PHYSICAL REVIEW C VOLUME 21, NUMBER 5 MAY 1980

Relativistic description of πd elastic scattering in the (3,3) resonance region

N. Giraud, C. Fayard, and G. H. Lamot

Institut de Physique Nucléaire (et IN2P3), Université Claude Bernard Lyon-I, 43, Bd du 11 Novembre 1918 - 69622 Villeurbanne, Cedex, France

(Received 28 December 1979)

The πd elastic scattering observables are calculated in the 142 to 256 MeV energy range within a relativistic three-body theory without π absorption. The nonresonant πN partial waves are included perturbatively to an excellent degree of accuracy. Various NN tensor forces are elaborated at different levels of quality. The sensitivity of elastic differential cross sections and polarization parameters to the description of the NN and πN channels is investigated. The importance of using a realistic deuteron wave function is demonstrated.

[NUCLEAR REACTIONS πd elastic scattering, $T_\pi = 140-260$ MeV, $d\sigma/d\Omega$, total cross section, vector and tensor polarizations, Faddeev calculation, relativistic NN tensor force.]

I. INTRODUCTION

Recently,[1] we have performed detailed calculations of πd elastic scattering at 142 and 180 MeV, based on the nonrelativistic three-body theory of Thomas,[2] with relativistic kinematics for the pion only (RPK theory). Various NN tensor forces were used, and we have considered either the P_{33} scheme, where only the P_{33} πN channel was retained, or the case where the "small" S and P πN partial waves were included in an exact manner, denoted as the SP scheme. The main conclusions we have drawn are the following:

(i) The SP scheme lowers the differential cross section $d\sigma/d\Omega$ at backward angles, thus improving the agreement with experimental data, and changes completely the structure of the vector polarization it_{11}, while the tensor polarization t_{20} is only slightly affected.

(ii) The use of a ${}^3S_1$-${}^3D_1$ parametrization giving a "realistic" deuteron wave function leads to a better agreement of $d\sigma/d\Omega$ with experimental data at backward angles. On the other hand, changing the value of P_D (D-state probability of the deuteron) has little influence on $d\sigma/d\Omega$ and it_{11}, but produces an appreciable decrease of the backward part of t_{20}.

At the same time, Rinat *et al.*[3] have reported on calculations of πd elastic scattering in the energy range 142 to 256 MeV, based on a fully relativistic (FR) three-body theory including the effects of genuine pion absorption and emission. These calculations, where the interference effects from the small πN waves are treated perturbatively, constitute an extension of the FR approach of Rinat and Thomas[4] limited to the P_{33} scheme. The changes in the observables due to the small πN waves are similar to those we have observed in the RPK approach, and the absorptive corrections are found to be important both in the backward part of $d\sigma/d\Omega$ and in the polarization parameters.

The following aspects provided the motivation for the present work. At first, we extend our RPK calculations on the basis of the FR theory, in order to ensure a correct treatment of the relativistic effects when energy increases. Next, we want to produce πd calculations which can serve as reference for further calculations including the genuine effects of π absorption. Indeed, we think it is more reasonable to investigate such effects insofar as the situation without π absorption is completely clear, namely with regard to the sensitivity of the πd observables to the description of the NN and πN channels. To this end, the $P_{11}\pi N$ channel being completely omitted, two sets of parametrizations for the S and P πN channels are used, and various sets of NN tensor forces are elaborated in order to improve the usual Yamaguchi-type interactions.

Of course, the various experiments which are in progress are also strongly motivating. At SIN, Gabathuler *et al.*[5] are analyzing the elastic differential cross-section data at seven energies from 80 to 300 MeV in the angular range 0° to 140°, and Grüebler *et al.*[6] plan to measure $t_{20}(180°)$ at 140 MeV; the LAMPF group[7] has recently observed $d\sigma/d\Omega$ (180°) and t_{20} (180°) at 140 MeV and has proposed measuring t_{20} as a function of angle, and the CERN group[8] has given preliminary results for the cross section at backward angles between 130° and 175° in the energy range 140 to 260 MeV.

The paper is organized as follows. In Sec. II, we describe the practical calculation, namely the numerical procedure and the perturbation scheme used to include the small πN channels. Section III is devoted to the description of the πN and NN interactions used as input. Our results are presented and discussed in Sec. IV, and we conclude the paper in Sec. V.

