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Pion electroproduction from the nucleon near threshold

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We investigate the dependence on production amplitude of the low energy scattering cross section for positive pion electroproduction from the nucleon. Calculations based on the threshold approximation to the full amplitudes of Dombey and Read are illustrated and compared to results for the full amplitudes. Inclusion of the momentum-dependent terms yields differential cross sections in the near-threshold region differing from those given by the threshold approximation by no more than 15% at any angle.

[NUCLEAR REACTIONS Dependence of $\sigma(\theta)$ for electron scattering on nucleon upon production amplitudes.]

It has long been realized that electromagnetic probes of the nucleus can be particularly sensitive to details of nuclear structure. This sensitivity persists at energies over pion production threshold where it becomes possible additionally to investigate the dynamics of the π-nucleus interaction over a broad energy range. This is particularly important in the near-threshold region ($5 \leq T_\pi \leq 25$ MeV) where little useful information can be obtained from elastic pion scattering experiments.

Calculations on threshold and near-threshold pion photoproduction and electroproduction are typically carried out in the framework of the distorted wave impulse approximation (DWIA), incorporating various models for the nuclear structure, single-particle $\gamma\pi N$ amplitudes, and π-nucleus optical potentials. Clearly, the reliability of the structure and particle production input determines the reliability of the conclusions that can be drawn from these processes regarding the π-nucleus interaction. In many cases, the nuclear matrix elements can be determined independently from analysis of related experiments.[1] For charged pion photoproduction, the single-particle amplitudes are likewise well determined from analysis of the reaction $\gamma N \rightarrow \pi N$ on a free nucleon.[2] Consequently, one is often able to make for photoproduction fairly unequivocal statements regarding the form of the pion optical potential and its relation to the basic πN interaction. The situation with respect to electroproduction is quite different. Here the greater complexity of the full dispersion-theoretic single-particle amplitudes has resulted in the uniform use[3-5,10] of threshold reductions in calculations on electroproduction from nuclei. In the course of carrying out detailed calculations on electropion production from nuclei in the threshold and near-threshold regions, we found that predictions for the electroproduction process on a single nucleon based on the full amplitudes can differ by 15% from those based on threshold approximations even at low energy. We think it important to present these intermediate results here because of the need to have accurate single nucleon input for the analysis of recent and projected nuclear electroproduction measurements, several of which are to be carried out at energies far above threshold.

The differential cross section for meson production in electron-nucleon collisions may be written

$$\frac{d^4\sigma}{ds_2 dp d\Omega_2 d\Omega_\pi} = (2\pi)^{-2} s_2{}^2 p^2 \delta(\epsilon_1 - \epsilon_2 - \omega_\pi + m - E_f) \left| T_{fi} \right|^2 , \quad (1)$$

where the initial and final electron four-momenta are $(\vec{s}_1, \epsilon_1)$ and $(\vec{s}_2, \epsilon_2)$ respectively; the pion momentum is $[\vec{p}, \omega_\pi = (p^2 + \mu^2)^{1/2}]$; the nucleon recoil momentum is $[\vec{q}, E_f = (q^2 + m^2)^{1/2}]$. All angles are measured with respect to the z axis, here taken to lie in the direction of propagation of the virtual photon of four-momentum $k = s_1 - s_2$, polarization $e_\mu = e\bar{u}(s_2)\gamma_\mu u(s_1)/k^2$. This choice of frame[6] explicitly separates the single-particle amplitudes from the nuclear physics, thereby allowing easy incorporation of the latter into calculations of nuclear electroproduction.

