# $L_{2.3}$ -subshell x-ray fluorescence and Coster-Kronig yields at Z = 64 and 67

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The total Coster-Kronig transition probability  $f_{23}$  for the  $L_2$ - $L_3$  transition and the  $L_2$  and  $L_3$  subshell x-ray fluorescence yields  $\omega_2$  and  $\omega_3$  were measured utilizing resolved *L*-*K* x-ray coincidence techniques for Z = 64 and 67 with radioactive sources of 4.68 yr <sup>155</sup>Eu and 10.4 h <sup>165</sup>Er, respectively. Improved accuracy in the results is achieved by advances in the experimental method. The values of  $f_{23}$  are  $0.130 \pm 0.012$  and  $0.115 \pm 0.011$  at Z = 64 and 67, respectively, and of  $\omega_2$  and  $\omega_3$  are  $0.165 \pm 0.022$  and  $0.161 \pm 0.019$  at Z = 64, and  $0.186 \pm 0.023$  and  $0.180 \pm 0.020$  at Z = 67, respectively. The quoted uncertainties are twice the average deviation from the mean of 12 values of  $f_{23}$  and two values each of  $\omega_2$  and  $\omega_3$ , and arise predominantly from systematic errors.

 $\begin{bmatrix} \text{RADIOACTIVITY} & ^{155}\text{Eu}; & ^{165}\text{Er}; & \text{measured } \omega_2, & \omega_3, f_{23} \text{ in } L_{2, 3} \text{ subshells}; & XX \cdot t \\ & \text{coin}; & \text{Ge(HP)-Si(Li)} & \text{detectors.} \end{bmatrix}$ 

#### I. INTRODUCTION

This investigation of the  $L_{2,3}$  subshell x-ray fluorescence yields and Coster-Kronig total transition probability at Z = 64 and 67 was carried out to provide a test of theory and improved values for experimental applications. Recently, some new relativistic calculations of radiationless transition rates to L-subshell vacancy states have been performed<sup>1</sup> for selected elements with Z = 70 to 96. In this region, these calculated relativistic Lsubshell widths agree with experimental values better than previous nonrelativistic calculations.<sup>2-4</sup> Relativistic calculations for lower-Z elements, however, are not yet available, but they are of interest in the light of the new experimental results reported here and our current effort to extend the measurements to Z = 54.

Accurate experimental values of *L*-subshell fluorescence and Coster-Kronig yields also are important in many practical applications, ranging from elemental analysis by x-ray emission techniques to basic studies of nuclear and atomic processes leading to emission of x rays and Auger electrons, such as electron capture, internal conversion, and ionization cross section measurements.

Previous measurements of the total  $L_2 - L_3$ Coster-Kronig transition probability  $f_{23}$  at Z = 65(Ref. 5) and (Refs. 6 and 7) resulted in higher values than those found in the present work, whereas the discrepancies between previous and present values of the *L*-subshell x-ray fluorescence yields  $\omega_2$  and  $\omega_3$  are small. The present work is based on significant improvements in the experimental technique and in data evaluation. These advances include (1) an improved electronics system and the use of much faster coincidence timing (made possible by using a three-parameter multichannel  $XX \cdot t$  coincidence configuration); (2) much better resolution and spectral quality of the K x rays through the use of a new, state-of-the-art ionimplanted Ge(HP) detector; (3) collimation to drastically reduce Compton backscattering from one detector to the other in the 180° coincidence geometry used; and (4) application of a new method<sup>8</sup> for the direct experimental determination of the absolute correction for tailing of  $K\alpha_1$  events into the  $K\alpha_2$  x-ray peak.

## **II. BASIS OF THE EXPERIMENTAL METHOD**

The basic physical relationships among the L subshells involved in the determination of  $\omega_2$ ,  $\omega_3$ , and  $f_{23}$  can be found in the 1972 review by Bambynek *et al.*,<sup>9</sup> but the working equations containing the detailed experimental corrections are not included. They appear in various forms scattered throughout the literature with occasional unfortunate errors. For this reason, we consolidate and present here in a standardized form all of the necessary working experimental equations for the determination of  $\omega_2$ ,  $\omega_3$ , and  $f_{23}$ . Where applicable, the notation follows that of Rao *et al.*<sup>10</sup>

