Elastic, charge exchange, and inelastic $\bar{p}p$ cross sections in the optical model

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The optical model of antiproton-proton scattering is examined critically. We show that the addition of an attractive real part to the annihilation potential allows one to reduce the imaginary part and to improve the fit to the data. However, the absorptive component must be still rather strong and also have a fairly large range. Such features are difficult to understand in terms of naive direct channel meson exchange processes. We confirm also in the local optical model that no narrow structures can survive annihilation, unless they lie very close to threshold. The optical model we propose, with the realistic Paris potential for the long range t-channel part, provides an effective description of background $\bar{N}N$ scattering.

[NUCLEAR REACTIONS Analysis of elastic, charge exchange and inelastic $p\bar{p}$ cross sections in optical model; discussion of theoretical and phenomenological annihilation potential.]

There has been considerable recent interest in the narrow structures[1] seen in nucleon-antinucleon ($\bar{N}N$) elastic and total cross sections near threshold, πN production experiments, spectator experiments involving a deuterium target, and γ-ray inclusive spectra. If one wishes to interpret these structures in terms of the bound states and resonances of an $\bar{N}N$ potential,[2-4] it is necessary to consider the effect of annihilation on the stability of such states. $\bar{N}N$ annihilation was first described with an optical model,[5,6] by adding an imaginary part $-iW(r)$ to the real potential $V_t(r)$ generated by t-channel meson exchange. Using an early form of the one boson exchange model[7] for $V_t(r)$, a good fit to the existing $\bar{N}N$ data was achieved in Ref. 6. Later, several authors[8] showed that this phenomenological local annihilation potential was too strong for narrow $\bar{N}N$ states to survive. The reliability of the local optical potential for estimating the width of bound $\bar{N}N$ potential states has been criticized by Shapiro.[2] To avoid the use of an optical potential, several more or less explicit coupled channel formalisms have been developed.[9,10] Some of these[10] claim to reconcile the large observed $\bar{N}N$ annihilation cross section σ_A at threshold with the existence of narrow mesons close to threshold. So far none of these analyses has dealt with the energy dependence of elastic (σ_{EL}), charge exchange (σ_{CE}), and total $\bar{N}N$ cross sections in the entire low energy region ($p_{lab} \leq 1$ GeV/c) where a potential model may be applicable. In the present paper, we reexamine the optical model description of $\bar{N}N$ scattering with the following specific questions in mind:

(1) Using a more realistic meson exchange potential than that of Ref. 7, can we fit the observed energy dependence of σ_{EL}, σ_{CE}, and σ_A with a substantially weaker absorption $W(r)$ than that of Ref. 6?

(2) Are the data better fitted by allowing the annihilation potential to have a real part? If so, is it necessarily attractive?

(3) Is the annihilation potential needed to fit the data understandable in terms of meson exchanges in the s channel?

(4) Is the absorptive potential compatible with the existence of narrow structures close to the $\bar{N}N$ threshold?

This paper is organized as follows: We first present the fit of the data with an optical model and later comment on the results.

To present the experimental data on integrated elastic and annihilation cross sections, we adopt the smoothly energy dependent parametrizations

$$\sigma_{EL} = 28 + 17/p_{lab}\,, \qquad \sigma_A = 38 + 35/p_{lab} \tag{1}$$

due to Kalogeropoulos,[11] where p_{lab} is the lab momentum in units of GeV/c, and σ is in mb. For charge exchange, we use the parametrization

$$\sigma_{CE} = \frac{18.15[1-(0.1/p_{lab})^2]^{1/2}}{(1-0.49p_{lab}+2.4p_{lab}^2)} \tag{2}$$

of Tripp and collaborators.[12] Equation (1) fits the data well for $p_{lab} \leq 2$ GeV/c and Eq. (2) is a fit to the data in the region $0.276 \leq p_{1ab} \leq 0.963$ GeV/c.

