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Interpretation of quasielastic pion scattering coincidence experiments

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The energy dependence of the *np* charge exchange cross section, the angular dependence of the free $\pi^{\pm}p$ cross section ratio, and nucleon "absorption" due to quasielastic scattering are significant in predicting $(\pi^{\pm},\pi^{\pm}N)$ ratios. When these effects are taken into account, recent $(\pi,\pi N)$ coincidence data can be understood in the impulse approximation picture, with the recoil nucleon subject to charge exchange on leaving the nucleus.

[NUCLEAR REACTIONS Ratios of $\sigma(\pi,\pi N)$ when final π and/or N observed: ^{12}C, ^{27}Al, ^{208}Pb at $T_{\pi}=180$, 190, 255 MeV. Differences from radiochemical experiments.]

I. INTRODUCTION

The quasielastic scattering of pions by nucleons bound in nuclei is interesting for several reasons. It offers a way to study the πN amplitude in the nuclear medium,[1] complements $(e,e'p)$ and $(p,2p)$ studies of single particle energies, and provides a potential probe of the nuclear surface. With the pion intensities now available at the meson factories, it is feasible to do $(\pi,\pi N)$ coincidence experiments, and the first such experiments have recently been completed.[2-4] Here we will show that simple models of quasielastic scattering are in qualitative agreement with observations when the kinematics is appropriate.

In $(\pi,\pi N)$ experiments π^+/π^- ratios should be predictable if the scattering is indeed quasielastic, i.e., almost the same as free πN scattering. In the impulse approximation (IA), these ratios are trivially related to the free πN cross section ratios. Radiochemistry neutron removal ratios,[5] however, differ substantially from the IA predictions. This can be explained with a simple, semiclassical model[6] which modifies the π^-/π^+ total knockout cross section from the IA value of 3 at the (3,3) resonance. If P is the probability the outgoing nucleon undergoes charge exchange on leaving the nucleus, then[6]

$$R_n=\frac{\sigma(\pi^-,\pi^-n)}{\sigma(\pi^+,\pi^+n+\pi^0p)}=\frac{9-8P}{3+6P}\,. \tag{1}$$

This formula and its generalization to energies away from the (3,3) resonance have been quite successful in explaining the departure of R_n from the IA for all light nuclei studied[5,7,8] with not unreasonable values of the charge exchange probability ($P\simeq 20\%$).

Subsequently, this model, developed originally for radiochemical measurements which detect the presence of the residual nucleus, was extended to apply to counter experiments which detect the outgoing nucleon[9] or, in coincidence, both the pion and nucleon.[10] In this latter case, let $P_{\pm}$ be the probability that a neutron struck by a $\pi^{\pm}$ will emerge as a proton, and let $Q_{\pm}$ be the probability a struck proton will emerge as a proton. Because the incoming and outgoing fluxes are attenuated by quasielastic scattering, $(P_{\pm}+Q_{\pm})<1$. Then, for example, the proton knockout ratio appropriate for a coincidence experiment is

$$R_p=\frac{\sigma(\pi^+,\pi^+p)}{\sigma(\pi^-,\pi^-p)}=\frac{ZrQ_++NP_+}{ZQ_-+NrP_-}\,, \tag{2}$$

where r is the ratio of the π^+p to π^-p free cross sections. For an $N=Z$ nucleus, $P_+=P_-\equiv P$, $Q_+=Q_-\equiv Q$. If, near the resonance, $r=9$, then Eq. (2) would reduce in this case to

$$R_p=\frac{9+x}{1+9x}\simeq\frac{9-8P}{1+8P}\,,\qquad x=P/Q\,, \tag{3}$$

where the last approximate equality would hold only if $Q\simeq 1-P$. We give that form of the equation only to show the connection with Eq. (1). The denominators differ because the πN charge exchange process does not contribute when an outgoing charged pion is detected.

To calculate the P's and Q's in our semiclassical model, we assume that particles travel in straight line paths except at the point where the πN collision occurs. The assumption of incoherence then leads to simple differential equations for the particle fluxes. Their solution gives the P's and Q's as definite integrals over the nuclear volume. Details are given in Refs. 4, 9, and 10. This procedure is a semiclassical (and far simpler) representation of the distortion of the scattering waves that would be calculated in a distorted-wave impulse approximation (DWIA) approach—such as that outlined in Ref. 1—modified to include an isospin-dependent nucleon optical potential to account for the nucleon charge exchange.

