PHYSICAL REVIEW C VOLUME 21, NUMBER 5 MAY 1980

Relative importance of neutron and proton components of nuclear transitions and comparative π^-/π^+ inelastic scattering

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(Received 14 September 1979)

Shell-model calculations for ^{18}O and ^{26}Mg yield predictions of strong state dependence of the isovector component of nuclear transition strengths, with results for ^{18}O being consistent with recent measurements of π^-/π^+ inelastic scattering ratios.

[NUCLEAR STRUCTURE ^{18}O, ^{26}Mg; calculated neutron and proton components of inelastic scattering excitations; estimates of π^-/π^+ cross section ratios; shell model.]

For a complete understanding of nuclear structure it is necessary to be able to distinguish between the neutron and proton components of nuclear transitions. This problem has been addressed with conventional nuclear probes by comparing the reduced electromagnetic strength of a transition in a stable ($N > Z$) nucleus to the strength of the corresponding transition in the analog ($Z > N$) nucleus[1,2] or, alternatively, to the reduced strengths of the same transition in the ($N > Z$) nucleus as induced by hadronic probes such as protons or alphas.[3] The first of these approaches is complicated[1] by the necessity of dealing with the (often significant) differences[4] in the radial wave functions of the protons and neutrons in the two different nuclear systems. The second approach faces the problem of quantitatively relating the reduced strengths of a transition induced by completely different types of probes.

The new "meson factories" offer a fresh alternative approach to this problem via the measurement of the relative cross sections for excitation of a given nuclear level by the inelastic scattering of π^+ and π^- projectiles. The sensitivity of this measurement to the differences in the neutron and proton components of the transition comes from the fact that the resonant cross sections for the interactions of π^+ with protons and π^- with neutrons are about nine times larger than for the corresponding interactions of π^+ with neutrons and π^- with protons. Experimental results of studies on ^{18}O which employ this technique have recently been published.[5,6]

We present here some shell-model predictions for the isoscalar and isovector, or in other terms the neutron and proton, components of nuclear transitions in ^{18}O and ^{26}Mg. These shell-model predictions are used, together with the simplest possible assumptions about the π-nucleus reaction mechanism, to predict the results of π^-/π^+ inelastic scattering cross section ratios. The results for ^{18}O seem consistent with existing data and suggest that measurement of additional transitions in this system should prove very interesting. The results for ^{26}Mg explain the anomaly of the phase of the isovector term of the 0_1^+-2_1^+ transition noted in Ref. 1 and predict a striking difference between the relative neutron-proton structures of the 0_1^+-2_1^+ and 0_1^+-2_2^+ transitions.

The shell-model wave functions employed in the present analysis are obtained from calculations in which the space of basis vectors is generated either by the $0p_{1/2}$, $1s_{1/2}$, and $0d_{5/2}$ single-nucleon orbits[7] (a "^{12}C-core model") or by the $0d_{5/2}$, $1s_{1/2}$, and $0d_{3/2}$ orbits[8] (an "^{16}O-core model"). For $A=18$, the ^{16}O-core model provides only a schematic accounting of the observed level structure. Accordingly, while such results are presented for ^{18}O, attention is principally focused upon the results of the ^{12}C-core model for this system. For ^{26}Mg, the ^{16}O-core model provides the appropriate degrees of freedom.

Our procedure is to condense the information contained in the nuclear wave functions which is relevant to a transition of angular momentum and isospin rank ΔJ and ΔT into the one-body-transition densities

$$D^{N,JT,J'T'}_{\Delta J,\Delta T}(j,j') = \frac{\langle \psi^{NJT} ||| (a_j^\dagger \otimes \tilde{a}_{j'})_{\Delta J,\Delta T} ||| \psi^{NJ'T'} \rangle}{(2\Delta J+1)^{1/2}(2\Delta T+1)^{1/2}}, \quad (1)$$

where $a_{j'}$ annihilates a nucleon in shell-model orbit j' and $a_j^\dagger$ creates a nucleon in orbit j. These matrix elements constitute the many-body component for analyses of transition strengths which employ any desired complexity of formulation of the nucleonic operators and reaction mechanisms by which the transition is effected, so long as the process is one-body in nature. For simplicity and uniformity, in our present application we combine the isovector and isoscalar D values into neutron and proton strength amplitudes A_p and A_n by taking the appropriate sums and differences and multiplying the resulting $D_{n,p}$ values by the single-particle matrix elements of $r^L Y^L$, assuming harmonic oscillator radial dependence for the single nucleon states. (The harmonic oscillator parameters used for ^{18}O and ^{26}Mg were chosen to reproduce their respective charge radii, taking into account the orbit occupations in the model spaces and making the conventional corrections for finite nucleon size and relativistic and center-of-mass effects.)