II. PRACTICAL CALCULATION

A. Basic equations

We use the three-body equations described by Rinat and Thomas[4] which satisfy two and three-body unitarity and covariance. In operator form, the equations read, with the notation of Ref. 1:

$$X_{nm}=\sum_{\alpha} Z_{n\alpha}R_{\alpha}X_{\alpha m},$$
$$X_{\alpha m}=Z_{\alpha m}+\sum_{\beta} Z_{\alpha\beta}R_{\beta}X_{\beta m}+\sum_{n} Z_{\alpha n}R_{n}X_{nm}, \tag{1}$$

where the α,β and n,m labels refer respectively to $N(\pi N)$ and $\pi(NN)$ three-body channels. In our case, the α,β labels do not contain the N-P_{11} channel. The driving terms Z and the propagators R are defined in terms of the form factors of the separable interactions, and for the relative momenta of the interacting pairs, we take the choice (a) of Ref. 4 corresponding to the exchanged particle on its mass shell.

The scattering amplitudes $2X_{dd}$ are obtained by solving the system of coupled one-dimensional integral equations which result from angular momentum reduction of Eq. (1). The singularities of the kernel are treated by contour rotation and the system is solved by the Padé approximant technique. The coupling of πd channels with $l,\ l'=J\pm 1$ orbital angular momentum is included exactly.

Compared with the RPK case, the FR calculation is much more time consuming because of the intricate coefficients which appear in the relative momenta [see Eqs. (2.20) and (2.21a) in Ref. 4]. In order to save computing time, we have (i) reduced the number of mesh points in the $[0,+\infty]$ domain, and (ii) treated perturbatively the small πN channels.

B. Numerical integration

In order to preserve the numerical accuracy when energy goes up to 300 MeV, without increasing too much the number of mesh points, we split the $[0,+\infty]$ domain of integration for the integral equation into two intervals $[0,k_c]$, $[k_c,+\infty]$. The energy-dependent parameter k_c is chosen to be in the middle of the interval delimited by the pole of the propagator and the extreme position of the logarithmic singularity in the driving term. In each interval, we use Gaussian quadratures mapped onto $[0,k_c]$ and $[k_c,+\infty]$ with, respectively, N_1 and N_2 mesh points. Because of the structure of the kernel, the convergence in the $[k_c,+\infty]$ domain is faster than in the $[0,k_c]$ region, and we take usually $N_2=N_1/2$. The two main advantages of the method are (i) the number of mesh points is large in the region around k_c where the kernel is rapidly varying, and (ii) we have observed that the convergence of the scattering amplitudes is monotonic when N_1 and N_2 increases. Therefore, the stability of the results can be obtained with a total number of mesh points appreciably lower than in the case where the Gaussian quadrature mapped directly onto $[0,+\infty]$ is used. We estimate the accuracy to be within 2% with $N_1=16$ and $N_2=8$ at 256 MeV, while 32 points are necessary if the interval is not split.

C. Perturbation scheme

In order to treat the small πN channels, we choose the first order Alt-Grassberger-Sandhas (AGS) approximation which was applied successfully in the n-d elastic scattering problem some years ago.[9] In this method, the kernel is split into a dominant part which is treated exactly and a weak part treated approximately. In Ref. 9, it is shown that the convergence condition for the AGS perturbation theory can be formulated entirely in terms of the weak part of the kernel. From a practical point of view, the first order AGS approximation is obtained when the weak part of the kernel is taken to be zero.

In our case, the dominant part corresponds to the π-d and N-Δ channels, and the weak part involves all the N-(πN) channels with the πN pair in the S or P states (except P_{11}) other than Δ. We therefore set equal to zero all the overlapping terms between these weak channels in the three-body kernel, i.e., we set $Z_{\alpha\beta}=0$ for α and β not equal to d or Δ. So, a large amount of computing time (~30%) is saved compared with the exact calculation.

Of course, the full advantage of this method can be exploited only in the case where the system of equations is solved by means of the Padé approximant technique. We give in Table I the comparison between the exact and AGS results for two dominant scattering amplitudes $T^J_{l'l}$, calculated at 142, 180, and 256 MeV with the $SF(6.7)$-$SP(S)$ interactions described hereafter in Sec. III. Clearly the AGS approximation is very good, the accuracy being of about 2%.

