Elastic and inelastic charge form factors for $^6_3$Li from electron scattering

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A harmonic oscillator hard core plus residual two-body potential has been used to obtain the independent-particle shell-model wave functions for $^6_3$Li. These wave functions have been utilized to calculate the theoretical elastic and inelastic electron scattering charge form factors. These are then compared with the experimental data.

[NUCLEAR REACTIONS Elastic and inelastic charge form factors for $^6_3$Li from electron scattering using harmonic hard core plus residual two-body potential.]

I. INTRODUCTION

There has been great improvement in the accuracy of measurements for high-energy electron scattering experiments; the analysis of these experiments may provide very useful information on the electromagnetic structure of nuclei. Among the lightest nuclei, it is known that the $^6_3$Li nucleus behaves anomalously as regards electron scattering. When applied to $^6_3$Li, the usual independent-particle shell model (IPSM) does not fit the elastic scattering data.[1-3]

It has been suggested[4,5] that the two-body interactions among nucleons cannot be represented by an average single body central potential, but instead, one must somehow take into account a substantial residual two-body interaction. Most workers have invoked a repulsive short-range correlation between nucleons which give rise to Jastrow-type correlation function.[3,6]

We have used a model that assumes that the core nucleons move in a harmonic-oscillator well. The valence nucleons are assumed to move in a harmonic-oscillator well and they interact with each other via a residual two-body potential. We have assumed two cases of this residual interaction potential. The resulting two-body interaction is given by, in Case 1, a harmonic-oscillator hard-core potential,

$$V(r_1,r_2)=\begin{cases}\infty, & 0\le r\le r_c\\ \frac{1}{2}K_1r_1^2+\frac{1}{2}K_2r_2^2, & r>r_c\end{cases} \tag{1}$$

where $\vec{r}=\vec{r}_2-\vec{r}_1$ and r_c is the hard-core radius.

In Case 2, the resulting two-body interaction is given by a harmonic oscillator hard core and a residual two-body potential that we have assumed to be of the Gaussian form

$$V(r_1,r_2)=\begin{cases}\infty, & 0\le r\le r_c\\ \frac{1}{2}K_1r_1^2+\frac{1}{2}K_2r_2^2+V_0e^{-r^2/r_0^2}, & r>r_c\end{cases} \tag{2}$$

where r is the relative coordinate between two nucleons. In each case the interaction potential depends upon r. Since this type of potential has no effect upon the center-of-mass wave functions, we use the harmonic-oscillator wave functions for the center-of-mass wave functions. We have calculated the relative radial wave functions for both cases and therefore the wave function explicitly includes the correlation introduced by the potential. We have expressed the electron scattering nuclear charge form factor in the relative and center-of-mass formalism, then inserted the appropriate nuclear wave functions (expressed in terms of the relative and center-of-mass coordinates) into the respective form factor. The resulting integrals over the relative coordinates and over the center-of-mass coordinates can be evaluated to give the form factors.

II. WAVE FUNCTIONS

We will calculate the ground state and excited state ($E=-2.18$ MeV) wave functions for $^6_3$Li in L-S coupling.

A. Ground state of $_3$Li^6 in L-S coupling

We assume that $^6_3$Li is in the $(1s_{1/2})^4(1p_{3/2})^2$ configuration. The $(1s_{1/2})$ nucleons form an inert core and the $(1p_{3/2})$ nucleons are coupled together such that the observed ground state total quantum numbers are as follows:

$$\vec{T}=0,\ \vec{J}=1,\ \vec{L}=0,\ \vec{S}=1, \tag{3}$$

where $\vec{T}$ is the total isospin given by $\vec{T}=\vec{t}_k+\vec{t}_j$ and

$\vec{J}$ is the total angular momentum given by $\vec{J}=\vec{L}+\vec{S}$.