In terms of the model-space matrix elements A_p and A_n, the full proton and neutron matrix elements M_p and M_n are given[9] by

$$M_p = A_p + A_p\delta_{pp} + A_n\delta_{pn}\,,$$
$$M_n = A_n + A_n\delta_{nn} + A_p\delta_{np}\,, \tag{2}$$

where the δ_{ab} account for the coupling of the nucleons b of the model space to the virtual excitations of the core nucleons a. The strength of a transition induced by a probe which couples to protons and neutrons, respectively, with strengths C_p and C_n is given by

$$B(\Delta J) = (2J'+1)^{-1}(C_pM_p + C_nM_n)^2\,. \tag{3}$$

For electromagnetic transitions, $C_p = 1$ and $C_n = 0$ in units of e. We are concerned here with the *relative* differences in the strengths of a given transition as it is excited by probes with differing *relative* values of C_p and C_n. To emphasize this, as opposed to becoming involved with the probe-specific details of *absolute* transition strengths, we deal with normalized strengths such that $C_p' + C_n' = 1$, that is, such that a purely isoscalar transition (one in an $N = Z$ nucleus) would have the same strength value (equal to the electromagnetic strength) independent of the probe which induced it. The relevant characteristics of different probes are then represented solely by the difference $C_p' - C_n'$. For electromagnetic excitation or decay, $C_p' - C_n' = 1$, for alpha-induced excitation $C_p' - C_n' = 0$, and for $\pi^\pm$ excitation $C_p' - C_n' = \pm 0.5$.

Since the model cores have $N = Z$, $\delta_{pn} = \delta_{np}$ and $\delta_{pp} = \delta_{nn}$ up to order $(N-Z)/A$. Hence the complete structure of the core-valence coupling can be expressed in terms of the conventional model-space total effective charges $e_p = 1 + \delta_{pp}$ and $e_n = \delta_{pn}$. The isoscalar effective charge can be easily established from electromagnetic transition rates and it is found that $\delta_{pp} + \delta_{pn} = 0.7 \pm 0.1$ serves quite well for strong $E2$ and $E4$ transitions in this region with these model spaces.[10] There is very little known, on the other hand, about the empirical value of the isovector effective change, as is implied, of course, by the focus of the present work. Previous comparison[4] of experimental and calculated $B(E2)$ values and quadrupole moments has shown that $\delta_{pp} \leq \delta_{pn}$. The results to be presented here have been calculated with the values $\delta_{pp} = \delta_{pn} = 0.35$, although the empirical foundation of this choice of the ratio is weak. Ultimately, the study of data such as are treated here should provide a clearer picture of the empirically correct ratio δ_{pp}/δ_{pn}.

Our calculations for ^{18}O and ^{26}Mg are presented in Table I. They are based on the shell-model values of A_p and A_n as therein listed, Eqs. (2) and (3), and the values of C_p', C_n', and δ_{ab} just noted. We show results for the total electromagnetic strengths $B(E^L)$ and the ratios of the π^- to π^+ strengths, $B(\pi^-)/B(\pi^+)$, of the alpha-induced to electromagnetically-induced strengths, $B(\alpha)/B(\gamma)$, and of the ratio of total neutron to total proton transition strengths, $(M_n/M_p)^2$. Other quantities bearing on the central problem of the relative importance of neutron and proton components in nuclear transitions can be calculated from the A_p and A_n in a similar fashion. We emphasize that the various predictions of transition strength ratios depend not only upon A_p and A_n but also upon δ_{pp} and δ_{pn}.

The ^{12}C-core calculations for ^{18}O predict dominant neutron components for the 0_1^+-2_1^+ and 0_1^+-4_1^+ transitions which, while differing in details, are consistent with the naive ^{16}O-core picture. The predicted ratios of excitation strengths are reasonably consistent with the various measured values. The 0_1^+-$2_{2,3}^+$ and 0_1^+-3_1^- transitions are predicted to be dominated by proton excitations, with the calculated 0_1^+-3_1^- $B(\pi^-)/B(\pi^+)$ ratio being close to that observed. The large differences in the $B(\pi^-)/B(\pi^+)$ values calculated for these five states suggest that extending the present experimental results to include the neighboring states would be of great interest.

Our calculations predict that the 0_1^+-2_1^+ and 0_1^+-4_1^+ transitions in ^{26}Mg have larger proton than neutron components. The corollary of this result is that the electromagnetic strengths of these transitions in ^{26}Mg are larger than in the proton-rich mirror nucleus of ^{26}Mg, ^{26}Si. While this might seem surprising at first glance, it is, of

TABLE I. Matrix elements and transition rates in ^{18}O and ^{26}Mg.

