Consistency between pion exchange currents and N-N potential in doubly radiative n-p capture

A. Cambi
Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, Italy

B. Mosconi
Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, Italy
and Istituto di Fisica Teorica dell'Università di Firenze, Firenze, Italy

P. Ricci
Istituto Nazionale di Fisica Nucleare, Sezione di Firenze, Italy
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The consistency between the exchange currents in the pionic range and the Reid soft-core potential, which has an asymptotic behavior like V_{OPE}, is studied in the doubly radiative n-p capture process. The cross section is evaluated using the interaction Hamiltonian expressed by means of the nuclear current, that includes the two-body terms deriving from one-pion exchange. The gauge invariance is preserved by the generalized contact term. This cross section is compared to that obtained with the Siegert form of the interaction Hamiltonian. The difference between the values of the cross section, which is of the order of 24%, constitutes a quantitative estimate of the lack of consistency between pion exchange currents and the nuclear potential considered. The use of purely phenomenological wave functions leads to a great disagreement. Finally, the existence of an exchange effect of the order of 30% is pointed out in the process under examination, which occurs via the emission of two $E1$ photons.

[NUCLEAR REACTIONS Meson exchange currents, Reid soft-core potential and Siegert's theorem; $^1H(n,\gamma\gamma)$, thermal n, calculated $\sigma_{2\gamma}$.]

I. INTRODUCTION

The principal aim of this work is to give a quantitative estimate of the consistency between pion exchange currents (MEC) and N-N potential, taking the process of the doubly radiative n-p capture into consideration. To this end we compare the value of the cross section obtained using the nuclear current operator with that obtained using the charge density operator, which are connected through Siegert's theorem.[1] This last value will be assumed as the "comparison one," being model independent, as it will be shown in the following.

This reaction seems to be a convenient one because the use of Siegert's theorem is valid, being a low-energy process. At the same time, as it is a second order process, it involves off-shell matrix elements and, thus, it allows us to investigate the short range behavior of MEC and N-N potential. MEC operators are of course consistent with the long range part of an N-N potential, as the Reid soft-core potential[2] (RSC), since its asymptotic part is given by one-pion-exchange potential V_{OPE}. Their lack of consistency at short range can be pointed out in the reaction chosen because the virtual intermediate states have unlimited momenta. A similar investigation in single photon processes requires high momentum transfer, so that Siegert's theorem is no longer applicable.

The experimental and theoretical situation about the reaction $n+d\rightarrow d+2\gamma$ is briefly as follows. The most recent experimental result, which lowers considerably the first one obtained by Dress *et al.*,[3] is that of Wüst *et al.*,[4] and gives a value $\sigma_{2\gamma}=(-5.2\pm6.4)$ μb for photons in the energy range 330–1890 keV. Moreover, lowering the photon threshold to 233 keV, these authors find that the energy spectrum agrees with that expected for an $E1$-$E1$ emission mode. In agreement with this result, Earle and McDonald[5] have obtained an upper limit of 1.6 μb, for photons in the energy range 700–1520 keV.

In the meantime the problem has been studied in detail, from the theoretical point of view, in the framework of the usual electromagnetic interaction theory, i.e., without Adler's[6] conjecture about the amount of the nonorthogonality between ^3S bound and continuum wave functions. Grechukhin[7] demonstrated in 1971 that the electric dipole is the fundamental emission mode. Then the cross section for the $E1$-$E1$ mode has been calculated by several authors[8-11] still more carefully up to the value (0.1176 ± 0.0003) μb given by Blomqvist and Ericson.[12]

Emission mechanisms different from the dominant $E1$-$E1$, have been also evaluated. Bernabeu and Tarrach[13] have studied the emission of an $M1$ photon followed by a bremsstrahlung photon. The present authors[14] have calculated the influence of the exchange currents on the cross section for the $M1$-$M1$ mode. All these processes give negligible contributions to the dominant $E1$-$E1$ mode.

It is well known that for low-energy transitions, like those which come into play in this process, the continuity equation for the current allows the interaction Hamiltonian H_I to be expressed in two equivalent forms: one the "current operator" H_J by means of the current density, and the other, the "charge operator" H_ρ by means of the charge density. The mathematical formulation of the equivalence between these two operators is Siegert's theorem.[1] H_ρ is commonly used to calculate low-energy $E1$ transition amplitudes, neglecting retardation effects, relativistic corrections, and two-body exchange contributions to the charge density. With these approximations the charge operator assumes the Siegert form (3).

The results obtained with the two equivalent forms of H_I should be identical in principle in the long wavelength approximation. It is clear that this occurs only as one uses wave functions which are eigenfunctions of the nuclear Hamiltonian H. Furthermore, H must have eigenvalues coincident with the experimental values of the energies to be utilized. In general these conditions are never satisfied because, commonly, use is made of nuclear potentials, which only approximately give the experimental energies, and of wave functions which are approximations of the true eigenfunctions of H.

A simplification occurs in the case of the nuclear two-body problem where the wave functions can be exactly calculated once the N-N potential is given. Then, in order to have equivalence between the two approaches, it is necessary to use a nuclear current consistent with the potential, so that the continuity equation is satisfied.

In the low-energy processes, like the one under examination, it is usually assumed that a nuclear potential with the correct behavior at large r is sufficient. This behavior is determined by the exchange of one pion and the corresponding exchange potential is V_{OPE}.

It has been demonstrated by several authors, see for example Thompson and Heller,[15] that, up to terms M^{-2} in the nonrelativistic expansion in the inverse of the nucleon mass, the continuity equation is satisfied with V_{OPE}, if the exchange currents, commonly labeled pion current and pair current,[16] are added to the one-body current, while the charge density keeps its one-body form.

As regards the Δ_{33}-isobar excitation current, which has the same range as the pionic currents, we remember that it is not connected to the nuclear potential by the continuity equation, being divergenceless.[17] However, this current need not be considered in our case because the corresponding $E1$ operator vanishes as k^2 in the limit $k \to 0$.

At this point the outline of potential and electromagnetic operators appears consistent, but it is not so because any realistic potential must be modified from its correct OPE tail to reproduce the properties of the n-p system. In the RSC potential, as well as in many other potentials widely used in the literature, the part at intermediate and short range, which takes into account the exchange of mesons heavier than the pion, is determined in a phenomenological way. The corresponding terms are missing in the expression of the exchange current commonly used in the low-energy processes. The problem of the consistency between MEC and N-N potential, which is the object of our work, arises just from the absence of these terms.

