_{pA}$ and $\vec{r}_{tB}$ and the remaining six-dimensional integration is then carried out using a suitably modified finite-range single particle transfer computer code.

The DWBA transition amplitude is

$$T(\vec{k}_t,\vec{k}_p)=\left[\binom{3}{2}\binom{A}{2}\right]^{1/2}\mathcal{J}\int d\vec{r}_{pA}\int d\vec{r}_{tB}\,\Phi^{(-)*}_{\vec{k}_t}(\vec{r}_{tB})\times\langle J_B T_B M_B M_{T_B};\, J_t T_t M_t M_{T_t}|V|J_A T_A M_A M_{T_A};\,\sigma_p\tau_p\rangle\Phi^{(+)}_{\vec{k}_p}(\vec{r}_{pA})\,, \tag{1}$$

where $\Phi^{(+)}_{\vec{k}_p}$ and $\Phi^{(-)}_{\vec{k}_t}$ are the elastic scattering wave functions in the initial and final channels, respectively. $\vec{k}_p$ and $\vec{k}_t$ are the initial and final channel momenta. The quantum numbers J,T,M,M_T with the appropriate suffixes denote the total angular momentum, the isospin, and their respective z-components of the corresponding nuclei A,B or triton. σ_p and τ_p are the z-components of the spin and isospin of the proton. $\mathcal{J}$ is the Jacobian of the transformation to the coordinates $\vec{r}$, $\vec{r}_{tB}$ and $\vec{r}_{pA}$, i.e.,

$$\mathcal{J}=(A/A+1)^3(\tfrac{3}{2})^{3/2}\,.$$

The constant $[\binom{3}{2}\binom{A}{2}]^{1/2}$ is a normalization factor for the direct (p,t) amplitude arising from the antisymmetrization of the initial and final states of the system.

Following Satchler,[16] we express the nuclear matrix element in (1) as

$$\left[\binom{3}{2}\binom{A}{2}\right]^{1/2}\mathcal{J}\langle J_B T_B M_B M_{T_B};\, J_t T_t M_t T_{M_t}|V|J_A T_A M_A M_{T_A};\,\sigma_p\tau_p\rangle$$
$$=\sum_{\substack{LSJ\\ M_L M_S M_J}}(-i)^L\langle J_B M_B JM_J|J_A M_A\rangle\langle T_B M_{T_B}\,1-1|\,T_A M_{T_A}\rangle(-1)^{1/2-\sigma_p}\langle J_t M_t\tfrac{1}{2}-\sigma_p|\,SM_s\rangle$$
$$\times\langle LM_L SM_S|JM_J\rangle G_{LSJM_L}(\vec{R}_p,\vec{R}_B)\,, \tag{2}$$

where we have assumed that the transfer interaction is spin independent. $G_{LSJM_L}(\vec{R}_p,\vec{R}_B)$ is the two-neutron transfer form factor characterized by the orbital, spin, and total transfer angular momenta L, S, and J which contribute incoherently to the cross section. M_L is the z-component of L.

In the analyses presented in this paper, the transfer interaction which in the prior form is given by

$$V = V_{1p} + V_{2p}, \tag{3}$$

i.e.; the sum of the interactions of the proton with the two neutrons being transferred is replaced by $(\epsilon_t - K - V_{12})$, where ϵ_t is the triton binding energy, K the triton kinetic energy operator, and V_{12} the neutron-neutron interaction. One should note that for a realistic interaction $V_{1p} + V_{2p}$ includes partial waves from the triplet even channels and hence have contributions from the short range N-N tensor force. These terms are in principle more complicated to evaluate. Whereas, V_{12}, the neutron-neutron interaction, is diagonal for all the two-body channels considered and much simpler in structure. The above replacement is justifiable provided one uses the same interaction to calculate the internal state of the triton. We have used the triton wave function of Strayer and Sauer,[17] who obtained it as a variational eigenfunction of a three-body Hamiltonian with the Reid[18] soft core potential. We use the variational binding energy for ϵ_t and the Reid soft core potential for V_{12}. The triton wave function of Strayer and Sauer[17] primarily consists of a spatially symmetric S state, and mixed symmetric S' and D states. The other components of the triton wave function have negligible amplitude. The classification of the states used in the description of the triton wave function is given in Table I, along with the variational binding energy and the rms charge radius. The wave function is obtained variationally as a linear combination of harmonic oscillator functions which refer to the internal coordinates of the triton $r/\sqrt{2}$, $\sqrt{2/3}\,R_p$. The details of the wave function are contained in the paper of Strayer and Sauer[17] as well as in Ref. 4.