The valence nucleons (a proton and neutron) are in the $(1p_{3/2})$ orbit and each has individual quantum numbers:

$$\begin{aligned} &n_k=0,\ l_k=1,\ s_k=\tfrac{1}{2},\ t_k=\tfrac{1}{2},\\ &n_j=0,\ l_j=1,\ s_j=\tfrac{1}{2},\ t_j=\tfrac{1}{2}. \end{aligned} \tag{4}$$

The ground state wave function for ${}^6_3$Li in L-S coupling is given by

$$\psi_g=\psi_{\text{core}}\eta_0^0\chi_1^M|n_k l_k n_j l_j;00\rangle, \tag{5}$$

where ψ_{core} is a product of the single nucleon states of the $2(1s_{1/2})$ protons and the $2(1s_{1/2})$ neutrons; η_0^0 is the isospinor for $\vec{T}=0$, $\vec{T}_3=0$; χ_1^M is the spin wave function for $\vec{S}=1$ and $M_s=M$; and $|n_k l_k n_j l_j;00\rangle$ is the radial eigenket corresponding to total orbital angular momentum $\vec{L}=0$, $M_L=0$.

If we assume that the two valence nucleons are in a common harmonic-oscillator potential, we are free to express the radial eigenfunction $|n_k l_k n_j l_j;00\rangle$ either in terms of the individual particle coordinates ($\vec{r}_k$ and $\vec{r}_j$) or in terms of the relative and center-of-mass coordinates ($\vec{r}$ and $\vec{R}$).[9]

For two particles moving in a central potential, we may write the radial eigenkets corresponding to the total orbital angular momentum $\vec{\lambda}=\vec{l}_k+\vec{l}_j$, as

$$|n_k l_k n_j l_j;\lambda\mu\rangle=\sum_{m_k+m_j=\mu}\langle l_k m_k l_j m_j|\lambda\mu\rangle \times\frac{u_{n_k l_k}(r_k)}{r_k}\frac{u_{n_j l_j}(r_j)}{r_j} \times Y_{l_j m_j}(\hat{r}_j)Y_{l_k m_k}(\hat{r}_k), \tag{6}$$

where μ is the magnetic quantum number corresponding to the total orbital quantum number $\vec{\lambda}$, $\langle l_k m_k l_j m_j|\lambda\mu\rangle$ are Clebsch-Gordan coefficients, $Y_{lm}(\hat{r})$ are spherical harmonics $(\hat{r})=(\theta,\phi)$, and $u_{n_k l_k}(r_k)$ are solutions to the appropriate radial equation that is written in terms of the individual nucleon quantum numbers and coordinates.

We may now write the L-S coupled two-nucleon total angular momentum wave function using

$$(\psi^{\lambda S})_J^M=\sum_{\mu+M_S=M}\langle\lambda\mu S M_S|JM\rangle\chi_S^{M_S}|n_k l_k n_j l_j;\lambda\mu\rangle. \tag{7}$$

For the ^{6}Li ground state ($\vec{J}=1$, $\vec{S}=1$, and $\vec{\lambda}=0$), this becomes

$$(\psi_g^{LS})^M=\psi_{\text{core}}\eta_0^0\chi_1^M\sum_{m_k+m_j=0}\langle 1m_k 1m_k|00\rangle \times Y_{1m_k}(\hat{r}_k)Y_{1m_j}(\hat{r}_j) \times[u_{01}(r_k)/r_k][u_{01}(r_j)/r_j], \tag{8}$$

which becomes

$$(\psi_g^{\lambda=0,S=1})_1^M=\psi_{\text{core}}\eta_0^0\chi_1^M\{\tfrac{1}{3}[Y_{11}(\hat{r}_k)Y_{1-1}(\hat{r}_j)+Y_{1-1}(\hat{r}_k)Y_{11}(\hat{r}_j) - Y_{10}(\hat{r}_k)Y_{10}(\hat{r}_j)]\} \times[u_{01}(r_k)/r_k][u_{01}(r_j)/r_j], \tag{9}$$

where u_{nl} are the oscillator wave functions.

