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Closed-form description of heavy-ion transfer reactions based on distorted-waves theory

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The transition amplitude for quasielastic transfer reactions between heavy ions, given by distorted-waves theory, is evaluated in closed form. By means of suitable approximations the transfer partial-wave amplitude is expressed in terms of the elastic S matrix elements in the initial and final channels and of known functions defined by the Coulomb radial integrals of the transfer form factor. The partial-wave summation is performed by using the Poisson sum formula in conjunction with Fourier-Bessel transform techniques for evaluating integrals over localized functions in l space. The resulting expressions cover uniformly the whole angular range including forward and backward directions. The general structure of the excitation functions at 0° and 180° is discussed, including possible backward-angle enhancements of the transfer cross section. A crucial step in the derivation is the introduction of a mean elastic S-matrix, whose dependence on the initial and final angular momenta allows the display in detail of the magnetic quantum number and Q-value dependences of the smooth and oscillatory parts of the cross section. Applications and extensions of the present formalism will be given in subsequent papers.

[NUCLEAR REACTIONS Closed-form S-matrix theory of heavy-ion transfer reactions based on DWBA. Uniform approximation for transfer angular distributions, 0° and 180° excitation functions. Explicit form of magnetic quantum number and Q-value dependences.]

I. INTRODUCTION

In this paper we give an analytic description of "quasi-elastic" (as distinct from "deeply-inelastic") transfer reactions between heavy ions, based on distorted-waves theory. The purpose is to display explicitly all significant features of the transition amplitude, including the finer details of the dependence on magnetic quantum numbers, l matching, Q values, etc.

The earliest closed-form treatments of heavy-ion transfer reactions above the Coulomb barrier[1,2] assumed simple phenomenological forms of the partial-wave amplitude and were restricted to $L=0$ transfer. Though grossly oversimplified, these descriptions emphasized the predominantly diffractive nature of the transfer process and were the first to predict the existence of regular oscillations in the transfer cross section at small angles, even though the data known at the time showed smooth "bell-shaped" angular distributions only. Also, the systematic variations of the main cross section characteristics with energy, charge, and mass numbers were at least qualitatively described. At first, however, the diffraction oscillations were not found at the predicted energies, which was attributed[3] to the neglect of recoil effects. Not before early in the present decade was the oscillatory structure of many transfer angular distributions firmly established by experiment.[4-8]

Closed-form descriptions based on the distorted-wave Born approximation were first developed by Dar and his co-workers,[9-11] using for the mean elastic S matrix the parametric form of the strong absorption model.[12] This formulation was extended[13] by employing a more general technique for the partial-wave summation.

A different approach, with emphasis on the classical-refractive rather than the quantal-diffractive aspects of heavy-ion reactions, was followed in the early semiclassical theory.[14] This was later amended to account more appropriately for the effects of strong absorption by the use of complex trajectories and saddle-point methods.[15,16] The most recent semiclassical treatment,[17] though similar in concept to that of Ref. 15, takes a significant step towards a closed-form representation by performing analytically the main partial-wave summation in the distorted-waves transition amplitude.

Much of the recent advance in a more detailed understanding of heavy-ion transfer reactions is due to the experimental and theoretical work of Kahana and his co-workers.[18-20] These authors have interpreted several significant features of the distorted-waves amplitude that were not accounted for in the earlier closed-form descriptions, by means of an extended version of the phenomenological transfer S matrix[2] in which the localization with respect to the difference between the initial and final orbital angular momenta is incorporated explicitly. A clear and useful presentation of these developments is given in the review by Kahana and Baltz.[21]

In the present paper we employ analytic techniques developed previously for elastic[22,23] and inelastic[24] heavy-ion scattering to evaluate the partial-wave representation of the transfer transition amplitude. Particular attention is given to a more accurate treatment of the small- and large-angle regions, and to show how the specific properties pointed out by Kahana *et al.* can be derived from the transfer S matrix of distorted-waves theory.

II. SMALL-ANGLE REPRESENTATION

A. Angular distribution

We assume that the reaction

$$A_i + B_i = (A_f + c) + B_i \rightarrow A_f + (B_i + c) = A_f + B_f\,, \tag{2.1}$$

in which a nucleon or cluster c is transferred between the (heavy) nuclei A_i and B_i, can be described in distorted-wave Born approximation (DWBA). In view of excellent accounts of basic distorted-waves theory[25,21] we may start directly from the partial-wave representation of the DWBA transfer amplitude. With all due reservations, especially for heavy ions, we adopt the no-recoil approximation for simplicity, leaving recoil corrections for consideration at a later stage. Then the differential cross section for angular momentum transfer L can be written

$$\sigma_L^{(T)}(\theta) = A\,\frac{\mu_i\mu_f}{(2\pi\hbar^2)^2}\,\frac{k_f}{k_i}\sum_{M=-L}^{L}|\beta_{LM}(\theta)|^2\,, \tag{2.2}$$

where μ_i, μ_f are the reduced masses, and k_i, k_f are the relative wave numbers in the initial and final channels. Disregarding spin effects, we have lumped all spin and spectroscopic factors together in the constant A. Then the reduced transition amplitude for multipolarity (L, M) is given by

$$\beta_{LM}(\theta) = \sum_{l_i l_f} i^{l_i - l_f - L}\langle l_f L; 00|l_i 0\rangle\langle l_f L\,; -MM|l_i 0\rangle(2l_f+1)^{1/2}R^L_{l_f l_i}(k_f, k_i)\exp\{i[\sigma_{l_f}(k_f)+\sigma_{l_i}(k_i)]\}Y_{l_f,-M}(\theta, 0)\,. \tag{2.3}$$

Here we have chosen a coordinate frame with z axis in the direction of $\vec{k}_i$ and y axis in the direction of $\vec{k}_i \times \vec{k}_f$. The radial integrals are defined as

$$R^L_{l_f l_i}(k_f, k_i) = \frac{4\pi}{\zeta k_f k_i}\int_0^\infty dr\, f^{(f)}_{l_f}(k_f, \zeta r) \times F^{(\kappa)}_L(r) f^{(i)}_{l_i}(k_i, r)\,, \tag{2.4}$$

where $f_l^{(i,f)}$ are the radial parts of the distorted-wave functions, and

$$F^{(\kappa)}_L(r) = -i^L h^{(1)}_L(i\kappa r) \cong \frac{e^{-\kappa r}}{\kappa r} \tag{2.5}$$

is the transfer form factor, $\kappa = (2\mu_T\epsilon_B)^{1/2}/\hbar$ being the imaginary wave number associated with the binding energy ϵ_B of the transferred cluster in the final nucleus. The scale factor ζ arises from the no-recoil approximation and can vary between $\zeta = B_i/B_f$ and $\zeta = (B_i/B_f)[1 + (c/\mu_i)]$ depending on the range of the interaction[21] (where the symbols of the particles stand for their masses), and $\sigma_l(k)$ denote the Rutherford phase shifts.

Our purpose is to evaluate the summations over the orbital angular momenta l_i, l_f in explicit analytic form by utilizing the characteristic simplifying features of heavy-ion interactions above the Coulomb barrier. These are (i) large grazing angular momenta $l_i^{(0)}$, $l_f^{(0)}$, (ii) strong Coulomb interaction indicated by high values of the Sommerfeld parameters n_i, n_f, and (iii) strong absorption of low-l partial waves.

Under these conditions we may replace the spherical harmonics in (2.3) by their asymptotic forms for large l_f. In this section we use the "small-angle asymptotics"

$$Y_{l_f,-M}(\theta, 0) \cong \left(\frac{\lambda_f}{2\pi}\right)^{1/2}\left(\frac{\theta}{\sin\theta}\right)^{1/2} J_M(\lambda_f\theta)\,, \tag{2.6}$$

where $\lambda_f = l_f + \frac{1}{2}$, which is valid for $0 \leq \theta \lesssim \pi - |M|/\lambda_f$ and exact in the forward direction $\theta = 0$. For $L \ll l_i$, l_f we may then also use the asymptotic form of the Clebsch-Gordan coefficients

$$\langle l_f L;\, -MM|l_i 0\rangle \cong d^L_{MK}(\tfrac{1}{2}\pi) \tag{2.7}$$

in terms of reduced rotation matrix elements, where

$$K = l_i - l_f\,. \tag{2.8}$$

With the definitions

$$R^L_{l_f, l_f+K}(k_f, k_i) \equiv R_{LK}(\lambda_f),$$
$$\sigma_{l_{f,i}}(k_{f,i}) \equiv \sigma^{(f,i)}(\lambda_{f,i}), \quad (2.9)$$

and

$$b^L_{KM} = i^K d^L_{0K}(\tfrac{1}{2}\pi) d^L_{MK}(\tfrac{1}{2}\pi), \quad (2.10)$$

we replace the summation over l_f in (2.3) by the Poisson series of integrals over λ_f with the result

$$\beta_{LM}(\theta) = \frac{i^{-L}}{\pi^{1/2}}\left(\frac{\theta}{\sin\theta}\right)^{1/2} \sum_K b^L_{KM} \sum_{m=-\infty}^{\infty} e^{-im\pi} \int_0^\infty d\lambda_f \lambda_f R_{LK}(\lambda_f) \exp\{i[\sigma^{(f)}(\lambda_f) + \sigma^{(i)}(\lambda_f + K)]\} e^{i2m\pi\lambda_f} J_M(\lambda_f \theta). \quad (2.11)$$

Now we take the crucial step by making a suitable approximation for the functions $R_{LK}(\lambda_f)$. First, we assume that the nuclear part of the distorting interaction in the initial and final channels is contained in these functions through the complex nuclear phases only, so that we can write

$$R_{LK}(\lambda_f) \approx [S^{(N)}_{l_f}(k_f)]^{1/2} \mathcal{R}^L_{l_f l_i}(k_f, k_i) [S^{(N)}_{l_i}(k_i)]^{1/2}, \quad (2.12)$$

where $S^{(N)}_l(k)$ is the nuclear part of the total elastic S matrix

$$S_l(k) = S^{(N)}_l(k) e^{i2\sigma_l(k)} = \eta_l(k) \exp\{i2[\delta^{(N)}_l(k) + \sigma_l(k)]\}$$
$$= \eta_l(k) e^{i2\delta_l(k)}, \quad (2.13)$$

and

$$\mathcal{R}^L_{l_f l_i}(k_f, k_i) = \frac{4\pi}{k_i k'_f} \int_0^\infty dr\, \mathfrak{F}_{l_f}(k'_f r) F^{(\kappa)}_L(r) \mathfrak{F}_{l_i}(k_i r), \quad (2.14)$$

where $k'_f = \xi k_f$, are the transfer radial integrals with *Coulomb* radial wave functions $\mathfrak{F}_l(kr)$ only. Approximations similar to (2.12), which is known as the "Sopkovich prescription,"[26] are usually made in semiclassical approaches[14,15,17]; the same device was used in our description of inelastic heavy-ion scattering[24] to approximate the Coulomb excitation part of the inelastic radial integrals.

