PHYSICAL REVIEW C VOLUME 21, NUMBER 4 APRIL 1980

Total muon capture rates in neon

Y. K. Gambhir
Department of Physics, Indian Institute of Technology, Bombay, India

R. Parthasarathy
MATSCIENCE, The Institute of Mathematical Sciences, Madras-20, India
(Received 28 November 1978)

We report here the results of the calculation of total muon capture rates ($\Lambda_{\mu c}$) in Ne isotopes using Hartree-Fock wave functions. These wave functions are generated from (a) a phenomenological set of interaction matrix elements and (b) a microscopic set derived from the Reid soft core potential. Satisfactory agreement with the experiment both for the spectra of and the $\Lambda_{\mu c}$ in ^{20}Ne is obtained. The trend of the variation of $\Lambda_{\mu c}$ in Ne isotopes as predicted by the empirical formulas is explained by incorporating the oblate admixture in the ground states of ^{22}Ne and ^{24}Ne.

[NUCLEAR STRUCTURE Hartree-Fock (HF), projected HF, energy spectra, total muon capture rates, Ne isotopes.]

In recent years the projected Hartree-Fock (PHF) theory has been quite successful in explaining nuclear properties such as spectra, static moments, and some of the transition rates for deformed nuclei. Since spectra are not very sensitive to the wave functions and in order to test their correctness, one should choose a probe which is sensitive to the structure wave functions. With this motivation, we use the muon capture process as a probe to test the compatibility of the Hartree-Fock (HF) wave functions for the Ne isotopes. Muon capture by protons is a semileptonic strangeness conserving weak process,[1] and the appropriate coupling constants are governed by the conserved vector current (CVC)[2] and partially conserved axial vector current (PCAC)[3] hypotheses. The standard weak Hamiltonian governing the capture of muons by nuclei[4] uses the impulse approximation where the weak charged nuclear (one-body) current is represented by a sum over the nucleon contributions. This treatment has been extensively used and has shown the sensitivity of muon capture transitions to the nuclear model and the weak coupling constants for partial[5] and total[6] capture rates and the recoil nuclear polarization.[7] The observable capture rate in Ne isotopes, chosen here for study, is the total muon capture rate ($\Lambda_{\mu c}$), as there are no experimental data on partial capture rates. $\Lambda_{\mu c}$ under closure approximation involves the expectation value of a two-body operator Q in the ground state of the capturing nucleus and so will be sensitive to the nuclear ground state wave functions. Total muon capture studies with this motivation have so far been carried out for closed spherical nuclei.[8] In this note, we present a calculation of $\Lambda_{\mu c}$ in Ne isotopes (light deformed nuclei) using the HF wave functions obtained by a microscopic set of interaction matrix elements.

The valance nucleons in Ne isotopes outside the ^{16}O core are distributed in the full $2s$-$1d$ space. The relevant single particle energies are taken from the ^{17}O experimental spectrum. In the present calculation, two sets of interaction matrix elements used are (i) the phenomenological set of Chung and Wildenthal (CW)[9] and (ii) a microscopic set of matrix elements derived from the Reid soft core (RSC) potential[10], incorporating the core polarization corrections involving 3p-1h, 4p-2h excitations, as reported by Vary and Yang.[11] The results of the calculation with the above two sets will be denoted by CW and RSC, respectively. As the interaction matrix elements of Chung and Wildenthal have been obtained by fitting the experimental data in the region $A = 18-22$, the wave functions so obtained will be most reliable. A comparison of the spectra and $\Lambda_{\mu c}$ in the two sets CW and RSC will certainly reveal the ability of the Reid soft core potential to predict the nuclear properties. This is one of the aims of the present investigation. The results of the Hartree-Fock (HF) calculation for Ne isotopes are summarized in Table I. This table gives the HF energies (E_{HF}), the intrinsic quadrupole moment (Q_{HF}), and the energy gap ΔE_p (ΔE_n) between the last occupied and the first unoccupied proton (neutron) HF states for both the prolate and oblate solutions. The table reveals a remarkable similarity between the results obtained by using the phenomenological set of interaction matrix elements and those of the microscopic Reid soft core. Further, the results indicate that the lowest rotational band of ^{20}Ne may

TABLE I. Results of the HF calculation for Ne isotopes. CW—results obtained with the phenomenological interaction matrix elements of Chung and Wildenthal. RSC—results obtained with the interaction matrix elements derived from the Reid soft core potential with appropriate core polarization corrections.

