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Local field corrections in π-nucleus scattering

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The local field correction to the π-nucleus optical potential is calculated taking recoil into account. The correction arises from the triple scattering matrix element $\langle 0;\vec{k}|\tau_1 G\tau_2 G\tau_1|0;\vec{k}'\rangle$, where τ_i are πN amplitudes. The recoil energy of nucleon 1 affects the pion Green's function G and the intermediate scattering amplitude τ_2. The recoil of nucleon 2 affects only τ_2 and has not been included to save computational time. Effects of antisymmetry of the two nucleons, spin flip, and charge exchange have been included, but not those of pair correlation and Pauli blocking. Inclusion of recoil is found to reduce the local field correction by a factor of 7 from the result of fixed scatterer calculations. Nevertheless, the local field correction remains sizable. In the π-^{16}O case it represents a correction 2/3 as large as the first order optical potential near the πN resonant energy. Nucleon 1 typically recoils through ~ 1 fm while in terms of the internucleon separation r_{12} the effect is very long ranged. Thus the P-wave nature of the πN interaction which enhances the reflection of the pion from nucleon 2 back to nucleon 1 is the main reason why the local field correction is so important in π-nucleus scattering. The imaginary part of the local field correction part of the optical potential is analyzed and its reactive content is shown to arise from two competing mechanisms. The first, and dominant, mechanism is a reduction of the single nucleon knockout contribution that is overestimated in the first order optical potential while the second mechanism is the two-nucleon knockout. The analysis suggests very slow convergence of multiple scattering expansions near the πN resonance energy.

[NUCLEAR REACTIONS Pion optical potential, local field corrections, reactive content of optical potential.]

I. INTRODUCTION

Nuclear pion scattering has the unique feature that the mean separation of nucleons, $d \sim 2$ fm, is comparable to the resonant πN scattering radius: $r_s \equiv (\sigma/2\pi)^{1/2} \sim 1.8$ fm. The scattering radius is defined so that a black disk of radius r_s produces the same total cross section as is observed in the $J=\frac{3}{2}$, $I=\frac{3}{2}$, πN scattering state at resonance. The scattering radius is quite distinct from, and much larger than, the interaction range $r_{\pi N} \sim 0.3$ fm, within which the πN interaction is large. The consequence of a large scattering radius is that pion waves reaching a nucleon inside a nucleus are strongly modified by the presence of other nucleons. The rescattered pion wave a distance d from a nucleon is approximately $(f/d)\cos\theta\, e^{ikd}$, where f is the P-wave scattering amplitude, θ is the polar angle measured from the beam direction, and k is the pion's wave number. Because $f/d \simeq i\, r_s/d$ has magnitude comparable to unity at resonance, rescattered waves are as important as incident waves reaching a nucleon.

A substantial part of this multiple scattering effect is summed into the first order optical potential because the small angle ($\theta \sim 0$) pion multiple scattering tends to leave the nucleus in its ground state between scatterings. However, there is a correspondingly large amplitude for pion reflections ($\theta \sim \pi$). The reflections involve large momentum transfer and they preferentially populate excited nuclear states between scatterings. Thus, the effects of reflections come in as higher order terms of the standard optical potential expansions.[1] With few exceptions, analyses of π-nucleus scattering have been performed by omitting the higher order terms on grounds that they are thought to be small.[2] Generally calculations are based on a πN off-shell amplitude that is only kinematically modified in the nuclear medium.

By way of contrast, isobar hole theories of pion-nucleus scattering[3] describe the same physics in a fashion that emphasizes the local field corrections to the πN scattering in the (3, 3) isobar state. The pion scattering is viewed as a sequence of Δ-hole states with pion propagation between the annihilation of one Δ-h state and creation of the next. Often rather substantial local potentials are used to represent the effect of the nuclear field on the isobar self-energy and these effective interactions modify both the mass and width of the isobar in the nuclear medium.

The effects of the nuclear medium on the πN interaction are organized into higher order terms in the multiple scattering optical potential, terms which have been called local field correction by Foldy and Walecka.[4] The simplest multiple scattering model in which the local field correction has been studied consists of pion waves scattering from fixed, nonoverlapping potentials. Even though a local potential model is not adequate to describe pion scattering, the results of Agassi and Gal[5] are instructive by way of showing how a projectile's interaction with any one nucleon can be strongly modified by the presence of others nearby. In this simple model, the local field corrections arise from multiple reflections of the incident pion from the neighboring nucleons, a feature which is relevant to pion-nucleus scattering due to the large back-scattering πN amplitude.

In more recent fixed scatterer estimates for nuclear matter,[6] very large effects due to the local field correction have been found based on the Goldstone diagrams of Fig. 1. The importance of these diagrams is that they are the first terms in the multiple scattering series which have the reflection property, and thus the local field correction is small or large depending on their contribution to the optical potential. In the triple scattering process of Fig. 1, the pion wave scattering from nucleon 1 is modified by an intermediate scattering from nucleon 2. At resonance, this local field correction, calculated in fixed scatterer approximation, is about three times as large as the "leading" $t\rho$ term of the optical potential, indicating a severe divergence of the multiple scattering description. It has been argued[7,8] that self-consistent treatments of the

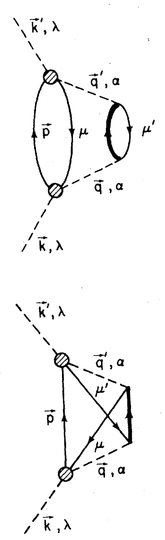

FIG. 1. Goldstone diagrams for local field correction due to triple scattering process. The contribution of the direct diagram (top) is divided into direct and spin-flip parts as discussed in the text. The contribution of the exchange diagram (bottom) is not divided. Heavy lines represent the isobar excitation.

multiple scattering process, which approximately sum many-body effects, ultimately regulate the local field correction to $\sim\frac{1}{2}t\rho$. However, this argument relies on a fixed scatterer analysis for infinite nuclear matter and it is recognized[7] that effects neglected in such analyses may regulate, or quench, the local field correction. Three-body analyses of π-d scattering[9] have shown that the fixed scatterer approximation overestimates the local field correction. In nuclei, however, the local field correction is long ranged[6,8] because the pion propagates at positive energy from one nucleon to another. The effect is thus proportional to $A(A-1)\langle 1/r^2\rangle$, i.e., to the number of pairs of nucleons and the mean value of r^{-2}. Clearly the deuteron case is not a decisive test of the local field effect since $\langle 1/r^2\rangle$ is not much different than for nuclei but the number of pairs is small. Because neither fixed scatterer nor three-body analyses have settled this issue, a detailed calculation of the diagrams in Fig. 1 is presented in this paper.

Because the pion twice scatters from nucleon 1 in Fig. 1, an internal loop exists through which large momentum flow is supported by the P-wave coupling of the pion to nucleons. The associated recoil energy of nucleon 1 is neglected in fixed scatterer analyses, and we shall show in this paper that omission of recoil causes serious overestimates of the leading local field correction. Nevertheless, the inclusion of recoil is not sufficient to quench the local field effect in nuclei such as ^{16}O, although it does suggest a more convergent multiple scattering analysis.

The strategy of this paper is to separate the local field correction from true pion absorption effects, which we do not consider here. This is done by excising the nucleon-pole part of the πN interaction with nucleon 2 in Fig. 1, thereby omitting the intermediate state where the pion is absent and two nucleons are excited. The analysis is described in detail in Sec. II and numerical results for the local field correction with and without recoil are presented in Sec. III. Section III also examines the reactive content of the local field correction to explicitly display the origin of the large effects. Conclusions are presented in Sec. IV.

II. RESCATTERING WITH RECOIL

The Goldstone diagrams of Fig. 1 can be evaluated using the rules for a pion propagator in a nucleus.[10] One may also derive the same result using the triple scattering term of Watson's multiple scattering expansion

$$\hat{T}^{(3)} = A(A-1)\langle 0;\vec{k}',\lambda|\tau_1 G \tau_2 G \tau_1|0;\vec{k},\lambda\rangle \cdots, \quad (1a)$$

where $\vec{k}$ and $\vec{k}'$ are the initial and final pion momenta and $\lambda(=1,2,3)$ the pion isospin. The ket $|0;\vec{k},\lambda\rangle$ is the product of a pion plane wave state and the target-nucleus ground state. The pion-nucleon scattering operator τ and the pion propagator G will be described later.