Using the Talmi[7]-Moshinsky-Brody[8,9] transformation brackets, the ${}^6_3$Li ground state wave functions in the center of mass and relative representation is given by

$$(\psi_g^{\lambda=0,S=1})_{J=1}^M=\frac{\psi_{\text{core}}}{\sqrt{2}}\eta_0^0\chi_1^M\left[\frac{u_{00}(r)}{r}\frac{u_{10}(R)}{R}-\frac{u_{10}(r)}{r}\frac{u_{00}(R)}{R}\right] \times Y_{00}(\hat{r})Y_{00}(\hat{R}), \tag{10}$$

where $\vec{r}=(\vec{r}_2-\vec{r}_1)$ and $\vec{R}=\frac{1}{2}(\vec{r}_1+\vec{r}_2)$.

B. Excited state in *L-S* coupling

The 2.18 MeV excited state of ${}^6_3$Li is again in the $(1s_{1/2})^4\,(1p_{3/2})^2$ configuration. The 4 $(1s_{1/2})$ nucleons are again coupled to form an inert core ($\psi_{\text{core}}=f_{p_1}f_{p_2}g_{n_1}g_{n_2}$). The two valence nucleons, $(1p_{3/2})$ proton and $(1p_{3/2})$ neutron, have quantum numbers

$$n_k'=0,\ l_k'=1,\ s_k'=\tfrac{1}{2},\ t_k'=\tfrac{1}{2}, \tag{11}$$

$$n_j'=0,\ l_j'=1,\ s_j'=\tfrac{1}{2},\ t_j'=\tfrac{1}{2}. \tag{12}$$

These individual nucleon quantum numbers are coupled to $\vec{J}'=3$, $\vec{L}'=2=\vec{\lambda}'$, $\vec{S}'=1$, $\vec{T}'=0$.

In L-S coupling, the excited state ($\vec{J}=3$) becomes

$$(\psi_e)_3^{M'}=\psi_{\text{core}}\eta_0^0\sum_{\mu'+M_S'=M'}\langle 2\mu' 1M_S'|3M'\rangle \times\chi_1^{M_S'}|n_k' l_k' n_j' l_j';2\mu'\rangle, \tag{13}$$

where μ' is the magnetic quantum number corresponding to $\vec{L}'=2$.

In terms of the individual particle coordinates $\vec{r}_k$ and $\vec{r}_j$ and their quantum numbers $n_k'=n_j'=0$, and $l_k'=l_j'$, we have

$$(\psi_e^{\lambda'=2,S'=1})_{J'=3}^{M'}=\left[\sum_{\mu'+M_S'=M'}\langle 2\mu' 1M_S'|3M'\rangle\chi_1^{M-\mu'}\left(\sum_{m_k'+m_j'=\mu'}\langle 1m_k' 1m_j'|2\mu'\rangle\frac{u_{01}(r_k)}{r_k}\frac{u_{01}(r_j)}{r_j}Y_{1m_k'}(\hat{r}_k)Y_{1m_j'}(\hat{r}_j)\right)\right]. \tag{14}$$

Using the Moshinsky-Talmi transformations the excited state ($E=-2.18$ MeV) L-S coupling wave function in relative and center-of-mass coordinates becomes

$$(\psi_e^{\lambda'=2,S'=1})_{J'=3}^{M'}=\psi_{\text{core}}\eta_0^0\left[\sum_{\mu'+M_S'=M'}\langle 2\mu' 1m_S'|3M'\rangle\frac{1}{\sqrt{2}}\left(\frac{u_{00}(r)}{r}\frac{u_{02}(R)}{R}Y_{00}(\hat{r})Y_{2\mu'}(\hat{R})-\frac{u_{02}(r)}{r}\frac{u_{00}(R)}{R}Y_{2\mu'}(\hat{r})Y_{00}(\hat{R})\right)\right]. \tag{15}$$

For both the ground state and the excited state the actual form of the wave functions will depend upon our choice of the interaction between the valence nucleons because it is this choice that determines the type of radial functions $u(r)$ that must be used in Eqs. (6) and (13).

We will assume that the core nucleons move in a harmonic well of frequency ν_{1s}. The valence nucleons, however, will be treated in more detail.