The transition matrix T_{fi} is proportional to the product of the leptonic and hadronic currents. The spatial part of the hadronic current can be written down from invariance requirements in the form[2]

$$\vec{\mathrm{H}} = i\vec{\sigma}F_1 + (\vec{\sigma}\cdot\hat{p})(\vec{\sigma}\times\hat{k})F_2 + i(\vec{\sigma}\cdot\hat{k})\hat{p}F_3 + i(\vec{\sigma}\cdot\hat{p})\hat{p}F_4 + i(\vec{\sigma}\cdot\hat{k})\hat{k}F_5 + i(\vec{\sigma}\cdot\hat{p})\hat{k}F_6, \quad (2)$$

where the F_i are the single-particle amplitudes and $\hat{p}(\hat{k})$ are unit vectors for the pion (photon). The time component can be obtained from the continuity equation $k_\mu H_\mu = 0$; its inclusion leads to a redefinition of F_5 and F_6. We may then write for the transition matrix ($\hbar = c = 1$)

$$|T_{fi}|^2 = |\epsilon_\mu H_\mu|^2 = \frac{8\alpha^2 f^2 m}{\mu^2\omega_\pi k^4 E_f}\,(M_{00}^{00} + M_{00}^{11} + 2M_{11}^{11}). \quad (3)$$

Here

$$M_{00}^{00} = A|2F_2\sin\theta_\pi|^2, \quad (4a)$$

$$M_{00}^{11} = A|\sin\theta_\pi(F_2 + F_3 + F_4\cos\theta_\pi)|^2 + B|(F_1 + F_5) + (F_3 + F_6)\cos\theta_\pi + F_4\cos^2\theta_\pi|^2, \quad (4b)$$

$$M_{11}^{11} = M_{-1-1}^{11} = A[|\tfrac{1}{2}\sin^2\theta_\pi F_4|^2 + |F_1 - F_2\cos\theta_\pi + \tfrac{1}{2}\sin^2\theta_\pi F_4|^2] + B(\tfrac{1}{2}\sin^2\theta_\pi)|F_4\cos\theta_\pi + F_6|^2. \quad (4c)$$

The terms in $A \equiv \frac{1}{2}\{1 - \cos\theta_2[1 - (s_2/s_1)^2\sin^2\theta_2]^{1/2}\}$ and $B \equiv \frac{1}{2}[1 + \cos\theta_2(1 - (s_2/s_1)^2\sin^2\theta_2)^{1/2} - s_2/s_1\sin^2\theta_2]$ correspond to transverse and longitudinal photons, respectively.[7] Again, all angles are measured with respect to the virtual photon. To include the four component of the current, we make the following replacements:

$$F_5 \to F_5 - \frac{k^2}{\omega_k^2}(F_1 + \hat{p}\cdot\hat{k}F_3 + F_5), \quad (5a)$$

$$F_6 \to F_6 - \frac{k^2}{\omega_k^2}(F_4\hat{p}\cdot\hat{k} + F_6). \quad (5b)$$

It remains only to insert expressions for the F_i. We have used the "full model" proposed for electroproduction by Dombey and Read.[8] This consists of a contact (seagull) term, two nucleon pole terms, and a pion pole term (Fig. 1), all calculated in pseudovector coupling, together with a $\Delta(1236)$ resonance contribution. It was shown[8] that this model was consistent with both partial conservation of axial-vector current (PCAC) and existing data 100 MeV above threshold. Nonrelativistic limits of the Born terms were used, retaining *all* terms[9] to order m^{-1}.

The cross sections we are interested in are those for which ω_π and the orientation of the pion with respect to the direction of the incident electron are measured. The integrals in Eq. (1) are

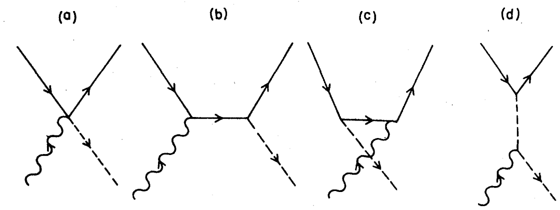

FIG. 1. Born diagrams included in present calculation: (a) contact term; (b) nucleon pole term; (c) crossed nucleon pole term; (d) pion current term. Solid lines are nucleons, dashed lines are pions, and wiggly lines are photons.

carried out in the forward peaking approximation (FPA) in which matrix elements of the elementary amplitudes are evaluated[5,10] at $\theta_2 \sim 0°$. The resulting cross sections are completely dominated by the transverse component of the amplitude with the various form factors replaced by their $k^2 = 0$ values.

