Low-energy kaon-nucleus interaction

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Low-energy kaon-nucleus interaction has been considered in the framework of multiple scattering theory as formulated by Watson and by Kerman, McManus, and Thaler. A kaon-nucleus optical potential of the shape of nuclear density is shown to follow from theory under the assumption that the nuclear excited states may be neglected and that the range of the underlying microscopic $\bar{K}N$ interaction is small in comparison with the nuclear radius. The depth of the effective potential is a nonlinear function of the $\bar{K}N$ scattering lengths and depends upon a single parameter related to the unknown range of the $\bar{K}N$ interaction. This parameter has been adjusted to the available kaonic atom data. The resulting potential has been used to evaluate the kaonic atom level shifts and widths and the calculated quantities are compared with experiment.

[NUCLEAR REACTIONS Optical potential derived and applied to evaluate kaonic atoms level shifts.]

I. INTRODUCTION

In recent years a variety of effective kaon-nucleus potentials have been proposed with the objective to be able to evaluate kaonic atom level shifts and widths via the appropriate two-body wave equation (a current review of the subject may be found in Ref. 1). The philosophy that underlies these schemes attempts at expressing the potentials in terms of the experimentally available low-energy $\bar{K}N$ scattering parameters. Thus, having derived such a potential one might correlate the observed level shifts and widths in kaonic atoms with the $\bar{K}N$ scattering data. Furthermore, since the potential necessarily has a nuclear structure factor built in, there was a hope at one time[2] that the kaonic atom data could bear directly on the neutron and proton distributions in nuclei. Unfortunately, the realization of the above program has proven to be quite difficult, which after all should not be too surprising in view of the fact that the kaonic atom presents a very complicated many-body problem of nuclear physics. As a matter of fact, it has been shown[3] that the task of calculating the level shifts and widths is equivalent to solving the K^--nucleus scattering problem, or, more precisely, the complex level shift may be tied up by a linear relation with the corresponding K^--nucleus scattering length (also complex).

The success of the optical potential approach in explaining the strong interaction effects in pionic atoms[4] has prompted investigations of the kaonic atoms using the same scheme in which the optical potential V is assumed to be a sum of all elementary two-body scattering amplitudes

$$V=\langle 0|\sum_{\alpha=1}^{A} t_\alpha|0\rangle , \tag{1}$$

where $|0\rangle$ denotes nuclear ground state and t_α are the free $\bar{K}N$ scattering operators. With zero-range, s-wave $\bar{K}N$ interaction, the potential (1) simplifies further and can be written in the form[5]

$$2\mu V(r)=-4\pi A(1+m_K/m_N)\bar{a}\rho(r) , \tag{2}$$

where μ is the K^--nucleus reduced mass, m_K and m_N are the kaon and nucleon masses, A is the mass number, $\rho(r)$ is the nuclear density normalized to one and $\bar{a}$ is the $\bar{K}N$ scattering amplitude averaged over the nuclear constituents. Taking for $\bar{a}$ the average value of K^-p and K^-n scattering lengths, the potential (2) underestimates the widths of kaonic atom levels by a factor of 2 to 3 and yields a poor approximation. On the other hand, if $\bar{a}$ is regarded as an adjustable parameter,[6] the potential (2) quite satisfactorily accounts for the kaonic atom data. It turns out that the real part of the fitted value of $\bar{a}$ is positive, whereas the simple model based on Eq. (1) predicts negative sign.

The attempts to explain this discrepancy can be divided into two categories. The first one groups papers where the starting point (1) is different and instead of summing the amplitudes t_α, the optical potential is assumed to be the expectation value of $\sum v_\alpha$, where v_α is the $\bar{K}N$ interaction (cf. Ref. 7, 8 and references therein). Clearly, for weak forces, for which Born approximation is valid, one may set $v_\alpha \simeq t_\alpha$ and it does not really matter[9] whether we sum v_α or t_α in (1). Since the $\bar{K}N$ forces are not weak, a potential of the form $\langle 0|\sum v_\alpha|0\rangle$ may be expected to be quite different

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from (1). Model calculations[6-8] have shown that such a potential does have the right sign and the failure of (1) has been attributed to the breakdown of the Born approximation. In papers which belong to the second category[10] a completely different attitude has been adopted. The authors[10] claim that formula (1) is correct, provided the t_α are properly continued below threshold, taking into account the many-body kinematics, binding effects, etc. Since the $I=0$ $\bar{K}N$ scattering amplitude has a resonance 27 MeV below threshold, the t_α are rapidly varying functions of energy and the values of $\bar{a}$ might therefore vary a good deal, depending at which energy value the t_α are evaluated. In principle this then provides a chance to restore the proper sign of $\mathrm{Re}(\bar{a})$, but with finite range forces the resulting potential is nonlocal and it is difficult to compare it with the simple expression (2). It should perhaps also be mentioned that some authors start with formula (1) stating that t_α are to be understood as "effective interaction" operators, rather than free scattering operators, and immediately introduce various approximations. However, in absence of a precise definition of t_α it is not always clear what are the basic assumptions of such models.

