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Mechanisms for deuteron production in relativistic nuclear collisions

Joseph I. Kapusta
Nuclear Science Division, Lawrence Berkeley Laboratory, University of California, Berkeley, California 94720
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A variety of mechanisms for deuteron production in relativistic nuclear collisions are considered. These include the coalescence model, the sudden approximation model, and the static thermal model. A new model based on time dependent perturbation theory is presented. A solution of the rate equations for a hydrodynamically expanding fireball suggests that chemical equilibrium might be achieved in central collisions. Emphasis is placed on the physical assumptions of the various models, their limitations, and their subtly different predictions. Of some importance is the effect of impact parameter averaging which, when written in terms of the usual power law relationship, introduces the necessity for measuring various two particle correlation functions to check the self-consistency of these models.

[NUCLEAR REACTIONS Relativistic heavy ions, models for deuteron production p-n correlations.]

I. INTRODUCTION

One of the more interesting aspects of relativistic heavy ion collisions is the huge number of reaction products which are observed. The list of particles starts with photons, pions, kaons, and protons, ranges through the light composite nuclei, and up to fragments as heavy as the beam and target nuclei themselves. Surely this variety of particles is a consequence of the variety of types of events which occur. In a large impact parameter peripheral type event the beam and target nuclei just tickle each other leading to low excitation energy and rapidity distributions centered on the beam and target rapidities. In a small impact parameter central type event the beam and target nuclei smash each other to pieces leading to high excitation energy and a rapidity distribution extending over the entire allowable phase space.

The purpose of this paper is to investigate some possible mechanisms for producing light nuclear fragments, in particular, deuterons. Deuterons were singled out for several reasons. As opposed to alpha particles, for instance, it is unlikely that knockout of preformed deuterons or deuteron evaporation from a heavy target residue will significantly contribute to the observed production cross section. Thus the dynamics which are unique to relativistic heavy ion collisions will be focused on. Also, the computational effort for some of the models to be discussed, and the applicability of all of them, are optimized for the smallest nuclear fragment.

We begin by reviewing, in Sec. II, the first model historically to be used to describe deuteron production in relativistic nuclear collisions, the momentum space coalescence model. Two dynamical models which were invoked to account for the coalescence phenomenon, the static thermodynamic model and the sudden approximation model, are discussed in Secs. III and IV. These are generalized to the relativistic domain where it is shown that they differ from each other and the coalescence model by powers of the relativistic dilation factor γ. In Sec. V, we introduce a new model based on time dependent perturbation theory. The formation probability in this model depends on the interaction time as well as on the volume of the producing system. Section VI contains the results of a numerical solution of the rate equation for deuteron formation in the context of a hydrodynamically expanding fireball. Clear experimental signatures of this mechanism are predicted. We discuss the effects of impact parameter averaging in Sec. VII and show how the measurement of two particle correlations can untangle them. Finally, Sec. VIII contains a summary of what we have learned so far. Present experimental data is referenced throughout the paper.

II. COALESCENCE MODEL

The coalescence model for relativistic nuclear collisions[1,2] was developed from the physical insight provided by proton-nucleus collisions. In those reactions Butler and Pearson[3] suggested that the deuteron production mechanism was the binding of cascade nucleons in the presence of the target nuclear optical potential. In their model the momentum space density of produced deuterons with momentum per nucleon $\vec{p}$ is

$$\frac{d^3n_d}{dp^3} \sim \frac{1}{p^2}\left(\frac{d^3n_p}{dp^3}\right)^2 , \tag{1}$$

where the momentum independent coefficient is not important for our purposes but can be calculated. Also, relativistic effects and differences between proton and neutron densities have been neglected. The important point about Eq. (1) is the factor $1/p^2$. This arises because the nucleon pair must transfer their excess energy momentum to the nucleus via the static optical potential before they can become a real deuteron in the final state. In the bulk of relativistic heavy ion collisions the projectile and target interact so strongly and quickly that one can no longer speak of a static nuclear optical potential. This production mechanism can probably be ruled out.

Schwarzschild and Zupančič[4] then pointed out that, independent of the detailed production mechanism, the deuteron density d^3n_d/dp^3 should be proportional to the square of the proton density $(d^3n_p/dp^3)^2$. The coefficient may be momentum dependent (perhaps only weakly so) and will depend on details of the mechanism. This statement of a "square law" behavior is just a reflection of the final state phase space assumed. A pair of independent nucleons in the final state somehow transfer energy momentum to the rest of the system to form a deuteron. See Fig. (1).

The derivation of the coalescence model for deuterons goes as follows.[1,2] Let $\gamma d^3\bar{n}_N/dp^3$ be the relativistically invariant momentum space density for nucleons before coalescence into deuterons. We assume that protons and neutrons have equal densities but the formulas can be generalized to include the nonequal cases. Consider a sphere in momentum space centered at $\vec{p}$ and with a radius p_0. The probability for finding one primary nucleon in this sphere is

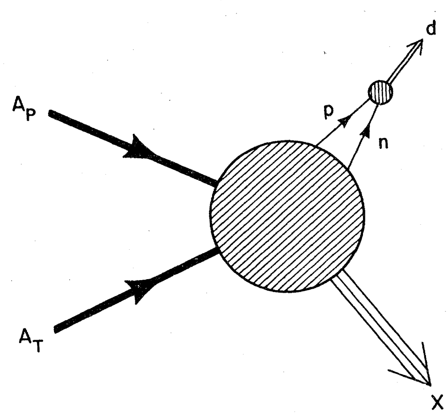

FIG. 1. Schematic for the production of a deuteron in the final state of a relativistic collision between two heavy nuclei.