Second, we replace the square root of the product of the nuclear S matrices by an average S matrix which may be regarded as the "geometrical mean" of $S^{(N)}_{l_i}$ and $S^{(N)}_{l_f}$. Assuming that $S_{l_{i,f}}(k_{i,f})$ may be interpolated by smooth functions $S(\lambda_{i,f})$ of the continuous variables $\lambda_{i,f} = l_{i,f} + \frac{1}{2}$, characterized by critical angular momentum parameters $\Lambda_{i,f} = l^{(0)}_{i,f} + \frac{1}{2}$ and l-window parameters $\Delta_{i,f}$ on which they depend through the combinations $\mu_{i,f} = (\lambda_{i,f} - \Lambda_{i,f})/\Delta_{i,f}$, we show in Appendix A that

$$[S^{(N)}_{l_f}(k_f) S^{(N)}_{l_i}(k_i)]^{1/2} \to [S^{(f)}_N(\lambda_f) S^{(i)}_N(\lambda_i)]^{1/2}$$
$$\approx S^{(K)}_N(\bar{\lambda}), \quad (2.15)$$

where

$$S^{(K)}_N(\bar{\lambda}) = \eta_K(\bar{\lambda}) e^{i2\delta^{(K)}_N(\bar{\lambda})}, \quad \bar{\lambda} = \tfrac{1}{2}(\lambda_i + \lambda_f) \quad (2.16)$$

with

$$2\delta^{(K)}_N(\bar{\lambda}) = \delta^{(i)}_N(\lambda_i) + \delta^{(f)}_N(\lambda_f)$$
$$= \delta^{(i)}_N(\bar{\lambda} + \tfrac{1}{2}K) + \delta^{(f)}_N(\bar{\lambda} - \tfrac{1}{2}K) \quad (2.17)$$

and

$$\eta_K(\bar{\lambda}) \equiv \hat{\eta}(\bar{\mu}_K), \quad \bar{\mu}_K = \frac{\bar{\lambda} - \Lambda_K}{\Delta}. \quad (2.18)$$

Here the critical angular momentum of the mean reflection function is given by

$$\Lambda_K = \bar{\Lambda} + \Delta_K,$$
$$\Delta_K = \Delta \ln\left\{\frac{1}{2}\left[\exp\left(\frac{K - K_0}{2\Delta_f}\right) + \exp\left(-\frac{K - K_0}{2\Delta_i}\right)\right]\right\}, \quad (2.19)$$

where

$$\bar{\Lambda} = \tfrac{1}{2}(\Lambda_i + \Lambda_f), \quad K_0 = \Lambda_i - \Lambda_f, \quad (2.20)$$

and Δ is the geometrical mean of the l-window parameters of the initial- and final-channel S matrices,

$$\Delta = \frac{2\Delta_i \Delta_f}{\Delta_i + \Delta_f}. \quad (2.21)$$

It will be seen later on that the K and K_0 dependences of the mean S matrix described by (2.18)–(2.20) determine the magnetic substate population, the l matching, and most of the Q-window effects in the transfer cross section.

Third, although the Coulomb radial integrals (2.14) can be expressed in terms of generalized hypergeometric functions of two variables known as Appell functions,[27] for practical purposes we use an approximation obtained by replacing the Coulomb radial wave functions $\mathfrak{F}_l(kr)$ by their WKB forms. Then we can write

$$\mathcal{R}^L_{l_f l_i}(k_f, k_i) = \frac{2\pi}{k_i k'_f K} I^{(\kappa)}_{L-K}(\alpha, \xi), \quad (2.22)$$

where the $I^{(\kappa)}_{L-K}$ are functions of the mean asymptotic Rutherford deflection function α and of the adiabaticity parameter ξ,

$$\alpha = \bar{\Theta}_R(\bar{\lambda}) = 2\arctan(n/\bar{\lambda}), \quad \xi = n_f - n_i \quad (2.23)$$

where

$$n = \tfrac{1}{2}(n_i + n_f), \quad (2.24)$$

$$n_i = \frac{\mu_i Z_{A_i} Z_{B_i} e^2}{\hbar^2 k_i}, \quad n_f = \frac{\mu_f Z_{A_f} Z_{B_f} e^2}{\hbar^2 k_f}. \quad (2.25)$$

The main properties of the functions $I_{L-K}^{(K)}$ and some further approximations are given in Appendix B.

Now we can write our approximation for the functions $R_{LK}(\lambda_f)$ as

$$R_{LK}(\lambda_f) = \frac{2\pi}{k_i k_f' \kappa} I_{L-K}^{(\kappa)}(\alpha, \xi)\,\eta_K(\lambda_f + \tfrac{1}{2}K) \times \exp[i2\delta_N^{(K)}(\lambda_f + \tfrac{1}{2}K)]\,, \tag{2.26}$$

and the amplitude (2.11) becomes

$$\beta_{LM}(\theta) = i^{-L}\frac{2\pi^{1/2}}{k_i k_f' \kappa}\left(\frac{\theta}{\sin\theta}\right)^{1/2}\sum_K b_{KM}^L \sum_{m=-\infty}^{\infty} e^{-im\pi}\int_0^\infty d\lambda_f \lambda_f I_{L-K}^{(\kappa)}(\alpha,\xi)\eta_K(\lambda_f + \tfrac{1}{2}K) \times \exp\{i[2\delta_K(\lambda_f + \tfrac{1}{2}K) + 2m\pi\lambda_f]\}J_M(\lambda_f\theta)\,, \tag{2.27}$$

where we have defined the mean total phase shift

$$2\delta_K(\bar\lambda) = \delta_N^{(i)}(\lambda_i) + \sigma^{(i)}(\lambda_i) + \delta_N^{(f)}(\lambda_f) + \sigma^{(f)}(\lambda_f) \equiv 2\delta_N^{(K)}(\lambda_f + \tfrac{1}{2}K) + 2\sigma^{(K)}(\lambda_f + \tfrac{1}{2}K)\,. \tag{2.28}$$

To evaluate the integrals over λ_f in (2.27), we first note that the nonoscillatory parts of the integrands are localized in l space. This is because the reflection functions η_K for heavy ions have a strong-absorption profile with very small values for low-l partial waves, and because the functions $I_{L-K}^{(\kappa)}$ fall off exponentially at large l with a rate determined by the binding parameter κ. Thus the "transfer partial-wave amplitude functions"

$$c_{LK}(\bar\lambda) = I_{L-K}^{(\kappa)}(\alpha,\xi)\eta_K(\bar\lambda)\,, \tag{2.29}$$

whose general shape is sketched in Fig. 1, have maxima at $\bar\lambda = \bar\Lambda_K^T$. We note that, in contrast to the simple parametric forms assumed in Refs. 1, 2, 18, and 20, the functions $c_{LK}(\bar\lambda)$ derived from distorted-waves theory are *essentially asymmetric* about their maximum positions, the asymmetry being determined by the difference between the binding parameter κ/k and the mean elastic window parameter Δ, in other words, by the difference between the range R_B of the bound state wave function of the transferred cluster and the mean surface diffuseness $\bar a$ semiclassically associated with Δ.

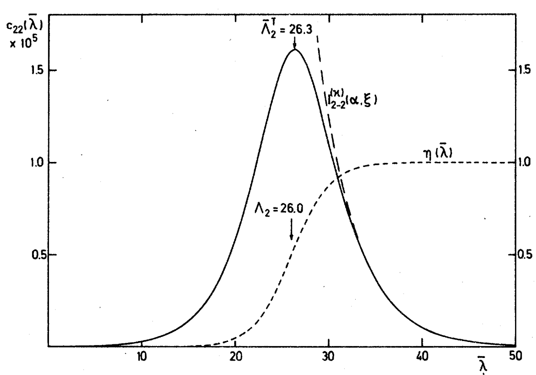

FIG. 1. Typical shape of the transfer partial-wave amplitude function $c_{LK}(\bar\lambda)$, defined by Eq. (2.29), as a function of the mean orbital angular momentum variable $\bar\lambda = \frac{1}{2}(\lambda_i + \lambda_f)$, for $L = K = 2$ (solid curve). The example shown pertains to the reaction $^{48}Ca(^{16}O, ^{12}C)^{50}Ti$ at $E_{lab} = 56$ MeV. Also shown are the functions $I_{L-K}^{(\kappa)}(\alpha,\xi)$ and $\eta_K(\bar\lambda)$ of which $c_{LK}(\bar\lambda)$ is the product: $I_{2-2}^{(\kappa)}(\alpha,\xi)$ with $\xi = -2.79$, $\kappa = 1.172$ and $\gamma = 0.2667$ (long-dashed curve), and $\eta_2(\bar\lambda)$ with $\Lambda_2 = 26.0$ and $\Delta = 2$ (short-dashed curve; note different scale on right-hand side).