Naive application of Eq. (2) to the recent coincidence data is dangerous, however, and that is the main thrust of this paper. There are a number of points of difference, not well recognized, between coincidence and radiochemistry experiments. We discuss these in Sec. II. With these differences in mind, we then go on to discuss each of the three new experiments[2-4] in some detail in Sec. III. A short summary concludes the paper.

II. DIFFERENCES BETWEEN COINCIDENCE AND RADIOCHEMICAL EXPERIMENTS

If we wish to apply Eq. (1) to the new experiments, we must take note of four simple but essential points which have been somewhat overlooked:

1. Kinematic limitations. If the struck nucleon does not recoil with an energy considerably larger than the minimum removal energy, multiple-step processes will swamp quasielastic scattering. (The NN cross section is large at very low energies.) Also, the Fermi motion of the struck nucleons will always smear the energies of the emerging particles. Hence the detectors must be able to observe particles with energies above and below the nominal quasielastic values.

2. Angular dependence of r. Unlike radiochemistry experiments, counter experiments observe outgoing pions and nucleons emerging at specific angles. Thus one should use *differential* cross section ratios for r. Despite the dominance of the (3,3) phase shift in the total cross sections over a large energy range, interference with other partial waves makes $r(\theta_\pi)$ strongly dependent on the scattering angle. In Fig. 1 we have plotted $r(\theta_\pi)$ for a number of incident pion energies, using a convenient energy-dependent representation of the πN experimental data.[11] There are big deviations from the naive ratio $r=9$ even at energies quite near the (3,3) resonance. Clearly (3,3) dominance is a good approximation for the total cross section ratio for a wide range of energies, but a bad approximation for the differential cross section ratio at all energies.

3. Energy dependence. Since the nucleon charge exchange cross section $\sigma_{N,\text{ex}}$ varies rapidly with

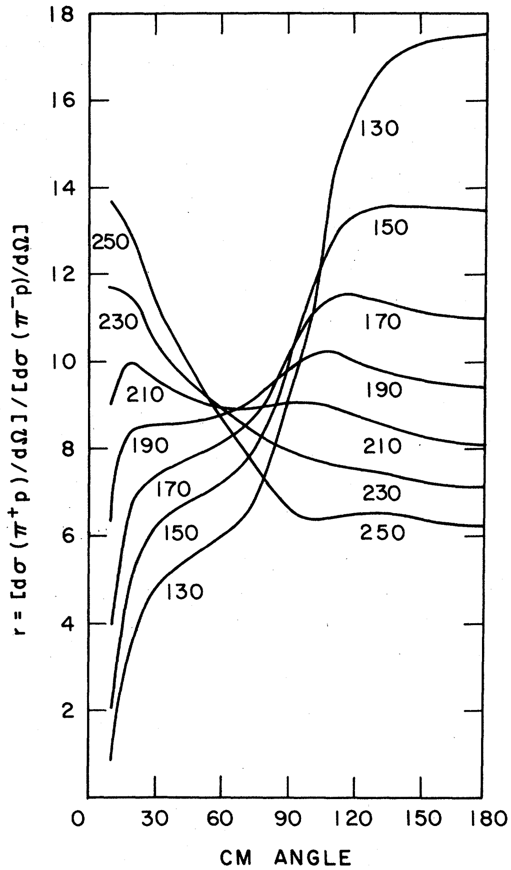

FIG. 1. The ratio of π^+p and π^-p differential cross sections as a function of the center of mass scattering angle. The laboratory energy in MeV is given next to each curve. The corresponding total cross section ratios change gradually from 9.8 at 130 MeV to 7.9 at 250 MeV.

energy, so does P. For small values of $\sigma_{N,\text{ex}}$,[6]

$$x \equiv P/Q \simeq (Z/A)\sigma_{N,\text{ex}}\rho d$$
$$\simeq \beta_N (Z/A) T_N^{-1.9} \rho d\,. \qquad (4)$$