		CW				RSC			
		$E_{\rm HF}$	$Q_{\rm HF}$	E_p	E_n	$E_{\rm HF}$	$Q_{\rm HF}$	E_p	E_n
^{20}Ne	prolate	−20.95	6.3	8.8	8.8	−20.58	6.57	8.6	8.6
	oblate	−12.95	−10.29	1.5	1.5	−10.75	−10.29	1.5	1.5
^{22}Ne	prolate	−29.82	5.45	7.8	3.6	−28.36	5.70	3.6	4.6
	oblate	−23.15	−8.42	2.0	1.5	−19.08	−8.9	2.2	2.1
^{24}Ne	prolate	−34.27	2.13	6.6	2.2	−28.47	6.13	6.1	0.7
	oblate	−31.93	−10.49	1.5	5.4	−25.44	−11.01	2.2	5.6

be accurately described in terms of the prolate HF solution, as the energy difference between the prolate and oblate solution is ~8 MeV, and ΔE_p and ΔE_n are fairly large (~8.8 MeV). In the case of ^{22}Ne, the prolate to oblate gap and ΔE_p are still large while ΔE_n is 3.6 MeV for CW (4.6 MeV for RSC) and, therefore, for the accurate description of even the lowest rotational band, the band mixing will be important.[12] However, the ground state of ^{22}Ne can still be described reasonably well by the prolate HF solution. For ^{24}Ne, one has to consider the band mixing and/or the generator-coordinate method with constrained HF basis, even for the description of the ground state.[13] Further, it is found that for a given isotope the calculated $\Lambda_{\mu c}$ are almost the same for the ground states corresponding to different prolate HF solutions while it ($\Lambda_{\mu c}$) is significantly smaller for the lowest oblate HF solutions. Guided by these considerations, we take the ground state for ^{22}Ne and ^{24}Ne isotopes as a mixture of the lowest prolate and the lowest oblate HF solutions, while for ^{20}Ne the ground state is obtained from the lowest prolate solution alone. The PHF spectra of ^{20}Ne, projected from the prolate and oblate solutions separately, is shown in Fig. 1 along with the experimental data for comparison. It is to be noted that the lowest rotational 0$^+$ band is well reproduced by both the sets of interaction matrix elements. It is found that the calculated spectra and the PHF wave functions, using the CW and RSC interaction matrix elements, are remarkably similar. This, along with the observation in Table I, implies that the interaction matrix elements derived from the free nucleon-nucleon Reid soft core potential[10] are equally successful as that of the phenomenological set in describing the nuclear properties. Now we proceed to calculate $\Lambda_{\mu c}$ using these HF wave functions.

The total muon capture rate $\Lambda_{\mu c}$ for an even-even nucleus (hyperfine complications do not arise) is given by[4,14]

$$\Lambda_{\mu c}=\frac{\langle\nu\rangle^2}{2\pi}\,|\phi_\mu|^2(G_V{}^2+3G_A{}^2+G_P{}^2-2G_PG_A)\times\left\langle a\left|\sum_{n,m=1}^{A}\tau_n^+\tau_m^-\exp[i\langle\vec{\nu}\rangle\cdot(\vec{r}_n-\vec{r}_m)]\right|a\right\rangle, \quad (1)$$

where $|a\rangle$ is the initial nuclear ground state, ϕ_μ is the muon wave function in the atomic K orbit averaged over the nuclear volume, G_V, G_A, and G_P are the muon capture coupling constants,[5] and $\langle\nu\rangle$ is the average neutrino momentum. In deriving the expression (1) for $\Lambda_{\mu c}$, three approximations have been invoked (1) the closure property, (2) the neglect of nucleon velocity dependent terms, and (3) the SU(4) symmetry.[15] It is known that the effect of (2) is to increase $\Lambda_{\mu c}$ by 10% while that of (3) is to decrease $\Lambda_{\mu c}$ by 10–20% so