From Appendix B it is seen that we can write

$$I_{L-K}^{(\kappa)}(\alpha,\xi) \equiv \tilde I_{L-K}^{(\kappa)}(\alpha,\xi)e^{-\gamma\bar\lambda} = \tilde I_{L-K}^{(\kappa)}(\alpha,\xi)e^{-\gamma K/2}e^{-\gamma\lambda_f}\,, \tag{2.30}$$

where $\tilde I_{L-K}^{(\kappa)}(\alpha,\xi)$ is a slowly varying function of $\bar\lambda$, and where

$$\gamma = (\rho^2 + \xi'^2)^{1/2}/n \tag{2.31}$$

with

$$\rho = \frac{\kappa}{k}n\,, \quad k = \tfrac{1}{2}(k_i + k_f)\,, \quad \xi' = \frac{k_i - k_f'}{k}n\,. \tag{2.32}$$

In terms of λ_f, the maximum of $c_K(\bar\lambda) = c_K(\lambda_f + \frac{1}{2}K)$ is at $\lambda_f = \Lambda_K^T = \bar\Lambda_K^T - \frac{1}{2}K$. Its value may be calculated approximately by determining the maximum of

$$\tau_K(\lambda_f) = \eta_K(\lambda_f + \tfrac{1}{2}K)e^{-\gamma\lambda_f}\,, \tag{2.33}$$

which yields

$$\Lambda_K^T = \Lambda_K - \tfrac{1}{2}K + \delta_T\,, \quad \bar\Lambda_K^T = \Lambda_K + \delta_T\,, \tag{2.34}$$

where Λ_K is given by (2.19) and δ_T is obtained by solving the equation

$$\frac{\hat D(\delta_T/\Delta)}{\hat\eta(\delta_T/\Delta)} = \gamma\Delta\,, \tag{2.35}$$

with the definition $\hat D(\mu) = d\hat\eta(\mu)/d\mu$. As an example, for the convenient parametric form $\hat\eta(\mu) = (1 + e^{-\mu})^{-1}$ we obtain

$$\delta_T = \Delta\ln\left(\frac{1}{\gamma\Delta} - 1\right), \tag{2.36}$$

under the condition $\gamma\Delta < 1$. Notice that, quite generally, the shift δ_T is independent of K.

Now we write

$$c_{LK}(\bar\lambda) = \tilde c_{LK}(\lambda_f)\tau_K(\lambda_f) = \tilde I_{L-K}^{(\kappa)}(\alpha,\xi)e^{-\gamma K/2}\eta_K(\lambda_f + \tfrac{1}{2}K)e^{-\gamma\lambda_f}\,, \tag{2.37}$$

and expand the phase functions in (2.27) linearly about $\lambda_f = \Lambda_K^T$,

$$2\delta_K(\lambda_f+\tfrac{1}{2}K)\approx 2\delta_K(\Lambda_K^T+\tfrac{1}{2}K)+(\lambda_f-\Lambda_K^T)\theta_K^T$$
$$=2\delta_K(\bar{\Lambda}_K^T)+(\lambda_f-\Lambda_K^T)\theta_K^T\,, \tag{2.38}$$

where

$$\theta_K^T=\Theta(\bar{\Lambda}_K^T)=\left[\frac{2\delta_K(\bar{\lambda})}{d\bar{\lambda}}\right]_{\bar{\lambda}=\bar{\Lambda}_K^T} \tag{2.39}$$

are the critical angles associated with the characteristic angular momenta $\bar{\Lambda}_K^T$. After taking the slowly varying functions $\tilde{c}_{LK}(\lambda_f)$ from under the integrals at the points $\lambda_f=\Lambda_K^T$, we obtain from (2.27)

$$\beta_{LM}(\theta)=i^{-L}\frac{2\pi^{1/2}}{k_i k_f' \kappa}\left(\frac{\theta}{\sin\theta}\right)^{1/2}\times\sum_K b_{KM}^L\tilde{c}_{LK}(\Lambda_K^T)e^{i2\delta_K(\bar{\Lambda}_K^T)}\times\sum_{m=-\infty}^{\infty}\exp[i2m\pi(\Lambda_K^T-\tfrac{1}{2})]T_{KM}(\theta_K^T+2m\pi)\,, \tag{2.40}$$

where

$$T_{KM}(x)=\int_0^\infty d\lambda\,\lambda\tau_K(\lambda)e^{i(\lambda-\Lambda_K^T)x}J_M(\lambda\theta)\,. \tag{2.41}$$

These integrals are of the type of combined Fourier and Bessel transforms that have been evaluated in Appendix A of Ref. 28 with the result

$$T_{KM}(x)=\tfrac{1}{2}\Lambda_K^T\{H_K^{(T)}(x-\theta)[J_M(\Lambda_K^T\theta)-iJ_{M+1}(\Lambda_K^T\theta)]+H_K^{(T)}(x+\theta)[J_M(\Lambda_K^T\theta)+iJ_{M+1}(\Lambda_K^T\theta)]\}\,, \tag{2.42}$$

where $H_K^{(T)}(z)$ is the Fourier transform of $\tau_K(\lambda)$,

$$H_K^{(T)}(z)=\int_{-\infty}^{\infty}d\lambda\,\tau_K(\lambda)e^{i(\lambda-\Lambda_K^T)z}\,. \tag{2.43}$$

With (2.33) we obtain

$$H_K^{(T)}(z)=\int_{-\infty}^{\infty}d\lambda\,\eta_K(\lambda+\tfrac{1}{2}K)e^{-\gamma\lambda}e^{i(\lambda-\Lambda_K^T)z}$$
$$=e^{-\gamma\Lambda_K^T}\int_{-\infty}^{\infty}d\bar{\lambda}\,\eta_K(\bar{\lambda})e^{i(\bar{\lambda}-\bar{\Lambda}_K^T)(z+i\gamma)}$$
$$=e^{-\gamma\Lambda_K^T}e^{i(\Lambda_K-\bar{\Lambda}_K^T)(z+i\gamma)}\int_{-\infty}^{\infty}d\bar{\lambda}\,\eta_K(\bar{\lambda})e^{i(\bar{\lambda}-\Lambda_K)(z+i\gamma)}$$
$$=ie^{\gamma K/2}e^{-\gamma\Lambda_K}e^{-i\delta_T z}\frac{F(\Delta(z+i\gamma))}{z+i\gamma}\,, \tag{2.44}$$

where

$$F(\Delta x)=\int_{-\infty}^{\infty}d\lambda\frac{d\eta_K(\lambda)}{d\lambda}e^{i(\lambda-\Lambda_K)x} \tag{2.45}$$

is the Fourier transform of the derivative of the mean reflection function.

Noting from (2.38) that

$$2\delta_K(\bar{\Lambda}_K^T)-\delta_T\theta_K^T\approx 2\delta_K(\Lambda_K)\,, \tag{2.46}$$

and from (2.30) and (2.37) that

$$\tilde{c}_{LK}(\Lambda_K^T)e^{\gamma K/2}e^{-\gamma\Lambda_K}=I_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)e^{\gamma\delta_T}\,, \tag{2.47}$$

where

$$\theta_{R,K}^T=2\arctan(n/\bar{\Lambda}_K^T)\,; \tag{2.48}$$

we can write the final result as

$$\beta_{LM}(\theta)=i^{1-L}\frac{\pi^{1/2}}{k_i k_f'\kappa}\left(\frac{\theta}{\sin\theta}\right)^{1/2}\sum_K b_{KM}^L\Lambda_K^T I_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)e^{\gamma\delta_T}e^{i2\delta_K(\Lambda_K)}\{\mathcal{H}_K^{(SA)}(\theta)[J_M(\Lambda_K^T\theta)-iJ_{M+1}(\Lambda_K^T\theta)]+\mathcal{H}_K^{(SA)}(-\theta)[J_M(\Lambda_K^T\theta)+iJ_{M+1}(\Lambda_K^T\theta)]\}\,, \tag{2.49}$$

where

$$\mathcal{H}_K^{(SA)}(\pm\theta)=e^{\pm i\delta_T\theta}\sum_{m=-\infty}^{\infty}\exp\{i2m\pi[\Lambda_K-\tfrac{1}{2}(K+1)]\}\times\frac{F(\Delta(\theta_K^T+2m\pi+i\gamma\mp\theta))}{\theta_K^T+2m\pi+i\gamma\mp\theta}\,. \tag{2.50}$$

Close to the forward direction we have approximately

$$\beta_{LM}(\theta)\approx i^{1-L}\frac{2\pi^{1/2}}{k_i k_f'\kappa}\sum_K b_{KM}^L\Lambda_K^T I_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)e^{\gamma\delta_T}\times e^{i2\delta_K(\Lambda_K)}\mathcal{H}_K^{(SA)}(0)J_M(\Lambda_K^T\theta)\,,\quad \theta\approx 0 \tag{2.51}$$

so that the near-forward cross section behaves like

$$\sigma_{LM}^{(T)}(\theta)\propto[J_M(\Lambda_K^T\theta)]^2\propto\theta^{2|M|}\,,\quad\theta\approx 0\,. \tag{2.52}$$