For each cross section, we minimize the quantity

$$\chi^2 = \sum_i \left(\frac{\sigma^{\exp}(E_i) - \sigma^{\text{th}}(E_i)}{\Delta\sigma(E_i)} \right)^2 , \tag{3}$$

where we have taken a nominal error $\Delta\sigma(E_i) = 0.05\sigma^{\exp}(E_i)$. We chose the lab kinetic energies E_i to range from 80 to 430 MeV in intervals of 50 MeV. This corresponds to a momentum range $0.4 \leq p_{1ab} \leq 0.9$ GeV/c; below 0.4 GeV/c, data are very uncertain, and above 0.9 GeV/c the simple nonrelativistic potential model used here is not appropriate.

The theoretical cross sections $\sigma^{\text{th}}(E_i)$ were generated by solving the Schrödinger equation with a local, complex $N\bar{N}$ potential $V_{N\bar{N}}(r)$ potential of the form

$$V_{N\bar{N}}(r) = V_t(r) + V_{ANN}(r) , \tag{4}$$

where $V_t(r)$ is the real potential arising from t-channel meson exchanges and $V_{ANN}(r)$ is a purely phenomenological annihilation potential arising from s-channel meson exchanges. We assume a Woods-Saxon form

$$V_{ANN}(r) = -(V_0 + iW_0)/(1 + e^{(r-R)/a}) , \tag{5}$$

where V_0, W_0, a, and R are adjustable parameters. The potential $V_t(r)$ is taken to be

$$V_t(r) = \begin{cases} V_{\text{Paris}}(r_0) , & r \leq r_0 \\ V_{\text{Paris}}(r) , & r > r_0 \end{cases} \tag{6}$$

where $V_{\text{Paris}}(r)$ is obtained from the nucleon-nucleon (NN) potential of the Paris group[13] by inverting the signs of terms corresponding to odd G-parity exchanges (π and ω). The two pion exchange in this model is calculated using dispersion techniques with πN and $\pi\pi$ scattering information as input. The theoretical medium and long range parts of the potential contain no free parameters, and supplemented by an ω exchange potential of reasonable strength and a simple prescription for the short range cutoff, provide an excellent fit to the NN phase shifts. $V_t(r)$ contains tensor, spin-orbit, and quadratic spin-orbit terms; we treat off-diagonal tensor coupling exactly by solving the coupled equations. The cutoff of the theoretical potential in Eq. (6) is *ad hoc*, but our results are essentially independent of the choice of r_0, taken to be 0.8 fm, since the strong absorption removes any sensitivity to the short range part of the potential $V_t(r)$.

We have investigated the parameter space $\{V_0, W_0, R, a\}$ and obtained a number of more or less equivalent fits to the data. Let us now discuss the role of each parameter.

We have found that the choice of R is not critical, i.e., whether the annihilation potential is flat or not for small r is inessential. Starting from any good fit with $0 < R < 0.8$ fm, one can always set $R = 0$ and readjust W_0 at fixed (V_0, a) in order to get a comparable fit. A noteworthy property of this readjustment is that it maintains about the same absorption in a crucial "surface" region $1 \leq r \leq 1.1$ fm. (The situation is somewhat analogous to the case of heavy ion reactions,[14] where "surface localization" also occurs.) The depth and the detailed shape of $\text{Im}V_{ANN}(r)$ for $r \leq 0.8$ fm are unimportant, as long as the absorption is sufficiently strong in the interior region ($\text{Im}V_{ANN} > 100$ MeV will suffice). In the results shown here, we have set $R = 0$ for simplicity, although solutions with $R > 0$ have the esthetic advantage of giving smaller values of the potential depths V_0 and W_0.

The choice of the surface thickness parameter "a" is more critical, since the rate of dropoff of $V_{ANN}(r)$ for radii outside the strong absorption region determines the relative contributions of various high partial waves and hence can influence the energy dependence of the cross sections. Reasonable fits were found with $4.5 \leq a^{-1} \leq 6$ fm^{-1}; the higher value was also found by Bryan and Phillips.[6] Most of the searches were done with $a^{-1} = 5$ fm^{-1}, a value we adopted for the results displayed in the figures.