A partial comparison between the results with H_ρ and H_J for the total cross section of this reaction has been made by Lee and Khanna,[10] considering only the convective current and the S part of the deuteron state.

A last consideration concerns the exchange effect in this reaction which occurs essentially with a double $E1$ emission. At first sight it can appear surprising that such an effect exists because Siegert's theorem, which allows us to replace the two body current density with the one-body charge density, is sometimes interpreted as demonstrating that there are no exchange effects in the low-energy $E1$ transitions. Formally, this interpretation immediately breaks down because the charge density also acquires two-body contributions[18] beginning with the second order terms in the nonrelativistic expansion in $1/M$ of the same OPE processes leading to the exchange currents. But just because of their dependence on the nuclear mass, we can neglect these two-body terms[19,20] and assume the usual one-body form of the charge density. Nevertheless, the matrix elements of the Siegert operator include the effects of the total (impulse and exchange) current because the meson exchange between the nucleons determines not only the electromagnetic operators but also the nuclear Hamiltonian and, thus, the wave functions. Comparing the cross section obtained with H_ρ and with H_J, we shall be able to distinguish the contributions of the one-body transition operators from the two-body ones, it being understood that the influence of mesons on the wave functions makes a clear distinction between one-body and two-body effects ultimately impossible.

Finally, we recall that an analogous comparison has been made by Bassani *et al.*[21] for the process of two photon absorption from the 1*s* to the 2*s* state of the atomic hydrogen. Their starting point was the problem of the choice of the electric dipole interaction $\vec{E}\cdot\vec{r}$ or $\vec{A}\cdot\vec{p}$, which are related by a gauge transformation of the electromagnetic potentials, in connection with the proper choice of the eigenstates of the unperturbed Hamiltonian. These authors have shown that both forms of H_I give the same transition amplitude for the considered process, using the same unperturbed wave functions. Hence their numerical problem is equivalent to ours, with the double advantage that the electron current is purely convective and the eigenstates of the unperturbed Hamiltonian are exactly known. In fact, the numerical results are the same, but the sum over the intermediate states converges to the final result in a very different way in the two cases. With the $\vec{E}\cdot\vec{r}$ operator, an excellent approximation is obtained considering just a few intermediate states of the discrete spectrum, the continuum giving negligible effects. On the contrary, when $\vec{p}\cdot\vec{A}$ is used, the continuum states give more than half the total value.

In Sec. II we have calculated $\sigma_{2\gamma}(E1\text{-}E1)$ for the n-p capture independently of the form of the interaction Hamiltonian. In Sec. III the calculation with H_ρ is outlined and in Sec. IV, that with H_J is more extensively reported. In Sec. V we have reported the numerical results obtained with H_J and we have discussed them in comparison with that obtained with H_ρ. In Sec. VI we have stated our conclusions.

II. $E1$-$E1$ CROSS SECTION

As is well known the interaction Hamiltonian for the emission of a photon with momentum $\vec{k}$, energy ω, and polarization λ, which is given in Coulomb gauge by

$$H_I(\vec{k})=-\left(\frac{2\pi}{\omega}\right)^{1/2}\int d^3x\,\vec{j}(\vec{x})\cdot\vec{\epsilon}_{\vec{k},\lambda}e^{-i\vec{k}\cdot\vec{x}}\,, \tag{1}$$

where $\vec{\epsilon}_{\vec{k},\lambda}$ is the polarization vector and $\vec{j}(\vec{x})$ is the nuclear electromagnetic current density, gives rise to transition amplitudes for second order processes which are not gauge invariant. In order to preserve the gauge invariance, a gauge (or contact) term must be added to the dispersive part of the amplitude coming from H_I in the second order perturbation expansion. Therefore, the amplitude for the transition from the initial state $|i\rangle$ to the final state $|f\rangle$ with emission of two photons has the form

$$M_{fi}=\sum_n (1+P_{12})\frac{\langle f|H_I(\vec{k}_2)|n\rangle\langle n|H_I(\vec{k}_1)|i\rangle}{E_i-E_n-\omega_1}+M^G_{fi}\,, \tag{2}$$

where the summation over n means sum over the discrete quantum numbers and integration over the continuous ones of the intermediate state $|n\rangle$; E_i and E_n are the energies of the state $|i\rangle$ and $|n\rangle$, respectively, and the operator P_{12} changes photon 1 into photon 2. Since we are considering the $E1$-$E1$ transitions in thermal n-p capture, the energies of the emitted photons are low ($\omega_1+\omega_2=B$, if we neglect nuclear recoil, where B is the deuteron binding energy). Therefore, we can take the long wavelength limit of the theory. When $k\to 0$ H_I can be transformed by means of the continuity equation so that the current density is replaced by the charge density $\rho(\vec{x})$. This is the content of Seigert's theorem,[1] while the $E1$ interaction Hamiltonian assumes the Siegert form

$$H_\rho=-i\left(\frac{2\pi}{\omega}\right)^{1/2}[H,\vec{\epsilon}_{\vec{k},\lambda}\cdot\vec{D}]\,, \tag{3}$$

where H is the nuclear Hamiltonian and $\vec{D}$ the nuclear electric-dipole momentum

$$\vec{D}=\int d^3x\,\vec{x}\rho(\vec{x})\,, \tag{4}$$

which, in the case of point nucleons of coordinate $\vec{r}_j$, become

$$\vec{D}=e\sum_j\frac{1+\tau_3(j)}{2}\vec{r}_j\,. \tag{5}$$

The current form H_J of the $E1$ interaction Hamiltonian in the limit $k\to 0$ follows simply from (1) as

$$H_J=-\left(\frac{2\pi}{\omega}\right)^{1/2}\vec{\epsilon}_{\vec{k},\lambda}\cdot\vec{J}\,, \tag{6}$$

where $\vec{J}$ is the Fourier transform of $\vec{j}(\vec{x})$ evaluated at $\vec{k}=0$. As said in the Introduction, the aim of this work is to compare the numerical results obtained with these two equivalent forms of H_I, when the nuclear potential contains V_{OPE} as exchange potential. Therefore, the nuclear current density in (6) must include, besides the convective current, the exchange current which is the sum of the two-body currents corresponding to the processes of pion exchange and pair excitation.[16] In fact, the magnetization part of the one-body current, as well as the two-body current corresponding to the excitation of the Δ_{33} resonance, goes to zero as k^2 in the limit $k\to 0$.