It should be noted that $\tilde{T}^{(3)}$ is not the matrix element of a part of the Klein-Gordon optical potential. In fact, $\tilde{T}^{(3)}$ does not even have the appropriate dimension. The discussions of Ref. 11 show that the contribution to the Klein-Gordon optical potential is given by

$$\begin{aligned} T^{(3)} &= \sqrt{2\omega_{k'}}\,\tilde{T}^{(3)}\sqrt{2\omega_k} \\ &= A(A-1)\sqrt{2\omega_{k'}} \\ &\quad\times\langle 0;\vec{k}',\lambda|\tau_1 G\tau_2 G\tau_1|0;\vec{k},\lambda\rangle\sqrt{2\omega_k}\,. \end{aligned} \tag{1b}$$

Equation (1) is a generalization of the usual Watson potential theory in that the pion propagator G and the πN amplitudes have appropriate relativistic properties. The full reduction of the relativistic amplitude to the form (1) is omitted as only minimal changes from the usual Watson multiple scattering theory arise. The scattering matrices τ_i are t matrices for pion scattering from nucleon i, wherein the energy variable is $\omega+H_T$, H_T being the Hamiltonian of the target nucleus and ω the pion's asymptotic energy. In this work, we approximate H_T by a constant average binding energy $(H_T\simeq -B)$ in which case the τ_i are simply related to the πN t matrix at a shifted energy, $\omega - B$.

The nuclear wave function is assumed to be a determinant of single particle wave functions ϕ_μ, and thus (1) reduces to standard two-nucleon forms which we expand as follows:

$$T^{(3)} = T_{\rm D}^{(3)} + T_{\rm SF}^{(3)} - T_{\rm EX}^{(3)}\,, \tag{2}$$

where direct plus spin flip and exchange terms are defined via

$$T_{\rm D}^{(3)} + T_{\rm SF}^{(3)} = \sqrt{2\omega_{k'}}\sum_\mu\sum_{\mu'}\langle\phi_\mu(1)\phi_{\mu'}(2);\vec{k}',\lambda|\tau_1 G\tau_2 G\tau_1|\phi_\mu(1)\phi_{\mu'}(2);\vec{k},\lambda\rangle\sqrt{2\omega_k}\,, \tag{3}$$

$$T_{\rm EX}^{(3)} = \sqrt{2\omega_{k'}}\sum_\mu\sum_{\mu'}\langle\phi_{\mu'}(1)\phi_\mu(2);\vec{k}',\lambda|\tau_1 G\tau_2 G\tau_1|\phi_\mu(1)\phi_{\mu'}(2);\vec{k},\lambda\rangle\sqrt{2\omega_k}\,. \tag{4}$$

In each of these expressions, the πN scattering matrices τ_1 and τ_2 are fully off energy shell. The πN matrix, τ_1, is evaluated at the shifted pion energy $\omega - B$. Expressed as a single nucleon operator, it has the form

$$\langle\vec{q}',\alpha'|\tau_1(\vec{r}_1;\omega)|\vec{q},\alpha\rangle = -4\pi\sum_{I,J} h_{2I,2J}(\omega)\Omega_{2J}(\vec{q}',\vec{q})\Lambda_{2I}(\alpha',\alpha)e^{i(\vec{q}-\vec{q}')\cdot\vec{r}_1}\frac{v(q')}{\sqrt{2\omega_{q'}}}\frac{v(q)}{\sqrt{2\omega_q}}. \tag{5}$$

In order to separate the local field correction from true pion absorption effects, the off-shell amplitude τ_2 is written in a form which omits the nucleon pole term, as follows:

$$\langle\vec{q}',\alpha'|\tau_2(\vec{r}_2;\omega')|\vec{q},\alpha\rangle = -4\pi\sum_{I,J}\tilde{h}_{2I,2J}(\omega')\Omega_{2J}(\vec{q}',\vec{q})\Lambda_{2I}(\alpha',\alpha)e^{i(\vec{q}-\vec{q}')\cdot\vec{r}_2}\frac{v(q')}{\sqrt{2\omega_{q'}}}\frac{v(q)}{\sqrt{2\omega_q}}. \tag{6}$$

The energy dependence of the off-shell amplitude is expressed as a dispersion integral, without crossing, as follows:

$$\tilde{h}_{2I,2J}(\omega') = -\frac{1}{\pi}\int_{m_\pi}^{\infty} dz\frac{{\rm Im}[h_{2I,2J}(z)]}{\omega' - z + i\eta}\,, \tag{7}$$

which is just the right-hand cut term of the Chew-Low form for the πN amplitude. The relationship of the energy variable ω' with the initial pion energy, ω, is discussed later. In these expressions, both the isospin (I) and spin (J) indices are understood to be summed over $\frac{1}{2}$ and $\frac{3}{2}$ values and the projection operators for spins $J=\frac{1}{2}$ and $\frac{3}{2}$ are

$$\Omega_1(\vec{q}',\vec{q}) = \vec{q}'\cdot\vec{q} + i\vec{\sigma}\cdot\vec{q}'\times\vec{q} = \vec{\sigma}\cdot\vec{q}'\vec{\sigma}\cdot\vec{q}\,, \tag{8a}$$

$$\Omega_3(\vec{q}',\vec{q}) = 2\vec{q}'\cdot\vec{q} - i\vec{\sigma}\cdot\vec{q}'\times\vec{q}\,, \tag{8b}$$

while those for isospins $I=\frac{1}{2}$ and $\frac{3}{2}$ are

$$\Lambda_1(\beta,\alpha) = \tfrac{1}{3}\tau_\beta\tau_\alpha\,, \tag{9a}$$

$$\Lambda_3(\beta,\alpha) = \delta_{\beta\alpha} - \tfrac{1}{3}\tau_\beta\tau_\alpha\,. \tag{9b}$$

Here $\beta,\vec{q}'$ denote the final state isospin and momentum while $\alpha,\vec{q}$ are similar labels of the initial state quantities. The πN energy dependent amplitudes, $h_{2I,2J}(\omega)$, are related to the phase shifts as follows:

$$h_{2I,2J}(\omega) = \frac{M+\omega}{M}\frac{\eta_{2I,2J}\exp(2i\delta_{2I,2J})-1}{2ik^3}. \tag{10}$$

The phase shift values used in the present calculations are based on Salomon's parametrization[12] up to 250 MeV pion kinetic energy. For the calculation of the right-hand cut amplitude, $\tilde{h}_{2I,2J}(\omega)$

based on Eq. (7), we employ in addition the CERN theoretical phase shifts[13] to extend the integration to 4 GeV πN center-of-mass total energy. The form factor $v(q)$ is specified later in Eq. (25).

The remaining ingredients of the triple scattering amplitude are the π-nucleus intermediate state Green's functions, $G=(E-\omega_q-H_T+i\eta)^{-1}$, where $E=\omega-E_0$ is the total energy of the external pion plus nuclear ground state. The first scattering of Fig. 1 excites the nucleon from an occupied (hole) state, ϕ_μ, to an unoccupied (particle) state which is approximated as a plane wave state of momentum $\vec{p}$. The energy of this one particle-one hole state is approximately represented as $E_0+B+p^2/(2M)$, where M is the nucleon mass, and $-B$ is the average hole state energy. Thus, the Green's function for momentum $\vec{q}$ of the pion takes the form $G=[\omega-\omega_q-p^2/(2M)-B]^{-1}$. A similar Green's function applies to the pion line of momentum $\vec{q}\,'$ in Fig. 1. The recoil effect in the pion Green's functions forces the propagation energy $\omega-p^2/(2M)-B$ negative for $p \gtrsim \sqrt{2M\omega}$. This causes two natural cutoffs for high momentum values which are not present in fixed scatterer analyses.

The off-shell πN amplitude, τ_2, may be viewed as an isobar-hole state with a continuous mass spectrum $M+z \geq M+m_\pi$. The energy denominator internal to this amplitude is thus $(\omega-p^2/2M-2B-z+i\eta)$, where $p^2/(2M)$ is the recoil energy of nucleon 1 and $2B$ represents the energy shift due to the two-hole states in the nucleus. The effect of the nucleon recoil energy is to cause a downward shift of the energy at which the off-shell πN amplitude is evaluated via Eq. (7). Because the πN amplitude, τ_2, is resonant with a width of $\Gamma \sim 100$ MeV, the recoil effect is to force τ_2 away from resonance except in a momentum range $\Delta p \sim \sqrt{2M\Gamma} \sim 2\ \mathrm{fm}^{-1}$. In the fixed scatterer analysis, the recoil energy is omitted permitting the τ_2 amplitude to remain large for an infinite range of momentum values.