B. Forward-angle excitation function

In the forward direction $\theta=0$ only the $M=0$ substate contributes, and we find

$$\beta_{LM}(0)=\beta_{L0}(0)\delta_{M0}\,,$$
$$\beta_{L0}(0)=\frac{2\pi^{1/2}}{k_i k_f'\kappa}\sum_K i^{K-L+1}[d_{0K}^L(\tfrac{1}{2}\pi)]^2\Lambda_K^T I_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)\times e^{\gamma\delta_T}e^{i2\delta_K(\Lambda_K)}\mathcal{H}_K^{(SA)}(0)\,, \tag{2.53}$$

with

$$\mathcal{H}_K^{(SA)}(0)=\sum_{m=-\infty}^{\infty}\exp\{i2m\pi[\Lambda_K-\tfrac{1}{2}(K+1)]\}\times\frac{F(\Delta(\theta_K^T+2m\pi+i\gamma))}{\theta_K^T+2m\pi+i\gamma}\,. \tag{2.54}$$

To discuss the energy dependence we consider

$L=0$ transfer only. Then the forward-angle cross section becomes

$$\sigma_0^{(T)}(0)=A\frac{\mu_i\mu_f}{\pi\hbar^4\zeta^2k_i^3k_f\kappa^2}(\Lambda_0^T)^2[I_{00}^{(\kappa)}(\theta_R^T,\xi)]^2 \times e^{2\gamma\delta_T}|\mathcal{K}_0^{(SA)}(0)|^2 . \qquad (2.55)$$

From the expression for $I_{00}^{(\kappa)}$ given in Appendix B it is seen that for $\Lambda_0^T \gg n$,

$$I_{00}^{(\kappa)}(\theta_R^T,\xi)\approx\left(\frac{\pi}{2\gamma\Lambda_0^T}\right)^{1/2}\exp[-(\rho+\xi\arctan\xi'/\rho)]e^{-\gamma\Lambda_0^T} \approx\left(\frac{\pi k}{2\kappa\Lambda_0^T}\right)^{1/2}e^{-\gamma\Lambda_0^T}, \qquad (2.56)$$

so that

$$\sigma_0^{(T)}(0)\approx A\frac{\mu_i\mu_f}{2\hbar^4\zeta^2\kappa^2k^3}\Lambda_0^Te^{-(2\kappa\Lambda_0/k)}|\mathcal{K}_0^{(SA)}(0)|^2 , \qquad (2.57)$$

with

$$\Lambda_0=\bar\Lambda+\Delta\ln\left[\cosh\left(\frac{K_0}{2\Delta}\right)\right], \qquad (2.58)$$

where for simplicity we have assumed $\Delta_i=\Delta_f=\Delta$.

Since well above the Coulomb barrier Λ_0/k depends only weakly on energy, the main energy dependence is described by $|\mathcal{K}_0^{(SA)}(0)/k|^2$.

Normally only the $m=0$ term gives a significant contribution to the Poisson sum (2.54) so that

$$\mathcal{K}_0^{(SA)}(0)=\frac{F(\Delta(\theta_0^T+i\gamma))}{\theta_0^T+i\gamma} . \qquad (2.59)$$

As a typical example, for the parametric form $\hat\eta(\mu)=(1+e^{-\mu})^{-1}$ we have $F(z)=\pi z/\sinh(\pi z)$, so that

$$|\mathcal{K}_0^{(SA)}(0)/k|^2=\frac{(\pi\Delta/k)^2}{[\sinh(\pi\Delta\theta_0^T)]^2+[\sin(\pi\Delta\gamma)]^2} . \qquad (2.60)$$

Since $\Delta\gamma\approx\kappa(\Delta/k)\approx$ const, the main energy dependence comes from $\Delta\theta_0^T\approx n(\Delta/\Lambda_0^T)$ and is thus chiefly determined by the energy variation of the Sommerfeld parameter n, a feature similar to what has been found for elastic and inelastic heavy-ion scattering.[22-24] In general, the structure of Eqs. (2.53) and (2.54) shows that as long as only a single ($m=0$) Poisson term is significant, the forward-angle transfer excitation function is smooth or at least not strongly oscillatory.

III. LARGE-ANGLE REPRESENTATION

A. Angular distribution

To evaluate the transfer amplitude for large angles θ we use instead of (2.6) the large-angle asymptotic form of the spherical harmonics,

$$Y_{l_f,-M}(\theta,0)\cong(-)^{l_f+M}\left(\frac{\lambda_f}{2\pi}\right)^{1/2}\left(\frac{\vartheta}{\sin\vartheta}\right)^{1/2}J_M(\lambda_f\vartheta) , \qquad (3.1)$$

where $\vartheta=\pi-\theta$, which is valid for $0\le\vartheta<\pi-|M|/\lambda_f$ and exact in the backward direction $\theta=\pi$. Then the Poisson representation becomes

$$\beta_{LM}(\theta)=\frac{i^{-L}}{\pi^{1/2}}(-)^M\left(\frac{\vartheta}{\sin\vartheta}\right)^{1/2}\sum_K b_{KM}^L\sum_{m=-\infty}^{\infty}e^{-i(m+1/2)\pi} \times\int_0^\infty d\lambda_f\lambda_fR_{LK}(\lambda_f)\exp\{i[\sigma^{(f)}(\lambda_f)+\sigma^{(i)}(\lambda_f+K)]\}e^{i(2m+1)\pi\lambda_f}J_M(\lambda_f\vartheta) . \qquad (3.2)$$

With the same approximations as in the previous section we obtain

$$\beta_{LM}(\theta)=i^{1-L}(-)^M\frac{\pi^{1/2}}{k_ik_f'\kappa}\left(\frac{\vartheta}{\sin\vartheta}\right)^{1/2}\sum_K b_{KM}^L\Lambda_K^TI_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)e^{\gamma\delta_T}e^{i2\delta_K(\Lambda_K)} \times\{\mathcal{K}_K^{(LA)}(\vartheta)[J_M(\Lambda_K^T\vartheta)-iJ_{M+1}(\Lambda_K^T\vartheta)]+\mathcal{K}_K^{(LA)}(-\vartheta)[J_M(\Lambda_K^T\vartheta)+iJ_{M+1}(\Lambda_K^T\vartheta)]\} , \qquad (3.3)$$

where

$$\mathcal{K}_K^{(LA)}(\pm\vartheta)=e^{\pm i\delta_T\vartheta}\sum_{m=-\infty}^{\infty}\exp\{i(2m+1)\pi[\Lambda_K-\tfrac{1}{2}(K+1)]\}\frac{F(\Delta(\theta_K^T+(2m+1)\pi+i\gamma\mp\vartheta))}{\theta_K^T+(2m+1)\pi+i\gamma\mp\vartheta} . \qquad (3.4)$$

For later use we note the relations, obtained by comparing (3.4) with (2.50),

$$\mathcal{K}_K^{(LA)}(\pm\vartheta)=e^{\pm i\pi(\Lambda_K^T-1/2)}\mathcal{K}_K^{(SA)}(\mp\theta) . \qquad (3.5)$$

Close to the backward direction we have approximately

$$\beta_{LM}(\theta)\approx i^{1-L}(-)^M\frac{2\pi^{1/2}}{k_ik_f'\kappa}\sum_K b_{KM}^L\Lambda_K^T I_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)e^{\gamma\delta_T}e^{i2\delta_K(\Lambda_K)}\mathcal{H}_K^{(LA)}(0)J_M[\Lambda_K^T(\pi-\theta)]\,,\quad \theta\approx\pi \tag{3.6}$$

so that the near-backward cross section behaves like

$$\sigma_{LM}^{(T)}(\theta)\propto\{J_M[\Lambda_K^T(\pi-\theta)]\}^2$$
$$\propto(\pi-\theta)^{2|M|}\,,\quad \theta\approx\pi\,. \tag{3.7}$$

B. Backward-angle excitation function

In the backward direction $\theta=\pi$ again only the $M=0$ substate contributes, and (3.6) reduces to

$$\beta_{LM}(\pi)=\beta_{L0}(\pi)\delta_{M0}\,,$$

$$\beta_{L0}(\pi)=\frac{2\pi^{1/2}}{k_ik_f'\kappa}\sum_K i^{K-L+1}[d_{0K}^L(\tfrac{1}{2}\pi)]^2\Lambda_K^T I_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)$$
$$\times e^{\gamma\delta_T}e^{i2\delta_K(\Lambda_K)}\mathcal{H}_K^{(LA)}(0)\,. \tag{3.8}$$

Comparison with (2.53) and using relation (3.5) shows that the relative magnitude of $\beta_{L0}(\pi)$ and $\beta_{L0}(0)$ is determined by the ratio $\mathcal{H}_K^{(SA)}(-\pi)/\mathcal{H}_K^{(SA)}(0)$. If, as is normally the case, the $m=0$ term dominates in the Poisson sum, this becomes

$$\left|\frac{\mathcal{H}_K^{(SA)}(-\pi)}{\mathcal{H}_K^{(SA)}(0)}\right|=\left(\frac{(\theta_K^T)^2+\gamma^2}{(\theta_K^T+\pi)^2+\gamma^2}\right)^{1/2}\left|\frac{F(\Delta(\theta_K^T+\pi+i\gamma))}{F(\Delta(\theta_K^T+i\gamma))}\right|\ll 1\,. \tag{3.9}$$

For example, with $F(z)=\pi z/\sinh(\pi z)$ we have for the $L=0$ backward-to-forward cross section ratio

$$\frac{\sigma_0^{(T)}(\pi)}{\sigma_0^{(T)}(0)}=\left|\frac{\mathcal{H}_0^{(SA)}(-\pi)}{\mathcal{H}_0^{(SA)}(0)}\right|^2$$
$$=\frac{4e^{-2\pi^2\Delta}}{|1+\coth[\pi\Delta(\theta_0^T+i\gamma)]|^2}\,. \tag{3.10}$$

This shows that the backward-angle transfer excitation function has an additional energy dependence: The ratio (3.10) decreases rapidly with increasing energy since $\Delta\propto E^{1/2}$.