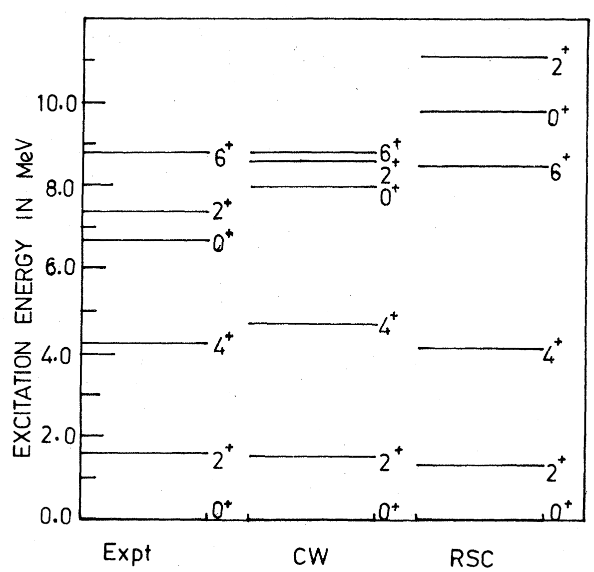

FIG. 1. Projected Hartree-Fock spectra from prolate and oblate solutions of Table I for ^{20}Ne. CW and RSC are the results of the phenomenological and microscopic Reid soft core interaction matrix elements, respectively.

that they compensate with a resultant correction[16] of ~10%. The average neutrino momentum $\langle\nu\rangle$ is a crucial quantity in the calculation of $\Lambda_{\mu c}$, and usually it is considered as a parameter to give the fit for $\Lambda_{\mu c}$ with experiment.[6] However, the analysis of Foldy and Walecka[15] clearly demonstrates that the dominant part of muon capture transition (~90%) leads to the giant dipole states, accounting for a major part of the total transition rate. In the case of ^{20}Ne, the location of the giant dipole resonance (GDR) has been studied in photo-neutron reactions by Fergusson *et al.*[17] and Woodworth *et al.*[18], and in photo-proton reactions by Dodge *et al.*[19] and Segel *et al.*[20] The conclusion of these experiments is that the GDR is centered around 20 MeV with a width of about 5 MeV. The theoretical "open shell" random phase approximation (RPA) calculation by Wong *et al.*[21] places GDR at about 22 MeV. The recent analysis by Ajzenberg-Selove[22] confirms that the GDR is centered around 20 MeV. Thus, the average nuclear excitation energy in muon capture by ^{20}Ne is about 20 ± 3 MeV, and so the average neutrino momentum is no longer a parameter but has the value

$$\langle\nu\rangle = m_\mu - \epsilon_\mu - (20 \pm 3)\ \text{MeV},$$

where ϵ_μ is the binding energy of the muon in the atomic K orbit. Equation (1) can be rewritten as

$$\Lambda_{\mu c} = \frac{\langle\nu\rangle^2}{2\pi}\,|\phi_\mu|^2(G_V{}^2 + 3G_A{}^2 + G_P{}^2 - 2G_PG_A)(Z-Q)\,, \tag{2}$$

where the $m=n$ part of the sum in Eq. (1) gives rise to Z, and

$$Q = -\left\langle a \left| \sum_{n\neq m=1}^{A} \tau_n^+\tau_m^- \exp[i\langle\hat{\nu}\rangle\cdot(\vec{r}_n - \vec{r}_m)] \right| a \right\rangle. \tag{3}$$

It should be noted that Q is the result of correlations in the ground state nuclear wave function $|a\rangle$, and hence it is sensitive to the structure wave functions. In the present calculation, we evaluate Q using rotational nuclear wave functions of the adiabatic form with HF intrinsic states, after transforming the operator in Eq. (3) from the laboratory to the intrinsic coordinate system. As the ground state $|a\rangle$ has total spin zero, the final expression for Q reduces to the expectation value of the scalar part of the operator between the HF states which have a single determinantal form consisting of occupied single particle (sp) HF orbitals. The final result is given by