In this analysis, the isobar recoil is neglected in order to keep the number of integrations manageable. The nucleon recoil affects both the pion propagators and the amplitude τ_2, while Δ-recoil affects only τ_2. Thus, it appears that the nucleon recoil provides the bulk of the recoil effect and omission of Δ recoil is a reasonable approximation, particularly considering the enormous amount of computational time required to take account of it. However, we recognize the possibility that isobar recoil could alter our results to some extent. If a nonzero momentum, say, $\vec{p}\,'$, were assigned to the isobar, then there would be an extra recoil energy, $-p'^2/2M_\Delta$, in the energy denominator. Note that the effect would be a further downward shift of the energy at which τ_2 is evaluated. The effect is qualitatively the same as that due to the nucleon recoil and in the same direction. When expressions (5) and (6) are substituted into (3), we find

$$\begin{aligned} T_{\mathrm{D}}^{(3)}+T_{\mathrm{SF}}^{(3)}=&\sum_{\mu,\mu'}\sum_{\alpha,\beta}(-4\pi)^3\int\frac{d^3p}{(2\pi)^9}\int\frac{d^3q}{2\omega_q}\int\frac{d^3q'}{2\omega_{q'}}\\ &\times\Big\langle\phi_\mu(1)\phi_{\mu'}(2)\Big|\Big[\sum_{I',J'}h_{2I',2J'}(\omega-B)\Omega_{2J'}(\vec{k}\,',\vec{q}\,')\Lambda_{2I'}(\lambda,\beta)e^{i(\vec{q}\,'-\vec{k}\,'-\vec{p})\cdot\vec{r}_1}\frac{v(k')v^2(q')}{\omega_{q'}-\omega-B-p^2/2M+i\eta}\\ &\times\sum_{I,J}\tilde{h}_{2I,2J}(\omega-2B-p^2/2M)\Omega_{2J}(\vec{q}\,',\vec{q})\Lambda_{2I}(\beta,\alpha)e^{i(\vec{q}-\vec{q}\,')\cdot\vec{r}_2}\frac{v^2(q)v(k)}{\omega_q-\omega-B-p^2/2M+i\eta}\\ &\times\sum_{I'',J''}h_{2I'',2J''}(\omega-B)\Omega_{2J''}(q,k)\Lambda_{2I''}(\alpha,\lambda)e^{i(\vec{k}-\vec{q}-\vec{p})\cdot\vec{r}_1}\Big]\Big|\phi_\mu(1)\phi_{\mu'}(2)\Big\rangle. \end{aligned} \tag{11}$$

The nucleon recoil effect resides in the $p^2/(2M)$ energy shifts and the $\vec{p}$ integration. In the $M\to\infty$ (fixed scatterer) limit, the $\vec{p}$ integration can be carried out to yield $(2\pi)^3\delta^{(3)}(\vec{r}_1-\vec{r}_1\,')$, which expresses the static nucleon propagation. In the recoil case, both $\vec{p}$ and $\vec{r}\equiv\vec{r}_1-\vec{r}_1\,'$ vector variables must be integrated. For a spin and isospin saturated nucleus, the following simplification holds [see Eq. (54)]:

$$-4\pi\sum_{\mu'}\Big\langle\phi_{\mu'}(2)\Big|\sum_{I,J}\tilde{h}_{2I,2J}(w)\Omega_{2J}(\vec{q}\,',\vec{q})\Lambda_{2I}(\beta,\alpha)\Big|\phi_{\mu'}(2)\Big\rangle=-4\pi\tilde{h}^{(+)}(w)4\rho_0(\vec{r}_2,\vec{r}_2)\delta_{\alpha\beta}\vec{q}\,'\cdot\vec{q}\,. \tag{12}$$

The sums over single particle states have been expressed in term of nuclear density matrices, $\rho_0(\vec{r},\vec{r}\,')$ normalized according to

$$\int d^3r\,\rho_0(\vec{r},\vec{r})=16\,, \tag{13}$$

the mass number for ^{16}O. The isospin sums simplify due to

$$\sum_{\alpha} \Lambda_{2I'}(\lambda,\alpha)\Lambda_{2I''}(\alpha,\lambda) = \delta_{I',I''}\Lambda_{2I'}(\lambda,\lambda)\,. \tag{14}$$

When the remaining spin and isospin sums are carried out, the result simplifies as follows:

$$T_{\rm D}^{(3)} + T_{\rm SF}^{(3)} = v(k)v(k')(2\pi)^{-3}\int d^3p\,[-4\pi\bar{h}^{(+)}(w)]\int d^3r_1\int d^3r_1'\int d^3r_2\, e^{i(\vec{k}\cdot\vec{r}_1 - \vec{k}'\cdot\vec{r}_1')}e^{i\vec{p}\cdot(\vec{r}_1-\vec{r}_1')}$$
$$\times 4\rho_0(\vec{r}_1,\vec{r}_1')4\rho_0(\vec{r}_2,\vec{r}_2)[\mathfrak{F}_{\rm D}(\omega-B)K_{\rm D}(\vec{k}',\vec{k},\vec{r}',\vec{r},w) - \mathfrak{F}_{\rm SF}(\omega-B)K_{\rm SF}(\vec{k}',\vec{k},\vec{r}',\vec{r},w)]\,, \tag{15a}$$

where

$$\vec{r} \equiv \vec{r}_1 - \vec{r}_2,\quad \vec{r}' \equiv \vec{r}_1' - \vec{r}_2,\quad w \equiv \omega - B - p^2/2M\,, \tag{15b}$$

$$K_i(\vec{k}',\vec{k},\vec{r}',\vec{r},w) = \frac{1}{(2\pi)^6}\int\frac{d^3q'}{2\omega_{q'}}\frac{v^2(q')e^{i\vec{q}'\cdot\vec{r}}}{\omega_{q'} - w + i\eta}\int\frac{d^3q\,e^{-i\vec{q}\cdot\vec{r}'}v^2(q)}{2\omega_q(\omega_q - w + i\eta)}N_i(\vec{k}',\vec{k},\vec{q}',\vec{q})\,, \tag{15c}$$

$$N_{\rm D}(\vec{k}',\vec{k},\vec{q}',\vec{q}) = \vec{k}'\cdot\vec{q}'\vec{q}'\cdot\vec{q}\vec{q}\cdot\vec{k}\,, \tag{15d}$$

$$N_{\rm SF}(\vec{k}',\vec{k},\vec{q}',\vec{q}) = \vec{q}'\cdot\vec{q}(\vec{k}'\cdot\vec{q}\vec{q}'\cdot\vec{k} - \vec{q}'\cdot\vec{q}\vec{k}'\cdot\vec{k})\,, \tag{15e}$$

and

$$\mathfrak{F}_{\rm D}(\omega) = \tfrac{1}{3}(-4\pi)^2\{[h_{11}(\omega) + 2h_{13}(\omega)]^2 + 2[h_{31}(\omega) + 2h_{33}(\omega)]^2\}\,, \tag{16a}$$

$$\mathfrak{F}_{\rm SF}(\omega) = \tfrac{1}{3}(-4\pi)^2\{[h_{11}(\omega) - h_{13}(\omega)]^2 + 2[h_{31}(\omega) - h_{33}(\omega)]^2\}\,. \tag{16b}$$

The term $\mathfrak{F}_{\rm D}$ is due to scatterings without spin-flip and it produces the direct contribution, $T_{\rm D}^{(3)}$, in (15). The term $\mathfrak{F}_{\rm SF}$ is due to double spin-flip scatterings and it produces the spin-flip contribution, $T_{\rm SF}^{(3)}$, in (15). In both (16a) and (16b), the possibility of charge exchange in the first and the last scatterings is included. Thus, when we define a pion propagation function by

$$G(r;w) \equiv \frac{1}{(2\pi)^3}\int\frac{d^3q}{2\omega_q}\frac{e^{i\vec{q}\cdot\vec{r}}v^2(q)}{\omega_q - w + i\eta}\,, \tag{17}$$

it is possible to reduce the kernel K to the sum of the following direct and spin-flip terms:

$$K_{\rm D}(\vec{k}',\vec{k},\vec{r}',\vec{r},w) = [\vec{k}'\cdot\vec{\nabla}\vec{\nabla}G(r;w)]\cdot[\vec{k}\cdot\vec{\nabla}'\vec{\nabla}'G(r';w)]\,, \tag{18}$$

$$K_{\rm SF}(\vec{k}',\vec{k},\vec{r}',\vec{r},w) = [\vec{k}\cdot\vec{\nabla}\vec{\nabla}G(\vec{r},w)]\cdot[\vec{k}'\cdot\vec{\nabla}'\vec{\nabla}'G(r',w)] - \vec{k}'\cdot\vec{k}\nabla_i\nabla_jG(r;w)\nabla_i'\nabla_j'G(r';w)\,. \tag{19}$$

Note that our form factor $v(q)$ does not absorb the $(2\omega_q)^{-1/2}$ factor and thus a hard form factor $v(q)$ is used in this paper, but there is no disagreement with "soft" form factors which do absorb the $(2\omega_q)^{-1/2}$ into $v(q)$. Further defining three pion propagation functions as follows:

$$G_0(r;w) \equiv \nabla_r^2G(r;w)\,, \tag{20}$$

$$G_1(r;w) \equiv r^{-1}(d/dr)G(r;w)\,, \tag{21}$$

$$G_2(r;w) \equiv G_0(r;w) - 3G_1(r;w)\,, \tag{22}$$

the direct and spin-flip kernels can rewritten as

$$K_{\rm D}(\vec{k}',\vec{k},\vec{r}',\vec{r},w) = \vec{k}\cdot\vec{k}'G_1(r)G_1(r') + \vec{k}'\cdot\hat{r}'\vec{k}\cdot\hat{r}'G_1(r)G_2(r')$$
$$+ \vec{k}\cdot\hat{r}\vec{k}'\cdot\hat{r}G_2(r)G_1(r') + \vec{k}'\cdot\hat{r}\vec{k}\cdot\hat{r}'\hat{r}\cdot\hat{r}'G_2(r)G_2(r')\,, \tag{23}$$

$$K_{\rm SF}(\vec{k}',\vec{k},\vec{r}',\vec{r},w) = \vec{k}\cdot\vec{k}'G_1(r)G_1(r') + \vec{k}'\cdot\hat{r}'\vec{k}\cdot\hat{r}'G_1(r)G_2(r') + \vec{k}\cdot\hat{r}\vec{k}'\cdot\hat{r}G_2(r)G_1(r')$$
$$+ \vec{k}\cdot\hat{r}'\hat{r}\cdot\hat{r}'\vec{k}'\cdot\hat{r}G_2(r)G_2(r') - \vec{k}\cdot\vec{k}'[3G_1(r)G_1(r') + G_1(r)G_2(r')$$
$$+ G_2(r)G_1(r') + G_2(r)G_2(r')]\,. \tag{24}$$

Here the parametric dependence of G_1 and G_2 on the recoil energy via $w = \omega - p^2/2M - B$ is implicit.