A different approximation is used in Ref. 3 where the small πN channels are introduced one at a time after the dominant part has been solved exactly. The choice of this method is imposed by the fact that the system of equations is solved by

TABLE I. Comparison of the first order AGS approximation with the exact calculation (in brackets) for two dominant scattering amplitudes. The T are dimensionless.

T_π (MeV)	T_{33}^{2+}	T_{11}^{2+}
142	$-12.976 - i28.190$ $(-12.999 - i28.172)$	$-865.39 - i1138.4$ $(-856.72 - i1142.7)$
180	$5.657 - i56.747$ $(5.630 - i56.733)$	$-275.36 - i1421.0$ $(-268.42 - i1420.4)$
256	$56.251 - i52.286$ $(56.245 - i52.261)$	$459.17 - i869.80$ $(460.53 - i869.32)$

matrix inversion, so that the storage problems are crucial. The accuracy was determined to be at the 5% level for the dominant scattering amplitudes.

In what follows, all the calculations including the small πN channels (*SP* scheme) are done with the AGS approximation.

III. TWO-BODY INTERACTIONS

We describe in this section the relativistic πN and NN separable interactions used as input in our calculations.

A. Pion-nucleon interactions

For the S_{11}, S_{31}, P_{13}, P_{31}, and P_{33} πN channels, we use two sets of parametrizations constructed by Rinat *et al.*[3] (denoted hereafter R) and by Schwarz *et al.*[10] (denoted S).

In the R case, the parameters are fitted in each channel to the experimental phase shifts taken from the recent analysis of Rowe *et al.*[11] which provides a smooth "best" fit to all modern πN phase shifts determined by various groups for energies below 400 MeV.

In the S case, the parameters are fitted to the "experimental" phase shifts and scattering lengths. These values are rather old and correspond to the data chosen by Thomas[2] for fitting the πN interactions in the RPK calculations. We note that in the P_{33} channel, the position of the resonance is imposed as an additional constraint in the fitting procedure.

The fits to the data are excellent as one can see in Fig. 3 of Ref. 3, and in Figs. 1 and 2 and Tables I–III of Ref. 10. We have recalculated the phase shifts and the scattering lengths for each set. The results are quite similar, the most apparent differences occurring in the P_{33} channel. This is illustrated in Table II where we give the scattering lengths for the two sets compared with the experimental values.

B. Nucleon-nucleon tensor forces

The simplest $^3S_1$-$^3D_1$ parametrizations are the relativistic generalizations of Yamaguchi-type interactions described in Ref. 3, with P_D values 4 and 6.7% (denoted hereafter $Y4$ and $Y6.7$). The parameters are fitted to the $^3S_1$ phase shift only, without any constraint on the low energy parameters a_t (scattering length) and Q (quadrupole moment). We have calculated these quantities, which are found to be far from the correct values. For $Y4$ and $Y7$ we get, respectively, $a_t = 5.63$ and 5.79 fm (accepted value 5.40 fm), and $Q = 0.37$ and 0.40 fm^2 (accepted value 0.280 to 0.286 fm^2).

In order to improve the situation, we have constructed three sets of rank-1 interactions (denoted YL, S, and SF), using more elaborate form factors, and introducing in the fitting procedure more and more constraints.

For the YL potentials, we take the usual Yamaguchi form factors:

$$g_L(p) = C_L p^L / (p^2 + \beta_L{}^2)^{(L+2)/2} . \tag{2}$$

For the S and SF potentials we define the form factors as a ratio of polynomials:

$$g_L(p) = C_L p^L (1 + \gamma_L p^2) \Big/ \prod_{i=0}^{L+2} (1 + \beta_{L i} p^2) . \tag{3}$$

The parameters are adjusted to fit the following

TABLE II. πN scattering lengths (in m_π^{-1} for S waves) and scattering volumes (in m_π^{-3} for P waves) for the Rinat (Ref. 4) and Schwarz (Ref. 10) parametrizations. The "new" experimental data are from Ref. 11, and the "old" are given in Refs. 9–11 of the Schwarz paper.