The present experiment measures the quantities  $\omega_3$ ,  $f_{23}$ , and  $\nu_2$ , the latter representing the total number of *L*-shell x rays (from all shells and not just from the radiative filling of an  $L_2$ -subshell vacancy) that result per primary vacancy in the  $L_2$  subshell.<sup>9,10</sup> Measurement of  $\nu_2$ ,  $f_{23}$ , and  $\omega_3$  proceeds directly from the relationships

$$\nu_2 = \frac{C_L(K\alpha_2)}{I_{K\alpha_2} \epsilon_L \epsilon_c} , \qquad (1)$$

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$$f_{23} = \frac{C_{L_{3X}(K\alpha_2)}C_{(K\alpha_1)}}{C_{L_{3X}(K\alpha_1)}(\theta)C_{(K\alpha_2)}},$$

$$\omega_3 = \frac{C_{L_{(K\alpha_1)}(\theta)}}{I_{K\alpha_1}\epsilon_L\epsilon_c},$$
(2)
(3)

from which the value of  $\omega_2$  is obtained through the relationship

$$\omega_2 = \nu_2 - f_{23}\omega_3 \,. \tag{4}$$

In Eqs. (1)-(4),  $C_{L(K\alpha_2)}$  is the total number of L x-ray counts gated by  $K\alpha_2$  x rays (after correction for continuum, nuclear cascading if present, chance coincidences, and for L x-ray coincidences arising from  $K\alpha_1$  tailing events falling within the  $K\alpha_2$  gate G2). Coincidence gates G1 through G5 and  $G(K_{\beta})$  are shown in Fig. 1.  $I_{K\alpha_2}$  is the intensity (counts) of  $K\alpha_2$  singles events falling within the energy region defined by G2 (corrected for background and singles  $K\alpha_1$  tail events). (Note that  $I_{K\alpha_1}$  and  $I_{K\alpha_2}$  are the intensities of  $K\alpha_1$  and  $K\alpha_2$ singles events falling within the energy region of the singles spectrum defined by G1 and G2, respectively, Fig. 1, and are not coincidence intensities.)  $\epsilon_L$  is the overall L x-ray detection efficiency averaged over the L x-ray energy region of interest and incorporates geometry and air-window attenuation, etc., and  $\epsilon_{c}$  is the coincidence efficiency determined experimentally.  $C_{L_3X(K\alpha_2)}$ represents the total number of L x-ray coincidence counts gated by  $K\alpha_2$  x rays (corrected for background, nuclear cascading if present, chance coincidences, and for  $K\alpha_1$  coincidence tail events in gate G2). The quantity  $C_{L_3X(K\alpha_2)}$  arises from  $L_3$ -X shell transitions, where X represents the  $M, N, \cdots$ higher shells, which give rise to the  $L_{t} + L\alpha_{1,2}$  $+L\beta_{2,15}$  components of the L x-ray spectrum at Z = 64 and 67. The factor  $C_{(K\alpha_1)}$  is the number of coincident  $K\alpha_1$  x-ray counts in the  $K\alpha_1$  gate G1, corrected for background. (The use of parentheses is to indicate a *coincidence* intensity, to distinguish the singles intensity  $I_{K\alpha_1}$  in the spectrum defined by gate G1.)

The factor  $C_{L_3X(K\alpha_1)}(\theta)$  is the total number of L x-ray counts, gated by  $K\alpha_1$  x rays arising from  $L_3$ -X transitions and corrected for background, nuclear cascading if present, chance coincidences, and for the directional correlation<sup>9</sup> in the 180° coincidence geometry used.  $C_{(K\alpha_2)}$  is the number of coincident  $K\alpha_2$  counts in gate G2, corrected for background and for the number of coincident  $K\alpha_1$  tail events falling in gate G2.  $C_{L(K\alpha_1)}(\theta)$  is the total number of L x-ray counts gated by  $K\alpha_1$  x rays, corrected for background, nuclear cascading if present, chance events, and directional correlation.<sup>9</sup>

The correction for nuclear cascading and chance



FIG. 1. (a) K x rays from  $4.68-y^{155}$ Eu in coincidence with L x rays. (b) K x rays from  $10.4-h^{165}$ Er in coincidence with L x rays. The coincidence gates G1, G2, G3, G4, G5, and  $G(K\beta)$ , indicated by the shaded areas, are set in the process of data analysis, since the threeparameter XX •t system stores all coincidences event by event on magnetic tape.