In order to permit some analytical reduction of the propagation functions, it is convenient to express the πN form factor in a monopole form

$$v(q) = (1 + k^2/\alpha^2)/(1 + q^2/\alpha^2)\,, \tag{25}$$

where k is the on-shell momentum. The cutoff parameter α is equal to 940 MeV/c in this work,[14] however, the sensitivity of our results to other values of α is tested (see Fig. 7).

Now employing the identity

$$\frac{1}{2\omega_q(w-\omega_q+i\eta)}=\frac{\theta(w)}{w^2-m_\pi{}^2-q^2+i\eta}+\frac{1}{\pi}\int_0^\infty\frac{dyw}{y^2+w^2}\frac{1}{y^2+m_\pi{}^2+q^2}, \tag{26}$$

we find the pion propagation function can be reduced to a quadrature of an analytic function as follows:

$$G(r;w)=\theta(w)g(r;Q)-\frac{1}{\pi}\int_0^\infty dy\frac{w}{y^2+w^2}g[r;i(y^2+m^2)^{1/2}], \tag{27}$$

where

$$g(r;Q)=\int\frac{d^3q}{(2\pi)^3}\frac{e^{i\vec{q}\cdot\vec{r}}}{Q^2-q^2+i\eta}\frac{(\alpha^2+k^2)^2}{(\alpha^2+q^2)^2} \tag{28a}$$

takes the analytic form

$$g(r;Q)=\frac{(\alpha^2+k^2)^2e^{-\alpha r}}{8\pi\alpha(Q^2+\alpha^2)}-\frac{(\alpha^2+k^2)^2}{4\pi(Q^2+\alpha^2)^2}\left(\frac{e^{iQr}-e^{-\alpha r}}{r}\right). \tag{28b}$$

In similar fashion, the propagation functions G_0, G_1, and G_2 are also expressed in the form of Eq. (27) through use of Eqs. (20)–(22). The explicit forms used in the calculations of this paper involve Eq. (27) with

$$g_0(r;Q)=\frac{\alpha(\alpha^2+k^2)^2e^{-\alpha r}}{8\pi(Q^2+\alpha^2)}[1-2/(\alpha r)]+\frac{(\alpha^2+k^2)^2}{4\pi(Q^2+\alpha^2)^2}\frac{Q^2e^{iQr}+\alpha^2e^{-\alpha r}}{r} \tag{29}$$

and

$$g_1(r;Q)=-\frac{\alpha(\alpha^2+k^2)^2}{8\pi(Q^2+\alpha^2)}\frac{e^{-\alpha r}}{\alpha r}+\frac{(\alpha^2+k^2)^2}{4\pi(Q^2+\alpha^2)^2}\left[\frac{e^{iQr}-e^{-\alpha r}}{r^3}-\frac{iQe^{iQr}+\alpha e^{-\alpha r}}{r^2}\right]. \tag{30}$$

We note in passing that the numerical evaluation of (27) requires some care near y values satisfying $i(y^2+m_\pi{}^2)^{1/2}\sim i\alpha$; however, straightforward analysis shows that the propagation functions $g_0(r;Q)$ and $g_1(r;Q)$ are not singular as $Q\to i\alpha$ or as $r\to 0$.

The next steps in the reduction of (15) to a tractable calculation consist of introducing new variables $\vec{r}\equiv\vec{r}_1-\vec{r}_2$; $\vec{r}'\equiv\vec{r}_1'-\vec{r}_2'$ and carrying out the $\vec{r}_2$ integration using a harmonic oscillator shell model density for ^{16}O. To this end we define

$$D(\vec{r},\vec{r}')=\int d^3r_2e^{i(\vec{k}-\vec{k}')\cdot\vec{r}_2}\rho_0(\vec{r}+\vec{r}_2,\vec{r}'+\vec{r}_2)\rho_0(\vec{r}_2,\vec{r}_2), \tag{31}$$

and consider the shell model density matrix

$$\rho_0(\vec{r}_1,\vec{r}_1')=\frac{4}{\pi^{3/2}b^3}(1+2\vec{r}_1\cdot\vec{r}_1'/b^2)\exp[-\tfrac{1}{2}(r_1{}^2+r_1'^2)/b^2], \tag{32}$$

with $b=1.786$ fm. When the momentum transfer, $\vec{k}-\vec{k}'$, of the pion is neglected, the following expression emerges:

$$D(\vec{r},\vec{r}')=\frac{4\sqrt{2}}{\pi^{3/2}b^3}\exp[-\tfrac{3}{8}(r^2+r'^2)/b^2+\tfrac{1}{4}\vec{r}\cdot\vec{r}'/b^2]$$
$$\times\left\{\left[1+\frac{(\vec{r}+\vec{r}')^2}{8b^2}\right]\left[1-\frac{3}{8}\frac{(\vec{r}+\vec{r}')^2}{b^2}+\frac{2\vec{r}\cdot\vec{r}'}{b^2}\right]+\frac{27}{4}-\frac{5}{8}\frac{(\vec{r}+\vec{r}')^2}{b^2}+\frac{3\vec{r}\cdot\vec{r}'}{b^2}\right\}. \tag{33}$$

Further specializing the analysis to the $\vec{k}=\vec{k}'$ (zero-momentum transfer) limit, it is observed that the scattering matrix element becomes a scalar function of a single vector, $\vec{k}$. The result is invariant with respect to rotations of $\vec{k}$ and we use this fact to simplify the expression by averaging over directions of $\vec{k}$. The results are expressed as follows:

$$T_D^{(3)}=k^2\mathfrak{F}_D(\omega-B)(2\pi^2)^{-1}\int_0^\infty dpp^2\left[-4\pi\tilde{h}^{(+)}\left(\omega-2B-\frac{p^2}{2M}\right)\right]$$
$$\times\int d^3r\int d^3r'j_0(ps)D(\vec{r},\vec{r}')\left(j_0(ks)\mathcal{G}_{D0}(\vec{r},\vec{r}';p)+\frac{j_2(ks)}{s^2}\mathcal{G}_{D2}(\vec{r},\vec{r}';p)\right), \tag{34}$$

$$T_{SF}^{(3)} = k^2 \mathfrak{F}_{SF}(\omega - B)(2\pi^2)^{-1} \int_0^\infty dp p^2 \left[-4\pi\tilde{h}^{(+)}\left(\omega - 2B - \frac{p^2}{2M}\right)\right] \times \int d^3r \int d^3r' j_0(ps) D(\vec{r}, \vec{r}')\left(j_0(ks)\mathcal{G}_{S0}(\vec{r}, \vec{r}'; p) + \frac{j_2(ks)}{s^2}\mathcal{G}_{S2}(\vec{r}, \vec{r}'; p)\right), \quad (35)$$

where

$$\mathcal{G}_{D0}(\vec{r}, \vec{r}'; p) = \tfrac{1}{9}G_0(r)G_0(r') + \tfrac{2}{9}G_2(r)G_2(r')P_2(\cos\theta), \quad (36)$$

$$\begin{aligned}\mathcal{G}_{D2}(\vec{r}, \vec{r}'; p) = &-\tfrac{1}{3}(2r^2 - r'^2)G_2(r)G_1(r') - \tfrac{1}{3}(2r'^2 - r^2)G_1(r)G_2(r') \\ &+ \tfrac{1}{3}rr'\cos\theta[4G_1(r)G_2(r') + 4G_2(r)G_1(r') + 3G_2(r)G_2(r')] \\ &- \cos^2\theta[r^2G_1(r)G_2(r') + r'^2G_2(r)G_1(r') + \tfrac{2}{3}(r^2 + r'^2)G_2(r)G_2(r')] + \tfrac{1}{3}\cos^3\theta\, rr'G_2(r)G_2(r'), \quad (37)\end{aligned}$$

$$\mathcal{G}_{S0}(\vec{r}, \vec{r}'; p) = \tfrac{2}{3}G_0(r)G_0(r'), \quad (38)$$

$$\begin{aligned}\mathcal{G}_{S2}(\vec{r}, \vec{r}'; p) = &-G_0(r)G_2(r')[r^2\cos^2\theta + \tfrac{1}{3}(2r'^2 - r^2) - \tfrac{4}{3}rr'\cos\theta] \\ &-G_2(r)G_0(r')[r'^2\cos^2\theta + \tfrac{1}{3}(2r^2 - r'^2) - \tfrac{4}{3}rr'\cos\theta], \quad (39)\end{aligned}$$

and

$$\cos\theta = \hat{r}\cdot\hat{r}', \quad s = |\vec{r} - \vec{r}'|. \quad (40)$$

The p dependence of the pion propagation functions, G_i, is implicit. The final forms (34) and (35) involve four integrations which are performed numerically, namely, p, r, r', and $\cos\theta$ integrations.