Explicitly we have for our two-body problem

$$\vec{J}_c=-i\frac{e}{4M}(\vec{\tau}_1-\vec{\tau}_2)_3(\vec{\nabla}-\overleftarrow{\nabla})\,, \tag{7a}$$

$$\vec{j}_{ex}=\tfrac{2}{3}ef^2_{\pi NN}\sqrt{\pi}(\vec{\tau}_1\times\vec{\tau}_2)_3 \times\left[e^{-\mu r}\Omega^{(1)}_{1,0}+\phi_2(\mu r)\sum_L(2\hat{L})^{1/2}\begin{pmatrix}2&1&L\\0&0&0\end{pmatrix}\Omega^{(1)}_{L,2}\right], \tag{7b}$$

where $\hat{L}=2L+1$, $\vec{r}$ is the relative coordinate, M and μ are the nucleon and pion masses, $f_{\pi NN}$ is the π-N coupling constant ($f^2_{\pi NN}\simeq 0.08$), the tensors $\Omega^{(j)}_{L,\rho}$ are defined by

$$\Omega^{(j)}_{L,\rho}=[Y^{(L)}(\hat{r})\otimes(\vec{\sigma}_1\otimes\vec{\sigma}_2)^{(\rho)}]^{(j)}, \tag{8}$$

$Y^{(L)}(\hat{r})$ being the spherical harmonics, $\hat{r}=\vec{r}/r$, and

$$\phi_2(x)=e^{-x}\left(1+\frac{3}{x}+\frac{3}{x^2}\right). \tag{9}$$

As far as the gauge amplitude is concerned, its complete expression generalized to the case in which the exchange currents are considered, and valid for any value of the photon momenta, has been obtained by Friar.[22] In the $E1$-$E1$ long wavelength limit, it reduces to the form given by Sachs and Austern[23]

$$M^G_{fi}=\langle f|[[H,\vec{\epsilon}_{\vec{k}_1\lambda_1}\cdot\vec{D}],\vec{\epsilon}_{\vec{k}_2\lambda_2}\cdot\vec{D}]|i\rangle, \tag{10}$$

if the nuclear Hamiltonian contains V_{OPE} as exchange potential.

Because the exchange current has a complicated dependence on nuclear variables, it is convenient for the calculation of the cross section to express the transition amplitude by means of the $E1$-$E1$ generalized polarizabilities[24] $\langle f|P_{jm}|i\rangle$ defined by

$$M_{fi}=\sum_{jmM_1M_2}(-)^m\sqrt{\hat{j}}\begin{pmatrix}1&1&j\\M_1&M_2&-m\end{pmatrix}D^{(1)}_{M_1\lambda_1}(R_1) \times D^{(2)}_{M_2\lambda_2}(R_2)\langle f|P_{jm}|i\rangle, \tag{11}$$

where R_i is the rotation which makes the z axis coincident with the $\vec{k}_i$ direction, and $D^{(1)}(R_i)$ is the corresponding rotation matrix. The advantage is that $\langle f|P_{jm}|i\rangle$ are the matrix elements of irreducible tensors of rank j. Therefore we can use the standard procedure, based on the Wigner-Eckart theorem, to factorize the dependence on geometrical factors while all the dependence on dynamics is included in the reduced matrix elements. Then we can easily perform the sum and the average over the magnetic quantum numbers of the nuclear states, and the sum over the polarization of the photons, by means of the Clebsch-Gordan series for the rotation matrices and the sum rules for the Racah coefficients. It follows for the total cross section, the expression

$$\sigma_{2\gamma}=\frac{1}{36\pi^3 v}\int_0^B d\omega_1(\omega_1\omega_2)^2\sum_j|\langle f\|P_j\|i\rangle|^2, \tag{12}$$

where v is the n-p relative velocity in the initial state.

Before concluding this general discussion which is independent of the form chosen for H_I, we have to remember that in the thermal n-p capture the initial state can be a spin singlet or triplet $l=0$ state, but only the triplet one can have $E1$-$E1$ transitions to the final deuteron state. Therefore, following a standard notation for the deuteron wave function and denoting with $z(r)$ the reduced radial wave function of the continuum $^3S_1$ state, the initial and final states are

$$|i\rangle=\frac{z(r)}{r}\mathcal{Y}^M_{011}\eta_0, \qquad |f\rangle=\sum_{l=0,2}\frac{u_l(r)}{r}\mathcal{Y}^M_{l11}\eta_0, \tag{13}$$

where η_0 is the singlet isospinor and $\mathcal{Y}^M_{LSJ}$ are the usual spin-angle functions.

For the sake of clarity, we must divide our treatment into two parts corresponding to the Siegert and "current" form of the $E1$ operator, because several details of the calculation are different in the two cases. We begin with the Siegert operator in order to obtain, shortly, in our formalism the results of Blomqvist and Ericson[12] and Lee and Khanna.[10] The numerical value of the cross section so obtained will be the comparison value in the next section where the cross section is evaluated with the "current" operator.

III. SIEGERT OPERATOR

When one works with the Siegert form of H_I, the dispersive part of the transition amplitude can be transformed, as noted by several authors,[8,10,12] in such a way that the gauge part is exactly canceled. It follows that the total amplitude is given by

$$M_{fi}=-2\pi(\omega_1\omega_2)^{1/2} \times\sum_n(1+P_{12})\frac{\langle f|\vec{\epsilon}_{\vec{k}_2\lambda_2}\cdot\vec{D}|n\rangle\langle n|\vec{\epsilon}_{\vec{k}_1\lambda_1}\cdot\vec{D}|i\rangle}{E_i-E_n-\omega_1} \tag{14}$$

and the corresponding reduced polarizabilities by

$$\langle f\|P_j\|i\rangle=(-)^{1+J_i+J_f}2\pi(\omega_1\omega_2)^{1/2}\sum_n[1+(-)^jP_{12}]\sqrt{\hat{j}}\begin{Bmatrix}1&1&j\\J_i&J_f&J_n\end{Bmatrix}\frac{\langle f\|\vec{D}\|n\rangle\langle n\|\vec{D}\|i\rangle}{E_i-E_n-\omega_1}. \tag{15}$$

Here J_i, J_f, and J_n are the total angular momentum of the initial, final, and intermediate states. Since for the two-body system the $\vec{\mathrm{D}}$ operator is a part the isospin factor, the relative coordinate, the possible intermediate states are spin triplet $l=1$ states. Assuming the effect of the interaction in these $|{}^3P_{J_n}\rangle$ states to be negligible, their wave functions become

$$|{}^3P_{J_n}\rangle = \sqrt{4\pi}\, j_1(pr)\mathcal{Y}^M_{11J_n}(\hat{r})\eta_{1,m}, \tag{16}$$