coincidences follows the method developed by McGeorge *et al.*<sup>11</sup> and is based on the fact that detection of  $K\beta$  x rays indicates K-shell vacancies which have been shifted to shells higher than the L shell, the only sources of L x rays in coincidence with a  $K\beta$  x-ray gate then being nuclear cascading if present and chance events. Unfortunately, in Ref. 11 an error in the equations led to an erroneous result for  $f_{23}$  at Z = 65, although the principle of the method was correct. The L x-ray coincidence spectra corrected properly for nuclear cascading and chance events are the differences.

$$C_{L(G1)} - C_{L(K\beta)} (I_{K\alpha_1} / I_{K\beta})$$

and

$$C_{L(G2)} - C_{L(K\beta)}(I_{K\alpha_0}/I_{K\beta}),$$

where  $C_{L(G1)}$  and  $C_{L(G2)}$  are the gross L x-ray coincidence spectra, corrected for background, gated by  $K\alpha_1$  and  $K\alpha_2$  gates G1 and G2, respectively, and  $I_{K\beta}$ ,  $I_{K\alpha_1}$ , and  $I_{K\alpha_2}$  are the respective singles intensities, corrected for background, in gates  $G(K_{\beta})$ , G1, and G2, respectively, as shown in Fig. 1. The ratios  $(I_{K\alpha_1}/I_{K\beta})$  and  $(I_{K\alpha_2}/I_{K\beta})$  are normalization factors needed to correct for the difference in the emission probabilities of  $K\alpha_1$  and  $K\beta$  and of  $K\alpha_2$  and  $K\beta$  x rays. These ratios are determined in the experiment and agree well with the theoretical values of Scofield.<sup>12</sup>

As indicated above, it is necessary to determine and correct for the number of  $K\alpha_1$  tail events falling in the  $K\alpha_2$  peak gate G2 in coincidence with the L x-ray spectrum. In all previous work reported in the literature, this correction has been made by normalizing a singles  $\gamma$ -ray line shape to the  $K\alpha_1$  peak and then subtracting its tail from the  $K\alpha_2$  peak. In the lower regions of Z where the  $K\alpha_1$  and  $K\alpha_2$  x rays exhibit appreciable overlap due to inadequate resolution, this procedure has been shown by us to overcorrect for the tailing,<sup>8</sup> because the tail from a singles peak is somewhat higher than that from a coincidence peak.

The working equation for determining the number of  $K\alpha_1$  tail events in the  $K\alpha_2$  gate in coincidence with L x rays, as applied in the present determination, is given by

$$N_{2}(2) = N_{2} - N_{3} \left[ \frac{N_{1}}{N_{2}(2)} \right] + N_{4} \left[ \frac{N_{1}}{N_{2}(2)} \right]^{2} - N_{5} \left[ \frac{N_{1}}{N_{2}(2)} \right]^{3} \left[ \frac{N_{2}(2)}{N_{1} + N_{2}(2)} \right],$$
(5)

where  $N_1$ ,  $N_2$ ,  $N_3$ ,  $N_4$ , and  $N_5$  are the total numbers of  $K\alpha_{(1+2)}$  events in gates G1, G2, G3, G4, and G5 shown in Fig. 1, respectively, and set in accordance with the exact procedure developed in Ref. 8. The quantity  $N_2(2)$  is the net number of coincident  $K\alpha_2$  events, after correction for  $K\alpha_1$ tailing, in gate G2, which is the one set at the  $K\alpha_2$  peak; i.e.,  $N_2(2) = N_2 - N_2(1)$ , where  $N_2(1)$  is the number of coincident  $K\alpha_1$  tail events falling in the  $K\alpha_2$  coincidence gate G2 and is the sum of the second, third, and fourth terms in Eq. (5).

