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Pion-nucleus optical potential including the nuclear Hamiltonian

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We use the fact that the mass of the pion is much smaller than the mass of the nucleon to derive an expression for the first-order pion-nucleus optical potential that includes the effect of the nuclear Hamiltonian through a single-particle model. We apply this potential to calculate the total and reaction cross sections of ^{12}C and the charge-exchange cross section to the isobaric analog state in ^{13}C for energies between 0 and 300 MeV, obtaining qualitative agreement with experiment.

[NUCLEAR REACTIONS $^{13}C(\pi^+,\pi^0)$, $^{12}C(\pi^+,\pi^+)$, $E=0-300$ MeV; calculated $\sigma_{tot}(E)$, $\sigma_{reac}(E)$, and $\sigma_{cex}(E)$.]

Recently, Tandy, Redish, and Bollé[1] have proposed a three-body model for the first-order optical potential of Kerman, McManus, and Thaler,[2] which in the case of pion-nucleus scattering takes the form

$$\langle \vec{k}'|V_{opt}(E)|\vec{k}\rangle = \frac{(A-1)}{A}\sum_{i=1}^{A}\langle \vec{k}'\phi_0|\bar{t}^{(i)}_{\pi N}(E)|\vec{k}\phi_0\rangle\,, \tag{1}$$

where A is the number of nucleons, ϕ_0 is the ground-state wave function of the nucleus, and the operator $\bar{t}^{(i)}_{\pi N}(E)$ is the pion-nucleon amplitude in the nuclear medium, which obeys the integral equation

$$\bar{t}^{(i)}_{\pi N}(E) = V^{(i)}_{\pi N} + V^{(i)}_{\pi N}\frac{1}{E-T_\pi-H_N+i\epsilon}\bar{t}^{(i)}_{\pi N}(E)\,, \tag{2}$$

where $V^{(i)}_{\pi N}$ is the potential between the pion and nucleon i, T_π is the kinetic energy operator of the pion, and H_N is the nuclear Hamiltonian. The simplest approximation to Eq. (2), which is known as the closure approximation,[3] consists in replacing the nuclear Hamiltonian H_N by a constant effective excitation $\bar{E}$, which is usually taken to be zero, so that the operator $\bar{t}^{(i)}_{\pi N}$ becomes the free pion-nucleon T matrix. Tandy, Redish, and Bollé[1] have proposed a systematic way to improve the description of Eq. (2) by taking for the nuclear Hamiltonian H_N the single-particle Hamiltonian $H_N=\sum_{i=1}^{A}T_N^{(i)}+V_N^{(i)}$, where $T_N^{(i)}$ is the kinetic energy operator of nucleon i, and $V_N^{(i)}$ is the potential of that nucleon in the nuclear well. This corresponds to the first term of an expansion in which the residual interaction appears only in second- and higher-order terms.[1] If we use the eigenstates of H_N to evaluate Eqs. (1) and (2), then when we introduce a complete set of these states in the propagator, the part of the wave functions describing all nucleons other than i collapses with the corresponding one in ϕ_0, so that ϕ_0 is left only with the part describing nucleon i, and the nuclear Hamiltonian H_N can effectively be taken to be $H_N=T_N^{(i)}+V_N^{(i)}$, so that if we now drop the index i, Eq. (2) can be written in the form

$$\begin{aligned}\bar{t}_{\pi N}(E) &= V_{\pi N}+V_{\pi N}\frac{1}{E-T_\pi-T_N-V_N+i\epsilon}\bar{t}_{\pi N}(E)\\ &= V_{\pi N}+V_{\pi N}\frac{1}{E-T_\pi-T_N-V_N-V_{\pi N}+i\epsilon}V_{\pi N}\,.\end{aligned} \tag{3}$$

If we now reexpress the kinetic energy operators in terms of relative and center of mass variables, we can write

$$\begin{aligned}T_\pi+T_N+V_N+V_{\pi N} &= T^{\rm rel}+T^{\rm c.m.}+V_N+V_{\pi N}\\ &= H^{\rm rel}_{\pi N}+T^{\rm c.m.}+V_N\\ &\simeq H^{\rm rel}_{\pi N}+H^{\rm c.m.}_N\,,\end{aligned} \tag{4}$$

where we have made the approximation that the single-nucleon potential V_N is a function of the pion-nucleon center of mass coordinate; that is, if we call μ the mass of the pion and M the mass of the nucleon, then

$$\vec{R}^{\text{c.m.}}=\frac{1}{M+\mu}(M\vec{r}_N+\mu\vec{r}_\pi)\simeq\vec{r}_N\,,\ \text{since}\ \frac{\mu}{M}\ll 1\,. \tag{5}$$

Substituting Eq. (4) into Eq. (3), we get the approximate expression

$$\bar{t}_{\pi N}(E)\simeq V_{\pi N}+V_{\pi N}\frac{1}{E-H_{\pi N}^{\text{rel}}-H_N^{\text{c.m.}}+i\epsilon}V_{\pi N}\,, \tag{6}$$

where $H_{\pi N}^{\text{rel}}$ is the Hamiltonian of the relative motion of the pion-nucleon system which depends on the relative variables, and $H_N^{\text{c.m.}}$ is the Hamiltonian of the nucleon which depends here on the pion-nucleon center of mass variables.