$\vec{\mathrm{p}}$ being the n-p relative momentum. In this approximation the radial matrix elements in (15) are independent of the angular momentum J_n, so that the sum over J_n can be easily performed with the result

$$\langle f \| P_j \| i\rangle = Me^2(\omega_1\omega_2)^{1/2}(1+P_{12}) \times[\delta_{j,0}K_0(\omega_1)+(\tfrac{2}{5})^{1/2}\delta_{j,2}K_2(\omega_1)], \tag{17}$$

where $\delta_{i,j}$ is the Kronecker symbol. The functions $K_l(\omega)$, with $l=0,2$, are defined by

$$K_l(\omega) = \int dp \frac{p^2}{p^2+M\omega} I_{u_l}(p) I_z(p) \tag{18}$$

with

$$I_f(p) = \int dr\, r^2 j_1(pr) f(r) . \tag{19}$$

From the expression (17) of the reduced matrix elements of P_{jm}, it is clear that the total cross section is the incoherent sum of $\sigma^S_{2\gamma}$ and $\sigma^D_{2\gamma}$ which correspond to the $l=0$ and $l=2$ parts of the deuteron state. When we take the asymptotic form of the wave functions

$$\begin{aligned} u_0(r) &= Ne^{-\alpha r}, \\ u_2(r) &= N\rho\phi_2(\alpha r), \\ z(r) &= \sqrt{4\pi}(r-a_t), \end{aligned} \tag{20}$$

where $\alpha=\sqrt{MB}$, N is the normalization constant, ρ the asymptotic D to S ratio in the deuteron wave function, and a_t the triplet scattering length, we obtain

$$\sigma^S_{2\gamma} = \bar{\sigma}_{2\gamma}\left[\frac{191}{10}+\pi-32\ln 2-\frac{3}{5}\frac{1}{\alpha a_t}+\frac{3}{2}\frac{1}{(\alpha a_t)^2}\right], \tag{21}$$

$$\sigma^D_{2\gamma} = \frac{\rho^2}{10}\bar{\sigma}_{2\gamma}\left[54+\frac{187}{16}\pi-128\ln 2 -\frac{120}{7}\frac{1}{\alpha a_t}+\frac{75}{2}\frac{1}{(\alpha a_t)^2}\right], \tag{22}$$

where

$$\bar{\sigma}_{2\gamma} = \frac{e^4}{v}\frac{N^2}{9\alpha}\left(\frac{B}{M}\right)^{5/2} a_t^{\,2} . \tag{23}$$

$\sigma^S_{2\gamma}$ reproduces the result given by Blomqvist and Ericson[12] and by Lee and Khanna,[10] while $\sigma^D_{2\gamma}$ is slightly different from the expression obtainable from formula (30) in Ref. 10. Because of the factor $\rho^2 \simeq 7\times 10^{-4}$ in $\sigma^D_{2\gamma}$, the value of the total cross section evaluated with the Siegert operator is determined by $\sigma^S_{2\gamma}$, and also by its model independent expression (21). In fact, as shown in Refs. 12 and 10, both the regularization of the wave functions at the origin and the n-p interaction in the intermediate states give an effect of the order of one part in 10^3. For all these reasons the cross section obtained with H_ρ can be considered the correct one. As for its numerical value, Blomqvist and Ericson,[12] using the best values[25] for the low-energy n-p parameters entering in (21), obtain the very accurate value quoted in the Introduction.

In the next section, in order to make the comparison with the value of the cross section calculated with H_J, we can assume $\sigma_{2\gamma}=0.118$ μb as that corresponding to H_ρ.

IV. "CURRENT" OPERATOR

The partial cancellation between the dispersive and the gauge part of the transition amplitude leading to (14) does not occur when the "current" form of H_I is used in (2). Therefore, also the generalized polarizabilities are composed of two terms

$$\langle f|P_{jm}|i\rangle = \langle f|P^D_{jm}|i\rangle + \langle f|P^G_{jm}|i\rangle . \tag{24}$$

The reduced matrix element of the first one, corresponding to the dispersive amplitude, derives from (15) with the substitution

$$\vec{\mathrm{D}} \to i\frac{\vec{\mathrm{J}}}{(\omega_1\omega_2)^{1/2}}, \tag{25}$$

and so we can avoid writing it explicitly. In order to have the generalized polarizabilities corresponding to the gauge amplitude (10) we must express the polarization vectors of the photons by means of the rotation matrices and develop the double commutator, recalling the working hypothesis that the nuclear potential contains V_{OPE}. Then with the standard recoupling techniques, the terms can be rearranged in irreducible tensors, dividing the angular from the spin variables, with the result

$$\langle f\|P^G_j\|i\rangle = \frac{4\pi^{3/2}e^2 f^2_{\pi NN}}{(3\omega_1\omega_2)^{1/2}}(\tau_{1x}\tau_{2x}+\tau_{1y}\tau_{2y})\begin{pmatrix}1&1&j\\0&0&0\end{pmatrix} \times\langle f\| re^{-\mu r}\Omega^{(j)}_{j,0} - \sum_\lambda (2\hat{\lambda})^{1/2}\begin{pmatrix}2&j&\lambda\\0&0&0\end{pmatrix} r\phi_2(\mu r)\Omega^{(j)}_{\lambda,2}\|i\rangle . \tag{26}$$

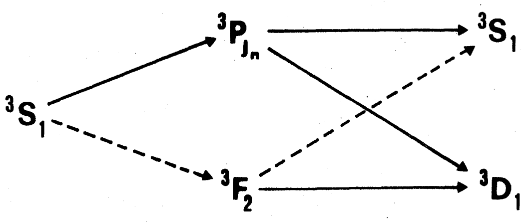

FIG. 1. $E1$-$E1$ transitions in $n+p\rightarrow d+2\gamma$. A full line indicates the transitions due to $\vec{j}_{ex}$ and $\vec{j}_c$, while a dashed line indicates those due to $\vec{j}_{ex}$ only.

We have not reported the term relative to the Thompson amplitude, which derives from the kinetic energy (and corresponding to the gauge term $A^2/2m$ of the atomic Hamiltonian), because it vanishes owing to the orthogonality between the wave functions of the initial and final states.

With respect to the calculation with H_ρ, another complication arises concerning the dispersive part in the amplitude, in addition to the need to consider the gauge term separately. The summation over the intermediate states extends to the $^3F_2$ channel besides the $^3P_{J_n}$ channels. In fact, the operator $\Omega^{(1)}_{3,2}$, included in the exchange current, allows the transition with $\Delta l=3$ from the initial $^3S_1$ state to the $^3F_2$ state, from which the S part of the deuteron can be reached through the same operator and the D part also through $\Omega^{(1)}_{1,0}$, $\Omega^{(1)}_{1,2}$ and the gradient operator.

