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Character of the low-lying 0^- and 0^+ states in the even Rn, Ra, Th, U, and Pu isotopes

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The systematics of the energies and inverse moments of inertia are compared for the ground state and low-lying $K=0^+$ and $K=0^-$ bands in the even Rn, Ra, Th, U, and Pu nuclei. This comparison suggests a structural relationship between the excited $K=0^+$ band and the well known $K=0^-$ band as expected from the calculations of Chasman.

[NUCLEAR STRUCTURE Systematics of energies and $h/2$ values of low-lying $K=0^+$ and $K=0^-$ bands in even Rn, Ra, Th, U, and Pu isotopes.]

In their generalization of shell structure for spherical systems Bohr and Mottelson[1] suggest that shell structure is expected when $(\partial e/\partial n):(\partial e/\partial l)=a:b$, where e is the energy of single particle levels characterized by radial quantum numbers n, and orbital quantum numbers l, and a and b are small integers. For light up to medium heavy nuclear systems, this ratio is known experimentally to be 2:1. This occurs because s and d, p and $f, \ldots$ orbitals are close together in energy. Since matrix elements of the type Y_{20} are large between these orbitals, reflection symmetric quadrupole shapes are expected. However, for heavy nuclei just beyond ^{208}Pb $f_{7/2}$, and $i_{13/2}$ protons and $g_{9/2}$ and $j_{15/2}$ neutrons lie close together in energy, giving rise to $(\partial e/\partial n):(\partial e/\partial l)=3:1$. Because the Y_{30} matrix elements are large between these orbitals, stable octupole reflection asymmetric shapes now compete with the quadrupole shapes. Because of the position of the orbitals relative to the other orbitals giving rise to the octupole shape, the main tendency toward octupole shape is expected early in the shell and is expected to maximize with configuration $v(g_{9/2})^{10}$. For spherical nuclei this occurs at neutron number 136.

In this interesting transition region where octupole and quadrupole shapes are competing there are at least three different macroscopic shape possibilities and corresponding spectra. In the first, the energy surface has a reflection symmetric ground state minimum giving rise to a $K=0^+$ ground state band with $I^\pi=0^+, 2^+, 4^+, \ldots$ with odd spins missing because of reflection symmetry. The $K=0^-$ states with $I^\pi=1^-, 3^-, 5^-, \ldots$ arise from an octupole vibration built on the ground state. This situation would correspond to an energy surface soft but stable with respect to an asymmetric degree of freedom. Excited 0^+ states would not be related to the octupole states except as possible two phonon octupole vibrations expected at double the excitation of the one phonon energy. This has been the accepted[2] interpretation of the low-lying positive and negative parity states in the even-even Rn-Pu nuclei in spite of the failure to find the expected two phonon octupole vibrations.

In the second case we assume a stable octupole deformation with a superimposed quadrupole deformation in which the potential barrier between one octupole minimum and its mirror image is very large. This gives rise to an alternating parity band, $I^\pi=0^+, 1^-, 2^+, 3^-, \ldots$. With a smaller barrier, tunneling between the mirror shapes gives two sets of levels $I^\pi=0^+, 2^+, 4^+$ and $I^\pi=1^-, 3^-, 5^-$, with the odd spin set displaced upward in energy relative to the even spin set. Theoretical calculations [3] have suggested, however, that nonzero octupole deformations are not expected for ground states for nuclei with $A \geqslant 218$.

A third possibility involves the coexistence[4] of *two* shape minima, one near spherical or prolate reflection symmetric, associated with an even parity ground state band, the other involving a stable octupole band. The positive parity states in the stable octupole band would tend to mix[5] with the ground state band and be pushed up in energy while achieving a mixed quadrupole-octupole deformation.

Recently Chasman[6] succeeded in treating both pairing and octupole-octupole interactions on an equal basis in the ground and excited states, thus entering a "transition" regime of mixed quadrupole and pairing deformations with octupole correlations. In these microscopic calculations he has shown that it is possible to account semiquantitatively for the energies and predict $B(E3)$ values for the mysterious low-lying $K=0^+$ excited states in ^{232,234}U and ^{236}U. His calculations also show that there is considerable octupole character in the excited low-lying 0^+ states although the condition for stable octupole deformation with a large barrier between the mirrored octupole minima has almost but not quite been met in these U nuclei.

In this study, the systematics of the energies and inverse moments of inertia ($\hbar^2/2\mathscr{I}$) of these excited $K=0^+$ bands are compared with the corresponding $K=0^-$ and ground state $K=0^+$ bands for nuclei from Rn through Pu. This comparison illuminates a number of important generic relationships between these bands which further verify the calculations of Chasman and suggests additional calculations and experiments.