The free pion-nucleon amplitude $t_{\pi N}$ differs from the amplitude $\bar{t}_{\pi N}$ by not containing the nuclear Hamiltonian, so that it is given in the language of Eq. (6) as

$$t_{\pi N}(E)=V_{\pi N}+V_{\pi N}\frac{1}{E-H_{\pi N}^{\text{rel}}+i\epsilon}V_{\pi N}\,. \tag{7}$$

Using Eq. (6), we can write the matrix elements of the operator $\bar{t}_{\pi N}$ that are needed in Eq. (1), as

$$\begin{aligned}\langle\vec{k}'\phi_0|\bar{t}_{\pi N}(E)|\vec{k}\phi_0\rangle=&\langle\vec{k}'\phi_0|V_{\pi N}|\vec{k}\phi_0\rangle\\&+\sum_{\alpha}\sum_{n}\langle\vec{k}'\phi_0|V_{\pi N}|\chi_\alpha\phi_n\rangle\\&\times\frac{1}{E-E_\alpha-E_n+i\epsilon}\\&\times\langle\chi_\alpha\phi_n|V_{\pi N}|\vec{k}\phi_0\rangle\,,\end{aligned} \tag{8}$$

where χ_α are relative pion-nucleon wave functions of energy eigenvalue E_α and ϕ_n are single-nucleon wave functions of energy eigenvalue E_n, considered here as functions of the pion-nucleon center of mass variable. Similarly, the free pion-nucleon T matrix given by Eq. (7) can be written as

$$\begin{aligned}\langle\vec{q}'|t_{\pi N}(E)|\vec{q}\rangle=&\langle\vec{q}'|V_{\pi N}|\vec{q}\rangle\\&+\sum_\alpha\langle\vec{q}'|V_{\pi N}|\chi_\alpha\rangle\frac{1}{E-E_\alpha+i\epsilon}\\&\times\langle\chi_\alpha|V_{\pi N}|\vec{q}\rangle\,,\end{aligned} \tag{9}$$

where $\vec{q}$ is the relative momentum of the pion-nucleon system which is given in terms of the pion and nucleon momenta $\vec{k}$ and $\vec{p}$ as

$$\vec{q}=\frac{1}{M+\mu}(M\vec{k}-\mu\vec{p})\simeq\vec{k}\,. \tag{10}$$

Using the separation into relative and center of mass components for the intermediate wave functions that enter in Eq. (8), we can write the term that appears there,

$$\begin{aligned}\langle\chi_\alpha\phi_n|V_{\pi N}|\vec{k}\phi_0\rangle&=\int d\vec{p}\langle\chi_\alpha\phi_n|V_{\pi N}|\vec{k}\vec{p}\rangle\langle\vec{p}|\phi_0\rangle\\&=\int d\vec{p}\langle\chi_\alpha|V_{\pi N}|\vec{q}\rangle\langle\phi_n|\vec{k}+\vec{p}\rangle\langle\vec{p}|\phi_0\rangle\\&\simeq\langle\chi_\alpha|V_{\pi N}|\vec{k}\rangle\Lambda_n(\vec{k})\,,\end{aligned} \tag{11}$$

where we have used Eq. (10), and we are defining

$$\begin{aligned}\Lambda_n(\vec{k})&\equiv\int d\vec{p}\langle\phi_n|\vec{k}+\vec{p}\rangle\langle\vec{p}|\phi_0\rangle\\&=\int d\vec{p}\,\phi_n^*(\vec{k}+\vec{p})\phi_0(\vec{p})\,.\end{aligned} \tag{12}$$