TABLE I. Classification of basis states and ground state properties of the triton using the wave function of Ref. 17.

State	Percent wave function	L_t	S_t	$[\lambda_{Lt}]$	(r_L)
S	89.8	0	$\frac{1}{2}$	[3]	(111)
S'	1.4	0	$\frac{1}{2}$	[21]	(211)
					(121)
D	8.8	2	$\frac{3}{2}$	[21]	(211)
					(121)

E_t = −6.7 MeV $\quad \langle r_{ch}^2\rangle^{1/2} = 1.85$ fm

Following Glendenning,[19] we define the generalized two particle fractional parentage coefficients

$$\beta([l_1 j_1 l_2 j_2]J) = \binom{A}{2}^{1/2} \langle [(J_B T_B)(l_1 j_1 l_2 j_2)T = 1 J] \times J_A T_A M_A M_{T_A} | J_A T_A M_A M_{T_A}\rangle, \tag{4}$$

and expand the two-neutron pair wave function in a complete set of antisymmetrized product of single neutron wave functions. In the actual calculation, we used the two-neutron amplitudes $\beta([l_1 j_1 l_2 j_2]J)$ obtained by shell model calculations and used the half-separation energy method[19] for the radial wave functions of the two neutrons.

The final expression of the form factor is

$$G_{LSJM_L}(\vec{R}_p, \vec{R}_B) = (i)^L \sum_{\substack{L_p L_B \\ M_p M_B}} \sum_{L_t [\lambda_{L_t}] S_t} \langle L_p M_p L_B M_B | L M_L\rangle \times \mathcal{G}^{L_t [\lambda_{L_t}] S_t}_{L_p L_B; LSJ}(R_p, R_B) Y_{L_p M_p}(\hat{R}_p) Y_{L_B M_B}(\hat{R}_B), \tag{5}$$

where L_t, $[\lambda_{L_t}]$, and S_t are, respectively, the orbital angular momentum, orbital permutation symmetry, and spin of the triton. L_p and L_B are, respectively, the orbital angular momentum of the proton relative to the c.m. of the two neutrons and that of the residual nucleus B relative to the c.m. of the two neutrons. The angular momenta L_p, L_B, L_t, and S_t contribute coherently to the cross section. Hence, it is easier to sum over their contributions as in (5). In reality, Eq. (5) contains an additional sum over l, the relative orbital angular momentum of the two neutrons. This sum has already been performed and is implicit in (5). The detailed expression for $\mathcal{G}^{L_t [\lambda_{L_t}] S_t}_{L_p L_B; LSJ}$ can be obtained from Ref. 4.

The differential cross section is given by

$$\frac{d\sigma}{d\Omega} = \frac{\mu_p \mu_t}{(2\pi\hbar^2)^2} \frac{k_p}{k_t} \sum_{LSJM_L} \left| \sum_{L_p L_B} B^{L_p L_B}_{LSJM_L}(\vec{k}_t, \vec{k}_p) \right|^2 \cdots, \tag{6}$$

where the reduced amplitude $B^{L_p L_B}_{LSJM_L}$ is defined as

$$(i)^L(2L+1)^{1/2}\,B^{L_pL_B}_{LSJM_L}(k_t,k_p)=\sum_{\substack{L_t\,[\lambda_{L_t}]S_t\\ M_pM_B}}\sum_{L_pM_pL_BM_BLM_L}\int dr_{pA}\int dr_{tB}\Phi^{(-)*}_{k_t}(r_{tB})\,\mathcal{G}^{L_t\,[\lambda_{L_t}]S_t}_{L_pL_B;LSJ}\times(R_pR_B)\,Y_{L_pM_p}(R_p)Y_{L_BM_B}(R_B)\Phi^{(+)}_{k_p}(r_{pA})\,. \tag{7}$$

The amplitudes $B^{L_pL_B}_{LSJM_L}$ are computed for fixed values of L, S, J, M_L, L_p, L_B and the allowed components are summed in the appropriate manner to compute the cross section.