$$P = \frac{1}{M}\frac{4\pi}{3}p_0^3\gamma\frac{d^3\bar{n}_N}{dp^3} , \tag{2}$$

where M is the mean nucleon multiplicity. The purely statistical probability for finding two nucleons in this sphere is

$$P_M(2) = \binom{M}{2}P^2(1-P)^{M-2} . \tag{3}$$

If $MP \ll 1$ and $M \gg 1$ then the last factor is approximately one. Hence

$$\gamma\frac{d^3n_d}{dp^3} = \frac{1}{2}\frac{4\pi}{3}p_0^3\left(\gamma\frac{d^3n_N}{dp^3}\right)^2 . \tag{4}$$

If we also take into account spin and isospin then the formula becomes

$$\gamma\frac{d^3n_d}{dp^3} = \frac{3}{4}\frac{4\pi}{3}p_0^3\left(\gamma\frac{d^3n_p}{dp^3}\right)\left(\gamma\frac{d^3n_n}{dp^3}\right) . \tag{5}$$

The unspecified parameter p_0 is a number to be taken from fits of this formula to experimental data. In principle p_0 could depend on $\vec{p}$, but then this simple momentum phase space model would have no predictive power. Note that Eq. (5) applies for a *single* impact parameter.

At this point one might ask what the mechanism is that allows a pair of free nucleons to coalesce into a deuteron. Mathematically this model says that whenever a proton and a neutron are within a momentum $\sim p_0$ of each other and in the correct spin state then they will coalesce. p_0 is not predicted by the model, but assumed to be on the order of the Fermi momentum of the deuteron. Energy-momentum conservation during coalescence is not considered to be a problem because the deuteron is so weakly bound. After all, in the initial state of the heavy ion collision the nucleons are off their mass shell by ~8 MeV, there may be multiple two or three body collisions in the intermediate state as well as virtual pions to boost a final state deuteron on to its mass shell.

An advantage of the momentum space coalescence model is its generality. It is pure phase space and statistics and makes no assumptions about the details of the production mechanism. This is also a limitation since it cannot predict how p_0 varies with projectile and target size, beam energy, or even whether p_0 is really independent of deuteron momentum.

III. THERMODYNAMIC MODEL

The thermodynamic model[5] accounts for light composite particle production by assuming that the projectile and target nuclei or portions thereof form an intermediate complex, or fireball,[2,6] in which thermal equilibrium, both kinetic and chemical, takes place. Given the baryon number, energy, charge, and density of the emitting system when it decays, one can calculate the volume V, temperature T, and neutron and proton chemical potentials μ_p and μ_p.[7] Alternatively, some of these parameters may be extracted from the data.

The distribution of particles of type i in momentum space is

$$\frac{d^3n_i}{dp^3} = \frac{2S_i+1}{(2\pi)^3} V \left[\exp\left(\frac{(p^2+m_i{}^2)^{1/2}-\mu_i}{T}\right) \pm 1\right]^{-1}, \qquad (6)$$

where S_i is the spin of the particle and $\pm$ refers to fermion/boson. *If* the density is low enough so that the particles can be treated as nondegenerate then we observe that

$$\frac{d^3n_d}{dp^3} = 8\frac{3}{4}\frac{(2\pi)^3}{V}\frac{d^3n_p}{dp^3}\frac{d^3n_n}{dp^3}, \qquad (7)$$

where $\vec{p}$ is the momentum per nucleon. Writing this in terms of Lorentz invariant densities we have

$$\gamma\frac{d^3n_d}{dp^3} = \tfrac{3}{4}8\frac{(2\pi)^3}{V}\frac{1}{\gamma}\left(\gamma\frac{d^3n_p}{dp^3}\right)\left(\gamma\frac{d^3n_n}{dp^3}\right). \qquad (8)$$

Comparing this with the coalescence formula Eq. (5) we identify

$$\tfrac{4}{3}\pi p_0{}^3 = \frac{8}{\gamma}\frac{(2\pi)^3}{V}. \qquad (9)$$

Thus p_0 is inversely proportional to the dimension of the emitting system. Notice also that Eqs. (8) and (9) are not Lorentz invariant statements since p_0 depends on γ measured *relative to the center of mass of the emitting system*. This can lead to problems when comparing Eq. (8) with data which covers a wide kinematic range since one does not know *a priori* what the center of mass frame is or if there may be more than one emitting system. One may invoke geometrical assumptions to determine center of mass frames,[6,8,9] but then one cannot focus on the thermodynamic assumption alone.[10]

One may question the wisdom of applying infinite matter, noninteracting gas formulas for the distribution of particles coming from a highly time dependent, strongly interacting finite size system. In response to this Mekjian[5] has estimated the reaction rates for deuteron production. It appears that they are larger than typical expansion rates by an order of magnitude so that thermal equilibrium may be a good first approximation for the more central collisions. This point is discussed further in Sec. VI. Das Gupta[11] has made some progress in estimating the errors involved when taking the infinite particle limit. The errors do not seem severe except near the boundaries of phase space.

Finally one might say that the application of thermodynamic formulas only require that the phase space for nucleons and deuterons be filled statistically. The mechanism for filling it, whether by two and three body collisions or by more exotic mechanisms, is not important to lowest order.

IV. SUDDEN APPROXIMATION

Another model[12] for light composite particle production is based on the following intuitive picture. During the intermediate stages of the collision the nuclear system will be at high density and excitation energy and there is strong interaction among the nucleons. Since they are such diffuse objects it probably does not make sense to speak of deuterons existing at high density. As the system expands and the density goes down, proton-neutron correlations develop which eventually lead to some proton-neutron pairs binding together to form deuterons. If this transition from a high density phase containing no deuterons to a low density phase containing some deuterons is fast enough, then one is tempted to apply the sudden approximation of quantum mechanics.