In conclusion, the possible $E1$-$E1$ transitions we must take into account in this case are reported in Fig. 1, where we have indicated with a dashed line the transitions due to $\vec{j}_{ex}$ and with a full line those possible with both $\vec{j}_{ex}$ and $\vec{j}_c$.

As far as the wave functions of the intermediate states are concerned, we remember that Lee and Khanna[10] have evaluated the effect of the n-p interaction in the $^3P_{J_n}$ states using only the convective part of the nuclear current. The enhancement of the cross section, which was unappreciable in the calculation with H_ρ, becomes about 5%, which is a small effect even if not completely negligible. Because it seems reasonable to assume that the interaction effect maintains this order of magnitude when the exchange current is added, we have considered noninteracting intermediate states only. With this hypothesis the reduced matrix elements of the nuclear current for the possible transitions become

$$\langle {}^3P_{J_n}\|\vec{\mathrm{j}}\|{}^3S_1\rangle = i(-)^{1+J_n}\frac{e}{M}\left(\frac{\hat{J}_n\pi}{3}\right)^{1/2}\left[A^1_z(p)+12(-)^{J_n}\begin{Bmatrix}2&1&1\\J_n&1&1\end{Bmatrix}B^1_z(p)\right], \tag{27a}$$

$$\langle d\|\vec{\mathrm{j}}\|{}^3P_{J_n}\rangle = -i\frac{e}{M}\left(\frac{\hat{J}_n\pi}{3}\right)^{1/2}\left\{A^1_{u_0}(p)+12(-)^{J_n}\begin{Bmatrix}2&1&1\\J_n&1&1\end{Bmatrix}B^1_{u_0}(p)-(-)^{J_n}3\sqrt{2}\right.$$
$$\left.\times\left[\begin{Bmatrix}2&1&1\\J_n&1&1\end{Bmatrix}A^1_{u_2}(p)+30\sum_L\hat{L}\begin{pmatrix}1&2&L\\0&0&0\end{pmatrix}^2\begin{Bmatrix}2&L&1\\1&2&1\\1&1&J_n\end{Bmatrix}B^1_{u_2}(p)\right]\right\}, \tag{27b}$$

$$\langle {}^3F_2\|\vec{\mathrm{j}}\|{}^3S_1\rangle = 6i\left(\frac{2\pi}{5}\right)^{1/2}\frac{e}{M}B^3_z(p), \tag{27c}$$

$$\langle d\|\vec{\mathrm{j}}\|{}^3F_2\rangle = 6i\left(\frac{2\pi}{5}\right)^{1/2}\frac{e}{M}\left[B^3_{u_0}(p)-\frac{1}{\sqrt{8}}[A^3_{u_2}(p)+2B^3_{u_2}(p)]\right]. \tag{27d}$$

The radial matrix element $B^l_f(p)$ is defined as

$$B^l_f(p)=\tfrac{4}{3}Mf^2_{\pi NN}\int dr\, rj_l(pr)\phi_2(\mu r)f(r), \tag{28}$$

while $A_f{}^l(p)$ is given by

$$A_f{}^l(p)=I_f{}^l(p)-J_f{}^l(p) \tag{29}$$

with

$$J_f{}^l(p)=\tfrac{4}{3}Mf^2_{\pi NN}\int dr\, rj_l(pr)e^{-\mu r}f(r),$$
$$I_f{}^l(p)=\int dr\left[rj_l(pr)\frac{df(r)}{dr}-f(r)\frac{d}{dr}[rj_l(pr)]+(3l_f-2)j_l(pr)f(r)\right], \tag{30}$$

where l_f indicates the orbital angular momentum of the state whose radial wave function is $f(r)$.

For the gauge term we obtain

$$\langle d\|P^G_j\|{}^3S_1\rangle = -\frac{4e^2\pi f^2_{\pi NN}}{3(\omega_1\omega_2)^{1/2}}\left[\delta_{j,0}(R_0+\sqrt{8}S_2)-\delta_{j,2}\frac{4}{\sqrt{5}}\left(S_0+\frac{1}{\sqrt{8}}(R_2-2S_2)\right)\right], \tag{31}$$

with the definitions

$$R_l=\int dr\, u_l(r)re^{-\mu r}z(r),$$
$$S_l=\int dr\, u_l(r)\phi_2(\mu r)z(r). \tag{32}$$

Since the radial wave functions of the intermediate states have been considered spin independent, all the dependence on the angular momentum J_n is contained in the (6-j) and (9-j) coefficients. Then by means of their sum rules we can make the summation over J_n in the dispersive part of the reduced generalized polarizabilities which transforms into the sum of three terms relative to the values of $j=0, 1, 2$:

$$\langle d\|P_j\|i\rangle = \frac{e^2}{(\omega_1\omega_2)^{1/2}}\frac{[1+(-)^j P_{12}]}{12\pi M} \times\{\delta_{j,0}[M_0(\omega_1)+2N(\omega_1)] + \delta_{j,1}[M_1(\omega_1)+\sqrt{3}N(\omega_1)] + \delta_{j,2}[M_2(\omega_1)+(1/\sqrt{5})N(\omega_1)]\}. \quad (33)$$

The quantity $N(\omega_1)$ corresponding to the intermediate states $^3F_2$ is given by

$$N(\omega_1) = \tfrac{36}{5}\sqrt{4\pi}\int dp\frac{p^2}{p^2+M\omega_1}B_z^3(p) \times\left[B_{u_0}^3(p) - \frac{1}{\sqrt{8}}[A_{u_2}^3(p)+2B_{u_2}^3(p)]\right] \quad (34)$$

and the $M(\omega_1)$, corresponding to the intermediate state $^3P_{J_n}$, are expressible as, omitting the dependence from ω_1,

$$\begin{aligned} M_0 &= 3F_0 + \tfrac{48}{5}K_0 - \frac{24}{\sqrt{8}}(G_2+\tfrac{2}{5}H_2+\tfrac{4}{5}K_2),\\ M_1 &= -\sqrt{3}\,\frac{36}{5}\left[K_0 - \frac{1}{\sqrt{8}}(H_2+2K_2)\right],\\ M_2 &= \frac{12}{\sqrt{5}}\left[G_0+H_0+\tfrac{7}{5}K_0 - \frac{1}{\sqrt{8}}(F_2+2G_2+\tfrac{7}{5}H_2+\tfrac{54}{5}K_2)\right], \end{aligned} \quad (35)$$