III. SELECTION RULES

The selection rules for (p,t) reactions using realistic triton wave functions that include mixed symmetry components have been reported in Ref. 20 and may be derived from the form factor. An additional constraint on the selection rules is that $l+S_{12}$ is even as a consequence of the antisymmetry of the two transferred neutrons where l is the relative orbital angular momentum and S_{12} the total spin of the two-neutron sybsystem. For a triton wave function composed of only a symmetric S state one has $S_{12}=0$ and l even. Most simple wave functions allow only $l=0$. For triton wave functions that contain mixed symmetry components it is possible for $S_{12}=1$ and for l to have odd values.

If π_A and π_B designate the intrinsic parities of the states of the target and residual nuclei A and B, respectively, then natural parity transitions are those which satisfy the condition

$$\pi_A\pi_B=(-1)^J\,,$$

while unnatural parity transitions satisfy the condition

$$\pi_A\pi_B=-(-1)^J\,.$$

The selection rules for each component of the triton can be obtained from the following equations and are given below.

$$\begin{aligned}&\vec{S}_{12}=\vec{L}_t+\vec{S}\,,\\ &(-1)^{l+S_{12}}=1\,,\\ &\vec{J}=\vec{L}+\vec{S}_{12},\\ &\vec{L}=\vec{L}_B+\vec{L}_p\,,\\ &\vec{L}_p=\vec{L}_t+\vec{l}\,,\\ &\vec{S}_t=\vec{S}_{12}+\tfrac{\vec{1}}{2}\,.\end{aligned} \tag{8}$$

A. Symmetric S state

For this state we have $L_t=0$, $S_t=\frac{1}{2}$, and $S_{12}=0$. From Eq. (8) we therefore have $S=0$, $l=L_p$, and $L=J$. The parity change is given by the relation

$$\Delta\pi=\pi_A\pi_B=(-1)^{l+L_B}\,.$$

However, since l is even for this case, then $\Delta\pi=(-1)^{L_B}$. Now for $l=0$ we have that $L_p=0$, $L=L_B=J$. Thus $\Delta\pi=(-1)^J$ and for $l=0$ one can only have natural parity transitions. However, if $l\neq0$ one has $\Delta\pi=(-1)^{|\vec{J}+\vec{1}|}$ and unnatural parity transitions can occur with a leading term corresponding to $l=2$.

B. Mixed symmetry S state

The mixed symmetry component of the triton can have $S_{12}=0,1$. For $S_{12}=0$ the selection rules are the same as for the symmetric state. For the $S_{12}=1$ component, one has $l=$ odd, and $\Delta\pi=-(-1)^{L_B}$. Thus unnatural parity transitions are possible with a leading term corresponding to $l=1$.

C. Mixed symmetry D state

Here the total intrinsic and orbital angular momentum of the triton are $S_t=\frac{3}{2}$ and $L_t=2$, respectively. Thus $S_{12}=1$ and l will always be odd. Therefore, this term can contribute to both natural and unnatural parity transitions with the leading component corresponding to $l=1$.

For a fixed angular momentum transfer J one can have values of orbital angular momentum transfer subject to the following conditions:

$$|J-S_{12}|\leq L\leq|J+S_{12}|$$

and

$$|L_B-L_p|\leq L\leq|L_B+L_p|\,.$$

The conditions on L_B are

$$|l_{n_1}-l_{n_2}|\leq L_B\leq|l_{n_1}+l_{n_2}|\,,$$

where l_{n_1} and l_{n_2} are the relative orbital angular momenta of the single particle states of neutrons n_1 and n_2 used in constructing the two particle states. Conditions on L_p are that $L_p\geq0$ for natural parity transitions and $L_p\geq1$ for unnatural parity transitions.