If we substitute Eq. (11) into Eq. (8), then using Eqs. (9) and (10) to recognize the free pion-nucleon T matrix, and the fact that the functions ϕ_n form a complete set, we get the expression

$$\langle\vec{k}'\phi_0|\bar{t}_{\pi N}(E)|\vec{k}\phi_0\rangle=\sum_n\langle\vec{k}'|t_{\pi N}(E-E_n)|\vec{k}\rangle\Lambda_n^*(\vec{k}')\Lambda_n(\vec{k})\,, \tag{13}$$

which together with Eqs. (1) and (12), is our result for the pion-nucleus optical potential including the effect of the nuclear Hamiltonian. Equations (12) and (13) provide a simple way to construct the matrix elements of the operator $\bar{t}_{\pi N}$ in terms of the free pion-nucleon T matrix $t_{\pi N}$ and the single-nucleon wave functions ϕ_n. Equation (13) represents also the solution to the problem of the first-order optical potential as formulated by Tandy, Redish, and Bollé, where one has to solve the three-body problem defined by the pion, the nucleon, and the residual nucleus, assuming that there is no interaction between the pion and the residual nucleus. If we call M_{res} the mass of the residual nucleus, then in the limit $M_{\text{res}}\gg M\gg\mu$, Eq. (13) is the solution of that simple three-body problem.

As we will show below, Eq. (13) reduces to the standard expression of the optical potential based in the closure approximation if we neglect the single-nucleon energies E_n in the argument of $t_{\pi N}$, since using the fact that the single-nucleon wave functions ϕ_n form a complete set,

$$\begin{aligned}\langle\vec{k}'\phi_0|\bar{t}_{\pi N}(E)|\vec{k}\phi_0\rangle&\simeq\langle\vec{k}'|t_{\pi N}(E)|\vec{k}\rangle\sum_n\Lambda_n^*(\vec{k}')\Lambda_n(\vec{k})\\&=\langle\vec{k}'|t_{\pi N}(E)|\vec{k}\rangle\sum_n\int d\vec{p}'d\vec{p}\langle\phi_0|\vec{p}'\rangle\langle\vec{k}'+\vec{p}'|\phi_n\rangle\langle\phi_n|\vec{k}+\vec{p}\rangle\langle\vec{p}|\phi_0\rangle\\&=\langle\vec{k}'|t_{\pi N}(E)|\vec{k}\rangle\int d\vec{p}'d\vec{p}\,\phi_0^*(\vec{p}')\delta(\vec{k}'+\vec{p}'-\vec{k}-\vec{p})\phi_0(\vec{p})\\&=\langle\vec{k}'|t_{\pi N}(E)|\vec{k}\rangle\int d\vec{r}e^{i(\vec{k}-\vec{k}')\cdot\vec{r}}|\phi_0(\vec{r})|^2\,,\end{aligned} \tag{14}$$

which is recognized as the standard expression of the first-order optical potential.[1-5]

As an example, we applied our expressions (1), (12), and (13) to calculate the total and reaction cross sections of ^{12}C up to 300 MeV. We can also apply our Eq. (13) to calculate the charge exchange reaction to the isobaric analog state $^{13}C(\pi^+, \pi^0)^{13}N$, since in this process the wave function of the valence nucleon is the same in the initial and final state except that its isospin projection has been flipped from $-\frac{1}{2}$ to $\frac{1}{2}$, so that using the distorted wave impulse approximation,[6,7] we can write

$$T^{\text{cex}}(E) = \int d\vec{k}' d\vec{k} \chi^{(+)*}(\vec{k}') \langle \vec{k}' \phi_0 | \bar{t}_{\pi N}^{\text{cex}}(E) | \vec{k} \phi_0 \rangle \chi^{(-)}(\vec{k}) , \tag{15}$$

where $\bar{t}_{\pi N}^{\text{cex}}$ is the elementary pion-nucleon charge exchange amplitude in the nuclear medium, ϕ_0 is the wave function of the valence nucleon, and $\chi^{(-)}$ and $\chi^{(+)}$ are the incoming and outgoing distorted pion waves.

In order to construct the optical potential given by Eqs. (1), (12), and (13), we need to know the free pion-nucleon T matrix $t_{\pi N}$, and the complete set of single-nucleon wave functions ϕ_n. We used for the free pion-nucleon T matrix the separable form with s and p waves,

$$\langle \vec{k}' | t_{\pi N}(E) | \vec{k} \rangle = \frac{\alpha^2 + k_0^2}{\alpha^2 + k'^2} [b_0(E) + b_1(E) \vec{k}' \cdot \vec{k}] \times \frac{\alpha^2 + k_0^2}{\alpha^2 + k^2} , \tag{16}$$

where k_0 is the on-shell momentum corresponding to the energy E, and the parameters $b_l(E)$ are given in terms of phase shifts.[4] We used the analytic parametrization of the phase shifts given in Ref. 8.