by means of the functions

$$\begin{aligned} F_l(\omega) &= 4\pi\int dp\frac{p^2}{p^2+M\omega}A_{u_l}^1(p)A_z^1(p),\\ G_l(\omega) &= 4\pi\int dp\frac{p^2}{p^2+M\omega}B_{u_l}^1(p)A_z^1(p),\\ H_l(\omega) &= 4\pi\int dp\frac{p^2}{p^2+M\omega}A_{u_l}^1(p)B_z^1(p),\\ K_l(\omega) &= 4\pi\int dp\frac{p^2}{p^2+M\omega}B_{u_l}^1(p)B_z^1(p), \end{aligned} \quad (36)$$

where the index $l=0, 2$ means transitions to the S, D part of the deuteron. From expressions (33) and (35) we can immediately observe that these transitions contribute simultaneously to the different j components of the generalized polarizabilities. Therefore the cross section has terms from S-D interference, unlike the Siegert operator. Furthermore, we note that because of the behavior of the operator $\vec{j}_{ex}$ for $r\to 0$, the use of the wave functions in their asymptotic form leads to divergent matrix elements. Therefore, we must consider wave functions with the correct behavior at the origin. With the aim of making as many calculations as possible analytically, we have assumed the deuteron wave functions to have the form

$$\begin{aligned} u_0(r) &= N\sum_i c_i e^{-\alpha_i r},\\ u_2(r) &= N\rho\sum_i d_i\phi_2(\beta_i r), \end{aligned} \quad (37)$$

which is the analytic form derived by Gourdin *et al.*[26] from the dispersive analysis of the n-p-d vertex in the nonrelativistic limit. The coefficients must satisfy the following sum rules:

$$\sum_i c_i = \sum_i d_i = \sum_i d_i\beta_i^2 = \sum_i \frac{d_i}{\beta_i^2} = 0. \quad (38)$$

As for the continuum $^3S_1$ state we have taken the parametrization

$$z(r) = \left(1+\sum_i g_i e^{-\gamma_i r}\right)(4\pi)^{1/2}(r-a_t) \quad (39)$$

with the condition $1+\sum_i g_i = 0$.

V. NUMERICAL RESULTS

The N-N potential considered in this work is the Reid soft-core potential.[2] Therefore, we have made a fit with the expression (37) to the deuteron wave function tabulated by the author, and with the form (39) to the scattering wave function obtained by numerical integration of the system of Schrödinger equations for the ^3S-^3D coupled channels at zero energy. We remember that the triplet scattering length given by the RSC potential is $a_t = 5.39$ fm, the D-state probability is $P_D = 6.47\%$ and the asymptotic D- to S-wave ratio is $\rho = 0.026\,223$. In Tables I and II we report the value

TABLE I. Parameters of the fit in expression (37) for the RSC deuteron wave function.

i	c_i	α_i (fm^{-1})	d_i	β_i (fm^{-1})
1	1.000	0.2316	1.000	0.2316
2	1115.640	4.6677	−11 686.137	3.2179
3	−101.330	4.3700	−399.674	4.7240
4	−42.167	4.1703	−81.167	1.4294
5	−0.366	1.2344	12 389.894	3.4068
6	−941.277	4.6854	82.495	7.1600
7	−142.490	6.6890	−1 382.949	4.2093
8	110.990	6.9821	1 076.538	2.4697

TABLE II. Parameters of the fit in expression (39) for the RSC $^3S_1$ scattering state.

i	g_i	γ_i (fm^{-1})
1	−3.388	2.125
2	2307.684	8.753
3	−2412.369	8.822
4	247.455	11.050
5	−140.382	11.720

of the other parameters of our fit to the Reid set of wave functions.

Because we are interested in seeing how much the results change with phenomenological wave functions, we have also used the wave functions existing in the literature and parametrized with the form (37) and (39). While the parametrization (37) of $u_0(r)$ as the sum of exponentials is nearly of general use and so would allow many different choices, that of $u_2(r)$ drastically reduces their number. In fact, besides the various sets of parameters of Gourdin *et al.*,[26] among which we have chosen No. 1, that provide a good fit to the Garthenaus wave function, we have found only the fit of McGee[27] to the Hamada-Johnston wave function, modified so that it vanishes at $r=0$, rather than at the hard-core radius. Since we must change the asymptotic behavior of the McGee fit in order to have the correct deuteron binding energy, we are really dealing with a phenomenological wave function in this case too. Moreover, the $^3S_1$-continuum wave function, which we must add to complete the set, is partially arbitrary because the condition of orthogonality with the $^3S_1$-bound wave function is not sufficient to define it completely. A satisfactory solution to this problem for the modified McGee wave function has been given by Lee and Khanna,[10] with the triplet scattering length fixed at the value $a_t = 5.41$ fm, which is very close to the best experimental value.[25] We refer to their Table II for the values of the other parameters. From now on we will call this set of deuteron and $^3S_1$-continuum wave functions the "McGee set." To complete the "Gourdin set" we have used in (39) the set of parameters obtained by Durand,[28] which gives a $z(r)$ orthogonal to the $u_0(r)$ of Gourdin *et al.* with $a_t = 5.406$ fm. The phenomenological wave functions chosen have a soft-core behavior at the origin and predict deuteron properties similar to those predicted by the Reid potential. In fact, the D-state probability corresponding to the Gourdin and McGee wave function is $P_D = 6.5\%$ and 7%, whereas the asymptotic D- to S-wave ratio is $\rho = 0.0265$ and 0.0269, respectively.

Our numerical results for the total cross section are reported in Table III for these three sets of wave functions, considering both the ^3S part of the deuteron state (normalized to $1-P_D$) and the complete $^3S+{}^3D$ state.

The values in the first column correspond to the contributions of the convective current alone, those in the other columns to the total (impulse + exchange) current. Furthermore, in the second column only the transitions through the $^3P_{J_n}$ states are considered, while in the last one those through the $^3F_2$ channel are also taken into account. In both cases we have included the contribution of the generalized gauge term which vanishes when the exchange currents are not considered. Before making any comment on the results in Table III, we recall that a comparison between the total cross section calculated with the Siegert operator and the "current" operator has been made by Lee and Khanna,[10] but neglecting the exchange currents and the deuteron D state. Their results

TABLE III. Total cross section in $n+p \to d+2\gamma$ reaction evaluated with the three sets of wave functions (Gourdin, McGee, Reid). The label ^3S indicates that only the S part of the deuteron wave function has been considered, and the label $^3S+{}^3D$ indicates that the D part is included. The values in the column headed ^3P correspond to transitions through the intermediate channels $^3P_{0,1,2}$ only, while those in the column $^3P+{}^3F$ include the $^3F_2$ intermediate state too.