The general many-body analysis of the problem is very complicated and cannot be handled without knowing the complete time development of the system. However, we can obtain a fair approximation to what we think the real physics might be in the following way. Consider the system as evolving from a high density phase described by the many-particle Hamiltonian H_i to a low density phase described by the many-particle Hamiltonian H_f. The spectrum of H_i consists of nucleons only, whereas the spectrum of H_f consists of nucleons and deuterons. The probability for a given proton-neutron pair in the high density phase to bind into a deuteron in the low density phase is given by the overlap of the proton-neutron wave function with the deuteron wave function. Both wave functions are evaluated at the volume V for which the high to low density phase transition is to occur. This is left as a parameter to be adjusted to fit experimental data, but is estimated to be that volume for which the average density is equal to or somewhat less than normal nuclear density.

Let the initial proton and neutron state be described by the plane wave

$$\psi_i = \frac{1}{L^3}e^{i(\vec{k}_1\cdot\vec{x}_1+\vec{k}_2\cdot\vec{x}_2)} = \frac{1}{L^3}e^{i(\vec{K}_1\cdot\vec{R}+\vec{k}\cdot\vec{r})}, \qquad (10)$$

where we use box normalization, and

$$\vec{K}_i=\vec{k}_1+\vec{k}_2\,,\quad \vec{R}=\tfrac{1}{2}(\vec{r}_1+\vec{r}_2)\,,$$
$$\vec{k}=\tfrac{1}{2}(\vec{k}_1-\vec{k}_2)\,,\quad \vec{r}=\vec{r}_1-\vec{r}_2\,. \tag{11}$$

Let the final deuteron state be described by an internal wave function $\chi(\vec{r})$ and an overall plane wave

$$\Psi_f=\frac{1}{L^{3/2}}e^{i\vec{K}\cdot\vec{R}}\chi(\vec{r})\,. \tag{12}$$

It is convenient to work in the rest frame of the deuteron, $\vec{K}=0$. Then the sudden approximation says that

$$\frac{1}{8}\frac{d^3n_d}{dp^3}(0)=\int|\langle i|f\rangle|^2\frac{d^3n_p}{dk_1^3}\frac{d^3n_n}{dk_2^3}dk_1^3dk_2^3\,. \tag{13}$$

Here $\vec{p}\equiv\frac{1}{2}\vec{K}$ is the momentum per nucleon. If we take into account spin factors and evaluate the integrals we find that

$$|\langle i|f\rangle|^2=\frac{3}{4}\frac{(2\pi)^3}{L^6}\delta(\vec{K}_i)\int dr^3dr'^3e^{i\vec{k}\cdot(\vec{r}'-\vec{r})}\times\chi^*(\vec{r})\chi(\vec{r}')\,. \tag{14}$$

We can change variables so that

$$dk_1^3dk_2^3=dK_i^3dk^3\,. \tag{15}$$

Furthermore, we neglect the variation of the nucleon densities over the range of integration of $\vec{k}$ to obtain

$$\frac{d^3n_d}{dp^3}(0)=8\frac{3}{4}\frac{(2\pi)^3}{L^6}\frac{d^3n_p}{dp^3}(0)\frac{d^3n_n}{dp^3}(0)\times\int dK_i^3\delta(\vec{K}_i)\int dr^3dr'^3dk^3\times e^{i\vec{k}\cdot(\vec{r}'-\vec{r})}\chi^*(\vec{r})\chi(\vec{r}')\,. \tag{16}$$

Since χ is normalized to unity we get

$$\frac{d^3n_d}{dp^3}(0)=8\frac{3}{4}\left(\frac{2\pi}{L}\right)^6\frac{d^3n_p}{dp^3}(0)\frac{d^3n_n}{dp^3}(0)\,. \tag{17}$$

Now we multiply by $(L/2\pi)^3$ to convert box normalization to continuum normalization (we had been using a mixture of the two for convenience, i.e., the Dirac deltas). Then we multiply by L^3/V_d because the proton-neutron to deuteron transition is occurring only in the *finite* volume V_d which is measured in the deuteron rest frame. Note that the sudden transition occurring in a finite volume is crucial, otherwise, on taking the limit $L\to\infty$ the number of deuterons would go to zero as a consequence of energy conservation. A nucleon pair cannot make a deuteron in free space.

So far our formula is

$$\frac{d^3n_d}{dp^3}(0)=8\frac{3}{4}\frac{(2\pi)^3}{V_d}\frac{d^3n_p}{dp^3}(0)\frac{d^3n_n}{dp^3}(0)\,. \tag{18}$$

Let us write this in terms of quantities as measured in the rest frame of the emitting system. Then

$$\frac{d^3n_d}{dp^3}(0)=\gamma\frac{d^3n_d}{dp^3}(\vec{p})\,,\ \text{etc.}, \tag{19}$$

and

$$V_d=V/\gamma\,, \tag{20}$$

where V is the proper volume of the system. The final formula is then

$$\gamma\frac{d^3n_d}{dp^3}=\tfrac{3}{4}8\frac{(2\pi)^3}{V}\gamma\left(\gamma\frac{d^3n_p}{dp^3}\right)\left(\gamma\frac{d^3n_n}{dp^3}\right). \tag{21}$$

Comparing this with the coalescence formula Eq. (5) we identify

$$\tfrac{4}{3}\pi p_0^{\,3}=8\gamma\frac{(2\pi)^3}{V}\,. \tag{22}$$

As in Eq. (9) for the thermal model we see that p_0 is inversely proportional to the dimension of the emitting system as expected from phase space. However, in the present model p_0 has the inverse dependence on γ compared to the thermal model. In the nondegenerate domain these models yield different predictions only because of relativistic effects.

