Relativistic calculation of radiative muon capture in hydrogen and ^{3}He

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In view of the fundamental importance of radiative muon capture, particularly in light nuclei, and of the somewhat confused theoretical situation, a new calculation of radiative muon capture on the proton and on ^{3}He has been made. This calculation is based on the standard set of diagrams but, unlike previous calculations, has been performed without making nonrelativistic approximations. Results are given for the rate and photon spectrum for both ^{1}H and ^{3}He. A detailed comparison of this calculation is made with the approach developed by Hwang and Primakoff which is based on general constraints of conserved vector current, partially conserved axial vector current, and gauge invariance and on a special linearity hypothesis to try to understand why the Hwang-Primakoff results differ markedly from all previous results. It is shown that this calculation, as well as other standard ones, satisfy the general constraints of Hwang and Primakoff and that the differences arise because of their linearity hypothesis and other approximations. These differences are examined in detail, and it is shown that the numerically most important one arises because the linearity hypothesis has been used in such a way that it leads to a Hwang-Primakoff amplitude which violates the Low soft photon theorem.

[RADIOACTIVITY $\mu^- p \to \nu n \gamma$ and $\mu^- {}^3\mathrm{He} \to \nu {}^3\mathrm{H}\gamma$; relativistic calculations of rate and photon spectrum.]

I. INTRODUCTION

Radiative muon capture (RMC) has long been recognized as one of the fundamental weak interaction processes which is particularly sensitive to g_P, the induced pseudoscalar coupling of the weak hadronic current. Several experiments have been performed in nuclei,[1-4] particularly ^{40}Ca, but more recently in ^{16}O,[4] to measure the photon spectrum and in some cases the photon asymmetry with the aim of extracting g_P. A number of theoretical calculations have also addressed the question of RMC in nuclei.[5-11] However, in a nucleus there are a number of uncertainties which make interpretation of the results in terms of g_P difficult.[6,8,12] These uncertainties arise in part because of the lack of knowledge of the nuclear structure or particularly lack of knowledge of the excitation spectrum of the final nucleus, which usually forces use of the closure approximation.

It is thus very important to look at the very light nuclei, e.g., the proton, deuteron, or ^{3}He, where the nuclear physics is well understood and where the final state is clearly defined so that closure is not necessary. Such experiments are quite difficult because of the low rate for RMC in light nuclei relative to the decay rate. However, higher muon fluxes available at the new meson facilities have made some such experiments feasible, and in particular an RMC experiment in ^{3}He has been proposed at LAMPF.[13]

Somewhat surprisingly the theoretical situation for such very light nuclei is confused. Older calculations for the proton exist, e.g., that of Opat,[14] and Beder[15] has calculated some specific correlations in both ^{1}H and ^{3}He. More recently Hwang and Primakoff[16] (HP) have made a new calculation of RMC in ^{1}H and ^{3}He, later applied also to ^{12}C,[11] using an approach which starts with a very general amplitude constrained by the conserved vector current (CVC), partially conserved axial current (PCAC), and current conservation or gauge invariance (GI) conditions plus a special "linearity hypothesis" rather than with a set of Feynman diagrams as used in essentially all other calculations of RMC in both light and heavy nuclei. Their results differ dramatically from the older calculations[14,15] for ^{1}H and ^{3}He and also from a new calculation[9] for ^{12}C (see Table I), which is somewhat surprising since the standard diagram approach seemingly also builds in currents satisfying CVC, PCAC, and GI, and contains roughly the same physics as the calculations of HP.

In view of the fundamental significance of these very simple reactions, and the implications they may have for understanding RMC in more complicated nuclear systems, it is extremely important to understand the reasons for the quite different results in the two approaches. Such understanding is the first purpose of this paper. The second is to provide some numerical results for ^{1}H and ^{3}He which may be useful in current[13] or future experiments.

To begin, the standard diagrammatic approach

TABLE I. Comparison of the results of Hwang and Primakoff with those of Opat (Ref. 14), Beder (Ref. 15), and Wullschleger and Scheck (Ref. 9). Listed are the RMC rates for capture from various spin states on the proton, the photon spectrum $d\Gamma/dk$ for several values of k for RMC on ^{3}He, and the ratio of total radiative to ordinary rates for capture on ^{12}C leading to the ground state of ^{12}B.