Taking as the starting point the multiple scattering theory developed by Watson, the optical potential V is given as an expansion in powers of the scattering operators describing scattering from the different nucleons imbedded in the nuclear medium. The lowest order term of this multiple scattering series represents scattering from uncorrelated single nucleons. Higher order terms describe scattering processes from at least two bound nucleons and depend on nucleon-nucleon correlations in the target. This formulation is exact and two approximations are required to reduce V to the form (1): (i) single scattering approximation which truncates the Watson series retaining only the first term, and (ii) impulse approximation (abbreviated hereafter as IA) where the scattering operator in medium is approximated by free scattering operator. On the other hand, the only approximation that is needed to reduce the exact V to the form $\langle 0|\sum v_\alpha|0\rangle$ is the coherent nucleus approximation (CNA hereafter) which neglects the excited states. With this approximation only the first term survives in the Watson series and the scattering operator in medium is exactly equal to v_α.

In the present paper we develop a model based on CNA and we will show that the optical potential of the same form as (2) can be derived from a scheme which neglects nuclear excitations. Furthermore, we explain also the origin of the complex depth $\bar{a}$ and show that $\bar{a}$ is a nonlinear function of $\bar{K}N$ scattering lengths. We demonstrate that the expansion of $\bar{a}$ in powers of free scattering length yields the wrong sign of $\mathrm{Re}(\bar{a})$ and is a poor approximation.

The organization of the paper is as follows. In Sec. II we briefly review Watson's theory of the optical potential specifying carefully the coherent nucleus approximation. In Sec. III we consider a class of local, spherically symmetric, short ranged potentials which are assumed to represent the $\bar{K}N$ interaction and employ them to construct the kaon-nucleus optical potential. To a good approximation, the shape of this potential follows the shape of the nuclear density and the complex depth $\bar{a}$ is a nonlinear function of the $\bar{K}N$ scattering lengths. Finally, in Sec. IV the resulting potential is used to calculate the kaonic atom level shifts and widths.

II. THEORY

In the standard treatment the optical potential is derived from multiple scattering theory developed by Watson[11] and collaborators, or from the completely equivalent formulation presented by Kerman, McManus, and Thaler.[12] These two approaches are identical in content and the resulting optical potentials may be readily transformed one into another.[13]

The optical potential V in the Watson scheme can be written in a form of an infinite series

$$V=\langle 0|\sum_\alpha \tau_\alpha+\sum_{\alpha\neq\beta}\sum \tau_\alpha G Q \tau_\beta+\cdots|0\rangle\,, \tag{3}$$

where $\alpha,\beta=1,2,\ldots,A$; Q is the excited state projection operator and G is the many-body Green's function

$$G(E)=(E-K-H_A+i\epsilon)^{-1}\,, \tag{4}$$

where E is parametric energy, K is the kinetic energy operator of the projectile, and H_A is the Hamiltonian of the isolated nuclear target. Usually, the τ_α in (3) are interpreted as the effective scattering operators in medium and they are related to the free space two-body scattering operators t_α, in the following way

$$\tau_\alpha(E)=t_\alpha(\omega)\{1+[G(E)Q-g_\alpha(\omega)]\tau_\alpha(E)\}. \tag{5}$$

In (5) $g_\alpha(\omega)$ is a two-body Green's function

$$g_\alpha(\omega)=(\omega-K-K_\alpha+i\epsilon)^{-1}\,, \tag{6}$$

where ω is parametric energy and K_α is kinetic energy operator of the nucleon. So far no approximation has been made and (3) together with (5) completes the Watson scheme.[14]

It goes without saying that the above optical model may render a useful scheme only if the rate of convergence of the series in (3) is sufficiently rapid. Regrettably, in most cases of

interest the convergence of (3) has been conjectured but not proved. It should perhaps also be mentioned that (3) is not the only form of the optical potential and some alternative, formally completely equivalent expressions for V have been derived in the literature.[13] By and large, however, the convergence problem remains an open question, no matter what the organization of the series expression for V is.

Unfortunately, the truncation of the series (3) alone is not enough to bring V to a tractable form since the τ_α are very complicated many-body operators. The ultimate goal is therefore to eliminate the τ_α in favor of t_α for the latter operators on-shell are related directly to the two-body scattering amplitude which may be inferred from experiment. This program is most readily effected at high energies[15] where the unitarity condition suppresses the scattering amplitude so that the t_α may be indeed regarded as convenient expansion parameters. At low energies, however, one has to be careful using prejudices borrowed from high-energy physics because the two-body scattering amplitudes may not be small any more and at threshold in a general case they may even be infinite as there is no unitarity bound to suppress their values. In principle, there is of course no reason why the optical model should not work at low energies provided one can come up with a reliable method of calculating the τ_α operators from the free two-body scattering amplitudes.