Figure 1 presents a comparison of the $K=0^-$ and the excited $K=0^+$ band heads[7–13] relative to the $K=0^+$ ground states as a function of neutron number. In most cases the only known excited 0^+ states have been used. When more than one excited 0^+ state is known in the low excitation, as occurs for ^{234,238}U and ^{240}Pu, the one with the smallest 0^+-2^+ rotational spacing is used in the comparison. It should be noted that the calculations of Chasman[6] give the 0^+ states in ^{234}U and ^{236}U at 870 and 1160 keV, respectively. This is somewhat nearer the observed state at 810 keV rather than the state at 1044 keV presented in Fig. 1. No excited 0^+ states have been observed in

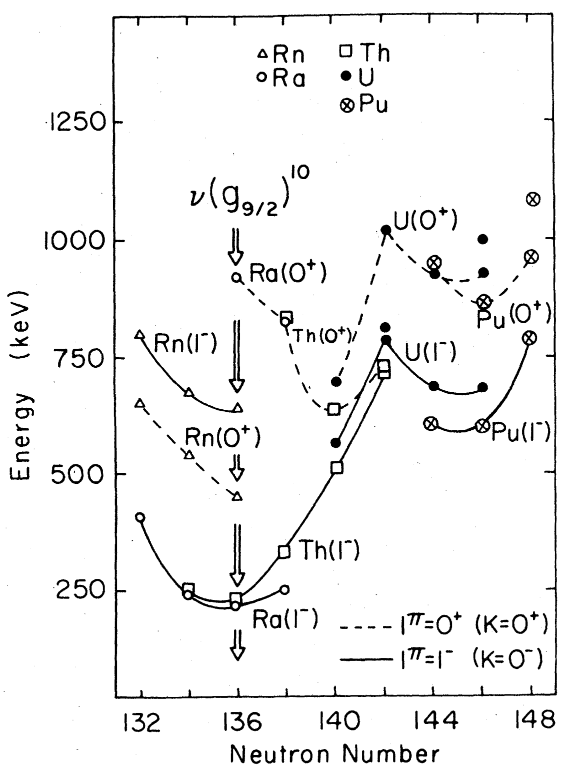

FIG. 1. Energy level systematics of the $K=0^-$, $I^\pi=1^-$ states (solid lines) and the $K=0^+$, $I^\pi=0^+$ excited states (dashed lines) relative to the ground states for Rn, Ra, Th, U, and Pu isotopes (Refs. 7–13) plotted against neutron number.

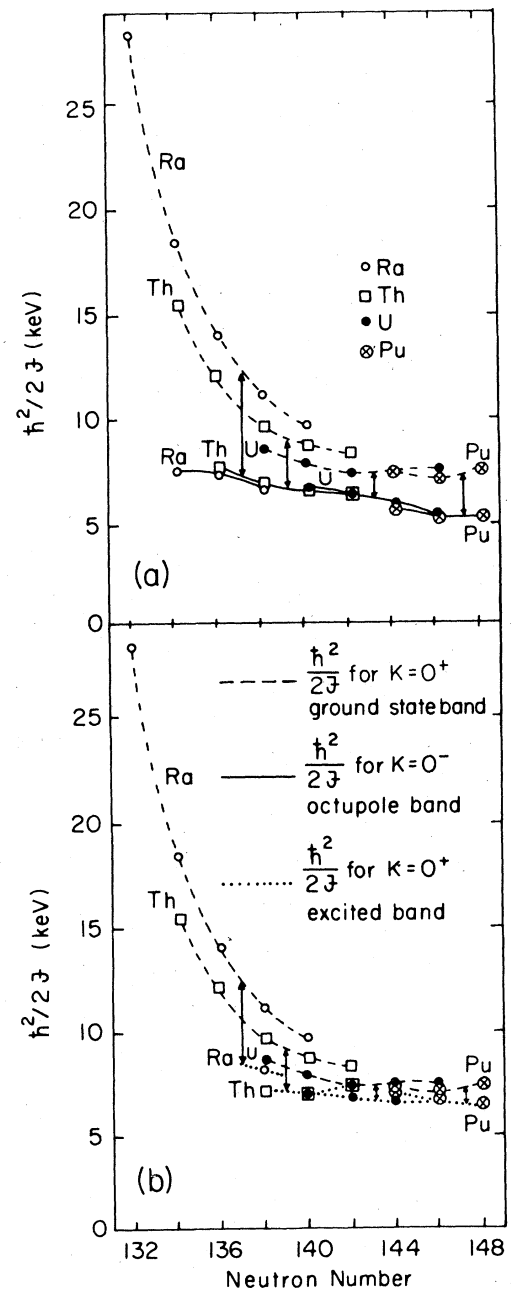

FIG. 2. Systematics of the values of $\hbar^2/2\mathscr{I}$ (a) for the $K=0^-$ bands (solid lines) and the $K=0^+$ ground state bands (dashed lines), and (b) for the $K=0^+$ excited bands (solid) and the $K=0^+$ ground state bands (dashed) in the Ra, Th, U, and Pu isotopes. Arrows facilitate comparisons between appropriate isotopic trends.