		HP	Others	Ref.
^{1}H:	Singlet rate	0.0180	0.0050	14
	Triplet rate	0.0377	0.0900	14
	Statistical rate	0.0328	0.0687	14
^{3}He:	$\frac{d\Gamma}{dk}$(70 MeV)	2.53×10^{-3}	4.58×10^{-3}	15
	$\frac{d\Gamma}{dk}$(80 MeV)	1.46×10^{-3}	2.65×10^{-3}	15
	$\frac{d\Gamma}{dk}$(90 MeV)	6.10×10^{-4}	1.07×10^{-3}	15
^{12}C:	Relative rate (to ^{12}B$_{\rm g.s.}$)	2.08×10^{-4}	3.09×10^{-4}	9

is reviewed and some details of our "standard" calculation (which differs from previous ones in that no nonrelativistic approximations are made) are described. The HP approach is then discussed and the explicit ways in which the final amplitude differs from the standard result and the numerical significance of these differences shown. The next step is to understand the origin of these differences in terms of the approximations and assumptions made by HP. Finally some numerical results are given for the proton and, after some discussion, for ^{3}He.

II. REVIEW OF STANDARD AND HP APPROACHES

The "standard" theory of radiative muon capture (see, e.g., Ref. 5) begins with a particular set of Feynman diagrams or equivalently a relativistic Hamiltonian which reproduces these diagrams.[7] A relativistic amplitude for RMC on the proton is constructed and an expansion of this amplitude in powers of $1/m$ leads to an effective nonrelativistic Hamiltonian which can be evaluated between the appropriate nuclear states or two-component spinors in the case of the free proton. This approach has been used for essentially all previous calculations of RMC except those of HP.

Five diagrams (Fig. 1) are usually used. The leading ones are the usual external radiation diagrams [Figs. 1(a), 1(b), 1(c)] in which the muon, proton, and neutron radiate. These are just the diagrams which are necessary to obtain the $1/k$ and k/k terms in an expansion of the radiative amplitude in powers of the photon momentum k and thus to satisfy the Low soft photon theorem.[17] To these are added two additional diagrams. Figure 1(d) corresponds to radiation from the internal pion in the diagram responsible for g_P. It is analogous to the type of contribution one would obtain by the minimal substitution $p \to p - eA$ in the momentum dependence of g_P. The final term, Fig. 1(e), is a gauge term obtained by letting $p \to p - eA$ in the explicit momentum-dependent terms in the weak hadronic current. Neglected are a number of additional terms such as gauge terms coming from the structure of form factors other than g_P, various structure terms, higher-order terms generated by PCAC, etc. Some of these were considered by Manacher and Wolfenstein[18] and by Adler and Dothan[19] and seem generally to be small.

From this set of diagrams one obtains a relativistic amplitude for RMC on a proton of the form

$$M_{fi} = M_a + M_b + M_c + M_d + M_e\,, \qquad (1)$$

where[20]

$$M_a = \epsilon_\beta \bar{u}_n \Gamma^\alpha(q_L) u_p \bar{u}_\nu \gamma_\alpha (1-\gamma_5) \frac{(\mu\cdot\gamma - k\cdot\gamma + m_\mu)}{2k\cdot\mu}\gamma^\beta u_\mu\,,$$

$$M_b = \epsilon_\beta L_\alpha \bar{u}_n \Gamma^\alpha(q_N) \frac{(p\cdot\gamma - k\cdot\gamma + m_p)}{-2k\cdot p} \times\left(Q_p\gamma^\beta - i\kappa_p \frac{\sigma^{\beta\delta}}{2m_p}k_\delta\right)u_p\,,$$

$$M_c = \epsilon_\beta L_\alpha \bar{u}_n \left(Q_n\gamma^\beta - i\kappa_n \frac{\sigma^{\beta\delta}}{2m_n}k_\delta\right) \times \frac{(n\cdot\gamma + k\cdot\gamma + m_n)}{2k\cdot n}\Gamma^\alpha(q_N)u_p\,,$$