In order to overcome this difficulty the tendency has been to use IA, which simply identifies the scattering in medium with free scattering (i.e., τ_α is set equal to t_α). Formally, IA assumes that Eq. (5) can be solved by iteration and expanding τ_α in powers of t_α, only the first term is retained. This unavoidably creates another convergence problem and since the kernel of the integral Eq. (5) is a complicated many-body-operator, it is very difficult to establish conditions under which an iterative solution leads to meaningful results. Thus, the proper evaluation of the scattering operator in medium remains one of the outstanding problems in all theories which represent V as a sum of scattering operators. The relation of the scattering in medium to free scattering operator involves the many-body aspect of the problem and is therefore extremely difficult to accomplish.

On the other hand, IA is not really necessary and the optical model may be reduced to a tractable form by making only *one* approximation which we called coherent nucleus approximation. This approximation is based on the assumption that in the processes considered the dominant contribution comes from the nuclear ground state, i.e., in the first approximation the excited nuclear states may be disregarded as if the nucleus were in the ground state at all times. Such procedure has been well known in atomic and nuclear physics and is sometimes called rigid or static approximation because one ignores nuclear polarization caused by the kaon-nucleus interaction.

Several arguments may be advanced in favor of this approximation. Firstly, due to the large absorptive part, the $\overline{K}N$ interaction takes place in a relatively diluted region of the nuclear medium where single scattering processes dominate. In these circumstances a single nucleon excitation is unlikely to be strongly coupled with excited nuclear states. Secondly, for kaon energies close to threshold most of the inelastic channels may be regarded as sufficiently distant and therefore effectively decoupled from the ground state. Thirdly, the presence of inelastic channels would lead to a correction to the effective interaction which is of the second order in the off-diagonal potential. This can be seen directly from (3). The resulting correction may therefore be expected to be of a shorter range and should eventually be suppressed by the centrifugal barrier. Finally, the ground state dominance ought to be viewed as a natural first approximation which, when need arises, may be improved by including excited states.

The coherent nucleus approximation brings major simplifications into the multiple scattering theory. Formally, the suppression of inelastic channels is achieved by setting $Q \to 0$ in which case only the first term survives in (3). Thus, CNA is equivalent to the single scattering approximation. But with $Q \to 0$ also (5) is simplified and reduces to the form

$$\tau_\alpha = t_\alpha(1 - g_\alpha \tau_\alpha) . \tag{7}$$

Now, there is no need to try to solve (7) by iteration (using IA) because the formal solution of (7) is

$$\tau_\alpha = (1 + t_\alpha g_\alpha)^{-1} t_\alpha \tag{8}$$

and the right hand side is nothing else but the corresponding two-body interaction v_α. The CNA alone then gives the following expression for the optical potential

$$V = \langle 0 | \sum \tau_\alpha | 0 \rangle = \langle 0 | \sum v_\alpha | 0 \rangle . \tag{9}$$

This equation will be of central importance for the rest of this paper. We would like to emphasize that for the coherent nucleus the effective scattering operator in medium is ***exactly*** equal to the underlying microscopic interaction. We wish to stress here that this has nothing to do with Born approximation because relation (9) holds true irrespective of the strength of the $\overline{K}N$ interaction,

being solely a consequence of the suppression of the excited states. This point is often confused in the literature.

III. KAON-NUCLEUS POTENTIAL

We have shown in the preceding section that the coherent nucleus approximation alone results in the kaon-nucleus optical potential (8). In the position representation V may be written as

$$\langle \vec{r}' \,|\, V \,|\, \vec{r} \rangle = \langle 0 \,|\, \Big[\sum_{\alpha} \langle \vec{r}' \,|\, v_{\alpha}(\vec{x}_{\alpha}) \,|\, \vec{r} \rangle \Big] \,|\, 0 \rangle \,, \tag{10}$$

where $\vec{r}$ is the kaon coordinate in the nuclear c. m. system and the $\vec{x}_{\alpha}$ ($\alpha = 1, 2, \ldots, A$) are the coordinates of the nucleons. In order to bring the potential (10) to a tractable form we now have to make a number of assumptions with regard to the microscopic interaction v_{α}. Our ultimate goal is to relate v_{α} with the $\bar{K}N$ scattering parameters. The experiment has revealed that below 300 MeV/c the three-body final states are strongly suppressed and the $\bar{K}N$ data are consistent with the interaction being purely s wave. Therefore, we suppose that without too much loss of generality we can assume that the v_{α} are local, central potentials. Then, the optical potential (10) will also be local, viz.,

$$\langle \vec{r}' \,|\, V \,|\, \vec{r} \rangle = V(r) \delta(\vec{r}' - \vec{r}) \,,$$

where

$$V(r) = A \int v(\vec{r} - \vec{x}) \rho(x) d^3x = A \int v(x) \rho(\vec{r} - \vec{x}) d^3x \tag{11}$$

represents the effective kaon-nucleus potential to be used in a two-body wave equation to yield the kaonic atom level shifts and widths. Here, $\rho(x)$ denotes the nuclear density normalized to one and $v(x)$ is the $\bar{K}N$ local potential. For the time being we have A identical nucleons and a more realistic case with neutrons and protons will be considered later on. To keep the number of parameters as low as possible, we preferred to deal with a one channel situation and the absorption in the $\bar{K}N$ channel due to the presence of inelastic processes is simulated by making complex the depth of the potential $v(x)$. For simplicity, we assume that apart from that, the function $v(x)$ is real. It should be noted, that if need arises, all these restrictions may be lifted at the expense of introducing some additional parameters to be adjusted to the data.