In Fig. 2 we illustrate our results for π^+ production using the parameters ($E_e = 280$ MeV, $T_\pi = 21.6$ MeV) of the recent Mainz experiment[4] on threshold electroproduction from ^{12}C and ^{16}O. Curve (a) is the result using the threshold reduction[3-5,10] of the operators [$\vec{\mathrm{H}} = \vec{\sigma}\cdot\vec{\epsilon}(1 - \mu/2m)$], corresponding to the $\vec{p} = 0$ limit of all diagrams in Fig. 1. Curve (b) contains the contact term and both nucleon pole terms. Curve (c) uses the

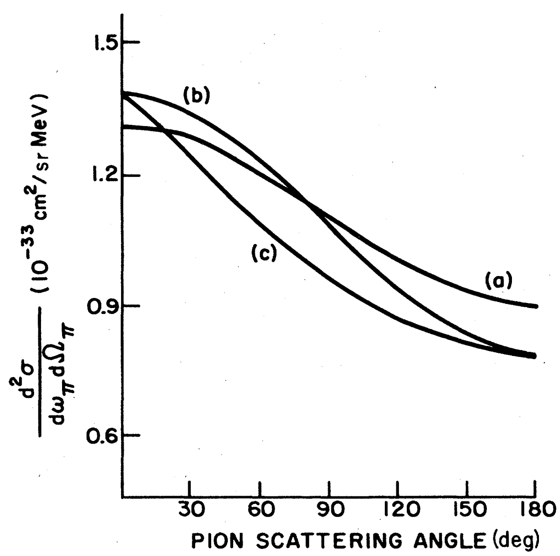

FIG. 2. Double-differential cross sections for positive pion production by 280 MeV electrons. Pion kinetic energy is 21.6 MeV. Curve (a) is for the threshold amplitudes (Refs. 3–5, 10); (b), for the contact and both nucleon pole terms [diagrams (a), (b), and (c) of Fig. 1]; (c), for the full amplitudes (all diagrams in Fig. 1 plus Δ resonance).

full form, consisting of all the Born terms and the Δ term (the latter contributes less than 1% at these energies).

The cross sections are dominated by the term proportional to $|F_1 - F_2\cos\theta_\pi + \frac{1}{2}\sin^2\theta_\pi F_4|^2$ in M_{11}^{11} and M_{-1-1}^{11} [Eq. (4c)]. The nucleon pole contributions (all of which are momentum dependent), entering through F_1 and F_2, yield in this region ~10% corrections to the momentum-independent terms and vary with $\vec{p}\cdot\vec{k}/2m\omega_k$. The $\cos\theta_\pi$ dependence leads to a rotation of the threshold curve ($F_1 \sim 1 - \mu/2m$) about $\theta_\pi = 90°$. A comparison of curves (b) and (c) illustrates the importance of the pion pole contribution. This term enters through $F_4 \sim -\vec{p}^2/(\omega_\pi\omega_k - \vec{p}\cdot\vec{k})$ and yields a 15–20% contribution. In the threshold approximation, all of the above momentum-dependent terms vanish identically. There is an additional pion pole term that does not vanish even at threshold. However, it enters only through the longitudinal amplitude F_5 and contributes little to the cross section. It should be noted in passing that although the F_5 term is negligible, its presence is formally required to maintain overall gauge invariance.

The chief conclusion to be drawn from this work is that the commonly made threshold approximation to the full electroproduction amplitudes leads to results for π^+ electroproduction on a free nucleon that can differ by ~15% from the predictions given by the full amplitudes. This finding is consistent with a recent result[11] on pion photoproduction from ^{7}Li and ^{12}C, a reaction equivalent to $(e,e'\pi)$ in the peaking approximation, where it was found that adding the momentum-dependent terms to the threshold operator yielded ~10% corrections for pion energies less than 50 MeV. Those electroproduction calculations on nuclei which employ threshold amplitudes within the framework of fairly sophisticated models of nuclear structure and final state interactions should include the correction terms, both for consistency of approach and for purposes of comparison with the accurate experimental data now becoming available.

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