Having thus determined the net number of pure  $K\alpha_2$  coincident events in the  $K\alpha_2$  gate G2,  $N_2(2)$ , the coincident L x-ray spectrum is then corrected for the presence of  $L_1 + L\alpha_{1,2} + L\beta_{2,15}$  x-ray components arising from coincidences with  $K\alpha_1$  tail events in the  $K\alpha_2$  gate. The remaining  $L_1 + L\alpha_{1,2} + L\beta_{2,15}$  x rays then must arise from  $L_3$  subshell vacancies produced only from Coster-Kronig

transitions shifting  $L_1$  and  $L_2$  vacancies into the  $L_3$  subshell. These remaining  $L_1 + L\alpha_{1,2} + L\beta_{2,15}$  x rays are the difference

$$C_{L_{3}X(G_{2})} - N_{2}(1) \left[ \frac{C_{L_{3}X(K\alpha_{1})}(\theta)}{N_{1}} \right].$$

Quantities that involve the  $L_3$ -K transition must also be corrected for directional correlation effects, whereas the  $L_2$ -K transition is isotropic.<sup>9</sup> For our 180° coincidence geometry, the corrected value of the number of L x-ray counts, gated by  $K\alpha_1$  x rays, arising from  $L_3$ -X transitions is

$$C_{L_{3}X(K\alpha_{1})}(\theta) = \frac{C_{L_{3}X(K\alpha_{1})}}{(1 + \overline{A}_{22}f_{\Omega})},$$
(6)

where  $C_{L_3X(K\alpha_1)}$  is the experimental value uncorrected for directional correlation.  $\overline{A}_{22}$  is the average directional coefficient,<sup>9</sup> the numerical value of which is given theoretically by Scofield,<sup>13</sup> and  $f_{\alpha}$  is the finite solid angle<sup>9</sup> given by

$$f_{\Omega} = \frac{1}{2} \cos \theta_m (1 + \cos \theta_m) , \qquad (7)$$

where  $\theta_m$  is the half angle subtended by the detector.

The final correction to be made arises from the unresolved  $L\eta$  x ray, which falls in the  $L\alpha$  x-ray peak, for which the method of McGeorge *et al.*<sup>14</sup> is applied. On the assumption that the detection efficiencies  $\epsilon_{L\alpha} \approx \epsilon_{L\eta}$ , the correct value of  $f_{23}$  may be calculated from the formula

$$f_{23} = f'_{23} - \left[\frac{(L\eta/L_2)(\omega_2/\omega_3)}{(L\alpha/L_3)}\right]k, \qquad (8)$$

where  $f'_{23}$  is the uncorrected value,  $(L\eta/L_2)$  is the intensity ratio of the  $L\eta$  component to all x rays emitted in transitions to the  $L_2$  subshell,  $(L\alpha/L_3)$ is the intensity ratio of the  $L\alpha$  component to all x-rays emitted in transitions to the  $L_3$  subshell, and k is the fraction of  $L\eta$  x rays included in the  $L\alpha$  x-ray peak. The value of k depends on the detector resolution, the energy separation between the  $L\eta$  and the  $L\alpha$  x-ray peaks (increasing with Z), and the method used to evaluate the  $L\alpha$  x-ray intensity. For  $Z \leq 70$ , with modern Si(Li) L x-ray detectors, the value  $k \approx 1$ . The theoretical values of Scofield<sup>15</sup> are used for the ratios  $(L\eta/L_2)$  and  $(L\alpha/L_3)$  and are in good agreement with experiment.

Two additional very small corrections to be considered are (1) subtraction of the continuum under the peaks and (2) subtraction of  $L\beta$  and  $L\gamma$ tail events from the  $L\alpha$  x-ray peak. (In the present work these corrections are negligible, e.g.,  $\leq 0.4\%$  and  $\leq 0.1\%$ , respectively, for both nuclides.)

#### III. EXPERIMENTAL

The radioactive sources used were carrier-free 4.68 y <sup>155</sup>Eu and high specific activity 10.4 h <sup>165</sup>Er. The latter was prepared by irradiating 2 mg of 67% enriched  ${}^{164}\text{Er}_2\text{O}_3$  for 4 h at  $10^{13} n/\text{cm}^2 \sec$  in the Georgia Tech Research Reactor. The irradiated sample was then dissolved in 1 ml of hot 3M HNO<sub>3</sub>, and 5 to 10  $\mu$ 1 aliquots were drop evaporated onto 0.25 mm thick Mylar discs, which were then lightly sealed with Krylon spray. The radioactive side was positioned to face the L x-ray detector in order to minimize attentuation of Lx rays. While counting the  $^{155}$ Eu source, a 100  $mg/cm^2$  plastic absorber was placed between the source and the K x-ray detector to remove most of the beta particles, to reduce the  $\beta^{-}$  continuum. This was unnecessary for <sup>165</sup>Er, which decays purely by electron capture to the ground state of <sup>165</sup>Ho.