The range of the potential $v(x)$ is presumably quite short as may be suggested by field theoretical arguments. Indeed, a one-meson-exchange mechanism would generate a Yukawa potential with a range parameter of about 0.25 fm, a value that corresponds to the average mass of the vector meson nonet which is the lightest known carrier of the $\bar{K}N$ interaction. Consequently, it seems reasonable to assume that the range of the $\bar{K}N$ potential is small in comparison with the nuclear radius. Thus, in the folding integral (11) the dominant contribution comes from small x values where $v(x)$ does not vanish. Owing to a large centrifugal barrier, in a kaonic atom situation the most important will be the tail of the potential $V(r)$ and we could afford some inaccuracies in the small r region. Accordingly, we shall evaluate the integral (11) by expanding $\rho(|\vec{r} - \vec{x}|)$ in a Taylor series about a fixed position r, large in comparison with the range of the potential $v(x)$. Only even powers of x give nonvanishing contributions to the integral and one ends up with the expression

$$V(r) = A \int v(x) d^3x \Big[1 + \sum_{n=1}^{\infty} \frac{\langle x^{2n} \rangle}{(2n+1)!} \Delta^n \Big] \rho(r) \,, \tag{12}$$

where $\langle x^{2n} \rangle$ is the $2n$ moment of $v(x)$ and Δ is Laplacian operator[16] acting on $\rho(r)$. Formula (12) can be expected to be valid for r values larger than the range of the potential $v(x)$ which should be sufficient for our purposes. For short ranged potentials $v(x)$, the sum containing moments in (12) may be dropped, and in the first approximation one obtaines a potential $V(r)$ which follows the shape of nuclear density. This shape has been adopted on a somewhat intuitive basis in all phenomenological models[6] and now we have presented a theoretical justification for it. As seen from (12), the strength of the potential $V(r)$ is determined by the volume integral of the microscopic potential $v(x)$, while in a phenomenological approach the volume integral has been replaced by a complex constant $\bar{a}$ to be adjusted to the kaonic atom data. Thus, formula (12) also provides theoretical explanation of the origin of the complex strength by relating it to the $\bar{K}N$ interaction. Since the latter is not known, it would be desirable to express the volume integral of $v(x)$ by the $\bar{K}N$ scattering parameters. For weak forces the volume integral of a potential is simply proportional to the appropriate scattering length (in Born approximation) but in a general case the two quantities are related by a more complicated expression.

In order to get further insight into the problem we are now going to eliminate the potential $v(x)$ in favor of the $\bar{K}N$ scattering parameters. To be able to do that we have to assume that $v(x)$ is a "well behaved" potential, which means that the function $v(x)$ does not change sign, is less singular for $x = 0$ than x^{-2} and has a sufficiently rapid falloff so that all the moments $\langle x^{2n} \rangle$ exist. It is

convenient to take the volume integral multiplied by the $\bar{K}N$ reduced mass μ_{KN}

$$\bar{v} = -2\mu_{KN}\int_0^\infty v(x)x^2dx \tag{13}$$

as a measure of the strength of $v(x)$ and use the intrinsic range b as a measure of the range of $v(x)$. The reason for this is quite obvious, using $\bar{v}$ and b rather than the somewhat more customary depth and range parameters is more general for we do not need to specify the shape of $v(x)$ [in particular $v(x)$ may be a sum of several terms each of which being characterized by a different set of depth and range parameters]. It can be shown[17] that the scattering length a_l [defined as $a_l = \lim_{k\to 0} k^{-2l-1}\tan\delta_l(k)$] is related to the underlying potential $v(x)$ by the expression

$$a_l = -2\mu_{KN}\int_0^\infty v(x)x^{2l+2}dx \times[(2l+1)!!]^{-2}\prod_{n=1}^{\infty}\frac{1-\bar{v}/br_n^{(l)}}{1-\bar{v}/bq_n^{(l)}}, \tag{14}$$

where $q_n^{(l)}$ and $r_n^{(l)}$ are real constants[18] which depend upon the shape of $v(x)$. The numbers $q_1^{(l)}$ ($l=0,1,\ldots$) determine the strength $\bar{v}$ of the potential (in units of b) necessary to support the lowest bound state in each partial wave l. Similarly, the $q_n^{(l)}$ with $n=2,3,\ldots$ determine the strength $\bar{v}$ necessary to yield all the higher (excited) levels, and at each $q_n^{(l)}$ value the appropriate scattering length a_l, regarded as a function of $\bar{v}$, has a pole. Between every two poles at $q_n^{(l)}$ and $q_{n+1}^{(l)}$ the function (14) necessarily has a zero at $\bar{v}=br_n^{(l)}$.