C. Enhancement effects

In our discussion so far we have assumed that the elastic S matrices in the initial and final channels have "normal strong-absorption profile," in the terminology of Ref. 28. As described there for elastic and inelastic scattering, backward-angle enhancement can occur if $\eta_K(\bar\lambda)$ contains rapidly varying deviations from the normal $\bar\lambda$ profile. If we write

$$\eta_K(\bar\lambda)=\bar\eta_K(\bar\lambda)+\tilde\eta_K(\bar\lambda)\,, \tag{3.11}$$

then the transfer amplitude has an additional contribution $\tilde\beta(\theta)$ of a form similar to (3.3), but with the functions $\mathcal{H}_K^{(LA)}(\pm\vartheta)$ replaced by

$$\tilde{\mathcal{H}}_K^{(LA)}(\pm\vartheta)=-ie^{\pm i\delta_T\vartheta}\sum_{m=-\infty}^{\infty}\exp\{i(2m+1)\pi[\Lambda_K-\tfrac{1}{2}(K+1)]\}$$
$$\times G[\theta_K^T+(2m+1)\pi+i\gamma\mp\vartheta]\,, \tag{3.12}$$

where

$$G(x)=\int_{-\infty}^{\infty}d\lambda\,\tilde\eta_K(\lambda)e^{i(\lambda-\Lambda_K)x}\,, \tag{3.13}$$

and where we have assumed for simplicity that the deviation $\tilde\eta_K(\bar\lambda)$ is centered at $\bar\lambda=\Lambda_K$. Then the condition for backward-angle enhancement is

$$\left|\frac{\tilde{\mathcal{H}}_K^{(LA)}(0)}{\mathcal{H}_K^{(LA)}(0)}\right|^2\approx\left|\frac{\theta_K^T-\pi+i\gamma}{F(\Delta(\theta_K^T-\pi+i\gamma))}G(\theta_K^T-\pi+i\gamma)\right|^2\gg 1\,, \tag{3.14}$$

if the $m=-1$ contributions to the Poisson sums in (3.12) and (3.4) are dominant. Thus, except for the effects of the transfer parameters γ and δ_T, the enhancement conditions are much the same as for elastic and inelastic scattering described in Ref. 28.

Interference between the $m=-1$ and $m=0$ terms in (3.12) gives rise to oscillations in the backward-angle excitation function. Assuming $L=0$ transfer for simplicity, we obtain

$$\tilde\sigma_0(\pi)=\tilde\sigma_0^{(0)}(\pi)\left|1-\frac{G(\theta_0^T+\pi+i\gamma)}{G(\theta_0^T-\pi+i\gamma)}e^{i2\pi\Lambda_0}\right|^2\,, \tag{3.15}$$

which shows that, aside from more slowly varying modulations due to γ and δ_T, the dominant oscillations are of period $\delta\Lambda_0=1$, as in the case of elastic and inelastic scattering. [Odd-even staggering effects in $\tilde\eta(\bar\lambda)$ would cause an additional modulation of period $\delta\Lambda_0=2$; see Ref. 28.]

IV. INTERMEDIATE ANGLES

A. Angular distribution

In this section we consider the intermediate-angle region for which we may use the asymptotic expressions

$$J_M(x)\pm iJ_{M+1}(x)\cong\left(\frac{2}{\pi x}\right)^{1/2}\exp[\pm i(x-\tfrac{1}{4}\pi-\tfrac{1}{2}M\pi)]\,. \tag{4.1}$$

Then (2.49) becomes

$$\beta_{LM}(\theta)=\frac{1}{(\sin\theta)^{1/2}}\sum_K B^L_{KM}[\mathcal{H}_K^{(SA)}(\theta)e^{-i[\Lambda_K^T\theta-(1/4)\pi]}+(-)^M\mathcal{H}_K^{(SA)}(-\theta)e^{i[\Lambda_K^T\theta-(1/4)\pi]}],\tag{4.2}$$

where

$$B^L_{KM}=\frac{i^{M-L+1}}{k_ik_f'\kappa}b^L_{KM}(2\Lambda_K^T)^{1/2}I^{(\kappa)}_{L-K}(\theta^T_{R,K},\xi)e^{\gamma\delta_T}e^{i2\delta_K(\Lambda_K)}.\tag{4.3}$$

Similarly, from (3.3) we obtain

$$\beta_{LM}(\theta)=\frac{1}{(\sin\theta)^{1/2}}\sum_K B^L_{KM}[\mathcal{H}_K^{(LA)}(-\vartheta)e^{i\pi(\Lambda_K^T-1/2)}e^{-i[\Lambda_K^T\vartheta-(1/4)\pi]}+(-)^M\mathcal{H}_K^{(LA)}(\vartheta)e^{-i\pi(\Lambda_K^T-1/2)}\times e^{i[\Lambda_K^T\vartheta-(1/4)\pi]}],\tag{4.4}$$

which is the same as (4.2) by virtue of the relations (3.5). Thus the expressions (2.49) and (3.3) cover the full angular range, with the common simplified formula (4.2) for the intermediate region.

Now it is convenient to change coordinate axes by transformation to a frame with the z axis in the direction of $\vec{k}_i\times\vec{k}_f$ and the x axis in the direction of $\vec{k}_i$. This rotation is performed by

$$\hat{\beta}_{LM}(\theta)=(-)^M\sum_{M'}i^{M'}d^L_{M'M}(\tfrac{1}{2}\pi)\beta_{LM'}(\theta).\tag{4.5}$$

Then the summations over M' and K can be carried out by using the symmetry and orthogonality relations of the rotation matrices

$$\sum_{M'}d^L_{M'M}(\tfrac{1}{2}\pi)d^L_{M'K}(\tfrac{1}{2}\pi)=\delta_{MK},\tag{4.6}$$

$$\sum_{M'}(-)^{M'}d^L_{M'M}(\tfrac{1}{2}\pi)d^L_{M'K}(\tfrac{1}{2}\pi)=(-)^L\delta_{M,-K}.\tag{4.7}$$

The resulting amplitude can be written

$$\hat{\beta}_{LM}(\theta)=\beta^{(+)}_{LM}(\theta)+(-)^M\beta^{(-)}_{LM}(\theta),\quad L+M\ \text{even}\tag{4.8}$$

(on the understanding that M from now on refers to the normal of the scattering plane as quantization axis), where

$$\beta^{(\pm)}_{LM}(\theta)=\mathcal{Y}_{LM}(\theta)B^{(\pm)}_{LM}\mathcal{H}^{(\pm)}_M(\theta)e^{\mp i[\Lambda_{\mp M}\theta-(1/4)\pi]},\tag{4.9}$$

with

$$\mathcal{Y}_{LM}(\theta)=\frac{d^L_{0M}(\frac{1}{2}\pi)e^{i(1/2)M(\pi-\theta)}}{(4\pi\sin\theta)^{1/2}}=\frac{Y_{LM}(\frac{1}{2}\pi,0)}{(2L+1)^{1/2}}\frac{e^{i(1/2)M(\pi-\theta)}}{(\sin\theta)^{1/2}},\tag{4.10}$$

$$B^{(\pm)}_{LM}=i^{1-L}\frac{(8\pi)^{1/2}}{k_ik_f'\kappa}(\Lambda^T_{\mp M})^{1/2}I^{(\kappa)}_{L,\pm M}(\theta^T_{R,\mp M},\xi)e^{\gamma\delta_T}e^{i2\delta_{\mp M}(\Lambda_{\mp M})},\tag{4.11}$$

$$\mathcal{H}^{(\pm)}_M(\theta)=\sum_{m=-\infty}^{\infty}(-)^{m(M+1)}e^{i2m\pi\Lambda_{\mp M}}\frac{F(\Delta(\theta^T_{\mp M}+2m\pi+i\gamma\mp\theta))}{\theta^T_{\mp M}+2m\pi+i\gamma\mp\theta}.\tag{4.12}$$

Thus the transfer cross section for multipolarity (L,M) can be written as

$$\sigma^{(T)}_{LM}(\theta)=\bar{\sigma}^{(T)}_{LM}(\theta)\{1+|t_{LM}(\theta)|^2+(-)^M2\,\mathrm{Im}[t_{LM}(\theta)e^{i2\bar{\Lambda}_M\theta}]\},\tag{4.13}$$

where

$$\bar{\sigma}^{(T)}_{LM}(\theta)=A\frac{\mu_i\mu_f}{2\pi^2\hbar^4\zeta^2k_0{}^3k_f\kappa^2}[d^L_{0M}(\tfrac{1}{2}\pi)]^2\Lambda^T_{-M}[I^{(\kappa)}_{LM}(\theta^T_{R,-M},\xi)]^2\times e^{2\gamma\delta_T}\frac{|\mathcal{H}^{(+)}_M(\theta)|^2}{\sin\theta},\tag{4.14}$$

$$t_{LM}(\theta)=\left(\frac{\Lambda^T_M}{\Lambda^T_{-M}}\right)^{1/2}\frac{I^{(\kappa)}_{L-M}(\theta^T_{R,M},\xi)}{I^{(\kappa)}_{LM}(\theta^T_{R,-M},\xi)}e^{i(\Delta_M-\Delta_{-M})\bar{\theta}}\frac{\mathcal{H}^{(-)}_M(\theta)}{\mathcal{H}^{(+)}_M(\theta)},\tag{4.15}$$

and

$$\bar{\Lambda}_M=\tfrac{1}{2}(\Lambda_M+\Lambda_{-M})=\bar{\Lambda}+\tfrac{1}{2}(\Delta_M+\Delta_{-M}).\tag{4.16}$$

In (4.15) we have used

$$2[\delta_M(\Lambda_M)-\delta_{-M}(\Lambda_{-M})]\approx(\Delta_M-\Delta_{-M})\bar{\theta},\tag{4.17}$$

where $\bar{\theta}=\Theta(\bar{\Lambda})$.