$$M_d = -\epsilon_\beta L_\alpha \bar{u}_n \frac{[2q_L^\beta + k^\beta]}{q_L^{\,2} - m_\pi^{\,2}} \frac{g_P^N}{m_\mu} q_N^\alpha \gamma_5 u_p\,,$$

$$M_e = \epsilon_\beta L_\alpha \bar{u}_n \left(\frac{ig_M}{2m}\sigma^{\alpha\beta} + \frac{g_S}{m_\mu}g^{\alpha\beta} + \frac{g_P^L}{m_\mu}\gamma_5 g^{\alpha\beta} + \frac{ig_T}{2m}\sigma^{\alpha\beta}\gamma_5\right)u_p\,,$$

$$\Gamma^\alpha(q) = g_V\gamma^\alpha + \frac{ig_M}{2m}\sigma^{\alpha\beta}q_\beta + \frac{g_S}{m_\mu}q^\alpha + g_A\gamma^\alpha\gamma_5 + \frac{g_P(q^2)}{m_\mu}q^\alpha\gamma_5 + \frac{ig_T}{2m}\sigma^{\alpha\beta}q_\beta\gamma_5\,,$$

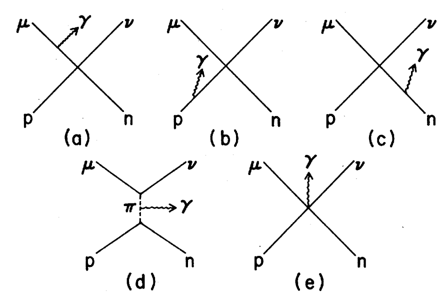

FIG. 1. Standard diagrams contributing to radiative muon capture on a proton.

$L_\alpha = \bar{u}_\nu \gamma_\alpha (1-\gamma_5) u_\mu$, $g_P^N = g_P(q_N^2)$, $g_P^L = g_P(q_L^2)$, and where the weak and electromagnetic couplings $Ge/\sqrt{2}$ have been extracted. In these expressions the four-momenta satisfy $\mu + p = n + \nu + k$, ϵ is the photon polarization vector, Q_p, κ_p, m_p, and Q_n, κ_n, m_n are, respectively, the initial and final nuclear charges, anomalous magnetic moments and masses, and m is the nucleon mass. The second-class terms g_S and g_T will be neglected from now on. The momentum dependence of $g_P(q^2)$ will be taken as $g_P(q^2) = g_P(m_\pi^2 + m_\mu^2)/(m_\pi^2 - q^2)$ with $g_P = m_\mu(m_p + m_n) g_A/(m_\pi^2 + m_\mu^2) \approx 6.6\, g_A$ being the Goldberger-Treiman[21] value. Unless otherwise noted, $g_V = 1.0$, $g_A = -1.25$, $g_M = 3.71$, $\kappa_p = 1.79$, and $\kappa_n = -1.91$.

Observe that there are two different momentum transfers in this expression: $q_L \equiv n - p$ and $q_N \equiv n - p + k$. Figures 1(a) and 1(d) depend on q_L, Figs. 1(b) and 1(c) depend on q_N, and Fig. 1(e) contains both. In the region of interest corresponding to large k, $q_L^2 \simeq -m_\mu^2$ whereas q_N^2 can approach $+m_\mu^2$. Thus g_P^N is evaluated nearly at the pion pole and hence is enhanced for large k by almost a factor of four over g_P^L.

In the usual approach, used by essentially all authors except Beder,[15] this amplitude is expanded in powers of p/m to obtain an effective Hamiltonian. Matrix elements of this Hamiltonian are taken and squared and multiplied by phase space which has also often been taken in some nonrelativistic approximation. The end result is a rate which is good through $O(1/m)$ but which generally contains only part of the $O(1/m^2)$ terms. (See, however, Ref. 7.)