For weak potentials, such that $\bar{v}\ll bq$, where from now on q stands for $q_1^{(0)}$, the product expression in (14) will be very close to 1 so that a_l is given to a good approximation by Born approximation [first order in $v(x)$]. For moderately strong potentials, i.e., those which are able to support no more than one bound state (1s state), we have to account for the corresponding pole term in (14). It turns out[17] that a simple one pole formula is in most cases sufficiently accurate. Since the $q_1^{(l)}$ numbers increase quite rapidly with l, the product expression in (14) for higher partial waves gives usually only a minor correction. Thus, the Born approximation is still reasonably good for higher partial waves ($l>0$), even if the forces are so strong that they can lead to binding in $l=0$ state.

In absence of any experimental evidence for other bound states in the $\bar{K}N$ system except for the $\Lambda(1405)$, it seems plausible that the $\bar{K}N$ forces are moderately strong, in the sense just explained. Accordingly, only the $l=0$ wave has to be corrected for a bound state and we are led to the following approximation scheme

$$a_0 \simeq \bar{v}(1-\bar{v}/bq)^{-1}; \tag{15a}$$

$$a_l \simeq -2\mu_{KN}\int_0^\infty v(x)x^{2l+2}dx;\ l=1,2,\ldots. \tag{15b}$$

The above is the minimal model but if better accuracy is required, one can always use formula (14) to bring back further correction factors at the expense of introducing new constants into the resulting potential $V(r)$.

Inserting (15) into (12), yields a formula

$$2\mu V(r) = -4\pi A\left(1+\frac{m_K}{m_N}\right)\left(1+\frac{m_K}{Am_N}\right)^{-1} \times\left[\frac{a_0}{1+a_0/bq}\rho(r)+\sum_{l=1}^{\infty}a_l\frac{(2l+1)!!}{(2l)!!}\Delta^l\rho(r)\right], \tag{16}$$

where the $\bar{K}N$ potential does not occur anymore. In principle, the scattering lengths a_l may be inferred from $\bar{K}N$ scattering experiments, so that the only unknown parameter left in (16) is the product bq. In Table I, for illustration we give numerical values of q and we relate b with the appropriate range parameters for six frequently used potential shapes. As seen from Table I the parameter q seems to be rather weakly dependent upon the shape of the potential and has a numerical value close to 0.8. Unfortunately, it does not seem possible to make a precise estimate of the value of b. In a model where the $\bar{K}N$ potential is generated by the exchange of vector mesons, the outer part of $v(x)$ will be a Yukawa potential with the range parameter of about 0.25 fm. From Table I the value of b then turns out to be about 0.5 fm but if we allow for a superposition of several Yukawa terms (lifting the degeneracy within the vector meson nonet), the value of b may be changed significantly. It seems that the upper limit for b should not be bigger than 0.5 to 1.0 fm. Since we really do not know q either, we preferred to leave the product bq as a free parameter to be adjusted

TABLE I. The values of the intrinsic range b and the parameter q (defined in the text). V_0 and d denote the depth and range of each of the listed potentials, the variable x is defined as $x\equiv r/d$.

Potential	$V(r)$	b/d	q
Square well	$-V_0\theta(x-1)$	1.0	0.822
Cutoff Coulomb	$-V_0\theta(x-1)/x$	0.872 26	0.829
Exponential	$-V_0\exp(-x)$	3.540 79	0.817
Hulthen	$-V_0[\exp(x)-1]^{-1}$	3.0	0.801
Yukawa	$-V_0\exp(-x)/x$	2.120 16	0.792
Gauss	$-V_0\exp(-x^2)$	1.435 23	0.829

to the kaonic atom data.

Unfortunately, the data on the higher partial wave scattering lengths are rather poor, and it is difficult to make a precise estimate of the contribution of the derivative terms in (16). The success of the phenomenological model[6] suggests that they are small. To obtain a rough estimate of the size of the correction due to the p-wave term in (16), we multiply (16) by r^2 and integrate over d^3r. The $a_1\Delta\rho(r)$ term in (16) gives rise to the correction $9(1+a_0/bq)a_1/\langle r^2\rangle a_0$ where $\langle r^2\rangle^{1/2}$ is rms radius of $\rho(r)$. Using the scattering lengths from Ref. 19 this correction can be estimated to contribute no more than about 10% to the leading $\rho(r)$ term. Thus, at least in the first approximation, the derivative terms can probably be neglected. On the other hand, it should be remembered that the terms containing derivatives of $\rho(r)$ are rapidly varying in the surface region which may enhance their importance. Ultimately, with a better quality of the data, the neglected terms may be reintroduced in due course at the expense of some extra parameters to be adjusted to the data.