We constructed the single-nucleon wave functions ϕ_n using a square well potential of radius $R = 2.9$ fm and depth $V_0 = 48$ MeV, which has only two bound states, one for $l = 0$ with energy -32.21 MeV and one for $l = 1$ with energy -16.57 MeV, so that they correspond approximately to the experimental single-particle energies of carbon.[9] The continuum wave functions can be expressed analytically in terms of spherical Bessel and Neumann functions.[10] We tested our integration routines by checking that using these wave functions in Eq. (13), together with a pion-nucleon T matrix that is independent of energy, gives indeed the closure result (14).

In order to calculate our results, we solved the Lippmann-Schwinger equation in momentum space[5] with the optical potential given by Eqs. (1), (12), and (13). We assumed that the ground-state wave function of ^{12}C consists of four particles in the s shell and eighth particles in the p shell, and evaluated the expression (13) separately for each shell, defining the energy parameter E in Eq. (13), as the external kinetic energy of the pion minus the binding energy of the nucleon. For the contribution of the bound states in Eq. (13), we included all possible transitions without regard of the Pauli principle in the case of closed shells. For the ^{13}C nucleus, we used the same well as for ^{12}C, with the wave function of the valence neutron being given as a p-wave bound state wave function.

We show our results in Fig. 1, where we also show for comparison the results of the closure approximation potential (14) for three different values of the off-shell parameter α in the pion-nucleon amplitude (16). We see that the predictions of the two models in the case of the total and reaction cross sections are not very different from each other or from the data, nor do they

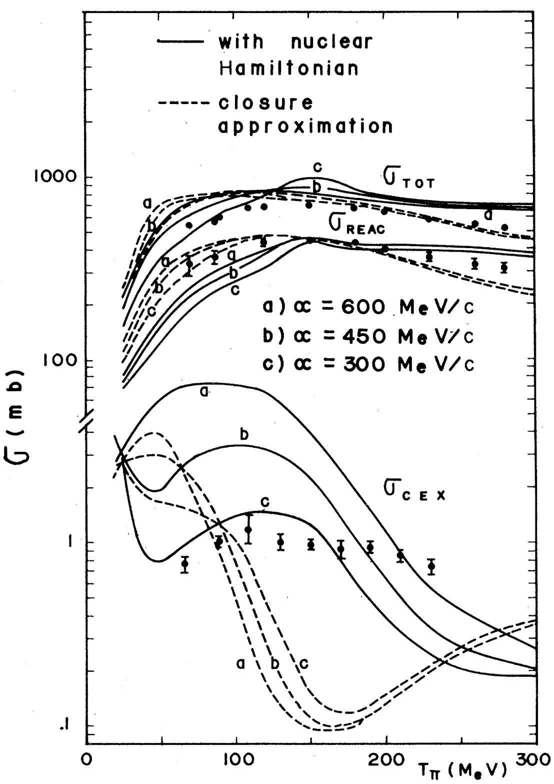

FIG. 1. Total (σ_{tot}) and reaction (σ_{reac}) cross sections for ^{12}C, and single charge exchange (σ_{cex}) cross section to the isobaric analog state of ^{13}C for incident pion kinetic energies from 0 to 300 MeV. The solid curves are the results of the potential containing the nuclear Hamiltonian (13), and the dashed curves are the results of the closure approximation potential (14). The experimental points are from Refs. 11 and 12.

depend very strongly on which off-shell extension of the pion-nucleon amplitude is used. The situation, however, is completely different for the charge exchange reaction, where first of all the results of the two models are very different both in shape and magnitude, and for the potential which includes the nuclear Hamiltonian very dependent on the value of α. In the case of the charge exchange reaction we observe the well-known result[6,7,13,14] that is practically impossible to fit the data with a model based in the closure approximation, while by including the nuclear Hamiltonian a great improvement is noticed. We see that we can explain the data reasonably well up to $T_\pi = 170$ MeV, with a value of α of 300 MeV/c, which is in striking agreement with the value of this parameter obtained by Goplen, Gibbs, and Lomon[15] in their analysis of the $\pi^+ + d \to p + p$ reaction. We interpret the disagreement at high energies as giving the range of validity of the theory, since relativistic effects will produce large corrections to the approximate result (13); we can see this roughly by replacing the mass of the pion by its relativistic energy $\omega = (\mu^2 + k_0^{\,2})^{1/2}$, and noticing that while at low energies the condition (5) $\mu/M \simeq \frac{1}{7} \ll 1$ is satisfied, for a 300 MeV pion $\omega/M \simeq \frac{1}{2}$, so that the approximation is very bad, and in this case Eq. (3) must be treated exactly as a relativistic three-body problem.

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