V. TIME DEPENDENT PERTURBATION THEORY

In this section we introduce a new model based on time dependent perturbation theory. In contrast to earlier models this one assumes that deuteron production is a small perturbation on the system so that deuteron saturation is *not* achieved. The number of deuterons increases with increasing interaction time available for production. Mathematically this model says that energy conservation in the formation $p+n\to d$ is not violated because of the uncertainty principle: The interaction is turned on only for a finite time. Physically we know that the rest of the system is well able to absorb excess energy momentum (see Sec. II).

Consider the quasi-initial neutron-proton state to be described by the plane wave

$$\Psi_i=\frac{1}{L^3}e^{i(\vec{k}_1\cdot\vec{x}_1+\vec{k}_2\cdot\vec{x}_2)}=\frac{1}{L^3}e^{i(\vec{K}_i\cdot\vec{R}+\vec{k}\cdot\vec{r})}\,. \tag{23}$$

The final state wave function is taken as

$$\Psi_f=\frac{1}{L^{3/2}}e^{i\vec{K}\cdot\vec{R}}\chi(\vec{r})\,. \tag{24}$$

The notation is the same as before. We work in the nonrelativistic limit for the moment and quote the relativistic result at the end.

The transition probability $\omega(\vec{K})dK^3$ to produce a deuteron via the neutron-proton potential $v(\vec{r})$ is

$$\omega(\vec{K}) = \left| \int_0^\infty dt\, P(t) H_{if} e^{-i\omega_{if} t} \right|^2 \rho(\vec{K}) , \tag{25}$$

where $P(t)$ describes the time dependence of the interaction, $\rho(\vec{K})$ is the density of final states, ω_{if} is the energy difference, and

$$H_{if} = \langle i | v | f \rangle . \tag{26}$$

In a more rigorous calculation we would use wave packets to describe the nucleons, but such an approach would imply that we know the detailed time development of the system. To implement what we think the real physics might be, we simulate the fact that the nucleonic wave functions overlap only for a finite time interval by the introduction of the factor $P(t)$. It is only within this finite time interval that a given proton-neutron pair are close enough together to interact. Qualitatively this corrects the glaring deficiency that plane waves overlap forever.[3] Also, it is only within this finite time interval that there are other nucleons close enough to absorb excess energy momentum, i.e., before the system blows apart. For the moment let us parameterize $P(t)$ by

$$P(t) = e^{-t/\tau} . \tag{27}$$

Then

$$\left| \int_0^\infty dt\, P(t) e^{-i\omega_{if} t} \right|^2 = \frac{1}{\omega_{if}^2 + \tau^{-2}} . \tag{28}$$

Let us also choose $\chi(\vec{r})$ to have the Hulthèn form

$$\chi(\vec{r}) = \frac{c}{r}(e^{-\eta r} - e^{-\xi r}) , \tag{29}$$

where $B = \eta^2/m$ is the deuteron binding energy:

$$\eta \approx 45.7 \text{ MeV}/c ,$$
$$\xi \approx 376 \text{ MeV}/c , \tag{30}$$
$$c^2 \approx \eta/2\pi .$$

The deuteron distribution for a given nucleus-nucleus collision is then

$$\frac{d^3 n_d}{dK^3} = \left(\frac{L}{2\pi}\right)^3 \frac{L^3}{V} \int dk_1{}^3 dk_2{}^3 \frac{d^3 n_p}{dk_1{}^3} \frac{d^3 n_n}{dk_2{}^3} \times \left| \int_0^\infty dt\, P(t) H_{if} e^{-i\omega_{if} t} \right|^2 . \tag{31}$$

Working out the integrals we find

$$\frac{d^3 n_d}{dp^3} = 8 \frac{3}{4} \frac{(2\pi)^3}{V} \frac{d^3 n_p}{dp^3} \frac{d^3 n_n}{dp^3} (2B\tau)^{1/2} \times \frac{1}{(1+x^4)^2} \left(1 - \frac{3}{\sqrt{2}} x + 2x^2 - x^4 + \frac{1}{\sqrt{2}} x^5\right) , \tag{32}$$

where $\vec{p} = \frac{1}{2}\vec{K}$, $x = (m/\tau)^{1/2}/\xi$, terms of order η^2/ξ^2 have been dropped, and spin has been accounted for.

This result, Eq. (32), is very interesting because in principle it allows one to determine from the data not only the volume of the interacting nuclear complex but also an effective lifetime. For currently available energies of 2 GeV/nucleon or less any reasonable estimate of τ gives $x \ll 1$. This is equivalent to letting $\xi \rightarrow \infty$ or using just the asymptotic exponential tail of the deuteron wave function. Comparing with the coalescence formula Eq. (5) we identify

$$\tfrac{4}{3}\pi p_0{}^3 = 8 \frac{(2\pi)^3}{V} (2B\tau)^{1/2} \gamma^{1/2} , \tag{33}$$

where we have inserted the correct relativistic factor. If we estimate τ as the time it takes the two nuclei to traverse each other in the nucleon-nucleon center of mass then $\tau \sim Rm/p_{\text{c.m.}}$, where R is a typical nuclear radius. For beam energies between 250 and 2100 MeV/nucleon, $(2B\tau)^{1/2}$ is of order unity. Also, p_0 should decrease as $T_{\text{LAB}}^{-1/12}$, which is a 14% decrease between the above quoted energies. Although some older data[1] suggest that sort of behavior, present experimental errors are probably too large to draw any conclusions. At still higher energies there is a more rapid beam energy dependence which unfortunately cannot be tested at present accelerators.

One might wonder about the dependence of the result on how the interactions are turned on and off. If we choose

$$P(t) = \theta(\tau - t) \tag{34}$$

and use just the asymptotic form of the deuteron wave function, valid for lower energies, then $(2B\tau)^{1/2}$ in Eq. (33) is replaced by $(32B\tau/\pi)^{1/2}$. Thus the extraction of an interaction time from the data will be slightly ambiguous in its numerical value due to its imprecise definition in this model.