The exchange contribution is given by Eq. (11) with particle labels 1, 2 interchanged in the final (or initial) state. However, this interchange gives rise to a more complicated spin-isospin sum, e.g., (14) no longer applies in a direct fashion. Consider the spin-isospin sums of Eq. (11) with final state particles exchanged in the following condensed notation:

$$\mathrm{SI} = \sum_{\mu,\mu'}\sum_{\alpha,\beta}(-4\pi)^3\left\langle\mu'\left|\sum_{I',J'}h_{2I',2J'}\Omega_{2J'}\Lambda_{2I'}(\lambda,\beta)\sum_{I'',J''}h_{2I'',2J''}\Omega_{2J''}\Lambda_{2I''}(\alpha,\lambda)\right|\mu\right\rangle \times\left\langle\mu\left|\sum_{I,J}\tilde{h}_{2I,2J}\Omega_{2J}\Lambda_{2I}(\beta,\alpha)\right|\mu'\right\rangle. \quad (41)$$

The projectors $\Omega_{2J''}$ and $\Lambda_{2I''}$ depend on spin and isospin operators of nucleon 1, in the notation of (11). Thus they have been commuted with the projectors Ω_{2J} and Λ_{2I} projectors that depend on nucleon 2 operators. The state $|\mu\rangle$, $|\mu'\rangle$ represent spin-isospin nucleon states that are summed for a closed shell nucleus. Thus, (41) involves a trace over nucleon spins and isospins and cannot depend on the external pion isospin projection, λ. If we average over the possible λ values, the spin-iosspin sums are rendered to the form

$$\Gamma = \tfrac{1}{3}\mathrm{Tr}\sum_{\alpha,\beta,\lambda}Q(\lambda,\beta)Q(\alpha,\lambda)\tilde{Q}(\beta,\alpha), \quad (42)$$

where

$$Q(\lambda,\beta) = -4\pi\sum_{I',J'}h_{2I',2J'}\Omega_{2J'}\Lambda_{2I'}(\lambda,\beta), \quad (43)$$

and $\tilde{Q}$ is a similar form involving the amplitudes $\tilde{h}$ in place of h. The projection property of Λ, namely

$$\sum_{\beta}\Lambda_{2I}(\lambda,\beta)\Lambda_{2I'}(\beta,\alpha) = \delta_{I,I'}\Lambda_{2I}(\lambda,\alpha), \quad (44)$$

can be used to evaluate the sums in (42) *provided* we first use the crossing matrix $C_{I',I''}$ to rearrange the order of indices as follows:

$$\begin{aligned}Q(\lambda,\beta) &= -4\pi\sum_{I',J'}h_{2I',2J'}\Omega_{2J'}\sum_{I''}C_{I',I''}\Lambda_{2I''}(\beta,\lambda) \\ &= H_1\Lambda_1(\beta,\lambda) + H_3\Lambda_3(\beta,\lambda), \quad (45a)\end{aligned}$$

where

$$H_{2I} = H_{2I}^{(+)}\vec{k}'\cdot\vec{q}' + H_{2I}^{(-)}i\vec{\sigma}\cdot\vec{k}'\times\vec{q}' \quad (45b)$$

and

$$H_1^{(+)} = -\tfrac{4}{3}\pi(-h_{11} + 4h_{31} - 2h_{13} + 8h_{33}), \quad (45c)$$

$$H_1^{(-)} = -\tfrac{4}{3}\pi(-h_{11} + 4h_{31} + h_{13} - 4h_{33}), \quad (45d)$$

$$H_3^{(+)} = -\tfrac{4}{3}\pi(2h_{11} + h_{31} + 4h_{13} + 2h_{33}), \quad (45e)$$

$$H_3^{(-)} = -\tfrac{4}{3}\pi(2h_{11} + h_{31} - 2h_{13} - h_{33}). \quad (45f)$$

Results similar to (45) hold for the $\tilde{Q}$ as well. In this fashion, we obtain from Eqs. (42), (44), and (45).

$$\Gamma = \tfrac{1}{3}\mathrm{Tr}\sum_{\beta} H_1 H_1 \tilde{H}_1 \Lambda_1(\beta,\beta) + H_3 H_3 \tilde{H}_3 \Lambda_3(\beta,\beta)\,, \quad (46)$$

where $\tilde{H}_1$ and $\tilde{H}_3$ are defined as in (45b) in terms of the $\tilde{h}_{2I,2J}$ pion-nucleon amplitudes with pole terms excised. The remaining trace in (46) is over the spin and it yields

$$\Gamma = \tfrac{4}{3}[\mathfrak{F}_1(\omega,p)\vec{k}'\cdot\vec{q}'\vec{q}'\cdot\vec{q}\vec{q}\cdot\vec{k} + \mathfrak{F}_2(\omega,p)(\vec{k}'\cdot\vec{q}'\,\vec{q}'\cdot\vec{k}q^2 + \vec{k}'\cdot\vec{q}\vec{q}\cdot\vec{k}q'^2) + \mathfrak{F}_3(\omega,p)\vec{k}'\cdot\vec{k}(\vec{q}'\cdot\vec{q})^2]\,, \quad (47a)$$

where

$$\mathfrak{F}_1(\omega,p) = (H_1^{(+)}H_1^{(+)} - H_1^{(-)}H_1^{(-)})\tilde{H}_1^{(+)} + 2(H_3^{(+)}H_3^{(+)} - H_3^{(-)}H_3^{(-)})\tilde{H}_3^{(+)} - 2H_1^{(+)}H_1^{(-)}\tilde{H}_1^{(-)} - 4H_3^{(+)}H_3^{(-)}\tilde{H}_3^{(-)}\,, \quad (47b)$$

$$\mathfrak{F}_2(\omega,p) = H_1^{(+)}H_1^{(-)}\tilde{H}_1^{(-)} + 2H_3^{(+)}H_3^{(-)}\tilde{H}_3^{(-)}\,, \quad (47c)$$

$$\mathfrak{F}_3(\omega,p) = H_1^{(-)}H_1^{(-)}\tilde{H}_1^{(+)} + 2H_3^{(-)}H_3^{(-)}\tilde{H}_3^{(+)}\,. \quad (47d)$$

In Eqs.(47), the amplitudes $H_{21}^{(+)}$ are evaluated via (45) in terms of $h_{2I,2J}(\omega - B)$. The amplitudes $\tilde{H}_{21}^{(+)}$ are evaluated in a similar fashion in terms of $\tilde{h}_{2I,2J}(\omega - 2B - p^2/2M)$, i.e., at the recoil shifted energy.

The remainder of the analysis proceeds in a straightforward fashion, rather similar to the analysis leading up to (34). For brevity, we omit the details and present the results in the following form in the $\vec{k}'=\vec{k}$ limit:

$$T_{\mathrm{EX}}^{(3)} = k^2(2\pi^2)^{-1}\int_0^\infty dp\,p^2\int d^3r\int d^3r'\,j_0(ps)D_{\mathrm{EX}}(\vec{r},\vec{r}')\frac{1}{12}\Big\{\mathfrak{F}_1(\omega,p)\Big[j_0(ks)\mathcal{G}_{D0}(\vec{r},\vec{r}';p) + \frac{j_2(ks)}{s^2}\mathcal{G}_{D2}(\vec{r},\vec{r}';p)\Big] + \mathfrak{F}_2(\omega,p)\Big[j_0(ks)\mathcal{G}_{S0}(\vec{r},\vec{r}';p) + \frac{j_2(ks)}{s^2}\mathcal{G}_{S2}(\vec{r},\vec{r}';p)\Big] + \mathfrak{F}_3(\omega,p)[3j_0(ks)\mathcal{G}_{D0}(\vec{r},\vec{r}';p)]\Big\}\,, \quad (48)$$

where

$$s = |\vec{r} - \vec{r}'|\,,$$

and

$$D_{\mathrm{EX}}(\vec{r},\vec{r}') = \int d^3r_2\,16\rho_0(\vec{r}'+\vec{r}_2,\vec{r}_2)\rho_0(\vec{r}+\vec{r}_2,\vec{r}_2)\,. \quad (49)$$

Employing the shell model density matrix of (32), we find

$$D_{\mathrm{EX}}(\vec{r},\vec{r}') = \frac{4\sqrt{2}}{\pi^{3/2}b^3}\exp[-\tfrac{3}{8}(r^2+r'^2)/b^2 + \tfrac{1}{4}\vec{r}\cdot\vec{r}'/b^2] \times\{[1+\tfrac{1}{8}s^2/b^2 - \tfrac{1}{4}(r^2+r'^2)/b^2]^2 - \tfrac{1}{16}(r^2-r'^2)^2/b^4 + \tfrac{27}{4} - \tfrac{3}{4}(r^2+r'^2)/b^2 + \tfrac{1}{8}s^2/b^2\}\,. \quad (50)$$

Equations (34), (35), and (48) have been numerically evaluated. As the calculation is nontrivial, two entirely independent computer codes were written, each employing different integration variables and numerical methods. The codes were checked against each other and thus a high degree of confidence in placed on the resulting numerical evaluations.