Transition	A_p	A_n	Excitation (MeV) th	Excitation (MeV) exp	$B(E^L)(e^2\ fm^{2L})$ th	$B(E^L)(e^2\ fm^{2L})$ exp	$B(\pi^-)/B(\pi^+)$ th	$B(\pi^-)/B(\pi^+)$ exp[d]	$B(\alpha)/B(\gamma)$ th	$B(\alpha)/B(\gamma)$ exp[e]	$(M_n/M_p)^2$ th	$(M_n/M_p)^2$ exp[f]
$^{18}O(^{12}C$ core, $\hbar\omega = 13.24$ MeV)												
$0^+ \to 2_1^+$	2.12	8.57	1.80	1.98	34.4	45 ± 5 [a]	2.05	1.67 ± 0.08	2.40	1.46 ± 0.45	4.41	5.5 ± 0.9
$0^+ \to 2_2^+$	1.58	−0.01	4.18	3.92	4.5	27 ± 10 [a]	0.29		0.40	0.72 ± 0.33	0.06	0.35 ± 0.21
$0^+ \to 2_3^+$	3.95	0.02	5.10	5.26	28.5	8 ± 2 [a]	0.30		0.40		0.07	
$0^+ \to 3_1^-$	17.5	10.0	4.90	5.10	736	1120 ± 100 [a]	0.72	0.89 ± 0.05	0.74		0.52	
$0^+ \to 4_1^+$	1.7	−65.3	3.89	3.56	423		3.60		6.91		18.1	
$^{18}O(^{16}O$ core, $\hbar\omega = 12.65$ MeV)											14.9	
$0^+ \to 2_1^+$	0	9.41	2.00	1.98	10.8	45 ± 5 [a]	3.36	1.67 ± 0.08	5.89	1.46 ± 0.45	14.9	
$0^+ \to 4_1^+$	0	104.6	3.52	3.56	1 340		3.36		5.89			
$^{26}Mg(^{16}O$ core, $\hbar\omega = 12.86$ MeV)												
$0^+ \to 2_1^+$	10.57	7.22	2.05	1.81	282	301 ± 13 [b]	0.80		0.81	1.00 ± 0.14	0.64	0.54 ± 0.25
$0^+ \to 2_2^+$	1.16	7.00	3.25	2.94	16	9 ± 2 [b]	2.35		2.99	6.0 ± 1.8	6.02	8.00 ± 6.00
$0^+ \to 4_1^+$	70.6	44.7	4.59	4.32	12 311		0.77		0.78		0.59	
$0^+ \to 4_2^+$	53.0	75.1	5.41	4.90	9 572	$(26 \pm 7) \times 10^3$ [c]	1.23		1.24		1.50	

[a] Reference 11.
[b] Reference 2.
[c] Reference 12.
[d] Cross section ratios from average of measurements at bombarding energies of 164, 180, and 230 MeV, Ref. 5.
[e] Based on the values of $\tilde{B}(IS)$ given in Ref. 3; $B(\alpha) = (A/2Z)^2 \tilde{B}(IS)$.
[f] Extracted from $B(E2)$ values of the mirror transitions as described in Ref. 1, but without corrections for Coulomb effects upon the single-particle wave functions.

course, consistent with the simplest shell-model expectations, in which the lowest 0^+, 2^+, and 4^+ states of ^{26}Mg are formed by the two *proton* $0d_{5/2}$ holes in ^{28}Si and the analogs in ^{26}Si by the corresponding two *neutron* holes. In contrast to the 0_1^+-2_1^+ transition, the 0_1^+-2_2^+ transition in ^{26}Mg is predicted to be dominated by neutron excitations, with a $B(\pi^-)/B(\pi^+)$ ratio three times bigger than that of the 0_1^+-2_1^+ transition. Since ^{26}Mg should present no target difficulties and these lowest two 2^+ states are well isolated in energy, the experimental test of our predictions should be precise.

We have illustrated in these calculations for ^{18}O and ^{26}Mg that current theory for the many-body aspects of nuclear structure predicts ratios of the neutron and proton components of nuclear transitions which differ markedly from state to state and that the analyses of these differences can yield revealing insight into the natures of the various different excitations. Comparison with some of the few existing relevant data suggests that our theoretical interpretation is correctly focused, even though at present the empirical foundation of our assumption for the isovector effective charge is quite weak. Calculations for the ratios for inelastic scattering strengths for π^- and π^+ show variations between different excited states which should be clearly identifiable with current experimental techniques and pursuit of this type of experiment should, therefore, prove very rewarding.

This material is based upon work supported in part by the U. S. National Science Foundation under Grant No. Phy-7822696. One of the authors (B. H. Wildenthal) wishes to thank Professor K. A. Allen and Dr. P. E. Hodgson for their hospitality during his stay in Oxford.

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