$$Q = \sum_{\lambda_p,\lambda_n}\sum_{j_p,j_n}(C^{\lambda_p}_{j_p}C^{\lambda_n}_{j_n})^2\sum_l \frac{(2j_p+1)(2j_n+1)}{(2l+1)}\tfrac{1}{2}[1+(-1)^{l_p+l_n-l}]\begin{bmatrix} j_p & j_n & l \\ \frac{1}{2} & -\frac{1}{2} & 0\end{bmatrix}^2\begin{bmatrix} j_p & j_n & l \\ \lambda_p & -\lambda_n & \lambda_p-\lambda_n\end{bmatrix}^2 \times\left|\int_0^\infty R_{n_nl_n}(r)j_l(\langle\nu\rangle r)R_{n_pl_p}(r)\,r^2dr\right|^2, \tag{4}$$

where $\lambda_p(\lambda_n)$ designates the occupied proton (neutron) HF states, and C^λ_j coefficients are the results of HF calculations. In the present investigation, owing to the observed width of the giant dipole resonance peak ($\Gamma \sim 5$ MeV), $\langle\nu\rangle$ is varied from 80 to 90 MeV. The oscillator well parameter b is chosen to be 1.65 fm in accordance with Kelson and Levinson,[23] and we also study the variation of $\Lambda_{\mu c}$ with b. The results for the variation of $\Lambda_{\mu c}$ in ^{20}Ne with $\langle\nu\rangle$ for various values of b are shown in Fig. 2, using the lowest prolate HF wave functions calculated from the microscopic interaction matrix elements derived from the Reid soft core potential. The phenomenological interaction matrix elements of Chung and Wildenthal give almost the same value for $\Lambda_{\mu c}$ as that of the RSC within 0.5%. Figure 2 shows that $\Lambda_{\mu c}$ is almost insensitive to the choice of b values but sensitive to $\langle\nu\rangle$ as also observed by Christillin *et al.*[16] However, the experimental region for $\Lambda_{\mu c}$ can be explained by choosing $\langle\nu\rangle$ from 78 to 84 MeV which corresponds to the experimentally observed giant dipole excitation region.[22] In Table II we present the calculated values of $\Lambda_{\mu c}$ using (a) the Fermi gas model, (b) the empirical formula of Primakoff with the best two parameter fit by Telegdi,[24] (c) the improved formula of Goulard and Primakoff[25] with a three parameter fit, and (d) the lowest prolate HF wave functions obtained from the microscopic interaction matrix elements derived from the Reid soft core potential along with the experimental data.[26] From Table II, it is evident that the Fermi gas model overestimates $\Lambda_{\mu c}$ by a factor ~2. A similar overestimate for spherical nuclei has been observed when a pure shell model is used.[8,27] The present calculation using the HF wave functions reproduces almost exactly the experimental value. The results of $\Lambda_{\mu c}$ for CW and RSC differ by less than 0.5%, which again implies the success of the Reid soft core potential in predicting nuclear properties. Furthermore, the present calculation predicts $\Lambda_{\mu c}$ closer to the experimental value than that of the empirical formula of Primakoff with two parameters. The empirical formula with three parameters predicts $\Lambda_{\mu c}$ which is almost the same as

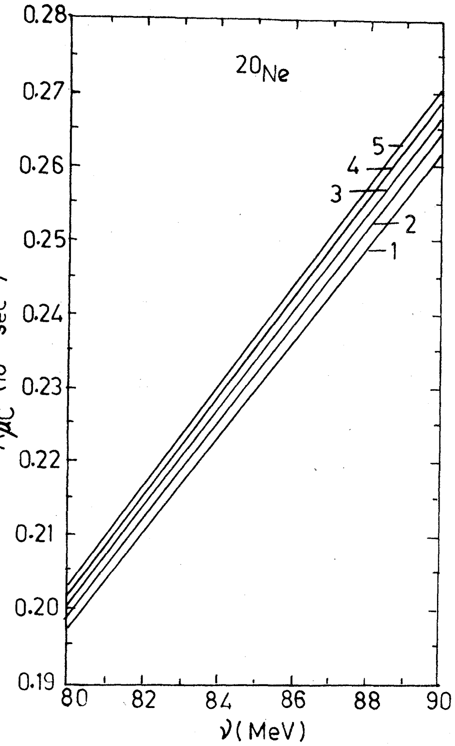

FIG. 2. Variation of total muon capture rate in ^{20}Ne with $\langle \nu \rangle$ for various b values. The results are obtained by using the lowest prolate HF wave functions generated from the interaction matrix elements derived from the Reid soft core potential. Curves 1, 2, 3, 4, and 5 are obtained with $b = 1.65$, 1.70, 1.75, 1.80, and 1.85 fm, respectively.