TABLE II. Optical parameters used in this study.

Target	Projectile	V_0	r_0	a_0	W	$4W_D$	r_I	a_I
^{18}O	p	56.0	1.17	0.75	1.70	0.0	1.32	0.588
^{18}O	t	146.0	1.40	0.44	14.70	0.0	1.40	0.551
^{208}Pb	p	47.9	1.25	0.65	0.0	40.0	1.25	0.750
^{208}Pb	t	149.8	1.24	0.68	12.0	0.0	1.43	0.870

IV. RESULTS

In this section we discuss details of the finite-range DWBA analysis of the reactions $^{18}O(p,t)^{16}O(0^+_{g.s.})$ and $^{208}Pb(p,t)^{206}Pb(0^+_{g.s.})$. These reactions are natural parity transitions and have been considered by other authors.[8] The particular target ^{208}Pb and ^{18}O were chosen since both the ^{208}Pb target and the ^{16}O residual nucleus are thought to be very good closed shell nuclei and thus are systems for which one would expect a shell model description to be valid.

The optical model potentials used to generate the distorted waves have the usual form

$$V(r) = V_c(r) - V_0 f_V(r) - iW_V f_w(r) + 4ia_s W_s f_s'(r) ,$$

where

$$f_\lambda(r) = [1 + \exp(r - R_\lambda)/a_\lambda]^{-1} ,$$

and where $V_c(r)$ is the Coulomb potential due to a uniformly charged sphere of radius $r_c A^{1/3}$. The radius R is often taken to have the form $R_\lambda = r_\lambda A_T^{1/3}$ or $R_\lambda = r_\lambda (A_T^{1/3} + A_p^{1/3})$, where A_T and A_p are the mass numbers of the target nucleus and projectile, respectively. In our analysis we employ the former of the two prescriptions since most of the optical model analyses with light projectiles utilize this form. The optical parameters are illustrated in Table II and were obtained from the literature. One should note that there is some sensitivity to the choice of these parameters for reactions in light nuclei. For reactions on ^{208}Pb, various combinations were employed with virtually no difference in the results obtained. However, for the reaction on ^{18}O, both the absolute magnitude and shape of the predicted angular distribution changed when different optical model parameters were employed. In this work, we chose that particular set which gave the best comparison between the theoretical and experimental angular distributions.

The two-neutron configurations were assumed to be identical to those predicted by shell model calculations[21,22] except that the radial wave functions were replaced by the eigenfunctions of a Woods-Saxon potential adjusted to yield eigenvalues equal to half the two-neutron separation energy. The radius and diffuseness parameters of these potentials are

$$R = 1.25A^{1/3},$$

$$a_0 = 0.65 \text{ fm} .$$

The two-neutron spectroscopic amplitudes for all of the reactions studied are given in Table III. For the $^{18}O(p,t)^{16}O$ transition the spectroscopic amplitudes were obtained from ^{18}O wave functions computed by Kahana[21] and are linear combinations of $[0d_{5/2}]^2$ and $[1S_{1/2}]^2$ configurations. Those for the reactions on ^{208}Pb are taken from the work of True[22] and are linear combinations of products of $2p$ and $1f$ hole states in the closed ^{208}Pb core.

Form factors for the reactions $^{18}O(p,t)^{16}O$ and $^{208}Pb(p,t)^{206}Pb$ are shown for oxygen in Figs. 2 and 3, and for lead in Figs. 4 and 5 for the ground state transitions. In Figs. 2 and 4 the fully two-dimensional form factors are shown while in Figs. 3 and 5 one-dimensional "slices" along the zero-range line $R_p = 0$ are given.

For the ground state transitions, the allowed values of the various angular momenta based on the previously discussed selection rules are given in Table IV. We find that *only* the component having $l = S = 0$ contributes in any significant way to the form factor. The corresponding form factors

TABLE III. Spectroscopic amplitudes defined in Eq. (4) and used in obtaining form factors in Figs. 2–5.