The electronic block diagram is shown in Fig. 2. The dashed portion is used to acquire simultaneous singles spectra for measurements of  $\omega_2$  and  $\omega_3$ , and the solid lines in the diagram represent the three-parameter coincidence experiment for determination of  $f_{23}$ . The logic gate width typically is 2  $\mu$ sec, and the system resolving time is 50 to 70 nsec (full width at half amplitude). The multiparameter analyzer is a computer-based Nuclear Data ND-4420 system that requires all input signals to arrive within an interval of 2  $\mu$ sec in order to be stored as coincidences, which is done event by event on magnetic tape. By using an



FIG. 2. The electronic block diagram for the threeparameter  $XX \cdot t$  coincidence system. The heavy lines indicate the path of the digital signals, the medium lines represent analog signals, and the fine lines carry the logic pulses. The dashed portion is used to acquire simultaneous singles spectra for measurement of  $\omega_2$ and  $\omega_3$ . The parameters of the system are described in the text.

 $XX \cdot t$  three-parameter arrangement, a broad TAC spectrum of approximately 500 nsec can be acquired, and then a narrow, prompt portion of the TAC spectrum can be selected in the data analysis. The ratio of the counts in the narrow, prompt TAC gate to those in the broad TAC spectrum is used to determine the coincidence efficiency, which is typically in the range of 60 to 65 % for the narrow TAC gate, assuming the coincidence efficiency to be unity for the broad TAC spectrum.

A very narrow TAC window (50-70 nsec FWHA), was used, in spite of the fact that this produced a coincidence efficiency of only 60 to 65%. This sacrifice of some coincidence efficiency was made in order to achieve a significant reduction in tailing in the K and L x-ray spectra, where rejection of slow pulses is achieved by using the very narrow TAC window.<sup>8</sup>

The K x-ray detector is a ruggedized ion-implanted planar ORTEC Ge(HP) (which exhibits a resolution under the coincidence conditions described above of 340 eV FWHM at 46 keV and a peak/tail ratio of 70:1) and the L x-ray detector is a planar Kevex Si(Li) (coincidence resolution of 240-eV FWHM at 5.9 keV), using a  $1.5-\mu$ sec amplifier time constant for each detector. Both detectors are fitted with 5-ml (0.125-mm) Be windows and were used in 180° coincidence geometry. In the experimental setup, collimation was employed to reduce drastically the Compton backscattering. This especially benefitted the L x-ray spectrum, almost completely removing the Compton continuum. The graded collimator consisted of 1.8-mm Pb, 0.6-mm Cd, 0.9-mm Cu, and 1-mm Al, placed back to back on each side of the radioactive source. The center hole was approximately 5.5 mm in diameter. An example of the K spectrum is shown in Fig. 1 and the L spectra in Figs. 3 and 4.

In order to determine  $\omega_2$  and  $\omega_3$ , it is necessary to establish the detection efficiency of the L x-ray detector. This was done by using calibration standards of <sup>57</sup>Co, from the Laboratoire de Métrologie des Rayonnements Ionisants, Gif-sur-Yvette, France, and <sup>54</sup>Mn from the Physikalisch-Technische Bundesanstalt, Braunschweig, West Germany. Campbell et al.<sup>16</sup> have demonstrated that the use of drop-evaporated radioactive sources as standards, calibrated as  $\gamma$ -ray standards, can result in the determination of efficiency points in the 5- to 40-keV region that may be as much as 3 to 16% too low, owing to self-absorption and the use of  $X/\gamma$  ray intensity ratios, the self-absorption being negligible for the much higher energy  $\gamma$  ray for which the calibration was done. Another major source of error in the efficiency calibration is the presence of small variations in



FIG. 3. (a) The L x-ray spectrum gated by  $K\alpha_1$  x rays at Z = 64 from decay of 4.68-y <sup>155</sup>Eu. (b) Similar L x-ray spectrum, but gated by  $K\alpha_2$  x rays.