		Total cross sections in μb		
Wave functions	Deuteron state	Without exchange currents	With exchange currents ^3P	With exchange currents $^3P+{}^3F$
Gourdin set	^3S	0.0876	0.266	0.339
	$^3S+{}^3D$	0.0878	0.206	0.273
McGee set	^3S	0.0902	0.216	0.269
	$^3S+{}^3D$	0.0906	0.153	0.195
Reid set	^3S	0.0830	0.155	0.183
	$^3S+{}^3D$	0.0831	0.118	0.146

are then equivalent to ours in the first column and, more precisely, to that labeled ^3S for every set of wave functions. In the common case of the McGee set we reproduce their value. From the other values in the first column we can see that the cross section changes very little when the deuteron D state also is taken into account, because there is not S-D interference when only the impulse current is considered and the corresponding contributions are in the same ratio $1/\rho^2$ as in the calculation with H_ρ.

Unlike the Siegert operator, the "current" operator produces a large model dependence of the cross section. In Ref. 10 the considerable variation between the value obtained with hard-core and soft-core wave functions was pointed out. Our results in the first column emphasize the model dependence of the cross section for different soft-core wave functions, and those in the other columns show that even more pronounced variations arise when the exchange currents are included in the calculation.

The second feature of the calculation with the Siegert operator we have to compare is the dependence of the result on the short range behavior of the wave functions. We think that a meaningful evaluation of this effect on the cross section calculated with the "current" operator is not possible. The reason is that any determination of the asymptotic cross section is in this case highly questionable. In fact, with the choice of the experimental values of the parameters the asymptotic cross section (0.21 μb) is grossly overestimated if at the same time the gauge term is neglected, as in Ref. 10. If, instead, the spurious contribution of the gauge term is considered, the asymptotic cross section drops to 0.05 μb. Since on the other hand it is also not sensible, in our opinion, to take $a_t = \alpha^{-1} = 4.31$ fm in order to preserve the orthogonality of the wave functions, we must conclude that there is not any acceptable way for calculating the asymptotic cross section. Furthermore, we want to point out that all we have said regards the convective current, because the matrix elements of the exchange currents are divergent with the asymptotic wave functions. We think rather that the values of the cross section obtained with the impulse current alone are interesting because they allow us to make an evaluation of the exchange effect in the process under examination which occurs with a double $E1$ emission. In analogy with the interaction effect in the single photon n-p capture, which has been explained with the contributions of the pionic exchange currents and the Δ_{33}-resonance excitation current,[29,30] we call here exchange effect the percentage difference between the total cross section and the contribution of the one-body current. This value follows obviously from the difference between the cross section calculated with the Siegert operator and the convective current alone. Therefore, the exchange effect exists also in this $E1$-$E1$ process and is about 30%, a value higher than the 10% of the single photon n-p capture, which occurs with an $M1$ emission.

If we observe the values reported in the last two columns of Table III, we note that the effect produced by the inclusion of the pionic exchange currents is extremely model dependent. The percentage variations among the three cases considered, which are less than 10% in the calculation with the impulse current, become 30% in the comparison between σ (Reid) and σ (McGee), 40% between σ (McGee) and σ (Gourdin), and even 80% between σ (Reid) and σ (Gourdin). Evidently the short range behavior of the exchange operators produces considerable differences in the cross section, even if the soft-core wave functions used give very similar properties of the two-body system.

Inside every set of wave functions we can observe the importance of the D part of the deuteron. In fact, because of the negative interference between the transition amplitudes to the S and D part of the deuteron, the cross section drops, on the average, more than 20%.

From the second column in Table III, which corresponds to transitions through the intermediate channels $^3P_{J_n}$, we see that with the complete deuteron wave function the cross section becomes 0.206 μb with the Gourdin set, 0.153 μb with the McGee set, and 0.118 μb with the Reid set. That is, with the wave functions derived from the Reid soft-core potential, we obtain again the value of 0.118 μb which is the model independent result of the calculation made with the Siegert operator.

As we can see from the third column, the inclusion of the intermediate state $^3F_2$ reduces the agreement between the values of the cross section calculated in the two different ways. In fact, $\sigma_{\rm Reid}$ changes from 0.118 to 0.146 μb and proportionally the cross section increases in the other two cases.

These values are the final result of our calculations and we think they can allow us to reach a quantitative conclusion on the consistency between the nuclear potential and the pionic exchange currents. In fact, the only effect we have neglected is the N-N interaction in the intermediate states that, as pointed out above, we can consider to be of the order of a few percent.

As regards the dependence of our results on the wave functions, we can observe that the value most different from the comparison one is that of the Gourdin set, which has purely phenomenological wave functions. A closer value is obtained with the McGee set where the ^3S scattering wave function is purely phenomenological, while the deuteron wave function corresponds to the Hamada-Johnston potential. The best value corresponds to the Reid set, where the ^3S scattering wave function has also been calculated by integrating the system of Schrödinger equations for the Reid soft-core potential.

It seems reasonable to conclude that the agreement is increasingly better as the wave functions are less independent of the nuclear Hamiltonian and the current density operators. In fact, we have a complete equivalence between the calculations made with the Siegert operator and those made with the two-body currents used, only if the exchange part of the N-N potential is V_{OPE}. On one hand this explains the better agreement obtained with the Reid set, on the other hand, it justifies the residual difference which can be ascribed to the medium and short range part of the interaction, owing to the exchange of heavier mesons. This part is taken into account in a phenomenological way by the Reid potential, but it is not contained in the exchange currents considered, which are those usually employed in low-energy processes. We must conclude that these short range effects are not negligible in the case under examination, because we must ascribe to them the residual difference of 20%.

VI. CONCLUSIONS

We have calculated the $E1$-$E1$ cross section for the doubly radiative n-p capture reaction, using the current operator, and taking into account the meson exchange currents in the pionic range and the generalized contact term, so that the transition amplitude is gauge invariant.

The principal aim of this work is to give a quantitative estimate of the consistency between exchange currents in the pionic range and N-N potential with the correct V_{OPE} behavior at large distances. To this end we compare the results obtained with H_J to that obtained with H_ρ. We have also briefly reported the calculation with H_ρ in the framework of our formalism to get a clear view of the principal differences between the two approaches. In the case of H_ρ the main feature is surely the model independence of the result $\sigma_{2\gamma}=0.118\ \mu$b, which is assumed to be the comparison value for that obtained with H_J.