Reaction	Ref.	$[0d_{5/2}]^2$	$[1S_{1/2}]^2$	$[2P_{1/2}]^{-2}$	$[2P_{3/2}]^{-2}$	$[1f_{5/2}]^{-2}$
$^{18}O(p,t)^{16}O(0^+_{g.s.})$	21	0.893	0.450			
$^{208}Pb(p,t)^{206}Pb(0^+_{g.s.})$	22			0.822	0.363	0.401

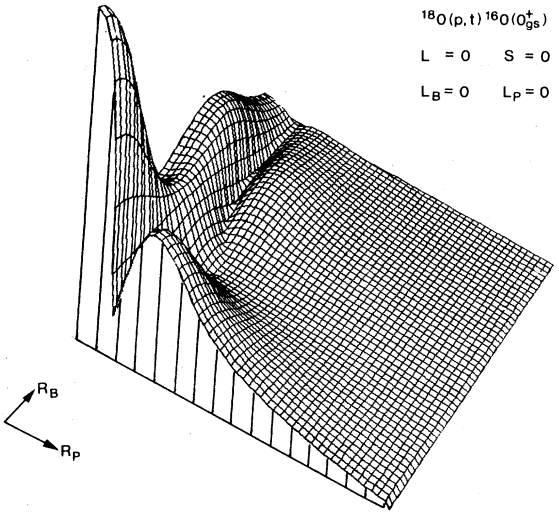

FIG. 2. The form factor $\mathfrak{F}^{LSJ}_{L_p L_B}(R_p, R_B)$, for the reaction $^{18}O(p,t)^{16}O(0^+_{g.s.})$, in arbitrary units, as a function of R_p and R_B, shown over the range 0 to 5 fm for R_p and R_B, respectively.

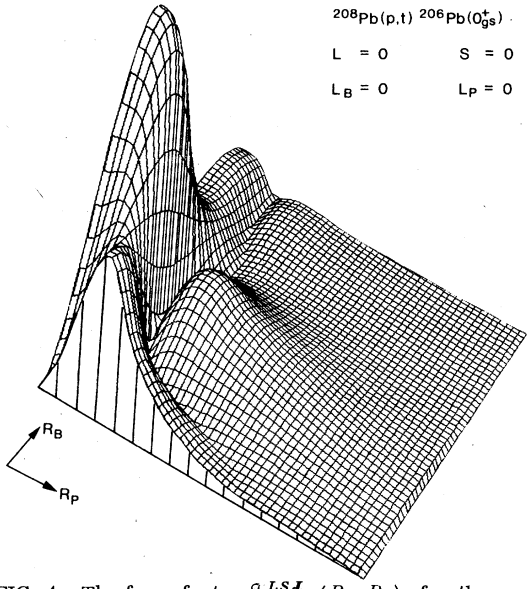

FIG. 4. The form factor $\mathfrak{F}^{LSJ}_{L_p L_B}(R_p, R_B)$, for the reaction $^{208}Pb(p,t)^{206}Pb(0^+_{g.s.})$ in arbitrary units, as a function of R_p and R_B, shown over the range 0 to 5 fm for R_p and R_B, respectively.

are shown in Figs. 2 and 4, respectively, for oxygen and lead. Both of these form factors comprise S and S' components and are extremely non-local, nonseparable functions. The form factors along the zero-range line in Figs. 3 and 5, respectively, for oxygen and lead display a structure significantly different from that away from the