Parametrizations	Scattering length or volume S_{11}	S_{31}	P_{13}	P_{31}	P_{33}
R	0.172	−0.092	−0.013	−0.039	0.170
S	0.170	−0.091	−0.016	−0.036	0.211
Exp. (old)	0.174	−0.092	−0.016	−0.037	0.220
Exp. (new)	0.185 ±0.008	−0.098 ±0.003	−0.013 ±0.002	−0.029 ±0.002	0.205 ±0.050

quantities

(i) For the YL potentials, we impose only the static parameters, namely the deuteron binding energy $E_D = 2.2245$ MeV, the triplet scattering length $a_t = 5.40$ fm, the quadrupole moment $Q = 0.285$ fm^2, the D-state probability P_D, and the ratio of the D to S asymptotic deuteron wave functions $\eta = 0.026$.

(ii) For the S parametrizations, the $^3S_1$ phase shift up to 200 MeV is added to the foregoing constraints.

(iii) For the SF interactions, we impose one more constraint, namely the monopole form factor of the deuteron $F_0(q^2)$ up to $q \sim 6$ fm^{-1}. We choose to fit the Reid soft-core (RSC) form factor which has a minimum at $q \sim 4.5$ fm^{-1}.

In each set, two parametrizations are constructed with $P_D = 4$ and 6.7%. The parameters for the S and SF interactions are given in Table III. For the YL potentials, the values of (C_0, β_0; C_2, β_2) are (124.707, 1.329; 220.795, 1.559) for $YL4$, and (98.941, 1.261; 446.940, 1.970) for $YL7$ (the C are in fm^{-2} and β in fm^{-1}).

We show in Fig. 1 the $^3S_1$ phase shift for the various interactions with $P_D = 6.7\%$, and in Table IV we give the monopole form factors compared with the Reid soft-core values. The phase shift for the SF interaction is not as good as for the S interaction above 100 MeV, but the form factor is much better for $q > 2$ fm^{-1}. We think that the SF interactions are therefore more "realistic" in the sense that the fitting procedure to $F_0(q)$ ensures the correct behavior of the deuteron wave function. Let us point out that the $SF6.7$ potential is the relativistic equivalent to the Pieper rank-1 potential used in our RPK calculations.[1]

IV. RESULTS AND DISCUSSIONS

We have performed detailed calculations at 142, 180, 230, and 256 MeV in order to investigate the sensitivity of πd observables to the NN and πN input. There are three subsections devoted, respectively, to the differential cross sections (A), the total cross sections (B), and the polarization parameters (C).

In what follows, each calculation is specified by two labels: The first one denotes the NN tensor force, and the second one refers to the πN channels which are retained (P_{33} or SP) and to the parametrization.

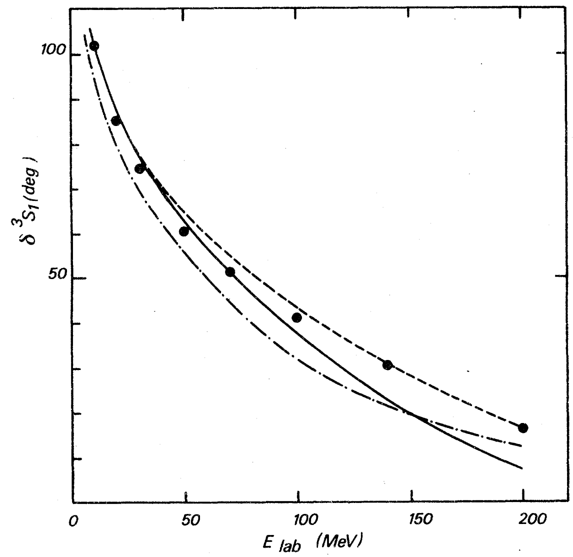

FIG. 1. Theoretical $^3S_1$ phase shifts given by the Y (—·—), YL (– – –), S (identical to YL), and SF (——) parametrizations for $P_D = 6.7\%$. The experimental data are from Ref. 12.

A. Differential cross sections

1. Effect of the small πN partial waves

The SP scheme consists in introducing with the AGS perturbation method the small πN channels S_{11}, S_{31}, P_{13}, and P_{31} in addition to P_{33}. As stated

TABLE III. Parameters for the S and SF tensor forces. The C_L are in fm^L and the β are in fm^2.