III. SPECIFICATION OF THE POTENTIALS

We consider two cases:

Case 1. We will assume that the valence nucleons move in a harmonic well of frequency ν_{1p} and that they also interact with each other via a hard-core potential, as given in Eq. (1). (Generally, for elastic scattering a better fit is obtained by choosing $\nu_{1p} \neq \nu_{1s}$.) The Schrödinger equation for two nucleons moving in a common harmonic-oscillator potential with a repulsive hard core has been solved.[10] The center-of-mass motion gives the usual harmonic-oscillator wave functions. The relative motion gives hard-core wave functions $u_{nl}^{\rm HC}(r)$ which are linear combinations of harmonic-oscillator wave functions.

The harmonic-oscillator frequency[11,12] for the relative motion (ν) is determined by equating the expectation values of the kinetic and potential energies. For the center-of-mass frequency[13] ($\nu_{\rm CM}$) we have[7] $\nu_{\rm CM}=4\nu$. We have picked the hard-core radius $r_c=0.4$ fm.

Case 2. We assume that the valence nucleons move in a harmonic well of frequency ν_{1p}, but that these valence nucleons interact with each other via a hard-core potential plus a residual two-body potential specified in Eq. (2). Once again the center-of-mass wave functions are harmonic-oscillator wave functions.

We have used the variational principle to determine the values for the Gaussian parameters V_0 and r_0. Using the perturbed hard-core wave functions we can calculate the correlation energy for the ground and the excited ($E=-2.18$ MeV) states of ^{6}Li using the effective Hamiltonian formalism.[3]

The correlation energies are defined as

$$W_{\rm ground}=\int_0^\infty (\psi_g^*)_1^M V_{12}(r)(\psi_g)_1^M r^2 dr \tag{16}$$

and

$$W_{\rm excited}=\int_0^\infty (\psi_e^*)_3^{M'} V_{12}(r)(\psi_e)_3^{M'} r^2 dr, \tag{17}$$

where $(\psi_e)_3^{M'}$ and $(\psi_g)_1^M$ are the excited and ground state wave functions for $^6_3$Li and $V_{12}(r)=V_0 e^{-r^2/r_0^2}$ is the two-body interaction potential.

The excitation energy ΔW can be estimated from the difference in the correlation energies of the two states.

$$\Delta W = W_{\rm excited} - W_{\rm ground}. \tag{18}$$

For $\nu=0.27$ fm^{-2} and $r_c=0.4$ fm we obtained the best value for the excitation energy when we assumed $V_0=-150$ MeV and $r_0=1.36$ fm. We varied r_0 between 1.3 and 1.4 fm, and V_0 between -100 and -200 MeV and found that the excitation energy lies within 10% to 20% of the experimental value.

We have obtained analytic forms for the relative radial wave functions corresponding to Case 2 using a perturbation method.[14] The perturbed hard-core relative wave functions are of the form

$$u'_{nl}(r)=u_{nl}^{\rm HC}(r)+K_{ln'}\,u_{n'l}^{\rm HC}(r) + K_{ln''}u_{n''l}^{\rm HC}(r)+\cdots, \tag{19}$$

where K_{ln} are coefficients obtained from the perturbation method and the n's are just successive values of n. Including an additional term in the wave function changed the coefficients of the $u_{nl}^{\rm HC}(r)$'s very slightly; the coefficient of the preceding changed by about 5% and of others by a much smaller amount. The additional term had practically no effect on the result.

IV. ELASTIC SCATTERING OF ELECTRONS FROM $^6_3$Li ASSUMING IPSM

It has been shown[15] that for shell-model calculations, the Born approximation is a very simple and accurate way of treating the elastic scattering of electrons from light nuclei. The shell-model elastic form factor $F_{\rm EL}^{\rm SM}(q)$ in the Born approximation may be written as

$$F_{\rm EL}^{\rm SM}(q)=\frac{1}{Z}\sum_{i=1}^{Z}\int\cdots\int(\psi_g^*)_J^M(\vec r_1,\ldots,\vec r_A) \times(\psi_g)_J^M(\vec r_1,\ldots,\vec r_A) \times e^{i\vec q\cdot\vec r_i}d\tau_1\cdots d\tau_Z d\tau_{Z+1}\cdots d\tau_A, \tag{20}$$

where ψ_g is the ground state nuclear wave function given by Eq. (9).