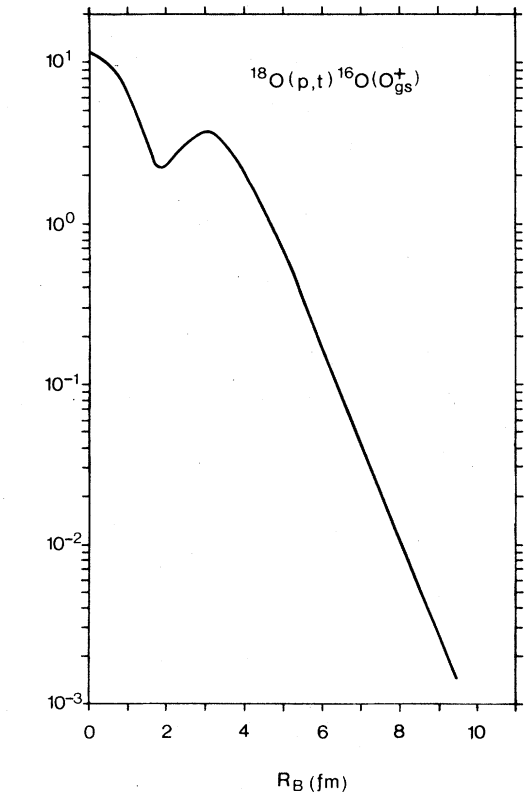

FIG. 3. The form factor in Fig. 2 plotted along the zero-range line $R_p = 0$ in units MeV fm^{-3}.

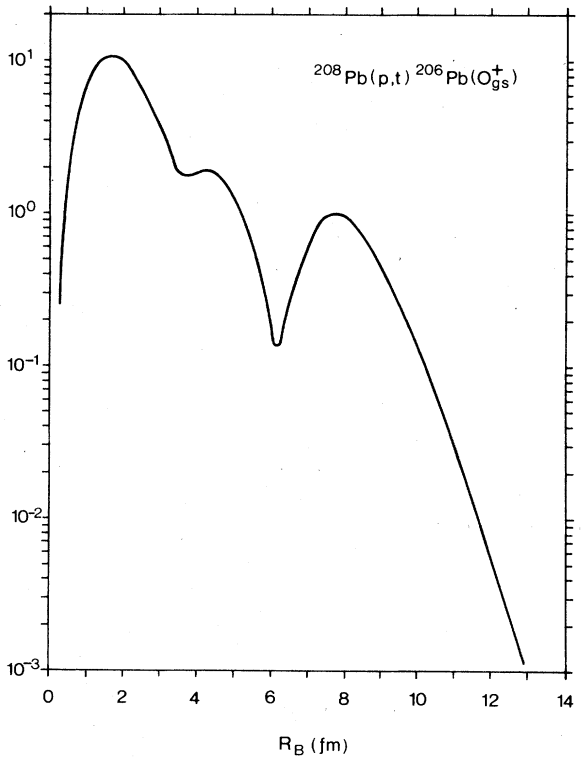

FIG. 5. The form factor in Fig. 4 plotted along the zero-range line $R_p = 0$ in units MeV fm^{-3}.

TABLE IV. Allowed values of the angular momentum quantum numbers for the ground state transitions in this work.

L	S	L_B	L_P	Triton and neutron pair components
0	0	0	0	S and S' states
		2	2	$S_{12}=S$, $l=L_P$
1	1	1	1	S' and D states
		3	3	$S_{12}=S$, $l=L_P\pm2, L_P$

zero-range line in Figs. 2 and 4. The zero-range form factors (Figs. 3 and 5) differ in the interior from those obtained by Ibarra, Feng, and Vallieres[23-27] using extended shell model basis techniques to account for the correlation of the neutrons being transferred. The half-separation energy method assumes that the neutrons act independently in a potential well and share the total separation energy equally. Any correlation between the two particles is introduced subsequently in the spectroscopic amplitudes.

In Fig. 6 a comparison is made between the finite-range DWBA prediction and the experimental data for the $^{18}O(p,t)^{16}O$ reaction at a proton energy of 20 MeV. The figure displays two sets of experimental data covering a wide range of angles. The theoretical calculation has been normalized by the factor

$$n=\sigma_{\text{expt}}/\sigma_{\text{cal}}=1.63\ .$$

An exact finite-range calculation of this reaction has been performed by Charlton.[8] In his analysis, Charlton uses two different optical model parameter sets and obtains results for one case which are comparable with those presented in this study. However, our calculation differs from Charlton's in the following respects: The triton wave function and transfer interaction are treated consistently in this work. The two-body interaction was the Reid soft core potential and the triton wave function was obtained variationally as given in Ref. 17. In Charlton's calculations the transfer interaction was *not* consistent with that used to obtain the triton wave function. The geometrical parameters of the bound neutron wave functions employed here are larger than those of Charlton and we have omitted spin-orbit terms from the potentials used to obtain these states. This omission has little effect on the wave function, however.