source position, not only in distance from the detector, but also in lateral deviation from the axial center.<sup>17</sup> A 5% variation in a 2-cm source-to-detector distance for a 20-mm diameter  $\times$  3-mm thick detector can cause a shift in the efficiency by

as much as 8%.<sup>18</sup> It is due to possible systematic errors of this magnitude that inherent errors of up to 11% in the efficiency calibration of the *L* x-ray detector may be present, and it is this uncertainty that is the major contribution to the un-

		<sup>155</sup> Eu			<sup>165</sup> Er			
Component	G1 (%)	G2 (%)	L <sub>(Kα1</sub> ) (%)	$L_{(K\alpha_2)}$ (%)	G1 (%)	G2 (%)	L <sub>(Kα1</sub> ) (%)	$\binom{L_{(K\alpha_2)}}{(\%)^2}$
Continuum	0.3	0.4			≤0.1	0.18		
Chance coincidences and								
nuclear cascades			17.9	13.8			0.5	0.4
Tailing of $K\alpha_1$ events into				<ul> <li>.</li> </ul>				
the $K\alpha_2$ gate		5.0				3.8		
Directional correlations			2.5				2.5	
$ll + L\alpha_{1,2} + L\beta_{2,15}$ (i.e., $L_3X$ ) counts in gate G2 caused by								
$K\alpha_1$ tail events				23.0				21.7
Tailing of $L\beta + L\gamma$ events into								
the $L^{\alpha}$ peak			≤1.0	2.0			≈0	≈0
Percentage of net true								
coincidence events	99.7	94.6	≥78.6	61.2	≥99.9	96.0	97.0	77.7

TABLE I. Corrections applied to the K x-ray gates and the gated L x-ray spectra.<sup>a</sup>

<sup>a</sup>G1 is the  $K\alpha_1$  gate and G2 is the  $K\alpha_2$  gate;  $L_{(K\alpha_1)}$  and  $L_{(K\alpha_2)}$  are the total L x-ray spectra produced by gates G1 and G2, respectively.



FIG. 4. (a) The L x-ray spectrum gated by  $K\alpha_1$  x-rays at Z = 67 from decay of 10.4-h<sup>165</sup>Er. (b) Similar L x-ray spectrum, but gated by  $K\alpha_2$  x rays.

certainty limits placed on the determination of  $\omega_2$ and  $\omega_3$  in the present work. Since the detection efficiencies are not involved in the determination of  $f_{23}$ , these systematic errors are absent in the measurement of  $f_{23}$ .

The coincidence experiment to determine  $f_{23}$  was repeated for each nuclide four times, the last two of which included the measurements of

 $\nu_2$  and  $\omega_3$ . Each run required approximately one week of counting time in order to reduce the statistical errors in counting to less than 1% in the coincidence gates and coincidence spectra.

## IV. RESULTS AND ANALYSIS OF ERRORS

The data were evaluated according to the procedure given in Sec. II and the various contribu-

	Z = 64		-		
Quantity <sup>a</sup>	Value	% error	Value	$\% \ \mathbf{error}$	
$C_{L(K\alpha_2)}$	78 000	0.4%	54850	0.4%	
IKan	$8.91 imes10^7$	0.1%	$5.05  imes 10^{7}$	0.1%	
$\epsilon_L$	$7.52 imes10^{-3}$	11%	$8.75 \times 10^{-3}$	11%	
€ <sub>c</sub>	0.6276	0.15%	0.6059	0.15%	
$C_{L_{2}X(K\alpha_{2})}$	9 037	2.5%	3475	2.7%	
$C_{(K\alpha_{\star})}$	104 150	0.3%	42 3 50	0.5%	
$C_{L_0X(K\alpha_4)}(\theta)$	78 860	1.0%	39 990	1.0%	
$C_{(K\alpha_{\alpha})}$	71 770	2.4%	27 925	2.6%	
$C_{L(K\alpha_{1})}(\theta)$	122 900	1.0%	86 510	1.1%	
$L_{K\alpha_1}$	$1.62 imes10^8$	0.08%	$9.06 imes10^7$	0.1%	

TABLE II. Quantities used to evaluate  $\nu_2$ ,  $f_{23}$ , and  $\omega_3$ .