Since we found it convenient to calculate the wave functions for $^6_3$Li in terms of the relative and center-of-mass coordinates, we will now express the elastic charge form factors in terms of the relative and center-of-mass coordinates of the valence nucleons. We assume that the core nucleons are in a harmonic well of frequency ν_{1s}.

The sum over the i protons consists of three terms, two for the core protons and one for the valence proton. For the first two terms we integrate over all nucleons except the core proton from which the scattering occurs, and in the third term integrate over all the core nucleons. The first two terms give equal contributions. Equation (20) becomes

$$F_{\rm EL}^{\rm SM}(q)=\frac{1}{Z}\left[\int |f_1^*(r_1)|^2 e^{i\vec{q}\cdot\vec{r}_1}d\tau_1+\int |f_2^*(r_2)|^2 e^{i\vec{q}\cdot\vec{r}_2}d\tau_2+|\eta_0^0|^2|\chi_1^M|^2\int\int\langle n_il_in_jl_j;00|e^{i\vec{q}\cdot\vec{r}_i}|n_il_in_jl_j;00\rangle\,d\tau_i d\tau_j\right], \tag{21}$$

where f_1 and f_2 are the single particle wave functions for the two core protons ($n=0$, $l=0$ harmonic-oscillator state).

Using the normality of the spinors and isospinors and rearranging Eq. (21) gives

$$F_{\rm EL}^{\rm SM}(q)=\frac{1}{Z}\left[Z_s\int_0^\infty\left|\frac{u_{00}(r_1)}{r_1}\right|^2 r_1^2dr_1\int e^{i\vec{q}\cdot\vec{r}_1}Y_{00}^*(\hat{r}_1)Y_{00}(\hat{r}_1)d\Omega_1+Z_p\int\int\langle n_il_in_jl_j;00|e^{i\vec{q}\cdot\vec{r}_i}|n_il_in_jl_j;00\rangle d\tau_id\tau_j\right]. \tag{22}$$

Carrying out the integration in the first term, we have

$$F_{\rm EL}^{\rm SM}(q)=\frac{1}{Z}Z_se^{-q^2/4\nu_{1s}}+\frac{Z_p}{Z}\int\int\langle n_il_i;n_jl_j;00|e^{i\vec{q}\cdot\vec{r}_i}|n_il_in_jl_j;00\rangle d\tau_id\tau_j\,, \tag{23}$$

where $Z_s=2$ and $Z_p=1$ and $|n_il_in_jl_j;00\rangle$ is given by Eq. (6) and ν_{1s} is the frequency of the harmonic well of the core nucleons.

We now write the second term of the above equation in terms of relative ($\vec{r}$) and center-of-mass ($\vec{R}$) coordinates using the well known transformation given by Eq. (10).

We have shown that the radial functions $|n_il_in_jl_j;00\rangle$ may be expressed in terms of $\vec{r}$ and $\vec{R}$ and corresponding quantum numbers by using the Moshinsky-Talmi transformations.[7-9]

The integral in term 2 of Eq. (23) becomes, upon substitution of Eq. (10),

$$I=\tfrac{1}{2}\int\int\left[\left(\frac{u_{00}(r)}{r}\frac{u_{10}(R)}{R}-\frac{u_{10}(r)}{r}\frac{u_{00}(r)}{R}\right)Y_{00}(\hat{r})Y_{00}(\hat{R})\right]^2 e^{i\vec{q}\cdot\vec{R}}e^{-i\vec{q}\cdot\vec{r}/2}d\tau_rd\tau_R, \tag{24}$$

where the u_{nl}'s represent the appropriate radial solutions corresponding to the assumed potential for the valence nucleons.

Case 1. The valence nucleons move in a hard-core harmonic-oscillator well of frequency ν_{1p}, where $\nu_{1p}\neq\nu_{1s}$.