With $R = 0$ and $a = 1/5$ fm, we first ignored the possibility of a real annihilation potential ($V_0 = 0$); a reasonable fit was obtained for $W_0 \approx 46$ GeV, a result similar to the value $W_0 \approx 62$ GeV of Bryan and Phillips[6] for their local potential version. We then kept W_0 fixed at various values from 2 to 50 GeV and searched on V_0. For $W_0 > 40$ GeV, a small *attractive* real part improved the fit slightly but V_0/W_0 remained small. As W_0 was decreased to about 20 GeV, the fit improved markedly (factor of 2 or more in χ^2) if we chose $V_0 \approx W_0$. The overall best fits were obtained in this region. Two typical examples are shown in Fig. 1. No fine tuning of the parameters was done. As W_0 is further decreased, acceptable fits were still obtainable with $W_0 \geq 10$ GeV, with correspondingly larger values of V_0. For $W_0 < 10$ GeV, the fits were no longer acceptable. Although we could reduce the strength of the absorptive part of the annihilation potential by a factor of 3 or 4 relative to Bryan and Phillips[6] by adding an *attractive* real part, the potential corresponding to the fits in Fig. 1 is still "strongly absorptive"; that is, $\text{Im}V_{ANN}(r) \approx 150$ MeV for $r \approx 1$ fm. The dependence of χ^2 on the choice of V_0 for fixed R, a, W_0 is shown in Fig. 2. The fit to inelastic and charge exchange cross sections clearly favors a large *positive* value of V_0 (attraction). The three components of the total