Here d is the mean distance traveled by the nucleon in the nucleus, ρ is the average nuclear density normalized to $\int \rho d\tau = A$, and $\beta_N = 7.06 \times 10^4$ mb $\times$ (MeV)$^{1.9}$ is a parameter representing the somewhat poorly determined Pauli exclusion principle inhibition on the charge exchange process. Its value was determined by fitting (within a restricted range) the ratio R_n in the ^{12}C radiochemistry experiment[5] at 180 MeV. The energy dependence of x (or P) is crucial in understanding the energy variation of R_n in that and other experiments.[5-7] For a coincidence experiment, which has the advantage of a better determination of the nucleon energy T_N, it is also important to take this energy dependence of x into account when comparing predictions with experiments.

4. Absorption effects. As noted above, quasielastic scattering reduces the fluxes. Nucleon "absorption" has a particularly large effect on the magnitude of R_p and on its A dependence. If the nucleon absorption cross section $\sigma_{N,\text{abs}}$ is large, d is essentially independent of A, while Z/A decreases with A; hence, by Eq. (4), P also *decreases* with A. On the other hand, if $\sigma_{N,\text{abs}} \simeq 0$, d scales with $A^{1/3}$, and P *increases* with A.

There are some ambiguities regarding this last point which deserve further comment. It is not obvious, for instance, what to use for $\sigma_{N,\text{abs}}$, since small angle scattering is inhibited by the Pauli principle, and large angle np scattering is already counted as charge exchange. In coincidence experiments, absorption refers to scatterings which deflect the outgoing particles from the detectors, i.e., which appreciably increase the angular spread of the outgoing flux over that due to Fermi motion. We elected to write $\sigma_{N,\text{ex}}$ as the integral of the Pauli corrected np cross section from a minimum angle $\pi - \theta_m$ to π, and took over from the radiochemistry experiments the fitted value of β_N. (One could argue for a smaller value.) This gave us a value of θ_m at each nucleon energy T_N. We then integrated the average of the pp and np cross sections from θ_m to $\pi - \theta_m$ to obtain $\sigma_{N,\text{abs}}$. We found a nearly constant $\sigma_{N,\text{abs}} = 35$ mb for T_N between 60 and 90 MeV, the energy range of interest.

In a radiochemistry experiment, on the other hand, nucleon absorption corresponds to NN scatterings which eject an additional nucleon. The energy of an outgoing nucleon is estimated[6] to average about $T_\pi/3$, but it varies over a large range. The slow nucleons, for which the nucleon charge exchange process is very large, have little or no phase space for ejection of a second nucleon. Thus $\sigma_{N,\text{abs}}$ is probably significantly less in a radiochemistry experiment as compared with a coincidence experiment. In Ref. 6 we made the convenient, albeit arbitrary, assumption that $\sigma_{N,\text{abs}} = 0$. As indicated in the previous paragraph, such a choice would be inappropriate for a coincidence experiment.

To conclude these remarks, we note that a DWIA description of the $(\pi, \pi N)$ process using an isospin dependent optical potential would account for nucleon "absorption" in a self-consistent way. However, we now believe that a Lane-type potential (having a term $\vec{\tau}_\pi \cdot \vec{T}$), such as was used by Hewson,[6] would probably not be sufficient, since *non*isobaric analog charge exchange transitions appear to be very important.[12]

III. DISCUSSION OF THREE RECENT COINCIDENCE EXPERIMENTS

A. $^{12}C(\pi^\pm, \pi^\pm p)^{11}B$ with energy resolution

The Virginia-SIN experiment[2] used 180 MeV π^+ and π^- incident on a ^{12}C target. Pions at 100° and 110° were detected in coincidence with protons at 30°. The proton spectrum was centered near 60 MeV, the free proton value of 76 MeV less the 16 MeV proton removal energy. Thus the kinematic requirements (point 1 of Sec. II) are satisfied. Using *all* events, these authors obtained $R_p^A(\text{exp}) = 5.27 \pm 0.45$. (We have taken a weighted average of the data at the two angles.) They note this is less than the IA value obtained from the free πp total cross sections, $R_p(\text{IA}) = r = 9$, but more than the 2.7 they anticipated from Eq. (3), using $r = 9$ and a value of x appropriate to the ^{12}C radiochemistry results.[5]