TABLE II. Results for $\Lambda_{\mu c}$, the toal muon capture rate in ^{20}Ne. Empirical formula (1) is the Primakoff's formula with Teledgi's fit. Empirical formula (2) is the result with the Goulard-Primakoff (Ref. 25) three parameter formula. The "present calculation" is the result of RSC interaction matrix elements. The three values along the row for each $\langle \nu \rangle$ are for $b = 1.65$, 1.70, and 1.75 fm, respectively.

Model			^{20}Ne in 10^6 sec^{-1}		
Fermi gas model	$\langle \nu \rangle$	82	0.3917		
	(MeV)	84	0.4227		
		86	0.4555		
Empirical		(1)	0.2772		
formula		(2)	0.2039		
Present	$\langle \nu \rangle$	82	0.209	0.211	0.213
calculation	(MeV)	84	0.222	0.224	0.226
		86	0.235	0.237	0.239
Experiment (Ref. 25)			0.20 ± 0.01		

that of the present microscopic calculation.

Experimentally, it is generally observed that, for a given Z, as A increases $\Lambda_{\mu c}$ decreases. The empirical formulas for $\Lambda_{\mu c}$ predict about a 30% decrease in going from ^{20}Ne to ^{22}Ne and ^{22}Ne to ^{24}Ne. When we used the pure prolate HF solutions for the ground states of ^{22}Ne and ^{24}Ne, we could obtain only about 3% decreases in $\Lambda_{\mu c}$ in going from ^{20}Ne to ^{22}Ne and from ^{22}Ne to ^{24}Ne. In order to examine the isotopic dependence of $\Lambda_{\mu c}$, we have improved our calculations by considering the oblate admixture in the ground states of ^{22}Ne and ^{24}Ne.

TABLE III. Results for $\Lambda_{\mu c}$ in ^{20}Ne, ^{22}Ne, ^{24}Ne in 10^6 sec^{-1} for various oblate admixtures in the ground state wave functions. The second set of results enclosed in parentheses for ^{24}Ne correspond to the second prolate solution. For all cases, $b = 1.65$ fm.

Nuclei	Oblate admixture %	82	$\langle \nu \rangle$ MeV 84	86
^{20}Ne	0	0.209	0.222	0.235
	5	0.205	0.217	0.230
	10	0.200	0.212	0.225
^{22}Ne	0	0.206	0.218	0.231
	10	0.197	0.209	0.221
	20	0.188	0.199	0.212
	30	0.178	0.190	0.202
^{24}Ne	0	0.200 (0.206)	0.212 (0.218)	0.224 (0.230)
	30	0.174 (0.177)	0.185 (0.189)	0.197 (0.201)
	40	0.165 (0.168)	0.176 (0.179)	0.188 (0.191)
	50	0.156 (0.159)	0.167 (0.170)	0.179 (0.181)

As the exact amount of oblate admixture in these cases had not been experimentally ascertained, we arbitrarily choose the admixtures, partially guided by Table I. The results are given in Table III for representative values of $\langle \nu \rangle$. We then find, with about 30% oblate admixture in the ^{22}Ne ground state and about 50% in the ^{24}Ne ground state, that $\Lambda_{\mu c}$ shows a 15% decrease in going from ^{20}Ne to ^{22}Ne and a 20% decrease in going from ^{22}Ne to ^{24}Ne, thus approximately explaining the empirical trend of $\Lambda_{\mu c}$.

The use of the PHF states in place of the rotational nuclear wave functions with intrinsic HF states may change $\Lambda_{\mu c}$. However, these changes are expected to be small, minimal for ^{20}Ne. This point is currently under investigation.

