Truncated pion-nucleon amplitudes and pion-nucleus interactions

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It is pointed out that there are considerable structural differences among the various types of truncated pion-nucleon amplitudes which have been proposed in order to avoid overcounting ambiguities in pion-nucleus interactions. A previously proposed prescription is shown to yield truncated amplitudes which are free of the usual problems involved with the proper representation of the nucleon-pole term. It is shown that these truncated amplitudes can also possess ghost pole singularities arising from the πNN vertex function. Some problems caused by the appearance of such ghost poles in approximations to pion-nucleus amplitudes are discussed.

[NUCLEAR REACTIONS Pion-nucleon amplitudes without nucleon propagator terms. Overcounting in pion-nucleus interactions. Pion-nucleon vertex function ghost poles in approximate pion-nucleus amplitudes.]

The possibility of pionic absorption on nuclei, whether real or virtual, significantly alters the picture of interactive processes familiar in potential scattering. For example, departures from the potential scattering picture are certainly required if one is to maintain a dynamical distinction between the interactive processes associated with the external pion(s) and the pionic processes involved in the formation of nuclear bound states, for instance. Such a distinction is of considerable calculational importance since independent information concerning these nuclear states can then be utilized. A clean separation of the description of the interactive process from the characterization of the initial and final nuclear states is obtained straightforwardly in potential scattering. Such a separation can also be obtained for a fully relativistic field-theoretic description of pionic interactions.[1] A crucial aspect of any theoretical description of pionic interactions is an unambiguous specification of the nuclear bound state. Only then can one avoid the ambiguities which can arise from including pionic interactions already accounted for in the dynamics of the nuclear bound states or perhaps in the initial and final distorted waves if these are employed.[1-6]

In this article we are concerned with elaboration of the analysis in Ref. 1 of a particular type of ambiguity which arises from an improper identification of the complete interaction of a pion with a nucleon in the nucleus under certain circumstances. What is required here in order to avoid overcounting is the subtraction of one or both parts of the off-mass-shell pion-nucleon amplitude which contain nucleon poles.[1,5,6] We show in detail here that the prescriptions of Ref. 1 for doing this differ in several significant respects from those proposed in Refs. 5 and 6 despite some superficial resemblances.[2] These differences highlight the importance of the characterization of the bound state as part of any such subtraction algorithm. We also show how practical questions, which are inherent in the prescriptions of Refs. 3–6, relating to the choice between the pseudoscalar and pseudovector (PV) representation of the nucleon-pole terms can be circumvented. Finally, we demonstrate how the structural aspects of the πNN vertex function enter into the subtraction prescriptions of Ref. 1, in contrast to those of Refs. 3–5, and we comment on their possible influence upon the transition amplitudes. The differences among these alternative subtraction methods may or may not be of quantitative importance in calculations of pion-nucleus interactions. However, in order to assess this it is first necessary to understand in detail what these differences are and what they do entail in the way of calculation and this is our objective at this time.[7,8]

In order to illustrate the essential aspects of the sort of overcounting ambiguity we have referred to in the preceding paragraph we review some of Ref. 1 for the specific example of pion absorption on deuterium:

$$\pi + d \rightarrow N + N\,. \tag{1}$$

The pion-rescattering mechanism (Fig. 1) makes an important contribution to the reaction (1). In Fig. 2 we show the detailed structure of the final-state nucleon-nucleon interaction. The central question in regard to Fig. 1 concerns the proper identification of the effective pion-nucleon scattering amplitude t_R. There is no *a priori* reason to expect that t_R can be identified, consistently, with the full πN amplitude and in fact it cannot be.[1] In

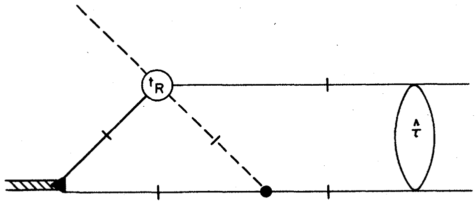

FIG. 1. Pion rescattering process where t_R represents the effective pion-nucleon scattering amplitude. The crossed solid internal lines represent full nucleon propagators S_F' while the crossed dashed internal line represents the full pion propagator Δ_F'. The final-state nucleon-nucleon interaction is denoted by $\hat{\tau}$. The dashed, solid, and cross-hatched external lines refer to the pion, nucleon, and deuteron, respectively. The dark circle corresponds to the proper πNN vertex function while the dark triangle represents the proper dNN vertex function. We follow these notational conventions in the other figures.