We insert [into Eq. (23)] harmonic-oscillator radial functions for the center-of-mass motion and the hard-core harmonic-oscillator wave functions $u_{nl}^{\rm HC}(r)$ for the relative motion

$$F_{\rm EL}^{\rm SM}(q)=\frac{Z_s}{Z}e^{-\gamma^2/\nu_{1s}}+\frac{1}{2Z}(T_1^{\rm HC}-T_2^{\rm HC}-T_3^{\rm HC}+T_4^{\rm HC}), \tag{25}$$

where

$$T_1^{\rm HC}=\left[1-\frac{2\gamma^2}{3\nu_{1p}}+\frac{\gamma^4}{6\nu_{1p}^2}\right]e^{-\gamma^2/2\nu_{1p}}\times\int_{r_c}^\infty\frac{[u_{00}^{\rm HC}(r)]^2}{r}j_0(\gamma r)r^2dr, \tag{26}$$

$$T_2^{\rm HC}=\frac{\gamma^2e^{-\gamma^2/2\nu_{1p}}}{\sqrt{6}\nu_{1p}}\int_{r_c}^\infty\frac{u_{10}^{*\rm HC}(r)}{r}\frac{u_{00}^{\rm HC}(r)}{r}j_0(\gamma r)r^2dr, \tag{27}$$

$$T_3^{\rm HC}=\frac{\gamma^2e^{-\gamma^2/2\nu_{1p}}}{\sqrt{6}\nu_{1p}}\int_{r_c}^\infty\frac{u_{00}^{*\rm HC}(r)}{r}\frac{u_{10}^{\rm HC}(r)}{r}j_0(\gamma r)r^2dr, \tag{28}$$

$$T_4^{\rm HC}=e^{-\gamma^2/2\nu_{1p}}\int_{r_c}^\infty\left(\frac{u_{10}^{\rm HC}(r)}{r}\right)^2j_0(\gamma r)r^2dr, \tag{29}$$

and $\gamma=q/2$ and $u_{nl}^{\rm HC}(r)$ are the unperturbed hard-core harmonic-oscillator wave functions that can be calculated in an analytic form.

The correction[16] for the motion of the center of mass of the $^6_3$Li nucleus is made by multiplying by terms of the form

$$e^{\gamma^2/A\nu_1}, \tag{30}$$

and the correction for the finite size of the proton[17] (assuming a Gaussian charge distribution model) by multiplying by

$$F_{\rm Gauss}(q)=\exp(-a_p^2q^2/6), \tag{31}$$

where $a_p^2=(0.653\text{ fm})^2$ is the square of the radius of the proton.

The corrected elastic charge form factor for $^6_3$Li is given as

$$F_{\rm EL}(q)=[\tfrac{2}{3}e^{-\gamma^2/\nu_{1s}}e^{\gamma^2/A\nu_{1s}}+\tfrac{1}{6}(T_1^{\rm HC}-T_2^{\rm HC}-T_3^{\rm HC}+T_4^{\rm HC})e^{\gamma^2/A\nu_{1p}}]e^{-2\gamma^2a_p^2/3}. \tag{32}$$

We have evaluated Eq. (32) using the Univac 1110 computer to evaluate the numerical integrals. The elastic form factor curve obtained (see Fig. 1) using the unperturbed hard-core wave function $[u_{nl}^{\rm HC}(r)]$ with $\nu_{1s}=0.42\text{ fm}^{-2}$, $\nu_{1p}=0.54\text{ fm}^{-2}$, and $r_c=0.4$ fm fits the experimental data for low q. For higher q, this curve has the proper shape. The curve, however, has its diffraction minimum and its maximum at too small a value of q.

Case 2. We again assume that the core nucleons move in a harmonic-oscillator well of frequency

ν_{1s}. The valence nucleons move in a hard-core harmonic-oscillator well of frequency ν_{1p} and interact with each other through a residual two-body interaction which we take to be a Gaussian potential, Eq. (2).