We conclude that the present calculation reveals the following: (1) The set of interaction matrix elements derived from the free nucleon-nucleon Reid soft core potential is successful in reproducing not only the spectra but also other nuclear properties which are sensitive to the nuclear wave functions; (2) the total muon capture rate in ^{20}Ne calculated with the lowest prolate intrinsic HF wave function is almost exactly the same as the experimental value; and (3) the trend of the variation of $\Lambda_{\mu c}$ in Ne isotopes as predicted by the empirical formulas is explained with 30% and 50% oblate admixture for the ground state of ^{22}Ne and ^{24}Ne, respectively.

ACKNOWLEDGMENTS

The authors thank Dr. S. B. Khadkikar and Dr. D. R. Kulkarni for lending their HF computer code. One of the authors (R.P.) wishes to acknowledge, with thanks, the warm hospitality and financial assistance of the Indian Institute of Technology and Professor Alladi Ramakrishnan for constant encouragement.

[1]R. J. Blin-Stoyle, *Fundamental Interactions and the Nucleus* (North-Holland, Amsterdam, 1973).
[2]R. P. Feynman and M. Gell-Mann, Phys. Rev. 109, 193 (1958).
[3]M. Gell-Mann and M. Levy, Nuovo Cimento 16, 705 (1960).
[4]A. Fujii and H. Primakoff, Nuovo Cimento 12, 327 (1959).
[5]J. D. Walecka, in *Muon Physics*, edited by C. S. Wu and V. Hughes (Academic, New York, 1973); N. C. Mukhopadhyay, Phys. Rep. 30C, 1 (1977).
[6]J. Bernabeu, Nucl. Phys. A201, 41 (1973), and Ref. 8.
[7]V. Devanathan, R. Parthasarathy, and P. R. Subramanian, Ann. Phys. (N.Y.) 73, 298 (1972); R. Parthasarathy and V. N. Sridhar, Phys. Lett. 82B, 167 (1979).
[8]J. S. Bell and C. H. Llewellyn-Smith, Nucl. Phys. 28B, 397 (1972), and Ref. 5.
[9]W. Chung and B. H. Wildenthal, quoted in Ref. 11.
[10]R. V. Reid, Ann. Phys. (N.Y.) 50, 411 (1968).
[11]J. P. Vary and S. W. Yang, Phys. Rev. C 15, 1545 (1977).
[12]I. P. Johnstone and B. Castel, Lett. Nuovo Cimento 3, 307 (1970).
[13]S. B. Khadkikar, D. R. Kulkarni, and S. P. Pandya, Pramana 2, 259 (1974).
[14]H. Primakoff, Rev. Mod. Phys. 31, 802 (1959).
[15]L. L. Foldy and J. D. Walecka, Nuovo Cimento 34, 1026 (1964).
[16]P. Christillin, A. Dellafiore, and M. Rosa-Clot, Phys. Rev. Lett. 31, 1012 (1973).
[17]G. A. Ferguson, J. Halpern, B. Nathans, and P. F. Yergin, Phys. Rev. 95, 776 (1954).
[18]J. G. Wood Worth, J. W. Jury, K. H. Lokan, and N. K. Sherman, Can. J. Phys. 53, 795 (1975).
[19]W. R. Dodge and W. C. Barber, Phys. Rev. 127, 1746 (1962).
[20]R. E. Segel *et al.*, Nucl. Phys. A93, 31 (1967).
[21]S. S. M. Wong, D. J. Rowe, and J. C. Parikh, Phys. Lett. 48B, 304 (1974).
[22]F. Ajzenberg-Selove, Nucl. Phys. A300, 1 (1978).
[23]I. Kelson and C. A. Levinson, Phys. Rev. 134, B269 (1964); W. H. Bassichis, B. Giraud, and G. Ripka, Phys. Rev. Lett. 13, 980 (1965).
[24]M. R. Gunye and C. S. Warke, Phys. Rev. 156, 1087 (1969).
[24]V. L. Telegdi, Phys. Rev. Lett. 8, 327 (1962).
[25]B. Goulard and H. Primakoff, Phys. Rev. C 10, 2035 (1974).
[26]J. L. Rosen *et al.*, Phys. Rev. 132, 2691 (1963).
[27]J. R. Luyten, H. P. C. Rood, and H. A. Tolhoek, Nucl. Phys. 41, 236 (1963).