<sup>a</sup>See Eqs. (1)-(3) in the text. These values represent results and error limits from a typical run on each nuclide.

tions to the coincidence gates and coincidence Lx-ray spectra are listed in Table I in the order in which the corrections are applied. The quantities needed for determining  $\nu_2$ ,  $f_{23}$ , and  $\omega_3$  from Eqs. 1, 2, and 3 are listed in Table II, with numerical results and percent errors for a typical run for each nuclide. Details of the error evaluations are given below.

In order to establish the independence of the values determined for  $\nu_2$ ,  $f_{23}$ , and  $\omega_3$  with respect to the location of the K x-ray gates G1 and G2, in each run G1 and G2 were analyzed for positions immediately below, at, and above the peak centroids. The resulting values exhibited a random distribution about the mean, thus assuring the validity of the tailing correction for  $K\alpha_1$  events in gate G2. However, tailing in the coincident Lx-ray spectra was at a minimum when gates G1 and G2 were chosen at the centroids of the K x-ray peaks. The percent errors listed in Table II from a typical run on each nuclide are purely statistical, except for  $\epsilon_L$ ,  $C_{L_3X(K\alpha_2)}$ , and  $C_{(K\alpha_2)}$ which are dominated by the presence of systematic uncertainty.

From the four runs, a total of 12 values of  $f_{23}$ was obtained, and their mean value is listed in Table III with an uncertainty of twice the average deviation from the mean, arising primarily from systematic error. All previous determinations of  $f_{23}$  contained estimated systematic errors that are two to three times larger than those in the present work. The uncertainty limits on  $\omega_2$  and  $\omega_3$  values are completely dominated by the systematic error arising in the *L* x-ray detection efficiency  $\epsilon_L$  (see Table II).

Previous measurements<sup>5,6</sup> of the L-subshell fluorescence yields,  $\omega_2$  and  $\omega_3$ , are in approximate agreement with the present results within the error limits. However, the previous values<sup>5,7</sup> of  $f_{23}$  lie significantly above the present ones and are outside experimental error limits. High values can result from the following experimental difficulties<sup>5,7</sup>: lack of sufficient energy resolution in the K and L x-ray detectors required to reduce the tailing correction to a minimum, use of a singles  $\gamma$ -ray line shape to estimate the tailing of  $K\alpha_1$  events in the  $K\alpha_2$  peak, apparent lack of correction for  $L\beta$  tailing into the  $L\alpha$  peak, lack of sufficiently narrow prompt time gating in the twoparameter coincidence technique used.<sup>5, 7</sup> and lack of collimation to reduce the Compton continuum from backscattered events.

TABLE III. Final values of  $\nu_2$ ,  $f_{23}$ ,  $\omega_3$  and  $\omega_2$  from the present work.

Quantity	Z = 64	Z = 67	
$\nu_2$	$0.184 \pm 0.022$	$0.204 \pm 0.023$	
$f_{23}$	$0.130 \pm 0.012$	$0.115 \pm 0.011$	
$\omega_3$	$0.161 \pm 0.019$	$0.180 \pm 0.020$	
$\omega_2$	$0.165 \pm 0.022$	$0.186 \pm 0.023$	

#### **V. DISCUSSION**

The nonrelativistic theoretical calculation of McGuire<sup>2</sup> at Z=67 predicts  $\omega_2=0.203$ ,  $\omega_3=0.201$ , and  $f_{23} = 0.138$ , all of which lie considerably above the present experimental values. Chen and Crasemann<sup>3</sup> predict from a nonrelativistic independentparticle model values at Z=67 of  $\omega_2=0.190$  and  $f_{23} = 0.147$ . Although the predicted value of  $\omega_2$ agrees well with the present experiment, the value of  $f_{23}$  is much too large, as has been found by Nix and Fink<sup>19</sup> in a systematic comparison of theory and experiment for  $f_{23}$  vs Z. The 1971 nonrelativistic calculations of Chen, Crasemann, and Kostroun<sup>4</sup> also predict values of  $f_{23}$  which exceed the trend of experimental results, although the theoretical predictions of  $\omega_2$  and  $\omega_3$  agree with the present results.

The 1979 relativistic calculations of Chen *et al.*,<sup>1</sup> do not extend below Z = 70. However, a linear extrapolation to Z = 64 indicates good agreement with the present experimental values of  $\omega_2$ . Extrapolation to  $Z \leq 70$  for  $f_{23}$  is not feasible, but it would appear to result in values much larger than the present experimental ones. We are extending the present measurement technique to Z = 54 and to Z = 82 to provide a more general critical test of theory.