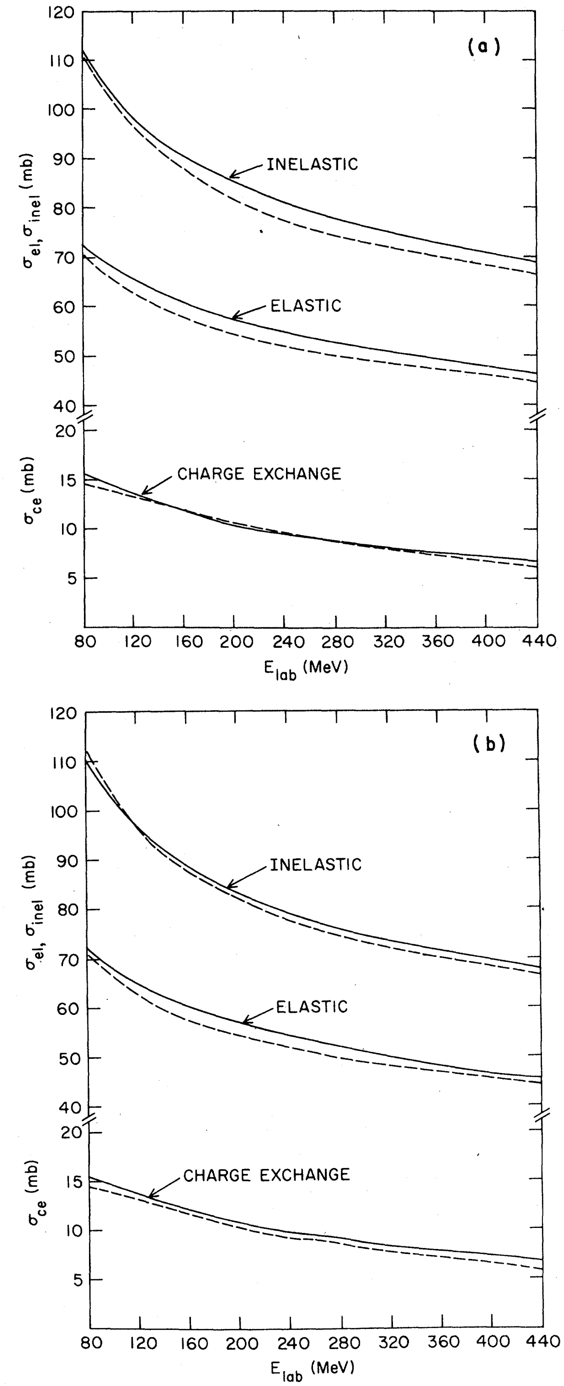

FIG. 1. (a) Optical model fit to elastic, inelastic, and charge exchange $\bar{p}p$ total cross sections in the energy region ($p_{lab} \leq 900$ MeV/c). The solid curves represent the theoretical calculation, using the Paris potential (Ref. 13) for the meson exchange part and the annihilation potential of Eq. (5), with $R=0$, $a=\frac{1}{5}$ fm, $V_0=21$ GeV, $W_0=20$ GeV. The dashed curves represent the parametrizations (1) and (2) of the experimental data. (b) The same as (a), with $R=0$, $a=\frac{1}{5}$ fm, $V_0=18$ GeV, and $W_0=20$ GeV.

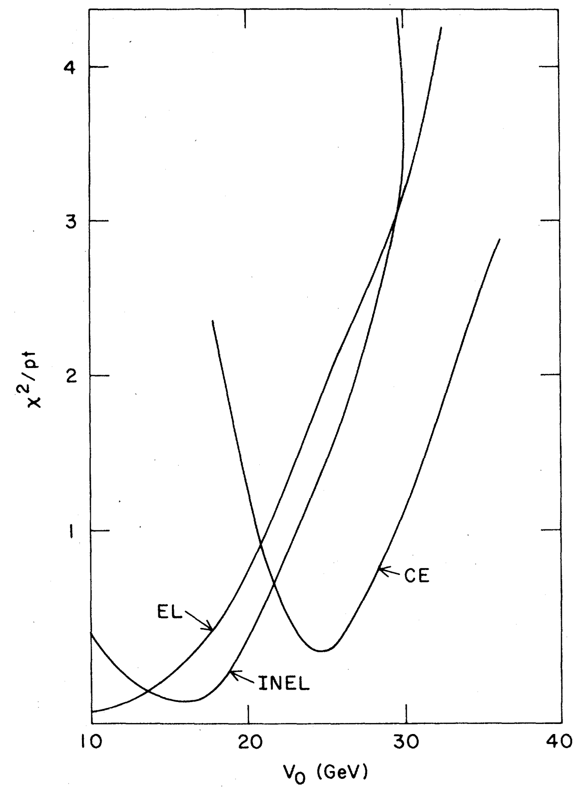

FIG. 2. Quality of fit (χ^2 per point) to the $\bar{p}p$ cross section data as a function of the depth V_0 of the real part of the annihilation potential, for fixed values $R=0$, $a=\frac{1}{5}$ fm, and $W_0=20$ GeV.

cross section are best fitted by quite different choices of V_0, indicating the shortcomings of the oversimplified local, state-independent annihilation potential of Eq. (6).

We now comment on the physical interpretation of our study.

I. SENSITIVITY TO THE MODEL OF NUCLEAR FORCES

Our results are somewhat deceiving. The improvement with respect to the early work of Bryan and Phillips[6] is due essentially to the addition of a real part to the annihilation potential. The use of a more elaborate treatment of meson exchanges at large distances seems to have only a minor influence. In the models which fit the data, the annihilation potential $V_{ANN}(r)$ is still significant in the crucial region $r \approx 1$ fm. The phenomenological freedom in $\mathrm{Re}V_{ANN}(r)$ precludes any test of the correctness of the meson exchange potential $V_t(r)$.