Ref. 1 it is shown that with a representation of the dNN vertex appropriate to the description of the deuteron bound state

$$t_R = t_{\pi N} - B_D - B_C \,. \tag{2}$$

Here $t_{\pi N}$ is the full πN amplitude and B_D and B_C comprise the sum of all connected $\pi + N \to \pi + N$ Feynman graphs which can be disconnected by cutting a single internal nucleon line. Such graphs consist of a single bare internal nucleon line with two external pion lines at its end points plus arbitrary self-energy insertions onto the nucleon lines and arbitrary (proper) vertex insertions at its end points.[9] The graphs which contribute to B_C and B_D are distinguished from each other as to whether their external pion lines are crossed or not, respectively (Fig. 3).

The analytic structure of B_D includes an s-channel nucleon pole while B_C has a u-channel nucleon pole and thus they are often referred to as the nucleon-pole or Born terms of $t_{\pi N}$.[10] These nucleon poles arise from the nucleon propagator S_F' which appears in the analytical expressions for B_D and B_C (Fig. 3):

$$B_D = \Gamma_{\pi NN}(k, k+p) S_F'(k+p) \Gamma_{\pi NN}(k', p') \,, \tag{3a}$$

$$B_C = \Gamma_{\pi NN}(k', p-k') S_F'(p-k') \Gamma_{\pi NN}(k, p') \,. \tag{3b}$$

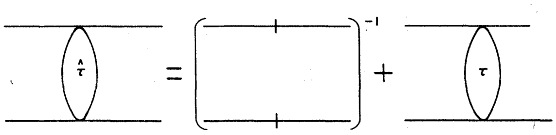

FIG. 2. Final-state nucleon-nucleon interaction. τ refers to the off-mass-shell nucleon-nucleon scattering amplitude.

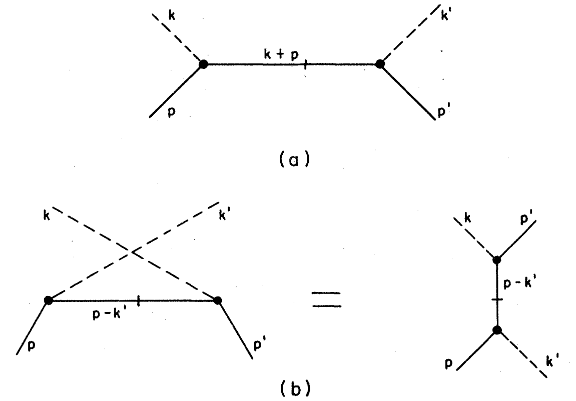

FIG. 3. (a) The full (direct) nucleon propagator term B_D. The internal nucleon line includes all self-energy corrections and the vertices (dark circles) represent fully dressed proper vertex functions. B_D contains the s-channel nucleon pole. (b) Two equivalent graphical representations for the full (crossed) nucleon propagator term B_C with the same internal nucleon line and vertex properties as in (a). B_C contains the u-channel nucleon pole.

We have suppressed in Eqs. (3) all inessential kinematical, isospin, and spinor features of B_D and B_C; the proper vertex function[11] is represented by $\Gamma_{\pi NN}$. It is important to keep in mind that $S_F'(p)$ possesses not only a pole at $p^2 = m_N^{\,2}$ but also a branch cut from $p^2 = (m_N + m_\pi)^2$ to $p^2 = +\infty$ which results from the multiparticle contributions to the nucleon two-point function.[12] The vertex function has a more complicated structure.[13-18]

If in Fig. 1 we were to use $t_{\pi N}$ instead of t_R, then Fig. 1 would include a contribution from B_C which is represented in Fig. 4. The internal pion line in Fig. 4 then gives rise to an ambiguous contribution to the reaction (1).[19] One can question the extent to which the internal pion exchange in Fig. 4 reproduces dynamical effects already included in the dNN vertex. In this regard, it is unclear to what degree Fig. 4 reproduces dynamics already included in the impulse term of Fig. 5 which combines coherently with the graph of Fig. 1 in its contribution to the reaction (1).

The conclusion of Ref. 1 is that Fig. 4 is ruled out as a legitimate contribution to the reaction (1)

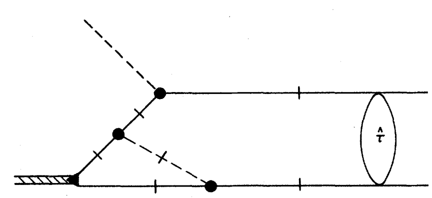

FIG. 4. Ambiguous contribution of B_C to the pion rescattering process.