	$S4$	$S6.7$	$SF4$	$SF6.7$
C_0	76.832	64.530	66.508	58.659
γ_0	−0.0404	−0.0381	−0.231	−0.240
β_{01}	0.227	0.351	0.121	0.0494
β_{02}	0.252	0.190	0.118	0.241
C_2	37.038	31.362	33.057	27.562
γ_2	1.299	0.340	0.813	0.688
β_{21}	0.364	0.0217	0.216	0.0268
β_{22}	0.503	0.428	0.414	0.580
β_{23}	0.468	0.384	0.301	0.254
β_{24}	0.271	0.0794	0.424	0.262

TABLE IV. Monopole charge form factor of the deuteron $F_0(q)$ for the new tensor forces SF, S, and YL with $P_D = 6.7\%$, compared with the Reid soft-core values.

q (fm^{-1})	RSC	SF	S	YL
0.2	0.975	0.975	0.975	0.975
1	0.616	0.614	0.618	0.623
2	0.257	0.254	0.269	0.283
3	0.0875	0.0877	0.112	0.129
4	0.0151	0.0190	0.0453	0.0614
5	−0.0120	−0.0600	0.0177	0.0308
6	−0.0188	−0.0127	0.0064	0.0163

in the Introduction, we omit completely the P_{11} channel. The reason is that the formalism we use here does not include the effect of pion absorption, so that the nucleon pole part which is the dominant part cannot be introduced. On the other hand, the P_{11} phase shift is very small up to 150 MeV, and it seems reasonable to neglect its background contribution. The latter assumption was checked at 142 and 256 MeV. Using a separable parametrization adjusted only to the P_{11} phase shift, the πd observables calculated in the *SP* scheme with and without the nonpole P_{11} contribution were found to be almost identical.

We compare in Fig. 2 the differential cross sections calculated at 142, 180, 230, and 256 MeV in the *SP* and P_{33} schemes, with the *SF*6.7 tensor force and the *S* parametrizations of the πN channels. The forward part of $d\sigma/d\Omega$ is lowered by the *SP* scheme at energies below the resonance, and is enhanced above the resonance, while the backward part is systematically lowered. The effect is very small at 180 MeV which is close to the resonance. At all energies the *SP* scheme improves significantly the agreement with experimental data[13-16] throughout the angular range, except at forward angles for $T_\pi = 230$ MeV. Nevertheless, the situation at 230 and 256 MeV is not satisfactory, especially for $\theta_{c.m.} > 80°$, where the broad minimum observed in the experiments is not reproduced by the theory, which remains too high. However, we do not understand the discrepancy at forward angles with the experiment of Cole *et al.*[15] at 230 MeV, since our theory describes correctly, in this angular range, the "old" data at 142, 180, and 256 MeV as well as the preliminary data of Gabathuler *et al.*[5] for $T_\pi = 140$ to 300 MeV. We can thus consider the possibility of an incorrect normalization. In fact, if we enhance Cole's results by a factor of 1.4, the theory becomes quite good. On the other hand, the recent experiments of Ref. 5 seem to indicate that the minimum at 256 MeV is not so deep, but the theory still remains higher than experiment.

Let us briefly compare these results with the RPK calculations using the Pieper tensor force at 142 and 180 MeV [see Figs. 3(b) and 4(b) of Ref. 1]. The FR and RPK differential cross sections look quite similar, demonstrating that relativistic effects are moderate at these energies. We also note that the interference effects from the small πN channels are smaller in the FR theory than in RPK.