Our approach, which was used to generate all numerical results given below, differs somewhat in that no nonrelativistic approximations or expansions in powers of $1/m$ were made. Instead the amplitude of Eq. (1) was evaluated numerically by direct multiplication of the various factors, squared and summed on spins, and used in the fully relativistic formula for the rate

$$\Gamma_{\rm rad} = \frac{\alpha G^2 |\phi_\mu|^2 m_n}{2(2\pi)^4} \int_0^{k_{\max}} k\,dk\, d\Omega_k\, d\Omega_\nu \frac{E_\nu^2}{W_0 - k(1-y)} \times \frac{1}{4} \sum_{\rm spins} |M_{fi}|^2 , \qquad (2)$$

where $y = \hat{k} \cdot \hat{\nu}$, $k_{\max} = (W_0^2 - m_n^2)/2W_0$, $W_0 = m_p + m_\mu$ − (binding energy of muon), $E_\nu = W_0(k_{\max} - k)/[W_0 - k(1-y)]$ and $|\phi_\mu|^2$ is the muon wave function averaged over the initial nucleus and is taken for the proton as the point Coulomb result at $r = 0$. The $\frac{1}{4}\sum_{\rm spins}$ is for the statistical spin mixture and must be replaced appropriately if singlet or triplet rates are desired.

Since no nonrelativistic approximations have been made here, the numerical results will contain all orders in powers of $1/m$ both in the phase space and matrix element and so may differ somewhat from previous calculations which have variously dropped terms of $O(1/m^2)$, often terms of $O(p/m)$ and even sometimes other $O(1/m)$ terms. In nuclei the size of these $O(1/m^2)$ terms has been investigated recently,[7] and it was found for ^{40}Ca that all $O(1/m^2)$ terms together contribute up to 20% of the rate, though the most important of these are obtained from the square of the usual $O(1/m)$ Hamiltonian. The $O(1/m)$ terms contribute 40–50%, depending on k. The velocity terms, which explicitly contain the momentum p and contribute in a nucleus as a result of the nonzero Fermi momentum of the initial proton, give roughly 10%. Thus there may be residual differences between the present results for which all terms have been kept and those of others in which some of these terms have been dropped. It has been verified that the present calculations, using comparable form factors and for HP the changes discussed below, agree in the $m \to \infty$ limit with results of both Opat[14] and of HP to a few percent. For finite mass these results agree with Opat[14] at the 10% level, the difference being presumedly due to these additional relativistic corrections. Numerical work has also been checked against analytic results in the $m \to \infty$ and in the $|M_{fi}|^2 = 1$ limits.

This standard approach can be contrasted with that of Hwang and Primakoff,[16] who attempt to go as far as possible in deriving the RMC amplitude using only very general properties such as CVC, PCAC, and GI. Their approach thus may potentially provide a quite useful framework for looking at corrections not given by the usual perturbation theory diagrams.

To begin, HP expand the hadronic part of the RMC amplitude in terms of the most general set of structures which can be formed from the available vectors. Each structure is multiplied by a form factor which can be a function of the available invariants and couplings. The leptonic part Fig. 1(a) is evaluated as in the standard theory, using the usual definitions of the hadronic weak current.

Next the general conditions CVC, PCAC, and GI are imposed on the full amplitude, thus generating a set of constraint equations among the general form factors and the weak form factors from the muon radiating diagram, Fig. 1(a). These constraints are not sufficient to determine all unknown form factors, so it is necessary to make an additional "linearity hypothesis." This hypothesis requires that the form factors be linear in the various couplings, which is just a reflection of the first order electromagnetic-first order weak nature of the process, but more importantly that each form factor have the specific form

$$\frac{R^+(q_L^2)}{k\cdot n}+\frac{R^-(q_L^2)}{k\cdot p}+R^0(q_L^2). \qquad (3)$$

This form is motivated by the expressions obtained in perturbation theory from diagrams 1(b) and 1(c). It involves two crucial assumptions, namely that the only denominators allowed are those which would be obtained from neutron or proton propagators as in Figs. 1(b) and 1(c) and that there is dependence on q_L^2 but not on q_N^2. Thus contributions such as M_d from the pion radiating diagram are ruled out by assumption, and g_P^N either cannot appear or is approximated by g_P^L.

Finally, it is still necessary to assume that some of the form factors have their perturbation theory values as obtained from Figs. 1(b) and 1(c). Then with the linearity hypothesis and the constraint equations derived from general principles it is possible to solve for all of the unknown form factors and obtain an amplitude analogous to that of Eq. (1). This amplitude is then expanded in powers of $1/m$ and all $O(1)$ terms, which all contain g_V or g_A, are kept but only those $O(1/m)$ terms involving g_M or g_P. Thus fewer terms are kept than in the usual nuclear calculations. This amplitude is then squared and the cross section obtained in the usual nonrelativistic manner.