The origin of the rather serious disagreement between theory and experiment for  $f_{23}$  can no longer be ascribed to error in the experiments, nor can it arise from the small admixture of double vacancies from *K*-*LL* and *K*-*LM* Auger transitions, since this effect has been shown at Z = 49,<sup>20</sup> 73,<sup>21</sup> and 82 (Ref. 22) to be less than about 10% on *L*-fluorescence yields and is probably altogether negligible for  $f_{23}$ .

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- <sup>1</sup>M. H. Chen, E. Laiman, B. Crasemann, M. Aoyagi, and Hans Mark, Phys. Rev. A <u>19</u>, 2053 (1979); Bull. Am. Phys. Soc. 24, 624 (1979).
- <sup>2</sup>E. J. McGuire, Phys. Rev. A <u>3</u>, 587 (1971).
- <sup>3</sup>M. H. Chen and B. Crasemann, in *Inner Shell Ionization Phenomena and Future Applications*, edited by R. W. Fink, S. T. Manson, J. M. Palms, and P. V. Rao (U.S. Atomic Energy Commission, 1973), p. 43.
- <sup>4</sup>M. H. Chen, B. Crasemann, and V. O. Kostroun, Phys. Rev. A 4, 1 (1971).
- <sup>5</sup>D. G. Douglas, Can. J. Phys. <u>50</u>, 1697 (1972).
- <sup>6</sup>C. P. Holmes and V. O. Kostroun, Bull. Am. Phys. Soc. 15, 561 (1970).
- <sup>7</sup>D. G. Douglas, Can. J. Phys. <u>54</u>, 1124 (1976).
- <sup>8</sup>B. E. Gnade, R. A. Braga, W. R. Western, J. L. Wood, and R. W. Fink, Nucl. Instrum. Methods <u>164</u>, 163 (1979).
- <sup>9</sup>W. Bambynek, B. Crasemann, R. W. Fink, H. U. Freund, Hans Mark, C. D. Swift, R. E. Price, and P. V. Rao, Rev. Mod. Phys. <u>44</u>, 716 (1972).
- <sup>10</sup>P. V. Rao, R. E. Wood, J. M. Palms, and R. W. Fink, Phys. Rev. 178, 1997 (1969).
- <sup>11</sup>J. C. McGeorge, H. U. Freund, and R. W. Fink, Nucl. Phys. A154, 526 (1970).

- <sup>12</sup>J. H. Scofield, At. Data Nucl. Data Tables <u>14</u>, 121 (1974).
- <sup>13</sup>J. H. Scofield, Lawrence Livermore Laboratory Report UCRL-51231, 1972 (unpublished).
- <sup>14</sup>J. C. McGeorge, S. Mohan, and R. W. Fink, Phys. Rev. A <u>4</u>, 1317 (1971).
- <sup>15</sup>J. H. Scofield, Phys. Rev. <u>179</u>, 9 (1969).
- <sup>16</sup>J. L. Campbell, H. H. Jorch, and J. A. Thompson, Nucl. Instrum. Methods <u>140</u>, 167 (1977); <u>143</u>, 551 (1977).
- <sup>17</sup>Z. B. Alfassi and R. Nothman, Nucl. Instrum. Methods <u>143</u>, 57 (1977).
- <sup>18</sup>G. Bertolini and G. Restelli, in *Atomic Inner-Shell Processes*, edited by B. Crasemann (Academic, New York, 1975), Vol. 2, p. 123.
- <sup>19</sup>D. W. Nix and R. W. Fink, Z. Phys. <u>A273</u>, 305 (1975).
- <sup>20</sup>P. A. Indira, I. J. Unus, P. V. Rao, and R. W. Fink, J. Phys. Atom. Molec. Phys. (London) <u>B12</u>, 1351 (1979).
- <sup>21</sup>J. L. Campbell, L. A. McNelles, J. S. Geiger, J. S. Merritt, and R. L. Graham, Can. J. Phys. <u>55</u>, 868 (1977).
- <sup>22</sup>P. V. Rao, R. E. Wood, and V. R. Veluri, J. Phys. Atom. Molec. Phys. (London) <u>B10</u>, 399 (1977).