II. THEORETICAL UNDERSTANDING OF THE ANNIHILATION POTENTIAL

A. Range of annihilation forces and size of nucleons

The fit of the data with an optical model requires a range $a^{-1} \lesssim 6$ fm^{-1}. As shown by Martin,[15] the an-

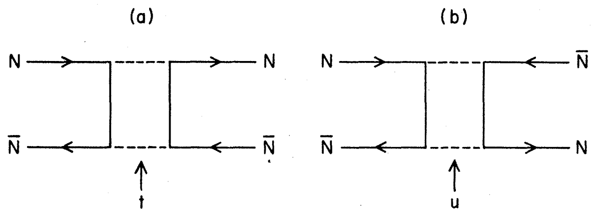

FIG. 3. Feynman graphs for $\bar{N}N$ annihilation which involve two meson intermediate states (dashed lines).

nihilation graphs for pointlike nucleon and antinucleon have a minimum inverse range $a^{-1} \geq 2M_N \simeq 10\ \text{fm}^{-1}$. Furthermore, if one computes explicitly the longest range diagrams of Fig. 3, one finds an effective value $a^{-1} \approx 13-15\ \text{fm}^{-1}$. In other words, annihilation forces do not propagate from nucleon to antinucleon. The spatial distribution of $V_{ANN}(r)$ is determined by the overlap of the hadronic matter of N and $\bar{N}$. Hence, one should not concentrate too much on the precise value of a^{-1}, but rather on the fact that fitting the data requires strong absorption up to $r \approx 1-1.2$ fm. In a naive spherical bag picture, this corresponds to a radius for N and $\bar{N}$, $R_N \geq 0.5$ fm, in agreement with the original MIT bag model estimate.[16] From our study, however, we cannot exclude a smaller radius such as $R_N \approx 0.2$ fm proposed by Brown and Rho.[17] In the "little bag" model, the meson cloud around the nucleon and the antinucleon will also influence the annihilation process.

B. The real part of the annihilation potential

In our experience, the overall quality of fit was always improved by the addition of an *attractive* real part. We did not find any case where added repulsion from annihilation improved the situation. The attraction in the surface region acts to focus the wave function to smaller distances, where annihilation is more effective, thus enabling us to decrease the strength W_0 of the imaginary part. Such a result is not expected from a naive model of elementary annihilation processes. Consider for instance the graph of Fig. 3(a), neglecting for simplicity spin and isospin. It satisfies a Mandelstam representation

$$V_{ANN}(s,t) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{dt'}{t'-t}\,\rho(s,t') \tag{7}$$

with

$$\rho(s,t') = \frac{1}{\pi}\int_{4\mu^2}^{\infty} \frac{ds'}{s'-s-i\epsilon}\,y(s',t')\,. \tag{8}$$

In position space, the corresponding (energy dependent) potential is a continuous superposition of Yukawa functions

$$V_{ANN}(r,s) \propto -\int_{4m^2}^{\infty} \frac{e^{-r\sqrt{t'}}}{r}\,\rho(s,t')\,dt'\,. \tag{9}$$

By standard methods,[18] one gets

$$y(s,t) = g^4\{st[(s-4\mu^2)(t-4m^2)-4\mu^4]\}^{-1/2}\,. \tag{10}$$

Thus, from Eq. (8), $\text{Im}\rho > 0$ as it should be and

$$\text{Re}\rho(s,t) = \frac{-2g^4}{\pi}[t(t-4m^2)s(s-s_0)]^{-1/2} \times \cosh^{-1}(s/s_0)^{1/2}\,, \tag{11}$$

where $s_0 = 4\mu^2 + 4\mu^4/(t-4m^2)$ is the Mandelstam boundary. Hence, we have $\text{Re}V_{ANN}(r,s) > 0$. The inclusion of spin and isospin is not expected to change the situation too much. Indeed, our result is simple $\text{Re}V_{ANN}/\text{Im}V_{ANN} < 0$ and $\text{Im}V_{ANN} < 0$ is a consequence of s-channel unitarity. There is also another version of the argument, in terms of naive two-channel quantum mechanics. Graphs of Fig. 3 arise because of the transition potential, which acts between $N\bar{N}$ and MM channels. Owing to the hermiticity of the Hamiltonian, such mixing shifts down the lightest mass channel (MM) and pushes up the heaviest one ($N\bar{N}$). Hence, annihilation would seem naively to induce some effective repulsion between N and $\bar{N}$.