III. COMPARISON OF STANDARD AND HP APPROACHES

Having reviewed the two different approaches the next step is to examine the differences to try to understand the reason for the large difference in numerical results. This comparison will be done in three stages, first by examining the explicit differences, then looking at the numerical effect of each of these differences, and then understanding the physics, i.e., the assumptions and approximations which lead to these differences.

It is perfectly straightforward, though tedious since the metric and coupling constant definitions are different, to compare directly the amplitude of HP, Eqs. (3)–(5), (26), and (33) with the result of Eq. (1) above. The conclusion is that to get the result of HP Eq. (1) must be modified in the following three ways:

(1) Drop the contribution M_d corresponding to the radiation from the intermediate pion.

(2) Neglect the difference between q_L and q_N in the form factors and evaluate all form factors at q_L^2 (actually at an average value of q_L^2). Thus the difference between g_P^L and g_P^N and the enhancement due to g_P^N is neglected everywhere.

(3) Change the sign of some of the g_P terms appearing in the hadronic amplitude, in particular

$$\frac{g_P^N(n-p+k)^\alpha}{-2k\cdot p}\to\frac{g_P^L(n-p-k)^\alpha}{-2k\cdot p}\ \text{in } M_b,$$
$$\frac{g_P^N(n-p+k)^\alpha}{2k\cdot n}\to\frac{g_P^L(n-p-k)^\alpha}{2k\cdot n}\ \text{in } M_c, \qquad (4)$$
$$g_P^L\to -g_P^L\ \text{in } M_e.$$

The set of changes (1) and (2) together or the set (1), (2), and (3) both lead to an amplitude which is gauge invariant so that it makes sense to look at the numerical differences brought about by these changes. Such results for the proton are shown in Table II. Observe first that the approxi-

TABLE II. Effect of the differences (1), (2), and (3) between standard and HP theories, as described in the text, on the singlet (Γ_s), triplet (Γ_t), statistical ($\frac{1}{4}\Gamma_s+\frac{3}{4}\Gamma_t$), and ortho $p\mu p$ molecular ($\frac{3}{4}\Gamma_s+\frac{1}{4}\Gamma_t$) rates for RMC on a proton. Case (a) is the standard result from Eqs. (1), (2) with $g_V=1$, $g_A=-1.25$, $g_M=3.71$, and $g_P=6.6g_A$. Case (b) is (a) with the pion radiating diagram Fig. 1(d) dropped and $q_N\to q_L$ corresponding to the modifications (1) and (2) of the text. Case (c) is the same as (b) but with the sign changes in certain g_P terms corresponding to modifications (3) of the text and thus has the same matrix element as HP. Case (d) is the same as (c) except that the slightly different form factors of HP have been used. Case (e) is the $m\to\infty$ limit of (a) and thus corresponds to neglecting all $O(1/m)$ and higher terms in the matrix element and phase space.

Case	Radiative rate Singlet	Triplet	Statistical	Ortho $p\mu p$
(a) Standard theory—using Eqs. (1), (2)	3.23×10^{-3}	9.98×10^{-2}	7.56×10^{-2}	2.74×10^{-2}
(b) As (a), but with changes (1) and (2), i.e., no π radiation and $q_N\to q_L$	1.80×10^{-3}	9.69×10^{-2}	7.32×10^{-2}	2.56×10^{-2}
(c) As (b), but with change (3) in addition, i.e., sign change in some g_P terms	19.97×10^{-3}	5.36×10^{-2}	4.52×10^{-2}	2.84×10^{-2}
(d) As (c) but using same couplings and form factors as HP	19.22×10^{-3}	5.14×10^{-2}	4.34×10^{-2}	2.73×10^{-2}
(e) As (a), but with $m\to\infty$ everywhere except in $\lvert\phi_\mu(0)\rvert^2$	0.426×10^{-3}	5.18×10^{-2}	3.90×10^{-2}	1.33×10^{-2}

mations (1) + (2) do not change the statistical or triplet rates very much, but do reduce the singlet rate by almost a factor of 2. Thus the pion radiating diagram seems to be relatively small and the enhancement coming from g_P^N also is much less important than might be expected. This probably just reflects the fact that the muon radiating diagram dominates. When the change (3) is made, however, both the statistical and triplet rates are reduced by a factor of almost 2, and the singlet rate is increased by more than an order of magnitude. Clearly it is the change in sign of some of the g_P terms which has the most important numerical effect.