In the calculation with the current operator, besides the wave functions deriving from the Reid soft-core potential, we have also used two essentially phenomenological sets of wave functions (Gourdin, McGee) in order to evaluate the importance of the fact that the wave functions are eigenstates of the nuclear Hamiltonian. The total cross section has a value which is extremely dependent on the kind of wave functions used. With the Reid set we obtain $\sigma_{2\gamma}=0.146\ \mu$b which is a value near the comparison one. In the other two cases the total cross section comes out to be 0.273 μb for the Gourdin set and 0.195 μb for the McGee set. Of particular interest is the fact that the phenomenological wave functions, which predict deuteron properties very similar to the Reid ones, give such a different value of the cross section. This is due to the short range behavior of the exchange operators which gives importance to the differences in the intermediate and short range of the wave functions. In the Reid set we have just eigenfunctions of the nuclear Hamiltonian, so that the residual difference between the cross sections calculated with H_J and with H_ρ must be ascribed to the fact that the exchange current considered does not contain the heavier meson exchange contributions and thus corresponds only to the asymptotic part of the Reid soft-core potential. In this potential V_{OPE} is in fact modified in a phenomenological way at intermediate and short range to take into account the meson exchange effect which differs from the OPE effects. In conclusion, we have an estimate of 24% for the lack of consistency at short range between the RSC potential and the pion exchange currents.

The use of the current operator places importance on the contributions coming from the D part of the deuteron wave function and from the intermediate channel $^3F_2$, which is now open because of the tensorial structure of the transition exchange operator. Furthermore, it is interesting to note that the contributions of the exchange currents are of the same order of magnitude as those of the impulse current. Therefore, if we express the interaction Hamiltonian by means of the current operators, it is impossible to distinguish a dominant contribution to the total cross section, as *a priori* one could be tempted to do, limiting the calculation to the impulse current and to the S part of the deuteron wave function.

Finally, comparing the value of the total cross section obtained with the impulse current ($\sigma_{2\gamma} \simeq 0.083\ \mu$b for the Reid set) to that obtained with the Siegert operator, we can estimate the overall effect of the exchange currents in the two-photons n-p capture, which occurs mainly via two $E1$ transitions. This effect is about 30% and thus greater than the 10% effect in the single photon capture which occurs via an $M1$ transition.

[1]A. J. F. Siegert, Phys. Rev. 52, 787 (1937).
[2]R. V. Reid, Jr., Ann. Phys. (N. Y.) 50, 411 (1968).
[3]W. B. Dress, C. Guet, P. Perrin, and P. D. Miller, Phys. Rev. Lett. 34, 7 (1975).
[4]N. Wüst, H. Seyfarth, and L. Aldea, Phys. Rev. C 19. 1153 (1979).
[5]E. D. Earle and A. B. McDonald, contribution to the Third International Symposium on Neutron Capture Gamma Ray Spectroscopy and Related Topics, BNL, Upton, N. Y., 1978 (unpublished).
[6]R. J. Adler, Phys. Rev. C 6, 1964 (1972).
[7]D. P. Grechukhin, Yad. Fiz. 14, 109 (1971) [Sov. J. Nucl. Phys. 14, 62 (1972)].
[8]H. Hyuga and M. Gari, Phys. Lett. 57B, 330 (1975).
[9]H. C. Lee and F. C. Khanna, Phys. Lett. 61B, 216 (1976).
[10]H. C. Lee and F. C. Khanna, Phys. Rev. C 14, 1306 (1976).
[11]J. Blomqvist and T. Ericson, Phys. Lett. 57B, 117 (1975).
[12]J. Blomqvist and T. Ericson, Phys. Lett. 61B, 219 (1976).
[13]J. Bernabeu and R. Tarrach, Phys. Lett. 58B, 1 (1975).
[14]A. Cambi, S. Craparo, B. Mosconi, and P. Ricci, Nuovo Cimento 47A, 421 (1978).
[15]R. H. Thompson and L. Heller, Phys. Rev. C 7, 2355 (1973).
[16]M. Chemtob and M. Rho, Nucl. Phys. A163, 1 (1971).
[17]J. A. Lock and L. L. Foldy, Ann. Phys. (N. Y.) 93, 276 (1975).
[18]J. L. Friar, Ann. Phys. (N. Y.) 104, 380 (1977); W. M. Kloet and J. A. Tjon, Phys. Lett. 61B, 356 (1976); E. Hadjimichael, Nucl. Phys. A294, 513 (1978); M. Gari and H. Hyuga, *ibid.* A278, 372 (1977).
[19]The two-body charge density gives, in fact, an appreciable contribution to the electromagnetic transitions only at photon energies higher than 20 MeV [see Ref. 18 and also M. Gari and B. Sommer, Phys. Rev. Lett. 41, 226 (1978); and E. Hadjimichael, Phys. Lett. 85B, 17 (1979)]. We have checked that the pair process, which is the dominant two-body term, gives a quite negligible contribution to the cross section of the doubly radiative thermal neutron capture by hydrogen (Ref. 20).
[20]A. Cambi, B. Mosconi, and P. Ricci (unpublished).
[21]F. Bassani, J. J. Forney, and A. Quattropani, Phys. Rev. Lett. 39, 1070 (1977).
[22]J. L. Friar, Phys. Rev. Lett. 36, 510 (1976).
[23]R. G. Sachs and N. Austern, Phys. Rev. 81, 705 (1951).
[24]J. M. Eisenberg and W. Greiner, *Excitation Mechanisms of the Nucleus* (North-Holland, Amsterdam, 1970), Nuclear Theory, Vol. II.
[25]H. P. Noyes, Annu. Rev. Nucl. Sci. 22, 465 (1972).
[26]M. Gourdin, M. Le Bellac, F. Renard, and J. Tran Thanh Van, Nuovo Cimento 37, 524 (1965).
[27]I. J. McGee, Phys. Rev. 151, 772 (1966).
[28]L. Durand, Phys. Rev. 123, 1393 (1961).
[29]D. O. Riska and G. E. Brown, Phys. Lett. 38B, 193 (1972).
[30]M. Colocci, B. Mosconi, and P. Ricci, Phys. Lett. 45B, 224 (1973); M. Gari and A. H. Huffman, Phys. Rev. C 7, 994 (1973); J. Thakur and L. L. Foldy, *ibid.* 8, 1957 (1973); E. T. Dressler and F. Gross, Nucl. Phys. A262, 516 (1976).