III. RESULTS AND DISCUSSION

The size of the leading order local field corrections,

$$U^{(3)} = T^{(3)} = T_D^{(3)} + T_{\mathrm{SF}}^{(3)} - T_{\mathrm{EX}}^{(3)}\cdots \quad (51)$$

can be discussed best in relation to the first order potential,

$$U^{(1)} = (2\omega_k')^{1/2}\Big\langle 0;\vec{k}',\lambda\Big|\sum_i \tau_i\Big|0;\vec{k},\lambda\Big\rangle(2\omega_k)^{1/2}\cdots\,. \quad (52)$$

We do this by comparing the $\vec{k}=\vec{k}'$ (diagonal) matrix elements $U^{(1)}$ and $U^{(3)}$. For $\pi - {}^{16}\mathrm{O}$ scattering the $\vec{k}=\vec{k}'$ element of the first order optical potential is

$$U^{(1)} = 16k^2[-4\pi h^{(+)}(\omega - B)]\cdots\,, \quad (53)$$

where

$$h^{(+)}(\omega) = \tfrac{1}{3}[h_{11}(\omega) + 2h_{31}(\omega) + 2h_{31}(\omega) + 4h_{33}(\omega)] \cdots . \quad (54)$$

The three parameters of the calculations are the binding shift $B = 21$ MeV, the πN form factor cutoff $\alpha = 940$ MeV/c of Eq. (25) and the ^{16}O oscillator parameter $b = 1.786$ fm of Eqs. (33) and (50).

Figure 2 shows the real and imaginary parts of $U^{(1)}$ and $U^{(3)}$ as functions of the kinetic energy of the pion in the laboratory frame. Below 100 MeV, the local field correction embodied by $U^{(3)}$ is seen to be a modest correction to the first order optical potential. However, in the range 120 to 240 MeV, the local field correction can be nearly as big as $U^{(1)}$. Both the real and imaginary parts of the optical potential are affected.

The local field correction $U^{(3)}$ is composed of direct, spin-slip, and exchange contributions as shown in Fig. 3. A notable coincidence is the near cancellation of spin-flip and exchange contributions. Thus the total $U^{(3)}$ is nearly equal to just the direct contribution $T_D^{(3)}$ of Eq. (34) as is seen by comparing the solid and dashed lines of Fig. 3.

An objective of this work was to test the validity of fixed scatterer estimates of the local field correction. In the fixed scatterer case, the nucleon mass $M \to \infty$, causing the nucleon recoil energy $p^2/(2M) \to 0$. In this event, the local field correction can be much more simply evaluated since the $\vec{p}$ integration produces $\delta^{(3)}(\vec{r}_1 - \vec{r}_1')$, i.e., the recoil distance is zero. For example, the direct contribution from Eq. (34) produces

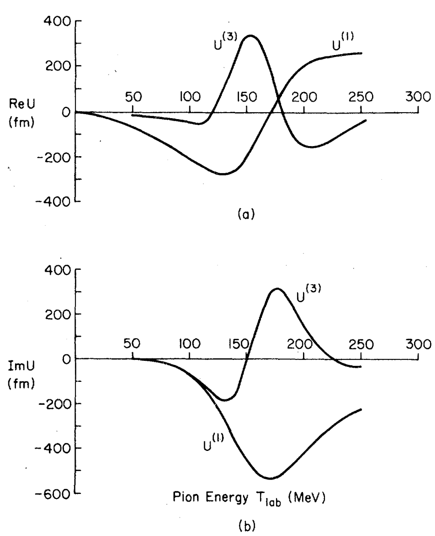

FIG. 2. Real and imaginary parts of the first order optical potential, $U^{(1)}$, and of the local field correction, $U^{(3)}$, both evaluated at $\vec{k} = \vec{k}'$, for π-^{16}O scattering.

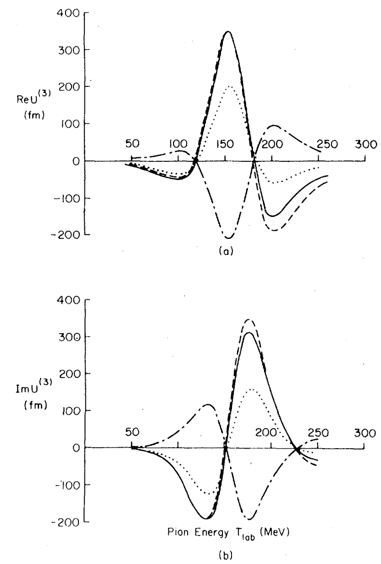

FIG. 3. Contributions to the real and imaginary parts of the local field correction due to direct (— — —), spin-flip (••••••), and exchange (—•—•—) terms for π-^{16}O scattering. The solid line shows the total which is approximately equal to just the direct contribution.

$$U_{\rm FSA}^{(3)} = k^2 \mathfrak{F}_D(\omega - B)[-4\pi \bar{h}^{(+)}(\omega - 2B)] \times \int d^3 r_{12} D(\vec{r}_{12}, \vec{r}_{12}) \mathcal{G}_{D0}(\vec{r}_{12}, \vec{r}_{12}, 0). \quad (55)$$

The function $\mathcal{G}_{D2}$ of (34) vanishes when $r = r'$ and thus does not contribute. Because of rotational invariance, a one-dimensional r_{12} integration suffices to evaluate this expression as opposed to the four-dimensional one needed when recoil is included. In the fixed scatterer calculation, the resonant amplitude $\bar{h}^{(+)}$ is unaffected by recoil energy and the pion Green's functions become free of recoil energy shifts.

The ratio R defined by

$$R \equiv U^{(3)}/U^{(1)} \quad (56)$$

is the complex ratio of local field correction to first order optical potential. The magnitude of the ratio with recoil effects included and for the fixed scatterer approximation (FSA) (55) are compared in Fig. 4. The inclusion of recoil typically reduces the local field correction by a factor 6 to 7. Nevertheless, the peak value $|R| = 0.66$ at 150 MeV shows that the local field correction remains sizeable compared to the first order optical potential. Because the resonant energy dependence of $U^{(1)}$ is divided out in the ratio R, Fig. 4 clearly shows the additional energy dependence due to the leading local field correction. Local field effects are most pronounced near the πN resonance energy and the fixed scat-

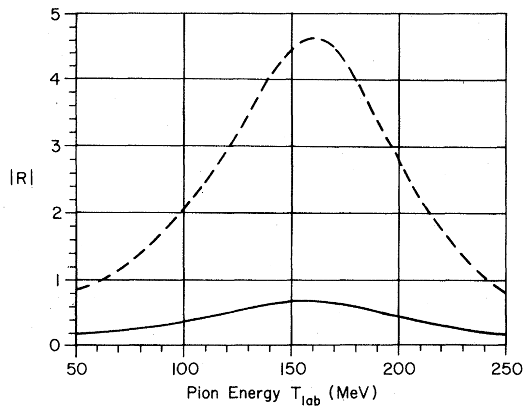

FIG. 4. Magnitude of the ratio R of $U^{(3)}$ to $U^{(1)}$ for π-^{16}O scattering. Solid line shows the recoil calculation and dashed line shows the fixed scatterer calculation.

terer limit is not a resonable approximation to them.

Figure 5 shows the ratio R as a function of M_N/M, where M is a variable nucleon mass and M_N is the physical nucleon mass. Results are shown for the local field correction with variable mass M at 150 MeV pion energy. There are two points to be noted. First, the ratio R drops rapidly in size as one moves away from the static limit, i.e., as M_N/M increases from zero. Second, the real and imaginary parts of R change differently as M_N/M goes from zero to unity, which corresponds to the physical case.