Shown also are the results obtained with the changes (1), (2), and (3) and in addition the form factors and couplings taken as nearly as possible to be the same as used by HP. The changes resulting from the slightly different choice of couplings are small. There still seem, however, to be some residual differences between these results and those actually quoted by HP, particularly for the triplet rate. Our results for the triplet rate are, however, in agreement with those of Opat[14] to within about 10% when evaluated at the appropriate values of the couplings. It may be that this residual difference with HP (and the smaller difference with Opat) is due to the higher-order relativistic corrections to both phase space and matrix element which are included in our result. Unfortunately as a result of the numerical evaluation of the amplitude used here it is impossible to sort out the contributions of each individual power of m. The $O(1)$ terms can be extracted numerically, however, by taking $m \to \infty$ everywhere except in $|\phi_\mu(0)|^2$. Such results are shown also in the table and agree in this limit with HP's results as calculated from Eq. (46a). Clearly the $O(1/m)$ and higher contributions are important, contributing almost half of the triplet rate and nearly all of the singlet rate. One further estimate can be made, albeit for the nucleus ^{40}Ca rather than for a proton, by using the work of Ref. 7 where terms in the matrix element were separated into various powers of m. Using a simple shell model-harmonic oscillator wave function approach one can test the approximation of HP of keeping only g_V, g_A, g_M/m, g_P/m terms in the Hamiltonian. For ^{40}Ca this approximation misses 20–30% of the photon spectrum for larger k and even more for small k. Thus it appears that these approximations may not be as good as was assumed, and it is at least possible that these higher-order corrections could account for the residual differences mentioned above.

Having seen that the differences between HP and the standard approach are numerically significant, it is important to understand the physics of the assumptions which lead to these differences so as to make some choice between the two approaches. Several important observations can be made.

(A) It is straightforward to show that *the standard theory of Eq. (1) satisfies all of the general constraint equations arising from CVC, PCAC, and GI* which were derived in HP. This is perhaps not unexpected as both approaches begin with the same basic weak currents which satisfy CVC and PCAC.

(B) Both approaches lead to amplitudes which are gauge invariant. Hence *they differ by terms which are by themselves explicitly gauge invariant*. This just reflects the nonuniqueness of amplitudes fixed by gauge invariance; it is always possible to add pieces which are separately gauge invariant.

(C) Since the standard result satisfies the same general constraints as the HP result *the differences must be traceable to the extra linearity hypothesis used by HP*.

It is clear, and in fact already has been mentioned, that the differences (1) and (2) are direct results of the linearity hypothesis since by assumption the form factors are functions of q_L^2 only and since the assumed form of the form factors does not admit a contribution from the pion radiating diagram which has a different denominator than those assumed.

The way in which difference (3), which is the numerically important one, arises from the linearity hypothesis is somewhat more subtle. Observe from Eqs. (1) and (4) that the effect of the sign change in these g_P terms is to add to the standard result a gauge-invariant piece which contains terms of the form $k/k\cdot p$ and $k/k\cdot n$, i.e., terms which are of $O(k^0)$ but which are not independent of k. Recall, however, the usual derivation of the Low soft photon theorem[17,19] which applies to any radiative process. There the amplitude is first expanded in powers of k. All $O(1/k)$ terms must arise from the external radiation diagrams, Figs. 1(a), 1(b), 1(c) in the present case, and all terms having $k\cdot p$ or $k\cdot n$ in the denominator must arise from the nucleon propagators appearing in these same external radiation diagrams. Terms which are of $O(k^0)$ and *independent of* k are then added to make the result gauge invariant. Thus in the context of the Low theorem the terms of the form $k/k\cdot p$ and $k/k\cdot n$ are *uniquely* determined by the perturbation theory diagrams Figs. 1(b), 1(c) and thus correspond to the standard result. HP's result contains different $k/k\cdot p$ and $k/k\cdot n$ terms and hence, even though it is a gauge invariant expansion containing $O(1/k)$ and $O(k^0)$ pieces, the $O(k^0)$ terms and the result violate the Low soft photon theorem.[22]