Equations (4.13)–(4.15) show that the transfer angular distribution has a smooth (bell-shaped) part determined by

$$\frac{1}{\sin\theta}(|\mathcal{H}^{(+)}_M(\theta)|^2+|\mathcal{H}^{(-)}_M(\theta)|^2),\tag{4.18}$$

and an oscillatory part described by the last term in the curly brackets. Under normal conditions only the $m=0$ term in the Poisson sum (4.12) is significant, so that

$$\mathcal{H}^{(+)}_M(\theta)\approx\frac{F(\Delta(\theta^T_{-M}-\theta+i\gamma))}{\theta^T_{-M}-\theta+i\gamma},\tag{4.19}$$

and for intermediate angles $|\mathcal{H}^{(-)}_M(\theta)|$ is much smaller than $|\mathcal{H}^{(+)}_M(\theta)|$. The maximum of the angular distribution is at $\theta=\theta^T_{-M}$, where the value of the cross section is

$$\sigma^{(T)}_{LM}(\max)=A\frac{\mu_i\mu_f}{2\pi^2\hbar^4\zeta^2k_i{}^3k_f\kappa^2}[d^L_{0M}(\tfrac{1}{2}\pi)]^2\frac{\Lambda^T_{-M}}{\sin\theta^T_{-M}}\times[I^{(\kappa)}_{LM}(\theta^T_{R,-M},\xi)]^2e^{2\gamma\delta_T}\frac{|F(i\Delta\gamma)|^2}{\gamma^2}.\tag{4.20}$$

If for example $F(z)=\pi z/\sinh(\pi z)$, we get

$$\frac{|F(i\Delta\gamma)|^2}{\gamma^2}=\left(\frac{\pi\Delta}{\sin(\pi\Delta\gamma)}\right)^2.\tag{4.21}$$

With the same parametric form we have

$$\frac{\mathcal{K}_M^{(-)}(\theta)}{\mathcal{K}_M^{(+)}(\theta)}=\frac{\sinh[\pi\Delta(\theta_{-M}^T-\theta+i\gamma)]}{\sinh[\pi\Delta(\theta_M^T+\theta+i\gamma)]}\approx e^{-\pi\Delta(\theta_M^T-\theta_{-M}^T)}e^{-2\pi\Delta\theta}. \quad (4.22)$$

This shows that the oscillations in the transfer cross section are most pronounced at smaller angles and become strongly damped with increasing θ. It is also seen that at a given angle the damping is controlled by the window parameter Δ. Thus if Δ is relatively small, as in the case of "surface transparency," the oscillations are enhanced. This is the effect first discovered by the Brookhaven group.[21]

Since the radial integrals for the asymptotic form of the transfer form factor (2.5) are real functions, we may write the cross section (4.13) more explicitly as

$$\sigma_{LM}^{(T)}(\theta)=\bar\sigma_{LM}^{(T)}(\theta)\{1+[\tilde t_{LM}(\theta)]^2+(-)^M 2\tilde t_{LM}(\theta)\sin[2\bar\Lambda_M\theta+(\Delta_M-\Delta_{-M})\bar\theta]\}, \quad (4.23)$$

with

$$\tilde t_{LM}(\theta)\approx\left(\frac{\Lambda_M^T}{\Lambda_{-M}^T}\right)^{1/2}\frac{I_{L-M}^{(\kappa)}(\theta_{R,M}^T,\xi)}{I_{LM}^{(\kappa)}(\theta_{R,-M}^T,\xi)}e^{-\pi\Delta(\theta_M^T-\theta_{-M}^T)}e^{-2\pi\Delta\theta}. \quad (4.24)$$

This shows that the diffraction oscillations have the angular period $\pi/\bar\Lambda_M$, which is M dependent because of

$$\tfrac{1}{2}(\Delta_M+\Delta_{-M})=\tfrac{1}{2}\Delta\ln\left(\tfrac{1}{4}\left[\exp\left(\frac{K_0}{\Delta_i}\right)+\exp\left(-\frac{K_0}{\Delta_f}\right)+2\exp\left(\frac{\Delta_f-\Delta_i}{2\Delta_i\Delta_f}K_0\right)\cosh\left(\frac{M}{\Delta}\right)\right]\right)\to\tfrac{1}{2}\Delta\ln\left(\tfrac{1}{2}\left[\cosh\left(\frac{K_0}{\Delta}\right)+\cosh\left(\frac{M}{\Delta}\right)\right]\right), \quad (4.25)$$

with the latter expression for $\Delta_i=\Delta_f$. The M dependence of the relative phase in (4.23) is given by

$$\Delta_M-\Delta_{-M}=\Delta\ln\left[\frac{\exp\left(\frac{K_0-M}{2\Delta_i}\right)+\exp\left(-\frac{K_0-M}{2\Delta_f}\right)}{\exp\left(\frac{K_0+M}{2\Delta_i}\right)+\exp\left(-\frac{K_0+M}{2\Delta_f}\right)}\right]\to\Delta\ln\left[\frac{\cosh\left(\frac{M-K_0}{2\Delta}\right)}{\cosh\left(\frac{M+K_0}{2\Delta}\right)}\right]\approx\tfrac{1}{2}(|M-K_0|-|M+K_0|). \quad (4.26)$$

This can modify the transfer analog of the Blair phase rule which would otherwise hold because of the factor $(-)^M$ in (4.23).

B. Magnetic substates and Q-value dependence

The dependence on the kinematic conditions of the transfer reaction can best be seen from the transfer partial-wave amplitudes (2.29).

$$c_{LK}(\bar\lambda)=\eta_K(\bar\lambda)I_{L-K}^{(\kappa)}(\alpha,\xi)=\eta_K(\bar\lambda)\tilde I_{L-K}^{(\kappa)}(\alpha,\xi)e^{-\gamma\bar\lambda}, \quad (4.27)$$

which describe the localization in $\bar\lambda$ space. As we have remarked earlier, these functions are essentially asymmetric about their maxima at $\bar\lambda=\bar\Lambda_K^T$: the falloff rate at large $\bar\lambda$ values is determined by γ and thus by the binding parameter κ, while the rate of decrease at low $\bar\lambda$ values is determined by the absorption and measured by Δ^{-1}.

The maximum of $c_{LK}(\bar\lambda)$ is given (approximately) by

$$c_{LK}(\max)\approx c_{LK}(\bar\Lambda_K^T)=\eta_K(\bar\Lambda_K^T)I_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)=\hat\eta\left(\frac{\delta_T}{\Delta}\right)e^{-\gamma\delta_T}\tilde I_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)e^{-\gamma\Lambda_K}. \quad (4.28)$$

Using expression (2.19) for Λ_K yields

$$c_{LK}(\max)=\frac{\hat\eta\left(\frac{\delta_T}{\Delta}\right)e^{-\gamma(\bar\Lambda+\delta_T)}\tilde I_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)}{\left\{\frac{1}{2}\left[\exp\left(\frac{K-K_0}{2\Delta_f}\right)+\exp\left(-\frac{K-K_0}{2\Delta_i}\right)\right]\right\}^{\gamma\Delta}}\approx 2^{\gamma\Delta}\hat\eta\left(\frac{\delta_T}{\Delta}\right)e^{-\gamma(\bar\Lambda+\delta_T)}\tilde I_{L-K}^{(\kappa)}(\theta_{R,K}^T,\xi)e^{-|K-K_0|/g^*}, \quad (4.29)$$

where

$$g^*=\frac{\Delta_i+\Delta_f}{\gamma\Delta_{i,f}} \quad (4.30)$$

with index i or f depending on whether $K-K_0$ is positive or negative.

It is instructive to compare the $\bar\lambda$ and K dependence of $c_{LK}(\bar\lambda)$ with the phenomenological ansatz used by Kahana *et al.*,[20] who assume the Gaussian form

$$c_{LK}^{\text{phen}}(\bar\lambda)=I_L\exp\left[-\left(\frac{\lambda_i-\Lambda_i}{\Gamma_i}\right)^2-\left(\frac{\lambda_f-\Lambda_f}{\Gamma_f}\right)^2-\left(\frac{K-K_0}{g_0}\right)^2\right]\approx I_L\exp\left[-2\left(\frac{\bar\lambda-\bar\Lambda}{\Gamma}\right)^2-\left(\frac{K-K_0}{g}\right)^2\right], \quad (4.31)$$

where the second expression is obtained for $\Gamma_i\approx\Gamma_f\approx\Gamma$ and where

$$g=\frac{2^{1/2}g_0\Gamma}{(g_0^2+2\Gamma^2)^{1/2}}. \quad (4.32)$$

Although in our $c_{LK}(\bar\lambda)$ the asymptotic dependence on $\bar\lambda$ and K is exponential rather than Gaussian, there is a correspondence between the parameter g and our $g^*\approx 2/\gamma$, while the parameter Γ corresponds to $\sqrt{2}/\gamma$ for large $\bar{\lambda}$ and to $\sqrt{2}\Delta$ for small $\bar{\lambda}$, or rather to the harmonic mean $\Gamma^* = \sqrt{2}\Delta/(1+\gamma\Delta)$.

Since the DWBA-based expression (4.27) has qualitatively similar features as the phenomenological form (4.31), we have in the present treatment removed the shortcomings of our earlier transfer formalism[13] which were rightly criticized in Refs. 20 and 21 for ignoring the significant K dependence of the partial-wave amplitude. The salient point where this is brought out here is the determination of the mean S matrix (2.16) by the method described in Appendix A. Most of the properties of transfer reactions discussed in Ref. 21 by means of (4.31) are described by the present formulation.