A calculation similar to that for Case 1 gives the elastic charge form factor

$$F_{EL}(q)=[\tfrac{2}{3}e^{-\gamma^2/\nu_{1s}}e^{\gamma^2/A\nu_{1s}}+\tfrac{1}{6}(T_1'-T_2'-T_3'+T_4')\times e^{-\gamma^2/\nu_{1p}}e^{\gamma^2/A\nu_{1p}}]e^{-2\gamma^2a_p^2/3}, \quad (33)$$

where the T_i' may be obtained from Eqs. (28) and (29) by replacing the unperturbed hard-core wave functions $u_{nl}^{HC}(r)$ by the perturbed hard-core wave functions $u_{nl}'(r)$ which include the effect of the residual potential.

We have evaluated Eq. (33) using the perturbed hard-core wave functions $u_{nl}'(r)$ corresponding to the following parameters (values of V_0 and r_0 which gave the best results for the excitation energy):

$$V_0=-150\text{ MeV},\ r_0=1.36\text{ fm},\ r_c=0.4\text{ fm},$$
$$\nu=0.27\text{ fm}^{-2},$$

and we have let

$$\nu_{1s}=0.42\text{ fm}^{-2}\text{ and }\nu_{1p}=2\nu=0.54\text{ fm}^{-2}.$$

The resulting curve $F_{EL}^2(g)$ vs q (see Fig. 2) fits

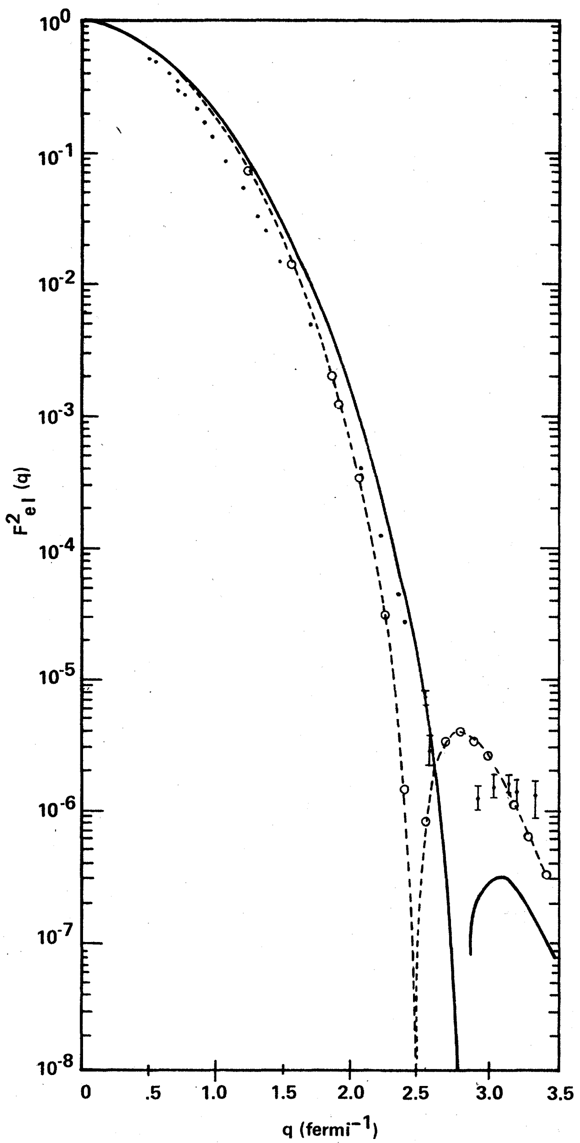

FIG. 1. Elastic form factor squared versus momentum transfer for the unperturbed hard-core wave function with $\nu_{1s}\neq\nu_{1p}$: —— $\nu_{1s}=0.45$, $\nu_{1p}=2\nu=0.54$; --o-- $\nu_{1s}=0.39$ $\nu_{1p}=2\nu=0.54$.