In Fig. 7, the theoretical and experimental angular distributions are compared for the reaction $^{208}Pb(p,t)^{206}Pb(0^+_{g.s.})$. The two-neutron spectroscopic factors are obtained from the shell model wave function of True. The normalization factor for this reaction is

$$n=\sigma_{\text{expt}}/\sigma_{\text{cal}}=1.23\ .$$

This reaction was also analyzed by Charlton[8] and although he found good agreement with the shape of the angular distribution, his calculation underpredicted the absolute magnitude by a factor of 1.6. In calculations presented here, both the symmetric and mixed symmetry triton S states contribute, and in Fig. 8, we compare the angular distributions calculated with and without the S' state. The difference between the two is never more than 5% except at the minimum near $\theta_{\text{c.m.}}$

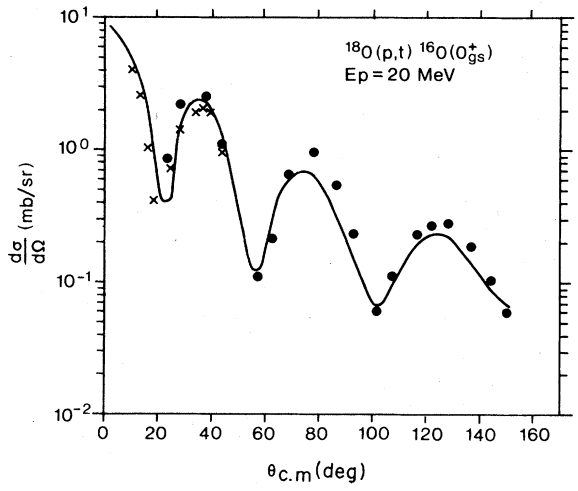

FIG. 6. The comparison of the theoretical and experimental angular distribution for the reaction $^{18}O(p,t)^{16}O(0^+_{g.s.})$ at the proton energy of 20 MeV. The experimental points $(x,0)$ are taken from Refs. 28, 29, respectively.

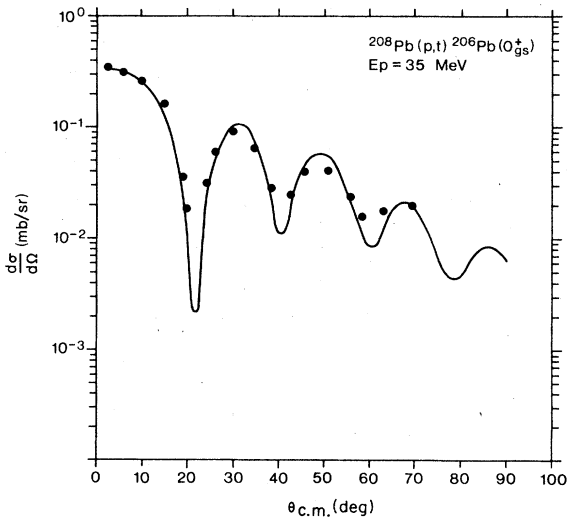

FIG. 7. The comparison of the theoretical and experimental angular distributions for the reaction $^{208}Pb(p,t)$ $^{206}Pb(0^+_{g.s.})$ at a proton energy of 35 MeV. The experimental points are taken from Ref. 30.