Again we can trace this difficulty to the linearity hypothesis. The form chosen is too general since it allows $1/k \cdot p$ and $1/k \cdot n$ terms in all form factors. The Low theorem, via the argument that such terms come only from external radiation diagrams, provides an additional constraint not used by HP which uniquely determines the $R^{\pm}$ coefficients in the linearity hypothesis. Had this constraint been used, the standard result would have been obtained for these $g_P k/k \cdot p$ and $g_P k/k \cdot n$ terms.

Thus to summarize this section, the HP results differ from results of the standard approach not because of any different use of the constraints deriving from the general principles of CVC, PCAC, and GI, but because of the specific assumptions made via the linearity hypothesis. In particular the pion radiating diagram and the dependence on $q_N{}^2$ are dropped by assumption, and the amplitude is made gauge invariant by adding terms which cause the result to violate the Low soft photon theorem. These differences are numerically significant for the rates, as was seen in Table II. They are important also for the photon spectrum, an example of which is given in Fig. 2. Since the Low theorem must be satisfied by any radiative process, and since the aspect being violated by the HP result essentially just expresses the requirement that the amplitude be obtainable from some set of field-theory-like interactions, it would seem that the standard result is the correct one and should be adopted. However, it should be relatively easy to impose the required additional constraint from the Low theorem on the HP form factors, and then the HP approach, since it is quite general up to the point at which the linearity hypothesis is imposed, may be quite powerful and useful for including a variety of corrections or improvements in the theory, such as contributions from additional diagrams, which do not arise in the standard approach.

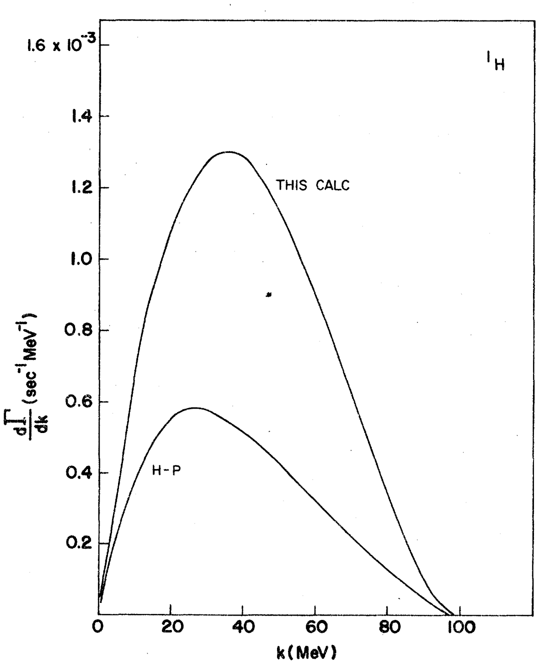

FIG. 2. Photon spectrum for radiative muon capture on a proton in the standard theory of this calculation and from HP, Eq. (48c).

IV. RMC IN ^{3}He

From an experimental point of view RMC in ^{3}He is somewhat easier than on a proton since the rate is $\sim Z^4$ and thus significantly higher. RMC in ^{3}He seems currently feasible and an experiment is planned.[13] It is thus important to have good theoretical estimates of the spectrum. Except for the work of HP and some results for specific correlations and for the upper tip of the photon spectrum by Beder,[15] such calculations in ^{3}He seem not to have been made.

However, if one uses the elementary particle picture, following for example Kim and Primakoff,[23] then it is very easy to adapt this calculation for the proton to the ^{3}He-^{3}H system, or for that matter to any spin $\frac{1}{2} \to \frac{1}{2}$ transition. The rate is thus evaluated in the standard approach using the formula obtained from Eqs. (1), (2) with the replacements $m_p \to$ mass of ^{3}He, $m_n \to$ mass of ^{3}H, $Q_p \to 2$, $Q_n \to 1$, and $\kappa_p \to -8.368$, $\kappa_n \to 7.918$. The weak couplings, as taken from Ref. 23, are $g_V = 1$, $g_M = -5.44$, $g_A = 1.22$, $g_P = 19.4\, g_A$. Observe that g_A, and hence g_P, has the opposite sign for ^{3}He than for the proton. Note also that the nuclear mass has been retained in the denominator in the definitions of the anomalous moments, but the nucleon mass is used for g_M. Finally, for ^{3}He averaging over the nuclear volume introduces a factor so that[23] $|\phi_\mu|^2 \to 0.965|\phi_\mu|^2$.