To display the M dependence of the transfer cross section (in the coordinate frame with quantization axis perpendicular to the reaction plane), we consider the maximum value (4.20) written as

$$\sigma_{LM}^T(\max) = \hat{A}[d_{0M}^L(\tfrac{1}{2}\pi)]^2 \frac{\Lambda_{-M}^T}{\sin\theta_{-M}^T}[I_{LM}^{(\kappa)}(\theta_{R,-M}^T,\xi)]^2 e^{2\gamma\delta_T}$$
$$= \hat{A}[d_{0M}^L(\tfrac{1}{2}\pi)]^2 \frac{\Lambda_{-M}^T}{\sin\theta_{-M}^T}[\tilde{I}_{LM}^{(\kappa)}(\theta_{R,-M}^T,\xi)]^2 e^{-2\gamma\Lambda_{-M}}, \tag{4.33}$$

where

$$\hat{A} = A\frac{\mu_i\mu_f}{2\pi^2\hbar^4\xi^2 k_i^3 k_f \kappa^2}\frac{|F(i\Delta\gamma)|^2}{\gamma^2}. \tag{4.34}$$

The M- and Q-value dependence of the functions $\tilde{I}_{LM}^{(\kappa)}$ is described by the expressions given in Appendix B. As discussed there, it is contained mainly in a factor $\exp(-b_{-M}^2/2\rho\epsilon_{-M})$, where

$$b_{-M} = \xi'\epsilon_{-M} + \xi + M\left(\frac{\epsilon_{-M}-1}{\epsilon_{-M}+1}\right)^{1/2},$$
$$\epsilon_{-M} = [(\Lambda_{-M}^T/n)^2 + 1]^{1/2}. \tag{4.35}$$

However, the most significant factor in (4.33) is

$$e^{-2\gamma\Lambda_{-M}} = e^{-2\gamma\bar{\Lambda}}e^{-2\gamma\Delta_{-M}}$$
$$= \frac{e^{-2\gamma\bar{\Lambda}}}{\left\{\frac{1}{2}\left[\exp\left(\frac{M-M_0}{2\Delta_i}\right)+\exp\left(-\frac{M-M_0}{2\Delta_f}\right)\right]\right\}^{2\gamma\Delta}}$$
$$\approx 4^{\gamma\Delta}e^{-2\gamma\Delta}\exp\left(-2\frac{|M-M_0|}{g^*}\right), \tag{4.36}$$

where $M_0 = -K_0 = \Lambda_f - \Lambda_i$, and g^* is defined by (4.30) with index i or f depending on whether $M > M_0$ or $M < M_0$. For $\Delta_i = \Delta_f$ the M dependence has the simple form $\exp(-\gamma|M-M_0|)$, showing that the cross section is largest for $M = M_0$.

Finally we consider the M dependence of the amplitude (4.24) of the diffraction oscillations, which can be written as

$$\tilde{t}_{LM}(\theta) = \left(\frac{\Lambda_M^T}{\Lambda_{-M}^T}\right)^{1/2}\frac{\tilde{I}_{L-M}^{(\kappa)}(\theta_{R,M}^T,\xi)}{\tilde{I}_{LM}^{(\kappa)}(\theta_{R,-M}^T,\xi)}$$
$$\times e^{-\gamma(\bar{\Lambda}_M^T-\bar{\Lambda}_{-M}^T)}e^{-\pi\Delta(\theta_M^T-\theta_{-M}^T)}e^{-2\pi\Delta\theta}. \tag{4.37}$$

Since $\Delta_{\pm M} \mp \frac{1}{2}M + \delta_T \ll \bar{\Lambda}$, and $\bar{\Lambda}_M^T - \bar{\Lambda}_{-M}^T = \Delta_M - \Delta_{-M}$, and further

$$\theta_M^T - \theta_{-M}^T = \Theta(\bar{\Lambda}_M^T) - \Theta(\Lambda_{-M}^T) \approx (\Delta_M - \Delta_{-M})\Theta'(\bar{\Lambda}), \tag{4.38}$$

this becomes

$$\tilde{t}_{LM}(\theta) \approx \frac{\tilde{I}_{L-M}^{(\kappa)}(\theta_{R,M}^T,\xi)}{\tilde{I}_{LM}^{(\kappa)}(\theta_{R,-M}^T,\xi)}\exp\{-[\gamma+\pi\Delta\Theta'(\bar{\Lambda})](\Delta_M-\Delta_{-M})\}$$
$$\times e^{-2\pi\Delta\theta}, \tag{4.39}$$

where $\Delta_M - \Delta_{-M}$ is given by (4.26). Aside from this difference, because of $\Theta'(\bar{\Lambda}) < 0$ the sign of the exponential depends on the relative magnitude of γ and $\pi\Delta|\Theta'(\bar{\Lambda})|$. An estimate of the latter quantity is given by the Rutherford value $\pi(\Delta/\Lambda)\sin\bar{\theta}_R$ which under strong-absorption conditions is small compared to γ. Thus the amplitudes are largest for good l matching (small K_0) in which case $\Delta_M - \Delta_{-M}$ is nearly independent of M, and the main M dependence of $t_{LM}(\theta)$ is given by the ratio of the functions $I_{LM}^{(\kappa)}$. Since from Appendix B we know that these are largest for $M = -L$ and smallest for $M = L$, the amplitude is maximal for $M = L$. Further the magnitude of $t_{LL}(\theta)$ decreases with increasing L, so that the strongest diffraction oscillations occur for $L = 0$ transfer and are weaker for higher multipolarities.

V. CONCLUSION AND OUTLOOK

The expressions derived in this paper give, in closed analytic form, a rather detailed description of the significant wave-mechanical aspects of heavy-ion transfer reactions as embodied implicitly in (first-order, no-recoil) DWBA calculations. By accounting for the important kinematical and dynamical effects of magnetic substate, angular momentum matching, and Q-value dependences, a considerable improvement is achieved over earlier closed-form evaluations of the DWBA amplitude which glossed over many subtle features by taking too rough averages over initial and final state energies and angular momentum projections. The main improving device is the introduction of the mean S matrix, which leads to an explicit representation of the partial-wave transfer amplitude that is more realistic than the *ad hoc* forms assumed in phenomenological treatments.

Of course, the formalism presented so far still has many shortcomings because of the simplifying assumptions made at the outset: (i) the no-recoil approximation, (ii) omission of higher-order and multi-step processes, and (ii) the neglect of spin and polarization effects. In subsequent work we endeavor to remove or remedy these limitations by suitable extensions of our method. One of several ways to account for recoil effects is a Taylor expansion of the final-channel distorted-wave function,[29,30] which results in the no-recoil transfer amplitude being amended by a linear combination of amplitudes pertaining to different values L_R of the transferred angular momentum and containing radial integrals with modified transfer form factors. The additional amplitudes can then be evaluated in closed form using similar approximations as for the no-recoil term. Further, the contributions of higher-order processes, such as inelastic scattering of the initial and final nuclei before and after the transfer takes place, can be taken into account by replacing the initial and final S matrices with the modified expressions derived recently[31] from a coupled-channels extension of the closed-formalism for elastic and inelastic heavy-ion scattering. Earlier, Udagawa and Tamura[32] gave a qualitative description by using a phenomenological form of the partial-wave transfer amplitude. Similar extensions are possible for multi-step, successive, or sequential transfers and will be treated in subsequent papers. Lastly, spin and polarization effects can be described by including spin-orbit interaction in the initial and final S matrices and taking the spin dependence of the bound-state wave function into account.

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APPENDIX A: DETERMINATION OF MEAN S MATRIX

Here we address the question of how to replace the product of square roots of the S matrices in the initial and final channels by a single mean S matrix. Assuming that $S_l(k)=\eta_l(k)\exp[i2\delta_l(k)]$ can be interpolated by a function of the continuous variable $\lambda=l+\frac{1}{2}$, with the reflection function $\eta_l(k)\to\eta(\lambda)=\hat{\eta}(\mu)$ characterized by the parameters Λ and Δ in the combination $\mu=(\lambda-\Lambda)/\Delta$, we require

$$[S_{l_f}(k_f)S_{l_i}(k_i)]^{1/2}\to[S^{(f)}(\lambda_f)S^{(i)}(\lambda_i)]^{1/2} \hat{=} S^{(K)}(\bar{\lambda})=\eta_K(\bar{\lambda})e^{i2\delta_K(\bar{\lambda})}, \quad (A.1)$$

where $\bar{\lambda}=\frac{1}{2}(\lambda_i+\lambda_f)$ and $K=\lambda_i-\lambda_f$. Here and in the following equations, $a\hat{=}b$ means "a is required to equal b." Then

$$2\delta_K(\bar{\lambda})=\delta^{(i)}(\lambda_i)+\delta^{(f)}(\lambda_f) =\delta^{(i)}(\bar{\lambda}+\tfrac{1}{2}K)+\delta^{(f)}(\bar{\lambda}-\tfrac{1}{2}K), \quad (A.2)$$

and our requirement for the reflection functions is

$$[\eta^{(f)}(\lambda_f)\eta^{(i)}(\lambda_i)]^{1/2}=\left[\hat{\eta}\left(\frac{\lambda_f-\Lambda_f}{\Delta_f}\right)\hat{\eta}\left(\frac{\lambda_i-\Lambda_i}{\Delta_i}\right)\right]^{1/2} \hat{=}\eta_K(\bar{\lambda})=\hat{\eta}\left(\frac{\bar{\lambda}-\Lambda_K}{\Delta}\right), \quad (A.3)$$

with Λ_K and Δ determined as functions of $\Lambda_i,\Lambda_f,\Delta_i,\Delta_f$ such that (A3) is optimally satisfied for functions $\hat{\eta}(\mu)$ of general "strong-absorption profile."

This is of course trivial in the sharp-cutoff limit where $\hat{\eta}(\mu)$ is a unit step function; then

$$\Lambda_K=\bar{\Lambda}+\tfrac{1}{2}|K-K_0| \quad \text{(sharp cutoff)}, \quad (A.4)$$

where $\bar{\Lambda}=\frac{1}{2}(\Lambda_i+\Lambda_f)$ and $K_0=\Lambda_i-\Lambda_f$.