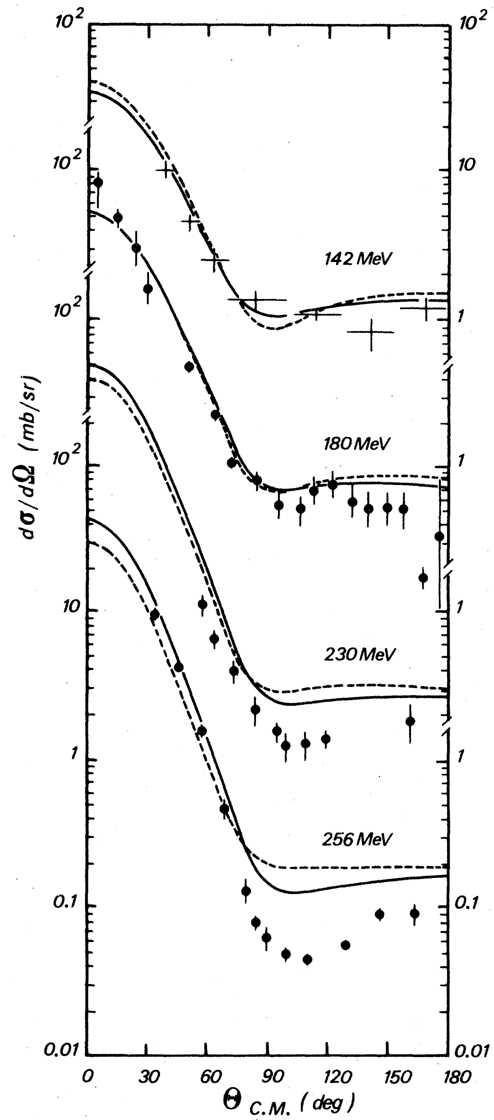

FIG. 2. πd elastic differential cross section at $T_\pi = 142$ (Ref. 13), 180 (Ref. 14), 230 (Ref. 15), and 256 (Ref. 16) MeV calculated with the *SF* 6.7 tensor force and the *S* parametrizations of the πN channels. – – – only P_{33}, —— P_{33} + small πN waves, except P_{11}.

2. *Sensitivity to the πN parametrizations*

We compare in Table V the values of $d\sigma/d\Omega$ obtained in the *SP* scheme with the *S* and *R* parametrizations of the πN channels and the *SF*6.7 tensor force, at $T_\pi = 180$ and 256 MeV. The *R* values are slightly higher than the *S* results throughout the angular range. However, the maximum variation does not exceed 13% at backward angles, reflecting that the two sets are nearly equivalent as shown in Sec. III A.

3. *Sensitivity to the NN tensor force*

Now, the calculations are done in the *SP* scheme with the *S* parametrizations of the πN channels.

TABLE V. Sensitivity of the differential cross section to the πN parametrizations. The numbers in each case are the values of $d\sigma/d\Omega$ in mb/sr, at $\theta_{c.m.}=0°$, 90°, and 180°.

T_π (MeV)	S	R
	53.6	53.9
180	0.74	0.77
	0.73	0.77
	41.9	44.7
256	0.15	0.17
	0.16	0.18

First, we consider the influence of D-state probability. In order to study a genuine effect of P_D, we use our parametrizations which, in a given set (YL, S, or SF), differ only by their P_D values as explained in Sec. III B. The differential cross sections calculated at 180 and 256 MeV with the SF4 and SF6.7 interactions are given in Table VI. When P_D goes from 4 to 6.7%, the forward part of $d\sigma/d\Omega$ decreases by about 3%, and the backward part increases by 4% at 180 MeV and by 14% at 256 MeV. Similar conclusions hold with the S4 and S6.7 interactions, and also with the YL4 and YL6.7 potentials. These results show that the overall effect of P_D is rather small, while the large variation at backward angles found in Ref. 4 is not a genuine effect, but is due to the fact that the Y4 and Y6.7 interactions give different values for the low energy parameters as mentioned in Sec. III B.

Next, we investigate the sensitivity to the description of the deuteron wave function for a fixed P_D value. We use the YL, S, and SF interactions with $P_D=6.7\%$. The variation of $d\sigma/d\Omega$ at forward angles is very small (~2%), but the backward part exhibits large variations (up to 25%). For example, the values of $d\sigma/d\Omega$ (180°) at 180 MeV are 0.97, 0.86, and 0.73 mb/sr for the YL, S, and SF tensor forces, respectively. Similar effects are observed at the other energies. If we remember how we have constructed the YL, S, and SF interactions in Sec. III B, we see that the agreement of theory with experimental data at backward angles becomes better as we improve the description of the $^3S_1$-$^3D_1$ channel, namely with regard to the deuteron wave function. These considerations justify the systematic use throughout this paper of the SF interactions, even if they fail to reproduce the minimum for $T_\pi=230$ and 256 MeV.

TABLE VI. Sensitivity of the differential cross section to the D-state probability. The numbers are ordered as in Table V.

T_π (MeV)	SF4	SF6.7
	54.9	53.60
180	0.78	0.74
	0.70	0.73
	43.3	41.9
256	0.16	0.15
	0.14	0.16

B. Elastic, reaction, and total cross sections

We have calculated the elastic (σ_{el}), reaction (σ_R), and total ($\sigma_T=\sigma_{el}+\sigma_R$) cross sections in the energy range 70 to 320 MeV with the SF6.7-$SP(S)$ interactions. In Fig. 3 we show our results and we compare the total cross section with the recent experimental data of Pedroni *et al.*[17] The agreement is fairly good, especially in the resonance region 140 MeV < T_π < 260 MeV where the theoretical curve goes through experiment. Outside this domain, the theoretical values are lower than experimental data, and the deviation increases when energy decreases from $T_\pi=120$ MeV or increases from $T_\pi=260$ MeV.