VI. RATE EQUATIONS IN HYDRODYNAMIC FLOW

Let us return now to the question of chemical equilibrium for deuterons in central collisions. Mekjian[5] has painted a scenario wherein the light nuclear elements are built up in a series of multibody reactions during the expansion stage of a fireball. For deuterons, typical reactions are

$$n + p + N \rightarrow d + N ,$$
$$d + n + p \rightarrow d + d , \tag{35}$$
$$n + n + p + p \rightarrow d + d .$$

One then has to solve a coupled set of rate equations to determine the deuteron concentrations. If we focus on the first reaction of Eq. (35) only, which will be the most important reaction initially, then

$$\frac{d\rho_d}{dt}=\left[\rho_p\rho_n\left(\frac{\rho_d}{\rho_p\rho_n}\right)_{\rm eq}-\rho_d\right]\times\rho_N\langle v_{\rm rel}\sigma(N+d\to n+p+d)\rangle\,. \qquad (36)$$

Here ρ_p, ρ_n, ρ_N and ρ_d are the proton, neutron, nucleon, and deuteron densities. The $(\rho_d/\rho_p\rho_n)_{\rm eq}$ is the equilibrium ratio:

$$\left(\frac{\rho_d}{\rho_n\rho_p}\right)_{\rm eq}=\frac{3}{4}\left(\frac{4\pi}{mT}\right)^{3/2}. \qquad (37)$$

The product $\sigma v_{\rm rel}$, where $v_{\rm rel}$ is the relative velocity of the nucleon and deuteron, is to be averaged over the thermal distribution of relative velocities:

$$\langle\sigma v_{\rm rel}\rangle=(16/\pi m)^{1/2}T^{-3/2}\int_0^\infty dE\,E\,e^{-E/T}\sigma(E)\,. \qquad (38)$$

All of these equations assume that kinetic thermal quilibrium for the deuterons and nucleons is achieved and that nonrelativistic, Boltzmann statistics are adequate.

A survey of available data[5] suggests that $\sigma(E)\approx 100$ mb for the energy ranges of interest to us. Mekjian then estimates that the deuteron formation rate is an order of magnitude greater than the expansion rate of a fireball. The purpose of this section is to solve the rate equation for a particular model of the fireball expansion to see if indeed there is sufficient time to build the deuterons up to equilibrium concentration.

Bondorf, Garpman, and Zimànyi´[13] have proposed a simple analytic hydrodynamic model for expanding fireballs which will be sufficient for our purpose. It is based on the expansion of a piece of hot nuclear matter into the vacuum. At time $t=0$ all of the energy is thermal. As the system expands an increasing fraction of the total energy is converted into collective hydrodynamic flow. The equation of state assumed is that of a monatomic ideal gas. We shall choose the initial baryon density distribution as $\rho_B(t=0)\theta(R-r)$, where the radius of the sphere R is determined by the total number of nucleons in the system. Then the system expands as a sphere of uniform density. The density evolves as

$$\rho_B(t)=\rho_B(0)[1+(t/t_0)^2]^{-3/2}\,, \qquad (39)$$

where the characteristic expansion time is

$$t_0=\left(\frac{3mN}{10E_{\rm tot}}\right)^{1/2}\left(\frac{3N}{4\pi\rho_B(0)}\right)^{1/3}, \qquad (40)$$

N is the number of nucleons, and $E_{\rm tot}$ is their total excitation energy. The total thermal energy is

$$E_{\rm tot}^{\rm therm}=\frac{E_{\rm tot}}{1+(t/t_0)^2} \qquad (41)$$

and the total hydrodynamic flow energy is

$$E_{\rm tot}^{\rm flow}=E_{\rm tot}\frac{t^2}{t^2+t_0^{\,2}}\,. \qquad (42)$$

Although the baryon density is uniform inside the expanding fireball the temperature is not. Since our only purpose here is to see how fast deuterons can be chemically equilibrated, and not to make detailed predictions, it is sufficient to solve the rate equations using an average temperature determined by the total thermal energy:

$$T(t)=\frac{2}{3}\frac{E_{\rm tot}}{N}\frac{1}{1+(t/t_0)^2}\frac{1}{1-\rho_d(t)/\rho_B(t)}\,. \qquad (43)$$

The nucleon, baryon, and deuteron densities are related by

$$\rho_B(t)=\rho_N(t)+2\rho_d(t)\,, \qquad (44)$$

where $\rho_B(t)$ is given in Eq. (39). We are of course neglecting the feedback of deuteron production on the expansion of the fireball. The full rate equation we must solve is thus

$$\frac{d\rho_d}{dt}(t)=\left[\frac{3}{16}\left(\frac{4\pi}{mT(t)}\right)^{3/2}[\rho_B(t)-2\rho_d(t)]^2-\rho_d(t)\right]\times[\rho_B(t)-2\rho_d(t)]\sigma_0\left(\frac{16}{\pi}\frac{T(t)}{m}\right)^{1/2}-\frac{3t}{t^2+t_0^{\,2}}\rho_d(t)\,. \qquad (45)$$

Here $\rho_B(t)$ and $T(t)$ are determined by Eqs. (39) and (43) and we take $\sigma_0=100$ mb. The last term in Eq. (45), which is not present in Eq. (36), is a dilution term reflecting the fact that the deuteron density will decrease due to expansion. This is familiar from astrophysics.[14]

We have solved Eq. (45) numerically for two cases. In case A the system consists of 160 nucleons initially (no deuterons) and has 37.5 MeV of excitation per nucleon. In case B the system consists of 20 nucleons initially (no deuterons) and has 150 MeV of excitation per nucleon. We start the expansion out at twice normal density in both cases. The t_0 for the two cases changes by a factor of four.