Before writing our final expression for the kaon-nucleus potential we have to introduce neutrons and protons. In order to reduce the number of parameters, we assume that the distributions of neutrons and protons are the same being identical with the nuclear density $\rho(r)$. Furthermore, we assume that the appropriate potentials $v(x)$ have the same shape for both K^--p and K^--n interactions and only their strengths are different. With these assumptions the potential (16) reduces to the form[20]

$$2\mu V(r) = -4\pi A(1+m_K/m_N)[(Z/A)a_p(1+a_p/bq)^{-1} + (N/A)a_n(1+a_n/bq)^{-1}]\rho(r), \tag{17}$$

where a_p and a_n denote the K^--p and K^--n scattering lengths, respectively.

Comparing (17) and (2), we can see that the strength $\bar{a}$ is a nonlinear function of the zero-energy free space $\bar{K}N$ scattering amplitude. The nonlinearity appears in the form of a characteristic denominator which is a consequence of the approximation which accounts for only the first factor in the product formula (14). For small $a_{p,n}$ one immediately recovers the IA result $\bar{a}\simeq(Z/A)a_p + (N/A)a_n$ but the expansion parameter is really $(a_{p,n}/bq)$. Since the latter quantity can hardly be expected to be small, the expansion of (17) in powers of $a_{p,n}$ is likely to be divergent, or at best slowly convergent precluding the correctness of IA where only the leading term is retained. This is also borne out by a numerical test which revealed that with IA the widths of the kaonic atom levels were systematically underestimated by a factor of 2 to 3.

As mentioned in the Introduction the failure of IA in predicting the observed kaonic atom level shifts and widths led some people to speculate[10] that this discrepancy might be removed by using an extended version of IA which would account for the presence of the $\Lambda(1405)$ resonance. As a matter of fact, the efforts to include the $\Lambda(1405)$ are not necessarily orthogonal to the procedure applied in this paper. Our K^--p and K^--n potentials have been adjusted in such a way that they give the correct values of a_p and a_n but one may wonder whether the resonance would also show up in our scheme. It is difficult to answer this question because the resonance is in the $I=0$ channel and it is not clear whether it should be present in the K^--p channel or not. Besides that, we do not really know the shape of the K^--p potential. In this situation the only reasonable thing one can do is to continue analytically the effective range expansion in the K^--p channel below threshold, neglecting the effective range term. The position of the pole in the t matrix can then be easily evaluated and related to the scattering length a_p. Writing a_p as $a_p=\alpha+i\beta$, we obtain the following expressions for the position (E_R) and the width (Γ_R) of the resonance:

$$E_R = (1/2\mu_{KN})(\alpha^2-\beta^2)(\alpha^2+\beta^2)^{-2},$$
$$\Gamma_R = (1/2\mu_{KN})(-4\alpha\beta)(\alpha^2+\beta^2)^{-2}.$$

As seen from the above expressions, for the $\Lambda(1405)$ to appear in the K^--p channel, the scattering length a_p must be such that the following inequalities should hold

$$|\mathrm{Re}a_p| > |\mathrm{Im}a_p|, \qquad (\mathrm{Re}a_p)\cdot(\mathrm{Im}a_p) < 0. \tag{18}$$

In this paper (cf. Sec. IV) we used two sets of values for the scattering lengths deduced from $\bar{K}N$ scattering data by different authors. Using the above inequalities, one set of data predicts a resonance in the K^--p channel, the other does not, which gives some indication of the importance of the resonance in explaining the kaonic atom data.

IV. COMPARISON WITH EXPERIMENT

Data. The bulk of kaonic atom data considered in this paper comprises of the level shifts and widths of the lower levels, supplemented in some cases by the values of the widths of the upper levels. The measured quantities are envisaged in Table II together with a reference to the appropriate experimental paper from which they have been taken. We believe that Table II presents rather complete mundane statistics.

TABLE II. Kaonic atom level shifts and widths. Theoretical values have been calculated from the potential (17) with $bq = 1.05$ fm and MAR scattering lengths. The nuclear densities were of Fermi shapes except for the two lightest elements where we used harmonic oscillator density (the corresponding entries in column a are marked by an asterisk).