For smooth profiles we first solve the problem by choosing a particular functional form of $\hat{\eta}(\mu)$ for which (A3) can be satisfied exactly, and then use the result in the general case. If $\Delta_i=\Delta_f=\Delta$, this can be done with Kauffmann's function[33]

$$\hat{\eta}(\mu)=\exp(-e^{-\mu}). \quad (A.5)$$

Then

$$\frac{1}{2}\left[\exp\left(-\frac{\lambda_f-\Lambda_f}{\Delta}\right)+\exp\left(-\frac{\lambda_i-\Lambda_i}{\Delta}\right)\right]=\exp\left(-\frac{\bar{\lambda}-\Lambda_K}{\Delta}\right) \quad (A.6)$$

yields

$$\Lambda_K=\bar{\Lambda}+\Delta\ln\left[\cosh\left(\frac{K-K_0}{2\Delta}\right)\right]. \quad (A.7)$$

For $\Delta_i\neq\Delta_f$ we obtain

$$\frac{1}{2}\left[\exp\left(\frac{K-K_0}{2\Delta_f}\right)\exp\left(-\frac{\bar{\lambda}-\bar{\Lambda}}{\Delta_f}\right) +\exp\left(-\frac{K-K_0}{2\Delta_i}\right)\exp\left(-\frac{\bar{\lambda}-\bar{\Lambda}}{\Delta_i}\right)\right]\hat{=}\exp\left(-\frac{\bar{\lambda}-\bar{\Lambda}}{\Delta}\right), \quad (A.8)$$

which can no longer be satisfied exactly. However, we can determine optimal values of Λ_K and Δ by the following requirements: First, (A8) should hold exactly at $\bar{\lambda}=\Lambda_K$. This yields

$$\Lambda_K=\bar{\Lambda}+\Delta\ln\left\{\frac{1}{2}\left[\exp\left(\frac{K-K_0}{2\Delta_f}\right)+\exp\left(-\frac{K-K_0}{2\Delta_i}\right)\right]\right\}. \tag{A.9}$$

Then we determine Δ such that in the case of K matching, $K=K_0$, where (A8) becomes

$$\frac{1}{2}\left\{\exp\left[\left(\frac{1}{\Delta}-\frac{1}{\Delta_f}\right)(\bar{\lambda}-\bar{\Lambda})\right]+\exp\left[\left(\frac{1}{\Delta}-\frac{1}{\Delta_i}\right)(\bar{\lambda}-\bar{\Lambda})\right]\right\}\hat{=}1, \tag{A.10}$$

it minimizes the difference between the exponentials,

$$\left(\frac{1}{\Delta}-\frac{1}{\Delta_f}\right)^2+\left(\frac{1}{\Delta}-\frac{1}{\Delta_i}\right)^2=\text{minimum}, \tag{A.11}$$

$$\Delta=\frac{2\Delta_i\Delta_f}{\Delta_i+\Delta_f}. \tag{A.12}$$

Clearly, for $\Delta_i=\Delta_f$, (A.9) reduces to (A.7), and both expressions reduce to (A.4) in the sharp-cutoff limit.

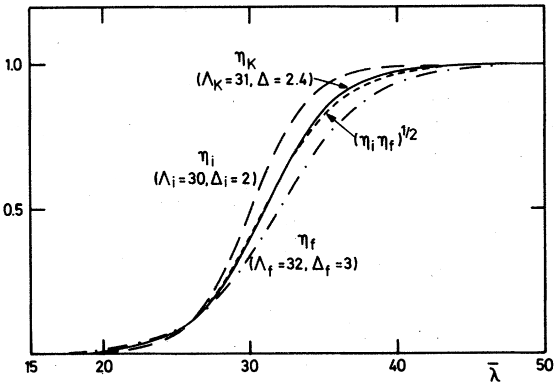

FIG. 2. Mean reflection function $\eta_K(\bar{\lambda})$ of Fermi function profile (solid curve), calculated from the initial-channel reflection function $\eta_i(\bar{\lambda})$ with $\Lambda_i=30$, $\Delta_i=2$ (long-dashed curve) and the final-channel reflection function $\eta_f(\bar{\lambda})$ with $\Lambda_f=32$, $\Delta_f=3$ (dot-dashed curve). $\eta_K(\bar{\lambda})$ represents the optimum symmetric approximation [Eq. (A3)] to the asymmetric function $[\eta_i(\bar{\lambda})\eta_f(\bar{\lambda})]^{1/2}$ (short-dashed curve), with parameters $\Lambda_K=31$ and $\Delta=2.4$ calculated from Eqs. (A9) and (A12), respectively.

An example of how relation (A.3) with (A.9) and (A.12) works for reflection functions of Fermi function profile $\hat{\eta}(\mu)=(1+e^{-\mu})^{-1}$ is shown in Fig. 2.

APPENDIX B: THE FUNCTIONS $I_{LM}^{(\kappa)}(\alpha,\xi)$

The functions $I_{LM}^{(\kappa)}(\alpha,\xi)$, defined in (2.22) as the WKB approximation to the Coulomb radial integrals (2.14) for the transfer form factor (2.5), have the integral representation

$$I_{LM}^{(\kappa)}(\alpha,\xi)=\int_0^\infty dw(-i^L)h_L^{(1)}[i\rho(1+\epsilon\cosh w)]\rho(1+\epsilon\cosh w)\cos\left\{\xi'\epsilon\sinh w+\xi w+M\arctan\left[\frac{(\epsilon^2-1)^{1/2}\sinh w}{\epsilon+\cosh w}\right]\right\}, \tag{B1}$$

where

$$\epsilon=(\sin\tfrac{1}{2}\alpha)^{-1}=[(\bar{\lambda}/n)^2+1]^{1/2}, \tag{B.2}$$

and ρ,ξ,ξ' are defined in (2.23) and (2.32). An approximate expansion, with an accuracy of about 10% for $|\xi'|<1.5$, is given in Ref. 14 as

$$I_{LM}^{(\kappa)}(\alpha,\xi)=\left(\frac{\pi}{2}\right)^{1/2}e^{-\rho(1+\epsilon)}\times\sum_{m=0}^{L}\frac{(L+m)!}{(L-m)!m!}\frac{1}{[2\rho(1+\epsilon)]^m}\frac{e^{-b^2/2a_m}}{a_m^{1/2}}, \tag{B.3}$$

with

$$a_m=\rho\epsilon+m\frac{\epsilon}{1+\epsilon},\quad b=\xi'\epsilon+\xi+M\left(\frac{\epsilon-1}{\epsilon+1}\right)^{1/2}. \tag{B.4}$$

For $\rho(1+\epsilon)>1$ and $b\lesssim 1$, the sum in (B.3) may be approximated by the $m=0$ term,

$$I_{LM}^{(\kappa)}(\alpha,\xi)\approx\left(\frac{\pi}{2\rho\epsilon}\right)^{1/2}\exp\left[-\frac{b^2}{2\rho\epsilon}-\rho(1+\epsilon)\right]. \tag{B.5}$$

As discussed in Ref. 14, this expression shows the Q value and M dependence of $I_{LM}^{(\kappa)}$ by the exponential of $b^2/2\rho\epsilon$. For a given angle α, the optimum Q value is determined by the adiabaticity parameters, giving maximum $I_{LM}^{(\kappa)}$ for $b=0$, or

$$\frac{\xi'}{\sin\frac{1}{2}\alpha}+\xi=-M\tan\tfrac{1}{4}(\pi-\alpha), \tag{B.6}$$

and the steepness with which $I_{LM}^{(\kappa)}$ decreases for increasing $|Q-Q_{\text{opt}}|$ is controlled by the binding parameter κ through $\rho\epsilon=\kappa n/k\sin\frac{1}{2}\alpha$ such that the falloff is faster for weakly than for strongly bound particles. The same relation shows that for positive Q values, $I_{LM}^{(\kappa)}$ is largest for $M=-L$ and smallest for $M=L$, and that the difference is more pronounced at smaller than at larger angles α.

For $L=0$, (B.1) can be evaluated exactly in terms of a modified Hankel function of imaginary order,[27]

$$I_{00}^{(\kappa)}(\alpha,\xi)=\exp\left[-\left(\rho+\xi\arctan\frac{\xi'}{\rho}\right)\right]K_{i\xi}(n\gamma\epsilon)$$

$$\cong\left(\frac{\pi}{2n\gamma\epsilon}\right)^{1/2}\exp\left[-\left(\rho+\xi\arctan\frac{\xi'}{\rho}+n\gamma\epsilon\right)\right], \quad (B.7)$$

where $\gamma=(\rho^2+\xi'^2)^{1/2}/n$, with the asymptotic form of (B.7) obtained for $n\gamma\epsilon\gg|\xi|$.

The main dependence of $I_{LM}^{(\kappa)}$ on $\bar{\lambda}$ is given by the factor $\exp(-n\gamma\epsilon)$, which becomes $\exp(-\gamma\bar{\lambda})$ for $\bar{\lambda}\gg n$. This enables us to define

$$I_{LM}^{(\kappa)}(\alpha,\xi)\equiv\tilde{I}_{LM}^{(\kappa)}(\alpha,\xi)e^{-\gamma\bar{\lambda}}, \quad (B.8)$$

where $\tilde{I}_{LM}^{(\kappa)}(\alpha,\xi)$ varies relatively slowly with $\bar{\lambda}$. Finally, using an amended notation, we note from (B.1) the symmetry relation

$$I_{L-M}^{(\kappa)}(\alpha,\xi,\xi')=I_{LM}^{(\kappa)}(\alpha,-\xi,-\xi'). \quad (B.9)$$

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