The results are plotted in Figs. 2 and 3. We define x_d to be the number of nucleons bound up in deuterons divided by the total baryon number in the system. Thus $0\leq x_d\leq 1$. The important point to notice is that the deuterons build up to their equilibrium value very quickly. If a central

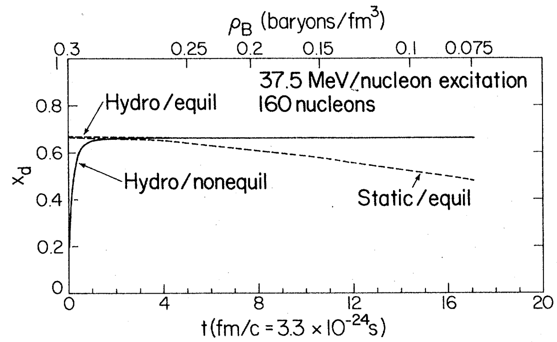

FIG. 2. The fraction x_d of nucleons which are bound in a deuteron as a function of time and density, for case A. The solid line is a solution of the rate equation in a hydrodynamically expanding fireball.

collision of two heavy ions can be at all modeled by hydrodynamics, then the deuterons can be expected to reach their equilibrium value. There is one objection to this conclusion: Does it make sense to speak of deuterons existing at normal nuclear density and above? No allowance was made for the extended size of the deuteron or nucleon. However, even if deuterons did not begin to be formed until nuclear matter densities were reached during the expansion, the deuterons would still have enough time to reach equilibrium in the cases we have considered.

Also shown in the figures is the value of x_d one would obtain in the static thermodynamic model. In that model all of the excitation energy goes into thermal motion, none into hydrodynamic flow. The deuteron yield in the static thermal model depends on the breakup density of the system whereas in the hydrodynamic model it does not. The reason is easy to see. From Eq. (45) the instantaneous equilibrium value $\hat{x}_d$ is determined

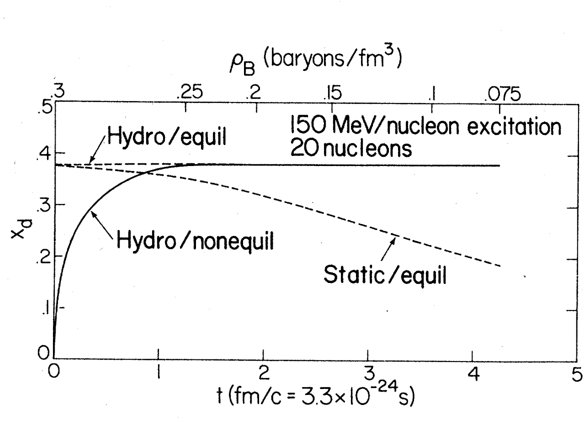

FIG. 3. The fraction x_d of nucleons which are bound in a deuteron as a function of time and density, for case B. The solid line is a solution of the rate equation in a hydrodynamically expanding fireball.

by the equation

$$\hat{x}_d = \frac{3}{8}\left(\frac{4\pi}{mT(t)}\right)^{3/2} \rho_B(t)(1-\hat{x}_d)^2 . \tag{46}$$

Then notice that, from Eqs. (39) and (43),

$$\rho_B(t)T(t)^{-3/2} = \left[\frac{3}{2}\frac{N}{E_{\text{tot}}}(1-\hat{x}_d)\right]^{3/2} \rho_B(0) . \tag{47}$$

Thus the instantaneous equilibrium value is independent of the breakup density. This is no accident of the particular expansion model we chose or the uniform temperature approximation. In true hydrodynamic flow the entropy per cell in phase space is constant. If the entropy is constant then $\rho_B(t)T(t)^{-3/2}$ is also constant and, since $\hat{x}_d$ depends only on that quantity, $\hat{x}_d$ is independent of time.

Finally, one might like to know how much the hydrodynamic model differs from the coalescence form of Eq. (5) or, alternatively, the momentum dependence of p_0. Consistent with our uniform temperature approximation we will assign a uniform radial flow velocity β to the system.[15] This velocity is determined by the total hydrodynamic flow energy, Eq. (42). It follows that the momentum space density of particles with momentum p and energy E is

$$\frac{d^3n}{dp^3} = \frac{N}{Z(T)}\exp[-\gamma_\beta(E/T)] \times\left[\left(\gamma_\beta+\frac{T}{E}\right)\frac{\sinh\alpha}{\alpha} - \frac{T}{E}\cosh\alpha\right] , \tag{48}$$

where N is the total number of particles, $Z(T)$ is a normalization factor, $\gamma_\beta = (1-\beta^2)^{1/2}$, and $\alpha = \gamma_\beta \beta p/T$. We choose a breakup density of one half normal density. This choice does not affect x_d but it does determine the fraction of excitation energy which ends up in hydrodynamic flow. The momentum dependence of $p_0{}^3$ can be read off Fig. 4. Notice that there is an order of magnitude in-

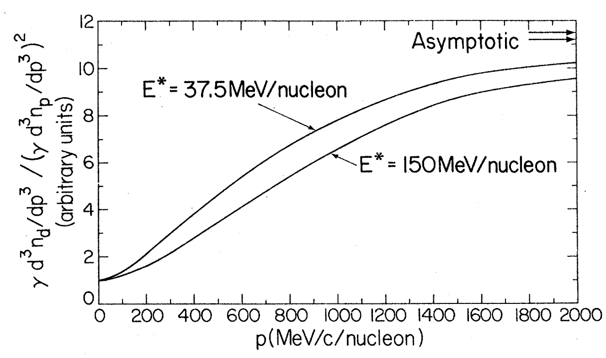

FIG. 4. Momentum dependence of the effective $p_0{}^3$ in a hydrodynamically expanding fireball, cases A and B.

crease in that quantity as p increases from 0 to 2 GeV/c, at least for the examples chosen. This qualitative behavior should be clearly seen in centrally selected collisions of equal mass heavy ions if the hydrodynamic models have any validity.