However, from Fig. 1, for pions which scatter through 100° or 110° in the lab (115° or 125° in the πN c.m. frame), r is 11 at 180 MeV (point 2). Furthermore, the spectrum shows that the average total energy of the final πN system is less than that of the incoming pion by about 35 MeV, suggesting the need to extrapolate the πN cross sections off-shell. Since the final pion and nucleon energies are measured, it is reasonable to evaluate the cross sections using the "final energy prescription", i.e., at $180 - 35 = 145$ MeV; then $r = 14$. Also, as mentioned above, in a radiochemistry experiment $\langle T_N \rangle \simeq T_\pi/3$, or 60 MeV in the present case. This is somewhat less than the 76 MeV peak quasielastic energy (point 3). Scaling the radiochemistry result according to Eq. (4) gives somewhat larger predictions, $R_p = 3.6$ ($r = 9$) or $R_p = 4.2$ ($r = 14$).

To study the effects of absorption (point 4), we must go beyond scaling arguments and calculate P and Q. We did this in our semiclassical model using the code QUASEX written by Varghese.[10] With $r = 14$ and $\sigma_{N,\text{abs}} = 35$ mb, we obtained $R_p^A(\text{calc}) = 6.0$, in good agreement with $R_p^A(\text{exp})$.

Ellis *et al.* also reported separate values of R_p for those events leaving the residual nucleus in relatively low states of excitation (<10 MeV) or in higher energy states:

$$R_p^L(\text{exp}) = 14.0 \pm 2.6, \quad R_p^H(\text{exp}) = 2.80 \pm 0.35 .$$

The first of these numbers is surprisingly large. However, proton energies in the R_p^L data average about 15 MeV higher than in R_p^A, and energies in R_p^H average 15 MeV lower. With nucleon energies of 76 ± 15 MeV, we find using QUASEX,

$$R_p^L(\text{calc}) = 7.1, \quad R_p^H(\text{calc}) = 4.8 .$$

Our result for R_p^H is reasonable, but R_p^L is still too small. Note that even a small value of P can greatly reduce the ratio R_p from $r=14$, since the product rP occurs in the denominator of Eq. (2).

B. $(\pi^{\pm},\pi^{\pm}p)$ on ^{27}Al in coincidence with nuclear γ rays

The Utah-Florida-LASL experiment[3] involved 190 MeV $\pi^{\pm}$ incident on ^{27}Al, and detected pions or protons at 35° in coincidence with a gamma ray. Free kinematics yields a proton energy at this angle of 74 MeV, very close to that in Ellis *et al.*[2] The observed proton spectrum is centered slightly *above* this energy, the part below 40 MeV being cut off. Thus the kinematic conditions for quasielastic scattering are moderately well satisfied when the *proton* is detected. For a proton in coincidence with the ^{26}Mg(I$\to$0) gamma ray, $R_p(\text{exp})=5.0\pm1.0$. Using $r=11$ and $\sigma_{N,\text{abs}}=35$ mb, we obtained with QUASEX $R_p=5.3$, which agrees very well.

When a *pion* is detected at 35°, free kinematics implies a nucleon recoil energy of only 15 MeV, not much more than the binding energy. Hence the assumption of quasielastic scattering is likely to fail (point 1). Here coincidence with ^{26}Mg(I$\to$0) gamma rays gives $R_p(\text{exp})=0.93\pm0.14$, which would require in Eq. (1) that $P/Q\simeq1$, a rather extreme value. By contrast, the π^-/π^+ neutron removal ratio obtained with coincidence with ^{26}Al(II$\to$0) gamma rays is $R_n(\text{exp})=3.23\pm0.82$, which would require a quite different P/Q, if the quasielastic scattering assumption and Eq. (2) were valid.