For a nucleus it may be necessary to include form factors in the g_V, g_A, and g_M couplings as well as g_P. This leads to a dilemma, however, since the two different momentum transfers involved will give different values of the form factors, e.g., $g_V(q_L{}^2)$ and $g_V(q_N{}^2)$, for the different diagrams and thus destroy gauge invariance unless some additional diagrams are added, analogous to Fig. 1(d), corresponding to radiation from the internal exchanged particles which generate the phenomenological form factor. In practice, however, such contributions are probably suppressed by the propagators of the (presumedly heavy) exchanged particles. Also the muon radiating dia-

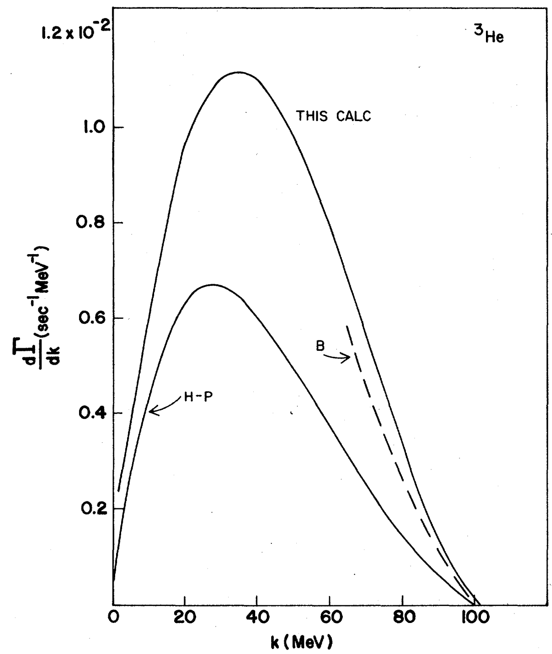

FIG. 3. Photon spectrum for radiative muon capture on ^{3}He in the standard theory of this calculation and from HP, Eq. (49q). Beder's result (perhaps with the wrong sign for g_P) is also shown. Reversing the sign of g_P would increase his result to be about 10% above the result of this calculation.

gram dominates. Thus it is probably a reasonable approximation to simply evaluate all form factors in the same way and take those values appropriate for the muon radiating diagram. Thus a simple linear interpolation of the form factors of Kim and Primakoff[23] was used for g_V, g_A, and g_M and evaluated at q_L^2, as appropriate for the muon radiating diagram. This is essentially the procedure of HP, except that they used an average value of q_L^2, independent of k.

Results for the photon spectrum as calculated in this model are shown in Fig. 3 as are the results of HP and Beder.[15] These spectra correspond to a total radiative rate of $\Lambda = 0.676$ sec^{-1} for the standard calculation vs $\Lambda = 0.373$ sec^{-1} from HP. Qualitatively the effects of the three differences in the amplitudes which arise from the linearity hypothesis of HP are the same for ^{3}He as for the proton, though for ^{3}He sensitivity to the spin state is not as great. The important effect is again (3), the change in sign of some g_P terms which cause the HP results to violate the Low theorem, which gives almost a factor of 2 reduction from the present results.

As a numerical check our results using the coupling constants and form factors of HP and the changes (1), (2), and (3) were compared with those quoted in HP. Agreement is at the 10–15% level, and hence if the higher-order terms are responsible for the differences in the proton triplet rate then they seem not to be as large here. Comparison was also made with results of Beder in the appropriate limit with the result that the standard theory seems to be about 20% higher than Beder's quoted results. If g_P is given the wrong sign, however, as we have been informed[24] may have been the case in Ref. 15, agreement is at the 10% level.

Finally, perhaps the most useful result with regard to ^{3}He is the larger rate, by a factor of 2, obtained in the standard theory, which may help make a difficult experiment somewhat easier.

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