VII. IMPACT PARAMETER AVERAGING

So far all of the discussion has been on deuteron production for fixed impact parameter. Of course central collisions are the most attractive to study but most of the deuteron data available involves impact parameter averaging. Now we shall investigate the consequences of impact parameter averaging on the various models studied.

Consider the coalescence model result, Eq. (5), with p_0 being independent of momentum and impact parameter. Define the proton-neutron momentum space distribution as

$$\frac{d^6n}{dp_1{}^3dp_2{}^3}(\vec{p}_1, \vec{p}_2, \vec{b}) = D(\vec{p}_1, \vec{p}_2, \vec{b})\,. \tag{49}$$

For simplicity we shall work in the nonrelativistic limit in this section. Generalization to the relativistic domain will be clear. The proton and neutron distributions are

$$\begin{aligned}\frac{d^3n_p}{dp_1{}^3}(\vec{p}_1, \vec{b}) &= D_p(\vec{p}_1, \vec{b}) = \frac{1}{N_n}\int dp_2{}^3 D(\vec{p}_1, \vec{p}_2, \vec{b})\,,\\ \frac{d^3n_n}{dp_2{}^3}(\vec{p}_2, \vec{b}) &= D_n(\vec{p}_2, \vec{b}) = \frac{1}{N_p}\int d^3p_1 D(\vec{p}_1, \vec{p}_2, \vec{b})\,.\end{aligned} \tag{50}$$

N_p and N_n are the total number of protons and neutrons. The coalescence formula, Eq. (5), becomes

$$\frac{d^3n_d}{dp^3}(\vec{p}, \vec{b}) = \pi p_0{}^3 D(\vec{p}, \vec{q}=0, \vec{b})\,. \tag{51}$$

The model actually assumes that

$$\frac{d^6n}{dp_1{}^3dp_2{}^3} = \frac{d^3n_p}{dp_1{}^3}\frac{d^3n_n}{dp_2{}^3}\,, \tag{52}$$

but this assumption is not essential for the present purpose, and we can imagine a more general model where Eq. (52) does not hold. Here we have defined $\vec{p} = (\vec{p}_1 + \vec{p}_2)/2$ and $\vec{q} = \vec{p}_1 - \vec{p}_2$. Integrating Eq. (51) over $\vec{b}$ we obtain

$$\begin{aligned}\frac{d^3\sigma_d}{dp^3} &= \int d^3b\, \frac{d^3n_d}{dp^3}(\vec{p}, \vec{b})\\ &= \pi p_0{}^3 \int d^2b\, D(\vec{p}, \vec{q}=0, \vec{b})\,.\end{aligned} \tag{53}$$

Now define a neutron-proton correlation function C by

$$\begin{aligned}C(\vec{p}, \vec{q}) &= \frac{\sigma \dfrac{d^6\sigma}{dp_1{}^3dp_2{}^3}}{\dfrac{d^3\sigma_p}{dp_1{}^3}\dfrac{d^3\sigma_n}{dp_2{}^3}} - 1\\ &= \frac{\sigma\int d^2b\, D(\vec{p}, \vec{q}, \vec{b})}{[\int d^2b\, D_p(\vec{p}_1, \vec{b})][\int d^2b\, D_n(\vec{p}_2, \vec{b})]} - 1\,.\end{aligned} \tag{54}$$

Here σ is the total reaction cross section. Combining Eqs. (53) and (54) we get

$$\frac{d^3\sigma_d}{dp^3} = \frac{\pi p_0{}^3}{\sigma}\frac{d^3\sigma_p}{dp^3}\frac{d^3\sigma_n}{dp^3}[C(\vec{p}, \vec{q}=0) + 1]\,. \tag{55}$$

Previous analyses of the data[1,10] have been done neglecting impact parameter averaging, i.e., setting $C(\vec{p}, \vec{q}=0) = 0$ in Eq. (55). In general this will not be correct. The presence of the correlation could introduce "anomalous" momentum dependence into the effective p_0. The fact that $\vec{q} = 0$ in the correlation function is an artifact of the approximations in the model. In practice $C(\vec{p}, \vec{q})$ should be smeared over some finite range of $\vec{q}$, say $|\vec{q}| < 100$ or 200 MeV/c.

Impact parameter averaging in a hydrodynamic model such as described in Sec. VI is probably not worthwhile since hydrodynamic effects would be greatly smeared out. However, we can ask what happens in the thermal, sudden approximation and perturbation-type models of the previous sections. Those models all have the general form

$$\frac{d^3n_d}{dp^3} = k\frac{1}{V(\vec{b})}D(\vec{p}, \vec{q}=0, \vec{b})\,, \tag{56}$$

where k is independent of momentum and impact parameter. The k in the perturbation model may have a very weak dependence on $\vec{b}$ because the interaction time will. The volume $V(\vec{b})$ in general will depend on $\vec{b}$. The deuteron cross section is

$$\frac{d^3\sigma_d}{dp^3} = k\int d^2b\, \frac{1}{V(\vec{b})}D(\vec{p}, \vec{q}=0, \vec{b})\,. \tag{57}$$

We cannot pull the same trick with the correlation function C because of the volume. To make progress we do some shuffling. Let $N(\vec{b})$ be the number of participant nucleons in the volume $V(\vec{b})$. This is essentially the multiplicity. Then Eq. (57) can be written

$$\frac{d^3\sigma_d}{dp^3} = k\rho\int d^2b\, \frac{1}{N(\vec{b})}D(\vec{p}, \vec{q}=0, \vec{b})\,. \tag{58}$$