Further internal evidence exists for the nonquasielastic nature of the π-coincidence data. The ratio of the laboratory frame free π^+p cross section for a π^+ scattered at 35° to that for a proton recoiling at 35° (the π^+ going off at 95°) is 3.5. In coincidence with the ^{26}Mg(I$\to$0) gammas, the ratio of the 35° π^+ and p cross sections is 0.38 $\pm$ 0.04, an order of magnitude smaller. To make another comparison, let $\sigma_p(\pi^+)$ be the cross section for proton removal when a π^+ is detected, etc. From the ^{26}Mg and ^{26}Al data, $\sigma_p(\pi^+)/\sigma_n(\pi^-)=5.1\pm0.7$, which is comparable to the spectroscopic factor ratio[13]; however, $\sigma_p(\pi^-)/\sigma_n(\pi^+)=18.3\pm4.7$. In the impulse approximation, neglecting Coulomb effects, both ratios are the same.

Clearly the impulse approximation and quasielastic scattering are not relevant when pions are detected at such forward angles. Similar conclusions have been reached concerning small angle (π^+,π^0) data.[14]

C. ^{27}Al$(\pi^{\pm},\pi^{\pm}p)X$ and ^{208}Pb$(\pi^{\pm},\pi^{\pm}p)X$ above the resonance

The Oregon State-Oregon-LASL-Florida-Texas experiment[4] had 255 MeV π^+ and π^- incident on ^{27}Al and ^{208}Pb targets. In their paper, they also report a preliminary ^{12}C result from another collaboration. For protons at 55° in coincidence with pions at 50°, they give

$$R_p(C)=7.9\pm1.0,\quad R_p(\text{Al})=6.9\pm0.7),$$
$$R_p(\text{Pb})=4.6\pm0.5\,.$$

Similar results were obtained with $\theta_p=64°$ and $\theta_\pi=37°$ for Al and Pb.

Using QUASEX, they were able to reproduce the measured R_p values if they assumed $\sigma_{N,\text{abs}}=0$ and an outgoing proton momentum of 650 MeV/c (204 MeV). However, if one uses the quasifree proton energies (50 MeV at $\theta_p=55°$, 28 MeV at 64°), the calculated ratios are much too low, since $\sigma_{N,\text{ex}}$ and hence P/Q increase by large factors. Furthermore, with reasonable values of $\sigma_{N,\text{abs}}$, the A dependence reverses, with R_p smallest for carbon. Thus, with $r=9$ (seen from Fig. 1 to correspond to this kinematical situation) and with $\sigma_{N,\text{abs}}=35$ mb we obtained for protons at 55°,

$$R_p(C)=4.2,\quad R_p(\text{Al})=4.2,\quad R_p(\text{Pb})=4.5\,.$$

One possible explanation for this disagreement between the calculations and the data is provided by the published spectra for Al. They start at 50 MeV and have tails extending beyond 125 MeV. Allowing for the 8 MeV binding energies, the quasielastic peaks should appear at about 42 and 20 MeV, both below the experimental cutoff. On the other hand, the fastest protons detected can be produced in quasielastic collisions only if the struck proton has a momentum greater than the Fermi momentum. Thus most of the quasielastic events are not observed, while some events far from the quasielastic peak are. Perhaps we should not be too surprised at the inapplicability of the quasielastic picture for this choice of kinematics and geometry.

IV. CONCLUSION

To summarize, we see that the energy dependence of the nucleon charge exchange cross section, the angular dependence of the impulse approximation ratio, and the absorption of nucleons due to quasielastic processes are all significant in predicting $(\pi,\pi N)$ ratios. Using plausible but uncertain estimates of various parameters, we can account for most of the data involving nucleons with enough energy for quasielastic scattering to apply. An exception is R_p^L of Ref. 2. As in the data of Ref. 4, this ratio involves protons with energies extending well above the nominal quasielastic value. Even with this taken into account, however, the ratio preducted by Eq. (1) is substantially smaller than observed. A large correction to the free πN amplitude may well be required to achieve agreement with the experimental data, but based on the work of Ref. 1, a large off-shell correction is not expected.

The general features of quasielastic $(\pi, \pi N)$ scattering appear to be understood, but the four points discussed here can only be sorted out by experiments with good energy resolution covering more angles and incident energies. It is also desirable now to carry out quantum mechanical calculations using DWBA codes which include pn charge exchange potentials.

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