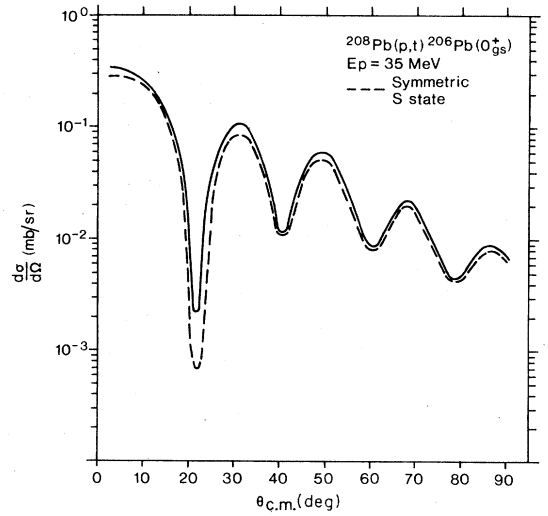

FIG. 8. The comparison of the total ($S+S'$) and S contributions to the reaction in Fig. 9.

$= 23°$, where the influence of the mixed symmetry S state is to fill in the minimum.

One uncertainty in fitting the absolute magnitude of these angular distributions can be ascribed to our use of the half-separation energy method. The half-separation energy method assumes that the transferred neutrons move independently in a potential well and share the removal energy equally. From the work of Ibarra *et al.*[23, 24] we know that this is not the case, and that treating the correlated neutron pair more precisely enhances the magnitude of the form factor in the surface of the nucleus. Accordingly, we can "effect" this enhancement within the framework of the half-separation energy method by determining larger geometrical parameters for the bound particles. One believes this kind of prescription only insofar as absorption and phase averaging of the distorted waves cause the reaction to be surface peaked. This procedure was used successfully for the reactions studied in Refs. 10–13, 20, with the geometrical parameters

$$R = 1.2(A^{1/3}+1) \text{ fm},$$

$$a_0 = 0.65 \text{ fm},$$

and produces the necessary enhancement needed to fit the reactions presented in this study.

V. SUMMARY AND CONCLUSIONS

We have presented a finite-range DWBA analysis of the reactions ^{208}Pb$(p,t)^{206}$Pb$(0^+_{g.s.})$ and ^{18}O$(p,t)^{16}$O$(0^+_{g.s.})$ and corresponding calculations of the two-neutron transfer form factor employing a realistic triton wave function. The transfer interaction in the prior formulation of DWBA is eliminated in favor of the binding energy, the internal kinetic energy, and the interaction between the neutron pair in the triton. Form factors calculated for these reactions demonstrate the importance of maintaining a manifestly finite-range theory.

Our work differs significantly from earlier finite-range calculations in our treatment of the triton wave function and of the transfer interaction. We have employed a variational triton wave function having a totally symmetric S state, and mixed symmetry S and D state components obtained using a realistic nucleon-nucleon potential. The same potential is used to construct the transfer interaction thereby maintaining a consistency between the interactions involving the transferred neutrons and the interaction between nucleons in the triton.

Selection rules are obtained from these form factors which have the following properties:

(i) For two-neutron states in nuclei which have uniquely $S_{12} = 0$ *only* the S and S' states can contribute, whereas for states with $S_{12} = 1$ *only* the S' and D states contribute.

(ii) The importance of the mixed symmetry states is enhanced in the two cases when the two-neutron configuration has natural parity and a dominant $S_{12} = 1$ component, or the two-neutron configuration has unnatural parity. For the above reactions the spatially symmetric S state of the triton contributes more than 90% of the differential cross section.

Our analysis of the ground state transitions has demonstrated very good agreement between the theoretical and experimental angular distribution shapes but absolute magnitudes which are too small by 60% and 23%, respectively, for the ^{18}O$(p,t)^{16}$O and ^{208}Pb$(p,t)^{206}$Pb reactions. However, as has already been discussed, we can expect enhancements of this order from the use of more lightly correlated wave functions.[31]

The results of our calculations indicate the importance of the use of realistic triton wave functions and consistent transfer interactions in the determination of the absolute magnitude of (p,t) or (t,p) cross sections.

We have recently received a report on similar analyses by Takemasa, Tamura, and Udgawa. We thank the authors for sending us the report.

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