Here ρ is the breakup density of the system. Define a new correlation function C' by

$$C'(\vec{p},\vec{q}) = \frac{\sigma_N \int d^2b \, \frac{1}{N(\vec{b})} D(\vec{p},\vec{q},\vec{b})}{[\int d^2b \, D_p(\vec{p}_1,\vec{b})][\int d^2b \, D_n(\vec{p}_2,\vec{b})]} - 1 \,, \tag{59}$$

where

$$\sigma_N = \int d^2b \, N(\vec{b}) = \langle N \rangle \sigma \tag{60}$$

is the total nucleon single particle cross section. A measurement of C' requires not only proton-neutron detection but also multiplicity determination event by event. In terms of C', Eq. (58) can be written

$$\frac{d^3\sigma_d}{dp^3} = \frac{k\rho}{\sigma_N} \frac{d^3\sigma_p}{dp^3} \frac{d^3\sigma_n}{dp^3} [C'(\vec{p},\vec{q}=0)+1] \,. \tag{61}$$

Again $C'(\vec{p},\vec{q}=0)$ is to be understood as $C'(\vec{p},\vec{q})$ smeared over a range $|\vec{q}| < 100$ or 200 MeV/c. This formula should be compared with Eqs. (5) and (55).

Reviewing the derivation of the influence of impact parameter averaging on the deuteron spectra, we notice that we could also have integrated over any finite interval in b. If an experiment had a trigger such that events with $b < 3$ fm were selected then the integration would only extend up to 3 fm. It is possible that an anomalous momentum dependence introduced by C or C' would help to discriminate among the models for deuteron production. At the very least they are necessary if meaningful values for such parameters as ρ or p_0 are to be determined from the data since the correlation functions could change the absolute normalizations. Of course proton-neutron correlations are hard to measure but proton-proton correlations would probably suffice.

VIII. SUMMARY AND CONCLUSION

In this paper we have tried to point out why the production of light nuclei, in particular deuterons, is an interesting aspect of relativistic nuclear collisions. Since it is not likely that there are many preformed deuterons to be knocked out, nor likely that many deuterons will be evaporated from a projectile or target residue, properties more directly related to the nucleus-nucleus collision dynamics will be focused on. The production of deuterons from nucleons should not greatly perturb the evolution of the collision but rather should carry away some information about the collision to the observer.

The coalescence model is a purely statistical model which merely figures the probability for finding a proton and neutron within a sphere of radius p_0 in momentum space. There is no explicit mechanism for turning them into a deuteron. However, it can be argued that statistical (or phase space) considerations are sufficient since it is easy to transfer a few MeV of energy momentum to neighboring nucleons in a relativistic nuclear collision. Unfortunately the precise value of p_0 and its variation with beam, etc., cannot be understood. The coalescence model yields essentially no new useful information about the dynamics.

The static thermal model assumes that thermal equilibrium, both kinetic and chemical, is achieved during the intermediate stage of the collision. Although there is controversy about the actual degree of thermalization, this model, together with some geometrical assumptions, allows one to calculate absolute differential cross sections. Alternatively, fits to the data allow one to extract effective temperatures and volumes for the emitting systems.

The sudden approximation model assumes that during the expansion of an intermediate complex there is a fast transition from a strongly interacting phase to a weakly interacting one. The sudden approximation of quantum mechanics is applied. The probability for deuteron formation involves the overlap of wave functions before and after the transition. The volume of the system at the transition point is the interesting quantity which is to be determined from the data.

The time dependent perturbation model assumes that there is a finite time during which protons and neutrons emerging from the system are allowed to interact to form deuterons. Energy conservation is not violated due to the uncertainty principle. Alternatively, the initial proton and neutron may be thought of as being off the mass shell. The process is definitely not of an equilibrium character since the number of deuterons formed increases with increasing interaction time. Assuming the validity of this model a comparison with experiment should allow a determination of both an interaction volume and an interaction time.

A solution of the rate equation for deuteron abundance in a hydrodynamically expanding fireball suggests that chemical equilibrium can be reached in the more central collisions of heavy nuclei. This conclusion depends to some extent on the approximate validity of kinetic equilibrium. Deuterons should provide a good experimental test of the validity of hydrodynamics in central collisions, since the ratio $(\gamma d^3n_d/dp^3)/(\gamma d^3n_p/dp^3)^2$ is much more strongly momentum dependent than the other models considered.

The effect of impact parameter averaging was considered and it was shown that the usual power law relationship between the deuteron and the nucleon spectra was modified by a correlation function. For the coalescence model, the correlation function $C(\vec{p}, \vec{q})$ between proton and neutron as customarily defined was the correct one. For the other models (except hydrodynamics), the proper correlation function $C'(\vec{p}, \vec{q})$ involved the measurement of the associated multiplicity of high energy particles. In either case only the knowledge of the correlations for low relative momenta is necessary. Comparisons of the various models with the data including correlations (not yet available) should provide important self-consistency tests. Measurement of proton-proton correlations, with perhaps some correction for identical particle and Coulomb effects, would probably suffice.

Although all of the models considered exhibit the power law relationship between deuterons and nucleons, which is just a reflection of the final state phase space, no pair of models have absolutely identical predictions. Hopefully a comprehensive experimental program involving good statistics, correlations, and multiplicity selections for a broad range of projectile, target, and beam energy combinations will be able to pin down the dominant mechanism for deuteron production and hence shed more light on the collision dynamics. It is also possible that in the real world several mechanisms are at work simultaneously.

At the conclusion of this work a paper by Remler and Sathe[16] was brought to my attention. They show how to incorporate deuteron production in the context of intranuclear cascade models. The interested reader is referred to their paper. One aspect of their work deserves comment here, however. It has been suggested[5] that if a third body N is necessary for deuteron formation in a nonequilibrium theory, then the deuteron spectra should be proportional to the cube of the proton spectra, not the square. The work of Remler and Sathe shows that this is not necessarily the case.

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