Compared with the RPK results (Fig. 5 or Ref. 1), the FR values of σ_T are better in a wider energy range. We also note a very good agreement with the calculations of Rinat *et al.*[3] without absorption. Our values for σ_T with the SF6.7-$SP(S)$ interactions at 142, 180, and 256 MeV are respectively 174, 230, and 135 mb, while the values of Rinat for $P_D=6.7\%$ are 176, 239, and 146 mb. In fact, we have observed that the total cross section is rather insensitive to the two-body input.

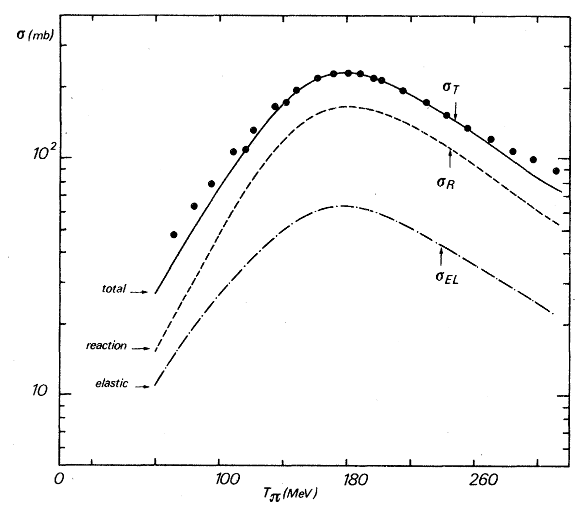

FIG. 3. Elastic (—·—), reaction (– – –), and total (——) cross sections calculated with the SF 6.7-$SP(S)$ interactions. The experimental results are from Pedroni *et al.* (Ref. 17).

C. Polarization parameters

The vector polarization it_{11} and the tensor polarizations t_{20}, t_{21}, and t_{22} for $\pi + d \to \pi + \vec{d}$ elastic scattering are easily calculated from the scattering amplitudes. The qualitative effects noted in the RPK approach are also observed in the FR calculations, only the numbers change more or less. We limit thus the present discussion to the most important aspects.

The complete change in structure of it_{11} due to the inclusion of the small πN partial waves in addition to P_{33} is observed at all energies. The FR curves look similar to the RPK curves shown in Fig. 6 of Ref. 1. The vector polarizations calculated in the SP scheme have a pronounced maximum which has the following characteristics: (i) the position is nearly independent of energy and of the two-body input and corresponds to $\theta_{c.m.} = 80°$, and (ii) the magnitude is practically independent of the πN and NN input, but varies with energy; namely, it is maximum around $T_\pi = 180$ MeV. For example, the values of it_{11} (80°) obtained with the $SF6.7$-$SP(S)$ interactions at 142, 180, 230, and 256 MeV are respectively 0.43, 0.45, 0.40, and 0.36.

In contrast with it_{11}, the tensor polarizations are only slightly affected when we include the small πN channels (the difference between the SP and P_{33} results is less than 5%) or when we change the πN parametrizations from S to R (the effect is ~2%). Of course, the most important variations in the tensor polarizations are due to the NN tensor force. We discuss only the quantity t_{20} which presents an immediate interest. The angular distributions of t_{20} have the same aspect as in the RPK theory (see Fig. 7 of Ref. 1), and the values of t_{20} (180°) calculated with the $SF6.7$-$SP(S)$ interactions at 142, 180, 230, and 256 MeV are respectively −0.74, −1.08, −1.26, and −1.27. The polarization t_{20}(180°) behaves as follows. (i) It decreases with increasing D-state probability; we give in Table VII the values obtained with the $SF4$ and $SF6.7$ tensor forces at 142, 180, and 256 MeV. (ii) It depends on the description of the deuteron wave function; for instance, the results at $T_\pi = 180$ MeV corresponding to the SF, S, and YL interactions with the same P_D value (6.7%) are respectively −1.08, −0.96, and −0.90. These variations are therefore of the same order as the variations coming from different P_D values. Recently, t_{20} (180°) was observed for the first time at $T_\pi = 140$ MeV by Holt *et al.*,[7] the value being -0.24 ± 0.15. The theoretical values obtained with the most elaborate tensor forces, namely the SF interactions, still remain far from this result since we find −0.74 for $P_D = 6.7\%$ and −0.61 for $P_D = 4\%$.