C. Locality

From the above expressions, it is clear that the annihilation amplitude is strongly energy dependent, since the physical region $s \geq 4m^2$ lies on several cuts due to multimeson thresholds. The Fourier transform is expected to be a highly nonlocal and/or energy dependent potential. It may be that simulating this interaction with a local operator is responsible for the rather large range needed to fit the data.

D. Channel dependence

In our numerical study, the annihilation potential was taken to be the same in all channels. Any naive annihilation graph similar to those of Fig. 3 has obvious channel dependence. For instance, two pion intermediate states require natural parity $\bar{N}N$ partial waves $J^{PC}(I^G) = 0^{++}(0^+)$, $1^{--}(1^+)$, etc. It can be argued that absorptive forces at low energy are dominated by two-meson intermediate states M_1M_2 where $M_i = (\pi, \eta, \rho, \omega, \ldots)$. This picture is compatible with experimental data. Since each M_1M_2 pair has obvious selection rules, one can ask whether or not a particular $\bar{N}N$ partial wave has only a few allowed decay channels and hence weak absorption. A bound state in this partial wave would then have a good chance to be very narrow (see next section). The answer is of course negative; any $\bar{N}N$ partial wave disposes of several annihilation channels. The approximation of a spin and isospin independent annihilation potential does not seem the worse defect of the optical model examined here.

III. CONSEQUENCES FOR THE QUASINUCLEAR MODEL OF BARYONIUM

It has been shown that the meson exchange $\bar{N}N$ potential $V_t(r)$ is attractive enough to produce several bound states and resonances. This provides a rather simple and natural description for the experimentally observed narrow states, which has the advantage of resting on the well established Yukawa theory. This quasinuclear model of baryonium has been extensively studied.[2-4] It has been shown, however, that within the context of the optical model all the structures due to the real potential are considerably broadened or washed out by annihilation.[8] From the present study, it is clear that no salvation can be expected in the framework of a complex local potential model. Although the inclusion of a real annihilation potential enables one to reduce the absorptive part by a factor of as much as 4–5, the imaginary component is still very strong at $r \approx 1$ fm, in the region where the wave functions of quasinuclear bound states or resonances are localized.[4] Note also that the depth V_0 is sufficiently large to form new types of bound states which would be absent without annihilation. The position and width of such states has recently been discussed in the context of a coupled channel picture.[9] Promising results have been found, i.e., some states remain rather narrow ($\Gamma \sim 30$ MeV) with a very short range annihilation ($a^{-1} \approx 10$ fm^{-1} in the work of Polikarpov and Simonov).[9] However, only the near threshold annihilation cross section was fitted in these models.[9] From our experience, it is always possible to reproduce the threshold value of $\nu\sigma_A$ with very short range annihilation. The difficulties come where one tries to fit the cross sections up to $E_{lab} \sim 200$–300 MeV (still reasonable for a potential picture). To accomplish this, one needs to increase dramatically the range of the annihilation potential, and thus the quasinuclear bound states become wider.

In conclusion, we have shown that, even if one uses realistic long range nuclear forces, describing the $\bar{N}N$ integrated cross sections with a complex annihilation potential requires a strong absorptive component acting out to $r \approx 1$ fm. The annihilation potential needed to fit the data seems difficult to understand, even qualitatively, in terms of naive Feynman diagrams involving a few mesons as intermediate states. The internal structure of the nucleon certainly plays a very important role in annihilation processes. The annihilation potential also has the unsettling property of making it difficult for narrow structures to survive. Clearly the quasinuclear model of baryonium cannot be ruled out by such a primitive picture of annihilation and the attempts made using a multichannel formalism must be pursued further. On the other hand, this optical model may provide a practical description of background cross sections which can be useful for experimental purposes, for instance, in understanding the energy dependence of backward elastic scattering.

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