Element	Level	a (fm)	c (fm)	$-\epsilon$ (eV) Expt.	$-\epsilon$ (eV) Theor.	Γ (eV) Expt.	Γ (eV) Theor.	Ref.
^{6}Li	2p	0.333*	1.854	−2 ± 26	3.15	55 ± 29	49	24
^{9}Be	2p	0.667*	1.739	79 ± 21	48.5	172 ± 58	243	24
^{10}B	2p	0.48	2.20	208 ± 35	188	810 ± 100	690	25
^{11}B	2p	0.48	2.17	167 ± 35	199	700 ± 80	704	25
^{12}C	2p	0.44	2.39	590 ± 80	590	1730 ± 150	1569	25
^{12}C	3d	0.44	2.39		−0.02	0.98 ± 0.19	0.68	25
^{27}Al	3d	0.52	3.07	130 ± 50	69	490 ± 160	441	26
^{28}Si	3d	0.59	3.01	240 ± 50	132	810 ± 120	881	26
^{31}P	3d	0.56	3.20	330 ± 80	269	1440 ± 120	1389	25
^{31}P	4f	0.56	3.20		−0.15	1.97 ± 0.33	2.16	25
^{32}S	3d	0.59	3.20	550 ± 60	466	2330 ± 200	2264	25
^{32}S	4f	0.59	3.20		−0.30	3.25 ± 0.41	4.48	25
^{35}Cl	3d	0.57	3.33	770 ± 400	808	3800 ± 1000	3293	25
^{35}Cl	4f	0.57	3.33		−0.34	5.69 ± 1.5	7.93	25
^{53}Co	4f	0.569	4.08	80 ± 50	85	980 ± 150	750	27
^{58}Ni	4f	0.565	4.19	180 ± 70	139	590 ± 210	1034	26
^{58}Ni	4f	0.565	4.19	260 ± 90	139	1340 ± 140	1034	27
^{58}Ni	5g	0.565	4.19		−0.25	6.0 ± 2.3	3.38	26
^{63}Cu	4f	0.50	4.29	240 ± 220	211	1650 ± 720	1160	26
^{63}Cu	5g	0.50	4.29		−0.14	7.1 ± 3.8	3.92	26
^{64}Zn	4f	0.55	4.32	−60 ± 300	319	1700 ± 400	1834	25
^{107}Ag	5g	0.503	5.38	500 ± 130	211	2420 ± 510	1279	27
^{114}Cd	5g	0.585	5.27	250 ± 120	255	3000 ± 350	1979	27
^{208}Pb	7i	0.549	6.62		5.9	370 ± 150	216	26
^{208}Pb	8j	0.549	6.62		−0.21	4.1 ± 2.0	1.92	26
^{238}U	7i	0.605	6.81	260 ± 400	74	1500 ± 750	1256	26
^{238}U	8j	0.605	6.81		−1.67	45.5 ± 24	17.4	26

Input. The values of the parameters entering our optical potential (17) have been specified as follows. The scattering lengths a_p and a_n have been taken from Refs. 19 and 21 and we have divided these data into two sets. In the first set (called OLD hereafter) we took an average from three different analyses[19] to obtain a_p and a_n. In the second set (called MAR), the a_p and a_n were adopted from a very recent analysis carried through by Martin[21] who uses dispersion relations to tie up the low- and high-energy $\bar{K}N$ scattering data to infer the K-matrix parameters. Hence, the scattering lengths used in this paper are

$$\text{OLD: } a_p = (-0.8865 + i\,0.6335)\ \text{fm},$$
$$a_n = (-0.09 + i\,0.56)\ \text{fm},$$
$$\text{MAR: } a_p = (-0.655 + i\,0.705)\ \text{fm},$$
$$a_n = (0.35 + i\,0.66)\ \text{fm}.$$

We wish to emphasize that, as follows from (18), the OLD data indicate a resonance below threshold in the K^--p channel, whereas MAR predict no resonance.[22] As will become apparent in a moment, this fact has virtually no effect on the quality of the fit achieved by means of our optical potential (17).

The adopted nuclear densities have been either of Fermi shape or Gaussian corresponding to a harmonic oscillator model. The current values of the parameters have been taken from a recent compilation in Ref. 23 and are listed in Table II. Finally, the kaon mass used in this paper was $m_K = 493.715$ MeV. As mentioned before, formula (17) contains one free parameter, viz., the product bq. The latter has been adjusted to the kaonic atoms data presented in Table II.

Results. The theoretical values of ϵ and Γ have been calculated from the appropriate Klein-Gordon equation with the potential (17), using a very rapid procedure described in Ref. 3. The free parameter bq has been varied from 0.5 fm to infinity and for each value of bq we computed ϵ and Γ for all entries listed in Table II together with the total χ^2. Setting $bq \to \infty$, formula (17) reduces to the IA limit of the optical potential. As might have been expected, the fit in this case is poor, the values of χ^2 were $\chi^2 = 186$ for MAR, and $\chi^2 = 384$ for OLD. Actually, the agreement for ϵ is not too bad, only the gammas are far off, being too small by a fac-

tor of 2 to 3. When bq is getting smaller, the fit rapidly improves, for both OLD and MAR inputs. The shifts and widths show an increasing tendency and the corresponding χ^2 reaches for either input a pronounced minimum for bq around 1 fm. For smaller bq, the shifts will grow and eventually change sign and become positive (for bq values around 0.5 fm). The best fits obtained for OLD and MAR inputs are of nearly the same quality, as may be seen from the corresponding χ^2 values

OLD: best fit for $bq = 0.95$ fm, $\chi^2 = 89.3$,

MAR: best fit for $bq = 1.05$ fm, $\chi^2 = 71.7$.

In Table II we have displayed our best fit values of ϵ and Γ obtained with MAR input, the other input gives essentially very similar results.