Figure 6 compares the fixed scatterer and recoil calculations as functions of the typical internucleon separations,

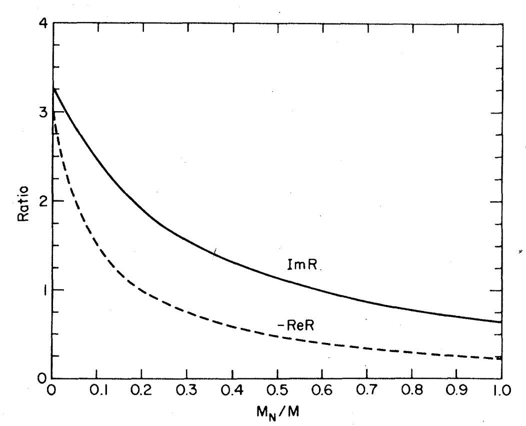

FIG. 5. Real and imaginary parts of ratio R are plotted versus inverse recoil mass, M for 150 MeV π-^{16}O scattering. (M_N is the physical nucleon mass.) Fixed scatterer results ($M_N/M=0$) are approached very slowly as the mass M increases.

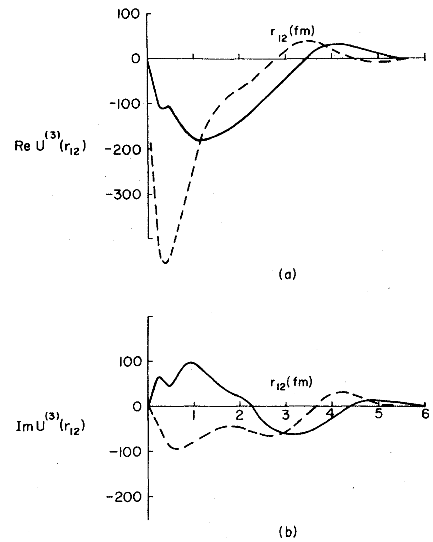

FIG. 6. Contributions to the local field correction, $U^{(3)}$, from different internucleon separations for 150 MeV π-^{16}O scattering. Solid line shows the recoil results and dashed line show one-fifth of the fixed scatterer results.

$$r_{12} \equiv \left| \frac{\vec{r}+\vec{r}\,'}{2} \right| = \left| \frac{\vec{r}_1+\vec{r}_1\,'}{2} - r_2 \right| ,$$

which give rise to the local field correction. The integrand of the r_{12} integration for $U^{(3)}$, other integrations having been done, is plotted versus r_{12} and it is immediately clear that the local field correction is a very long range effect. Most of the local field correction arises from nucleons 1 to 3 fm away from the subject nucleon, with non-negligible contributions arising from nucleons up to 5 fm away (i.e., on the other side of the nucleus). The failure of the fixed scatterer approximation, which is shown at one-fifth actual magnitude is Fig. 6, is most pronounced for small internucleon separations ($\lesssim 1.5$ fm). The recoil effects tend to increase the range of the local field correction.

A simple interpretation of the long range of the local field correction is made in terms of the resonant scattering amplitude mentioned in the Introduction. Pion scattered waves originating at nucleon 1 can freely propagate a substantial distance to a second nucleon, backscatter, and return to scatter once again from nucleon 1 with probability amplitude of order unity. Thus, the interaction of the pion with any nucleon is influenced by essentially all the other nucleons in the nucleus near the πN resonance energy.

It is also apparent that higher order effects will modify the local field correction. In a self-consistent treatment, the pion would not freely propagate large distances without experiencing scattering and absorption through its interaction with the surrounding nucleons.[6] Our results motivate a self-consistent description of the pion-nucleus scattering, however, that is beyond the scope of the present paper.

Because the local field correction is long range, there is not a great sensitivity to short range correlations or to πN form factor cutoffs. The dependence of the ratio R on the πN form factor parameter α of Eq. (25) is shown in Fig. 7. At 150 MeV pion energy, the ratio R reduces from 0.62 to 0.5 when the πN cutoff is reduced from 940 MeV/c to 600 MeV/c. Because of the complex interplay of direct, spin-flip, and exchange contributions, we find that further reduction of α has little effect on the local field correction.

The relative insensitivity to the πN form factor deserves some explanation. If one approximates the pion propagation functions by neglecting the tensor coupling [dropping $G_2(r)$ terms in Eqs. (36) and (37), for example], then the local field correction is found to be much shorter ranged and more sensitive to πN form factor cutoff. The long range of the actual effect is in the tensor coupling term,[15] which accounts for four-fifths of the local field correction. Thus, we find that an angular average over pion directions, such as is customarily used in derivations of the Lorentz-Lorenz correction, fails in the local field correction since it omits the tensor coupling.

Figure 8 shows the dependence of the local field correction on the recoil momentum variable, p, which is the momentum of the plane wave excited states. The integrand of the p integration for 150 MeV pion energy is plotted versus momentum p including direct, spin-flip, and exchange contributions and with all other integrations carried out. Owing to recoil effects, the contributions typically arise from excited nucleon states of momentum 0.5 to 3 fm^{-1}. We have tested the sensitivity of the calculation to Pauli exclusion of nucleon momenta in the range 0 to 1.4 fm^{-1} by excluding that range of p integration. As Fig. 8 suggests, it is found that the imaginary part of $U^{(3)}$ increases in magnitude while the real part of $U^{(3)}$ decreases in magnitude. At 150 MeV pion laboratory energy, the absolute magnitude of the ratio R does not change much due to Pauli blocking.

Figure 9 shows that the excited nucleon typically recoils a distance 0.5 to 1 fm as opposed to the zero recoil distance of the fixed scatterer approximation. Thus, it is seen that the local field correction mainly arises from backward angle scatterings in the intermediate nucleon interaction and the usual interpretation in terms of multiple reflections is verified.

Because the local field corrections substantially change the imaginary part of the optical

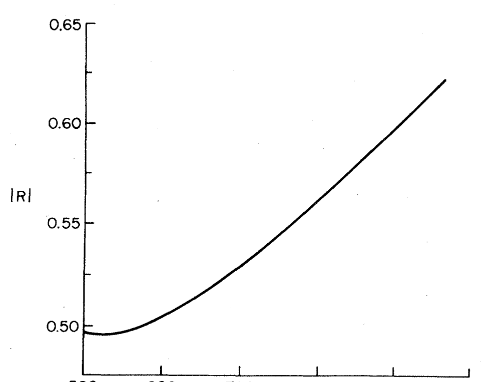

FIG. 7. Dependence of the ratio R on the πN off-shell form factor parameter [see Eq. (25)] for 150 MeV π-^{16}O scattering.

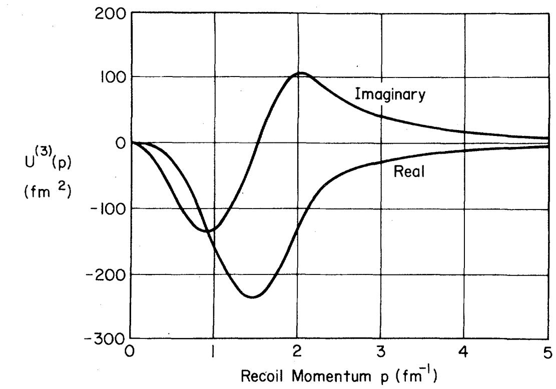

FIG. 8. Dependence of the local field correction on the nucleon recoil momentum p for 150 MeV π-^{16}O scattering.

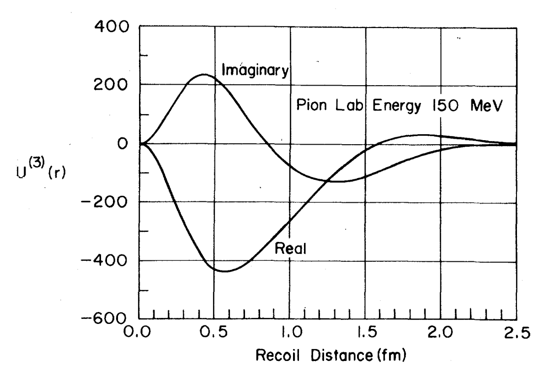

FIG. 9. Dependence of the local field correction on the nucleon recoil distance, $|\vec{r}_1 - \vec{r}_1'|$, for 150 MeV π-^{16}O scattering.

potential, it is important to identify the physical processes which are responsible. The reactive content of the optical potential is considered along the lines of the work of Tandy, Redish, and Bollé.[16] The discontinuity operator

$$\Delta U(\omega) \equiv U(\omega + i\epsilon) - U(\omega - i\epsilon) \qquad (57)$$

is useful in this regard since

$$\lim_{\epsilon \to 0} \Delta U(\omega) = 2i\,\mathrm{Im}U(\omega)\,. \qquad (58)$$

Multiple scattering expansions generally involve operator strings of the form

$$\Gamma = ABABA\cdots, \qquad (59)$$

where $A \equiv A(\omega + i\epsilon)$ and $B \equiv B(\omega + i\epsilon)$ depend on the energy variable ω and ϵ is an infinitesimal imaginary part. The discontinuity of such operator strings is given by the following theorem:

$$\Delta\Gamma = (\Delta A)BABA\cdots + A^*(\Delta B)ABA\cdots + A^*B^*(\Delta A)BA\cdots + A^*B^*A^*(\Delta B)A\cdots + A^*B^*A^*B^*(\Delta A)\cdots + \cdots, \qquad (60)$$

where $A^* \equiv A(\omega - i\epsilon)$ and $B^* = B(\omega - i\epsilon)$. A deductive proof can readily be constructed. The discontinuity operator Δ obeys a chain rule with the exception that it conjugates the complex energy variable of all operators to its left.