TABLE VII. Sensitivity of t_{20} (180°) to the D-state probability.

T_π (MeV)	$SF4$	$SF6.7$
142	−0.63	−0.74
180	−0.92	−1.08
256	−1.17	−1.27

V. CONCLUDING REMARKS

We have presented an extensive analysis of πd elastic scattering observables in the resonance region within a fully relativistic three-body theory. The small πN channels have been included to an excellent degree of accuracy through the AGS perturbation method, and the numerical integration has been refined in order to save computing time without losing accuracy. Since the theory we used did not include the effects of pion absorption, the P_{11} πN channel was completely omitted.

The great sensitivity of πd observables (namely the differential cross section and the t_{20} polarization at large angles) to the details of the two-body input that was demonstrated in previous RPK calculations[1] is still observed in the FR approach. The agreement between the theoretical and experimental differential cross sections is significantly improved when the small πN channels are included and when a tensor force giving a realistic deuteron wave function is used.

Let us note here that the differences between the present calculations and those of Rinat *et al.*,[3] without absorption, are moderate. Seeing that the two sets of πN interactions used in each calculation are nearly equivalent, it is clear that the differences must be attributed to the tensor forces. Figure 6 of Ref. 3 gives a spectacular illustration of the model dependence of $d\sigma/d\Omega$ at backward angles relative to the deuteron wave function. The variations corresponding to different tensor forces are found to be as important as the variations due to the inclusion of absorption effects. Our main effort in the present work was to produce pure three-body calculations based on a tensor force having the maximum degree of quality. So, we have now a sound reference for further theoretical investigations. We think that the SF parametrizations that we have constructed represent a real improvement compared with the tensor forces used up to now. Of course, they have some defects deriving from their structure: The SF interactions are of rank 1, and therefore the $^3S_1$ phase shift remains positive and the $^3D_1$ phase shift has the wrong sign. These difficulties should be removed by considering rank 2 parametrizations. However, from our experience in the RPK approach with the Pieper rank-1 and rank-2 interactions (they have the same deuteron wave function but differ by their $^3D_1$ phase shifts, and they give nearly the same πd observables), we think that this step is not essential.

None of our calculations do reproduce the minimum in the differential cross sections which is observed in the experiments at 230 and 256 MeV, and it seems now clear that the two-body input is not responsible for this discrepancy. Besides the apparent need to reconfirm the experimental data, we have now to consider further theoretical investigations. At first, we must include the effects of absorption. Detailed calculations have been recently performed in this direction by Rinat *et al.*[3] within a field-theoretical formulation including genuine pion absorption and ρ-meson exchange. The effects of absorption induce significant changes in the large angle differential cross section, but they do not produce the deep minimum at 256 MeV. On the other hand, absorption has a tremendous influence on all polarization parameters; for example, t_{20} (180°) becomes small positive at 142 MeV. An independent calculation of πd elastic scattering including π absorption is now in progress in our group, based on the approach to the theory of coupled πNN-NN systems recently developed by Avishai and Mizutani.[18] Also the effects of inelasticity in the πN partial waves may be important[19] (mainly in the P_{11} channel) in accounting for the minimum in the cross section at 256 MeV. Another direction concerns the inclusion of dibaryon resonances, but here the situation is not clear. For some people,[20] the resonant amplitude (the $^3F_3\,NN$ for instance) must be added to the πd elastic scattering amplitude, while for others[21] the πNN system itself is resonant and the inclusion of the NN ($^3P_2$) and πN (P_{11} and P_{33}) leads to a resonant $^3F_3$ wave.

Finally, we think that the new experimental data which ought to appear in the near future will be decisive for further theoretical work.

ACKNOWLEDGMENTS

We would like to thank A. S. Rinat and A. W. Thomas for very helpful discussions and correspondence. It is a pleasure to thank F. Myhrer for his stimulating remarks. The constant interest of E. Elbaz in the achievement of the present work is highly appreciated.

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