It is perhaps worth mentioning that the model developed in this paper agrees better with experiment than our previous folding model[7,8] where we kept the shape of the $\bar{K}N$ potential fixed (Gaussian) and adjusted the range of the $\bar{K}N$ potential to the K^--^{4}He scattering data. Qualitatively, in the folding model the calculated widths were systematically slightly overestimated and for the kaonic atom data presented in Table II we obtained $\chi^2 = 209$ for OLD, and, respectively $\chi^2 = 149$ for MAR input.

It seems of interest to compare our model with the purely phenomenological approach.[6] Setting for simplicity $2Z = 2N = A$ and comparing (2) with (17), the effective strength of the potential (17) may be written as

$$\bar{a} = \frac{1}{2}\left(\frac{a_p}{1 + a_p/bq} + \frac{a_n}{1 + a_n/bq}\right). \tag{19}$$

By contrast, in the phenomenological model the strength parameter $\bar{a}$ was not related to the $\bar{K}N$ scattering data and had to be fitted to the kaonic atom data, e.g. Koch and Sternheim[6] obtained

$$\bar{a} = [(0.44 \pm 0.04) + i(0.83 \pm 0.07)]\ \text{fm}, \tag{20}$$

which differs significantly from the value predicted by the IA version of the optical model. The latter yields

$$\bar{a} = \tfrac{1}{2}(a_p + a_n) = \begin{cases} (-0.488 + i\,0.596)\ \text{fm, for OLD} \\ (-0.15 + i\,0.68)\ \text{fm, for MAR}. \end{cases}$$

Hence, for either input the real part of $\bar{a}$ is of the opposite sign than (20) which, as was discussed in Sec. III, simply means that IA is inadequate. However, many authors seem to be puzzled by this "change" of sign and suggest various explanations within the IA, or extended IA models. In particular, as mentioned before, the $\Lambda(1405)$ mechanism has been popular in certain quarters. On the other hand, a careful application of the optical model effected in this work results in a nonlinear relation between $\bar{a}$ and the $\bar{K}N$ scattering lengths which in consequence leads to the correct sign and magnitude of $\bar{a}$. Indeed, setting $bq \simeq 1$ fm, formula (19) gives

$\bar{a} = (0.46 + i\,1.01)$ fm, for OLD

$\bar{a} = (0.42 + i\,0.74)$ fm, for MAR.

Thus, in our model the real part of $\bar{a}$ is positive and quite close to the values obtained in Ref. 6.

Summarizing, in the presented approach our main incentive was to provide a better understanding of the qualitative features of kaon-nucleus interaction. The derivation of our optical potential rests on two basic assumptions, viz., (i) that the nuclear excited states may be neglected, and, (ii) that the range of the microscopic $\bar{K}N$ interaction is small as compared with the nuclear radius. The first assumption implies that to obtain the optical potential one has to sum up the microscopic potentials rather than the free scattering amplitudes. In consequence, the strength of the effective potential turns out to be a nonlinear function of the elementary $\bar{K}N$ scattering amplitude. The second assumption bears directly on the shape of the optical potential which, in the first approximation, follows the shape of nuclear density.

A palatable feature of the presented approach is that we not only provide a theoretical justification for the phenomenological potential, but we have also managed to explain the origin of the complex depth $\bar{a}$ in (2) by relating it to the $\bar{K}N$ scattering lengths. Another important advantage over the phenomenological model is that we halved the number of free parameters, i.e., the two parameters of the phenomenological model (real and imaginary parts of $\bar{a}$ have been expressed in terms of a single parameter related to the unknown range of the $\bar{K}N$ interaction).

Concluding, we are of course aware that there would be a number of secondary effects which deserve a more careful treatment and might improve the agreement with experiment. To mention but a few, the coherent approximation may be extended by adding some of the excited states, the $\bar{K}N$ dynamics may be generalized by employing a many channel formalism and taking into account effective ranges, or higher partial waves, the derivative terms $\Delta^n \rho(r)$ may be included assuming different distributions for neutrons and protons, the product formula (14) may be truncated at the next factor making the strength a more complicated function of the scattering amplitude, etc. On the other hand, since every extension unavoidably introduces unknown parameters, we felt that it would be premature to include all those refinements with the present quality of the data.

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[16]Formula (12) may be further simplified in view of the relation

$$\Delta^n \rho(r) = [(d/dr)^{2n} + (2n/r)(d/dr)^{2n-1}]\rho(r).$$

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[18]In Ref. 17 we used the first moment of the potential (denoted as s) as a measure of the strength. Since $\bar{v} = Cs$, where C has been defined in Ref. 17, the poles and zeros [$s_n^{(l)}$ and $z_n^{(l)}$] obtained in Ref. 17 have to be rescaled to find $r_n^{(l)}$ and $q_n^{(l)}$, viz., $r_n^{(l)} = (C/b) z_n^{(l)}$ and $q_n^{(l)} = (C/b) s_n^{(l)}$.

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