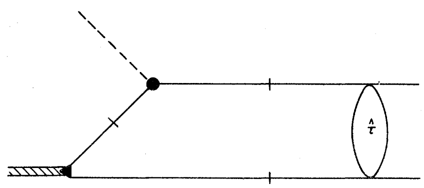

FIG. 5. Impulse graph for pion absorption with a final-state interaction.

and so is a similar graph generated by B_D in which the internal pion has its upper insertion to the right of the external pion's attachment vertex. The essential point is that it is the *full* contribution of both B_D and B_C as given by Eqs. (3) which must be subtracted from $t_{\pi N}$ to obtain the correct t_R.[20] It is exactly in this respect that Ref. 1 differs from the prescriptions of Refs. 5 and 6 for choosing t_R which involve subtracting off only the s-channel and u-channel *pole* terms from $t_{\pi N}$. These pole terms are then represented by PV coupling[21] with point vertices and a free nucleon propagator so that t_R is identified with the background term studied elsewhere by Liu and Shakin.[22]

Evidently one can decompose $t_{\pi N}$ in a variety of ways into a sum of "Born terms" which include the s- and u-channel poles plus a background term analogous to t_R.[21-24] The essential constraint upon any of these choices for the Born terms is that the residues of the poles be properly described. The conventional PV-coupling choice for the representation of the Born terms is motivated by the good approximation they provide to the threshold amplitudes.[21] However, because one has then lost the Feynman graph correspondence there is no compelling justification for identifying these Born terms with the sum of Feynman graphs which B_D and B_C represent and therefore the model proposed by Liu and Shakin[5,6,22] for the background amplitude should not be identified with t_R. There may be circumstances where this identification is adequate, e.g., in the neighborhood of the nucleon poles or when the ultimate effects of any of the various subtractions are small.

However, it is possible that the behavior of B_D and B_C is markedly different from that of say the Born terms resulting from PV-coupling with point vertices and a free nucleon propagator. We refer here, for example, to the fact that the vertex functions $\Gamma_{\pi NN}$ may possess ghost poles.[14,18,25] These are poles of $\Gamma_{\pi NN}$ coupled to the zeros of the full nucleon propagator S_F'. These ghost poles appear in B_D and B_C by Eqs. (3); however, they must be canceled by compensating poles in t_R in order to yield a $t_{\pi N}$ without these unphysical singularities. This can be shown to be the case.[14,25] In a situation where, e.g., only t_R enters into an amplitude (Fig. 1) the compensating ghost poles in t_R will *not* be canceled *explicitly* and this may produce effects of importance under certain kinematical conditions and in an approximation where, e.g., only the amplitude corresponding to Fig. 1 is retained. We remark that the calculation of Mizutani and Rochus of $\Gamma_{\pi NN}$ with only one nucleon off its mass shell has yielded two ghost poles at unphysical energies.[18]

In Ref. 18 it is conjectured that it may be possible to observe the effects of these poles in pion-nucleus interactions, although no specific examples are proposed there nor are any reasons given why it should be possible to observe such effects. The pion rescattering process as represented by Fig. 1 may constitute such an example. However, in order that this be the case it is necessary to show that no other collection of graphs which also contributes to the process (1), such as those represented by Fig. 5, will cancel the effects of the poles in t_R. If such a cancellation does occur, analogous to what happens for free $t_{\pi N}$ amplitudes,[14,25] then under circumstances where ghost poles appear in t_R consistent approximations to (1) would consist of such "ghost-free" combinations of amplitudes. We remark that any poles in t_R will be converted into branch singularities of the amplitude represented in Fig. 1, e.g., as a consequence of the integration over the loop momenta.

According to the prescriptions of Ref. 1 truncated pion-nucleus amplitudes appear in the contributions to an arbitrary pion-nucleus amplitude even those for elastic scattering.[20] It is not at all clear how the presence of ghost poles would alter the formulation of consistent approximations for these processes. These points deserve further investigation.

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[1]K. L. Kowalski, E. R. Siciliano, and R. M. Thaler, Phys. Rev. C 19, 1843 (1979).
[2]A rather large literature exists concerning some of these overcounting ambiguities which predates Ref. 1. (We remark that the receipt date of Ref. 1 is misprinted and was actually 2 January 1979.) Most of the published works are cited in Ref. 1. Three unpublished works also warrant mention, namely Refs. 3–5; these

investigations have been cited frequently in connection with one or another overcounting ambiguity. The work of Liu and Shakin (Ref. 5) bears considerable resemblance to some aspects of Ref. 1, especially to those parts concerned with nucleon-pole-term subtractions from the constituent pion-nucleon amplitudes. However, Ref. 5, along with several other previous studies (including Refs. 3 and 4), does not contain an unambiguous specification of the nuclear bound state and as a consequence the subtraction prescriptions advocated there do not necessarily resolve the overcounting problems.