^{236}U except the state presented in Fig. 1 at 919 keV. Furthermore, the excited states in ^{218}Rn and ^{220}Rn, originally tentatively assigned[7] as the $I^{\pi} = 0^+$ state and presented that way in Fig. 1 have been left unassigned in the most recent compilation.[14,15] Neutron number 136, where for spherical nuclei the $g_{9/2}$ orbital is filled, has been noted on Fig. 1. Just at his neutron number the energy of the $K = 1^-$ band reaches a minimum. Indeed, this is the lowest energy of any excited state in nuclei other than rotational states.

Not only is there a very regular relationship between the excited 0^+ and 1^- states for the Rn nuclei, but also for the U and Pu isotopes of Fig. 1. Now, however, the 1^- state is systematically below the 0^+ state, although mirroring accurately the discontinuity at the well known gap at 142 neutrons. The fact that the excited 0^+ and 1^- states track each other argues strongly for a generic structural relationship and for the greater octupole content in the wave function of the excited $K = 0^+$ states of the U isotopes calculated by Chasman.[6]

However, the excited 0^+ states of the Ra and Th nuclei, although known for only five of the nine possible nuclei, do not track the 1^- bands. The experimental information on these nuclei is quite limited so it is not clear whether this is an experimental difficulty.

In a parallel comparison of the inverse moments of inertia, we again find a striking similarity in the values of $\hbar^2/2\mathscr{I}$ for the excited $K = 0^+$ and $K = 0^-$ bands and a considerable deviation from the value of $\hbar^2/2\mathscr{I}$ for the ground state $K = 0^+$ band. The comparisons are shown in Figs. 2(a) and 2(b). Plots somewhat similar to both Figs. 1 and 2(a) have been published previously (see, e.g., Fig. 1 of Ref. 4 and Fig. 4 of Ref. 16). In order to make the broadest possible comparison, $0^+ - 2^+$ and $1^- - 3^-$ energy differences[7–13] *only* were used in the calculation of $\hbar^2/2\mathscr{I}$. In Fig. 2(a) it is immediately obvious that the value of $\hbar^2/2\mathscr{I}$ is systematically smaller and more constant for the $K = 0^-$ octupole band than for the $K = 0^+$ ground state band. Furthermore, there is an obvious change in trend in the value of $\hbar^2/2\mathscr{I}$ for the ground state band beginning at $N \leq 136$. Thus with $N > 136$ the value of $\hbar^2/2\mathscr{I}$ for the ground state band is approximately constant and parallel but larger than the trend in the $K = 0^-$ states. With $N \leq 136$ a sharp deviation away from the $K = 0^-$ trend is clearly developing.

In Fig. 2(b) we find a similar pattern for the $K = 0^+$ excited band. The value of $\hbar^2/2\mathscr{I}$ is approximately constant and always less than the value for the ground state band. Actually it is intermediate in value between the $K = 0^+$ ground state and the $K = 0^-$ band although closer and more nearly mirroring the systematics of the $K = 0^-$ band. Unfortunately, because of a lack of data it is impossible to follow the $K = 0^+$ band over as great a range of neutron numbers as the $K = 0^-$ band. To facilitate comparisons arrows are used to relate appropriate isotopic trends in the values of $\hbar^2/2\mathscr{I}$.

Although the macroscopic picture presented here cannot mirror completely the complexity of the microscopic calculations of Chasman, the systematics of the energies (Fig. 1) and the inverse moments of inertia [Figs. 2(a) and 2(b)] of the $K = 0^+$ (ground state and excited) and the $K = 0^-$ bands strongly implies that the excited $K = 0^+$ and 0^- bands are structurally more strongly related to each other than to the ground state band. The intermediate values of $\hbar^2/2\mathscr{I}$ for the excited $K = 0^+$ band, between that of $K = 0^+$ ground state and the $K = 0^-$ band, suggest that the excited $K = 0^+$ band has a mixed nature at least qualitatively like that calculated by Chasman[6] containing both reflection symmetric and reflection asymmetric character. Thus it seems likely that the recent calculations of Chasman may provide the theoretical framework which Kurcewicz *et al.*[16] found to be missing.

This study also points up the need for additional experimental study of the excited states of ^{218}Rn and ^{220}Rn to see if the originally assigned spin sequence, $0^+, 1^-, 2^+, \ldots$, expected for a coexistence octupole shape is confirmed. It is also important to look for low-lying $I^{\pi} = 0^+$ states in the Ra and Th isotopes to try to understand the transition from Rn to U and Pu, and to measure the $B(E3)$ values populating the excited 0^+ states as proposed by Chasman. Finally, it is clear that additional calculations of the type of Chasman for the Rn, Ra, and Th isotopes will be of decisive importance in understanding this interesting transition region.

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