The imaginary part of the triple scattering optical potential $U^{(3)}$, or of the T matrix $T^{(3)}$ of Eq. (1), arises from five separate discontinuities as follows:

$$\Delta U^{(3)} = \langle 0;\vec{k},\lambda | [\Delta\tau_1 G\tau_2 G\tau_1 + \tau_1^*\Delta G\tau_2 G\tau_1 + \tau_1^* G^*\Delta\tau_2 G\tau_1 + \tau_1^* G^*\tau_2^*\Delta G\tau_1 + \tau_1^* G^*\tau_2^* G^*\Delta\tau_1] | 0;\vec{k},\lambda\rangle\,. \qquad (61)$$

Figure 10 indicates the five cuts corresponding to this equation. The notation τ_1^* designates $\tau_1(\omega - i\eta)$, which differs from the Hermitian conjugate of $\tau_1(\omega + i\eta)$ due to the spin and isospin projections of Eqs. (5) and (6). When the spin and isospin sums are evaluated as in Sec. II, the various contributions to the discontinuity, $\Delta U^{(3)}$, can be shown to involve Hermitian forms, however, those details are omitted. For the present, we note that the third term of Eq. (61) represents the cut corresponding to a nucleon, a Δ and two hole states in the residual nucleus in the intermediate state. Because the Δ mass is greater than $M_N + m_\pi$, this discontinuity corresponds to production of πNN intermediate states and must contribute a negative imaginary part to the optical potential. Note that omission of the pole term from τ_2 in this work means that the true pion absorption cut corresponding to NN intermediate

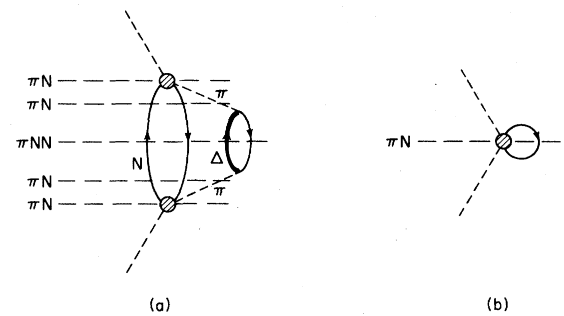

FIG. 10. Absorptive cuts of the local field correction (a) and the first order optical potential (b). The imaginary part of $U^{(1)}$ arises from plane wave impulse approximation of the quasifree knockout process ($\pi, \pi N$). The four πN cuts of the local field correction serve to modify the absorption of the first order optical potential by inclusion of initial and final state distortions to the quasifree knockout.

states is not present. A complete analysis, of course, must include the true absorption; however, the present focus is on the local field effect which we logically consider as a separate issue.

The remaining four cuts of Fig. 10 and Eq. (61) correspond to intermediate πN states. These cuts are corrections to the absorptive cut of the first order optical potential,[17]

$$U^{(1)} = A\langle 0;k,\lambda | \Delta\tau_1 | 0;k,\lambda\rangle\,,$$

which also corresponds to πN intermediate states as indicated in Fig. 10. However, the nucleon knockout to πN intermediate states is overestimated in the first order optical potential because its discontinuity corresponds to plane wave knockout. The corrections to this discontinuity present in $\Delta U^{(3)}$ are distortion effects of the medium on the outgoing pion which generally cause a reduction of the πN reaction cross section. For this reason, the πN distortion cuts of $U^{(3)}$ tend to be emissive in that they act to reduce to absorption present in $U^{(1)}$. The imaginary part of $U^{(3)}$ can thus be either positive or negative depending on the predominance of the πN or πNN cuts.

Figure 11 shows a comparison of the various contributions to the imaginary part of the optical potential as functions of the pion energy. The imaginary part of $U^{(3)}$ is seen to consist of a large emissive part due to πN cuts and an absorptive part due to the πNN cut. The figure reemphasizes the slow convergence of the multiple scattering expansion of the optical potential in the vicinity of the πN resonance energy. It should be noted, however, that Pauli exclusion effects that are not included here will reduce the absorptive parts significantly.

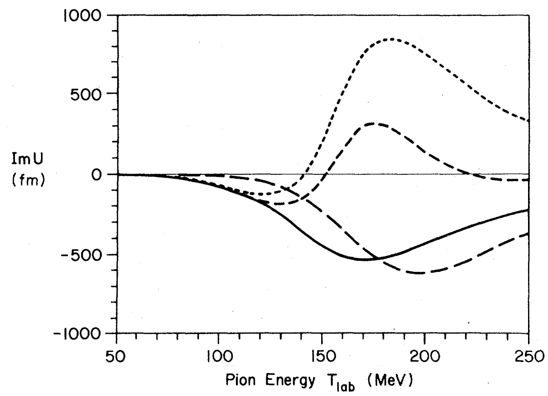

FIG. 11. Imaginary part of first order optical potential $\mathrm{Im}U^{(1)}$ (——) is compared to the imaginary part of the local field correction $\mathrm{Im}U^{(3)}$ (– – – –). The πNN cut contribution to $\mathrm{Im}U^{(3)}$ (— — —) is purely absorptive, but is conterbalanced by the sum of the four πN cut contributions (- - - -), which tend to be emissive.

IV. CONCLUSIONS

The leading order local field corrections in pion-nucleus scattering are quite large near the πN resonant energy. The conventional multiple scattering theories thus converge, at best, quite slowly. The previous work on the local field corrections employing the fixed scatterer approximation seriously overestimates the effect; however, the present analysis including recoil effects basically points to the same conclusion. The pion-nucleus interaction involves very high order multiple scattering processes which may be better organized into the theory if one insists on a self-consistent optical potential theory in which the pion propagation between scatterings is modified by the effects of the medium. Inclusion of the pion absorption mechanism is not likely to alter this conclusion.

Recent papers by Oset and Weise[18] considered several higher order corrections to the optical potential in an isobar-hole model. The isobar self-energy in the nucleus was shown to be substantially altered by the isobar-nucleus Hartree potential, by Pauli blocking effects, and by pion absorption and reflection contributions. Of these effects, the pion reflection contribution to the isobar self-energy is the agent for the local field correction considered in this paper. Many differences in the approaches deter any simple numerical comparison of our results for the local field correction with the isobar self-energy results of Ref. 18. However, several remarks are in order. An important conclusion of Ref. 18 was that partial cancellation between various corrections takes place in the calculation of total cross sections for π-nucleus scattering. Furthermore, the results show that all of the above-mentioned medium corrections are large and they must be incorporated in an equal footing. Our results for the local field correction also argue strongly for its inclusion into microscopic theories of π-nucleus scattering.

We further emphasize that the local field effect should be incorporated in all orders and in a self-consistent manner. Incorporation in all orders requires that all internal pion propagators contain the pion optical potential. Self-consistency requires that the π-nucleon t matrix, so modified, be able to reproduce the pion optical potential in the relevant energy range. This conclusion is based on the very large, and opposite in sign, contributions to the imaginary part of the local-field correction discussed in relation to Figure 11. The quasifree reactive content of the optical potential is incorrect if the intermediate state pions are not distorted by the optical potential.

Because the pion-nucleus elastic scattering is nearly black-disk scattering near the πN resonance energy, the inclusion of local field corrections is not expected to induce dramatic changes in forward angle elastic scattering predictions. Back angle scattering is expected to be sensitive to the local field correction. However, the chief importance of the local field corrections may lie in their implications for distorted wave calculations for reactions. Local field corrections tend to increase the nuclear transparency. The anomaly in predictions of single pion, charge-exchange cross sections[19] when standard optical potentials are used in DWIA calculations may be an experimental indication of increased transparency of the nucleus to pions. A second pion application which may be sensitive to knowledge of the local field correction is the extraction of matter radii from pion-nucleus elastic scattering. Although the data very precisely determine a strong absorption radius for "the optical potential," it is a difficult problem to relate this information to matter radii using a microscopic theory of pion multiple scattering. Local field corrections are expected to further complicate such attempts.

As noted previously, the predominance of the first order optical potential implies that the pion-nucleus reaction cross section is strongly dominated by quasifree knockout of nucleons which is a single nucleon process. The presence of large local field corrections suggests increased importance of multinucleon processes. As shown by Morris *et al.*,[20] there are data which support this view. The ratio of nucleon knockout by π^+ and π^- beams, when the nucleus is left in a definite final state, is found to differ significantly from prediction based on quasifree scattering.

A principal conclusion of this work is that pion multiple scattering requires a self-consistent treatment of the pion self-energy in the nucleus. It is reasonable to expect that local field correction will be reduced by the self-consistency as has already been shown in the fixed scatterer case by Johnson and Bethe.[7]

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