[3]A. W. Thomas, Ph.D. thesis, Flinders University, 1973 (unpublished).

[4]T. Mizutani, Ph.D. thesis, University of Rochester, 1976 (unpublished).

[5]L. C. Liu and C. M. Shakin, Brooklyn College Report No. B.C.I.N.T.-77/121, 1977 (unpublished).

[6]The prescriptions of Ref. 5 concerning overcounting are proposed again by R. S. Bhalerao, L. C. Liu, and C. M. Shakin, Brooklyn College-C.U.N.Y. Report No. B.C.I.N.T.-79/041/93, (unpublished).

[7]Reference 8 contains a preliminary calculation of the quantitative effects of nucleon-pole-term subtractions. The exact nature of the subtractions actually employed in this work is unclear, but it appears to be more in accord with the prescription of Refs. 5 and 6 rather than those of Ref. 1.

[8]D. R. Giebink, W. R. Gibbs, and E. R. Siciliano, *Meson-Nuclear Physics—Houston, 1979*, Proceedings of the 2nd International Topical Conference on Meson-Nuclear Physics, edited by E. V. Hungerford III (AIP, New York, 1979), p. 205.

[9]We keep in mind the nucleon propagator, πNN vertex, and external line renormalizations in connection with these graphical designations.

[10]Unfortunately, these appellations are not very precise. With the momentum labels of Fig. 3, $s=(k+p)^2$ and $u=(p-k')^2$.

[11]By this we mean that $\Gamma_{\pi NN}$ represents the sum of all connected graphs with two external nucleon lines and one external pion line which cannot be made disconnected by cutting either a single internal nucleon line or a single internal pion line.

[12]J. B. Bjorken and S. D. Drell, *Relativistic Quantum Fields* (McGraw-Hill, New York, 1965), p. 151. See also M. Gell-Mann and F. E. Low, Phys. Rev. 97, 1300 (1954).

[13]M. Ida, Phys. Rev. 135, B449 (1964); 136, B1767 (1964).

[14]Y. S. Jin and S. W. MacDowell, Phys. Rev. 137, B688 (1965).

[15]W. Nutt and B. Loiseau, Nucl. Phys. B104, 98 (1976).

[16]W. T. Nutt and C. M. Shakin, Phys. Rev. C 16, 1107 (1977).

[17]G. N. Epstein, Phys. Lett. 79B, 195 (1978).

[18]T. Mizutani and P. Rochus, Phys. Rev. C 19, 958 (1979).

[19]It may be thought that the internal pion in Fig. 4, for example, can somehow be distinguished kinematically from the pion exchanges which contribute to the *dNN* vertex function Γ_{dNN} which in the nonrelativistic limit is related to the deuteron wave function. This appears to be an ill-defined expectation; fortunately, it also appears to be an irrelevant one. The correct way to phrase these questions is to begin from a consistent description of the scattering process which includes a precise characterization of the composite nuclear state. When this is done diagrams such as Fig. 4 are simply ruled out from consideration at the start and the question of kinematic distinction never arises. It is also relevant to keep in mind that for appreciable momentum transfers carried by the internal pion in Fig. 1, Γ_{dNN} will not necessarily be represented adequately in terms of a nonrelativistic wave function.

[20]Other types of graphs contributing to (1) which involve effective pion-nucleon scattering insertions may require the subtraction of one or the other, or neither, of B_D and B_C from $t_{\pi N}$ to obtain proper amplitude. Several examples are explored in Ref. 1. See also Ref. 5.

[21]B. H. Bransden and R. G. Morehouse, *The Pion-Nucleon System* (Princeton University Press, Princeton, New Jersey, 1973), p. 286.

[22]L. C. Liu and C. M. Shakin, Phys. Rev. C 18, 604 (1978).

[23]M. G. Olsson and E. T. Osypowski, Nucl. Phys. B101, 136 (1975).

[24]B. H. Wilde, S. A. Coon, and M. D. Scadron, Phys. Rev. D 18, 4489 (1978).

[25]C. J. Goebel and B. Sakita, Phys. Rev. Lett. 11, 293 (1963).