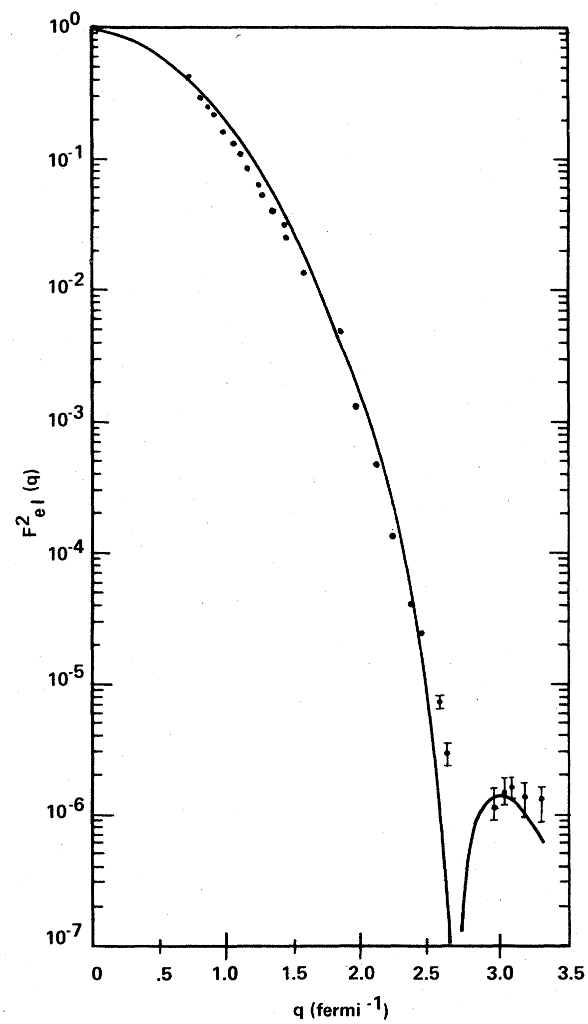

FIG. 2. Elastic form factor squared versus momentum transfer for the perturbed ($V_0=-150$ MeV, $R_0=1.36$ fm, $R_c=0.4$ fm, and $\nu=0.27$ fm^{-2}) hard-core wave functions with $\nu_{1s}\neq\nu_{1p}$; —— $V_0=150$, $\nu_{1s}=0.42$, $\nu_{1p}=2\nu=0.54$.

the data for both small and large q (up to $q=3.2$ fm^{-1}).

It is clear from Fig. 2 that we are able to fit both the low momentum transfer and high momentum transfer data[18] using the perturbed hard-core wave functions and assuming that the valence and core nucleons move in different harmonic wells ($\nu_{1s} \neq \nu_{1p}$). We are able to predict the diffraction minimum ($q \simeq 2.6-2.9$ fm^{-1}) and come very close to fitting the maximum at $q \simeq (3.1-3.2$ fm$^{-1})$.

V. CALCULATION OF THE INELASTIC FORM FACTORS IN THE RELATIVE AND CENTER-OF-MASS REPRESENTATION

Studies on the structure of the $^6_3$Li nucleus have shown that electron scattering data cannot be consistently explained by the simple IPSM. When using the IPSM to explain the $^6_3$Li electron scattering data, one must pick a larger harmonic-oscillator parameter than for the other $1p$-shell nuclei.[2] This large value does not, however, consistently explain other experiments. It has been suggested that for $^6_3$Li, the two-body interactions among the nucleons cannot be represented by an average single body central potential, but instead, one must somehow take into account a substantial residual two-body interaction.

For inelastic scattering, the form factor in the Born approximation is given by[19]

$$|F_{\text{in}}(q)|^2 = \frac{1}{2J+1} \sum_{M} \sum_{M'} |F_{\text{in}}^{MM'}(q)|^2, \qquad (34)$$

where

$$F_{\text{in}}^{MM'}(q) = \frac{1}{Z} \sum_{i=1}^{Z} \int \cdots \int (\psi_e^*)_{J'}^{M'} (\psi_g)_J^M e^{i\vec{q}\cdot\vec{r}} \times d\tau_1 \cdots d\tau_Z d\tau_{Z+1} \cdots d\tau_A \qquad (35)$$

is the contribution due to scattering from the ground state $(\psi_g)_J^M$ having quantum numbers (J, M) to the excited state $(\psi_e)_{J'}^{M'}$ having quantum numbers (J', M'), the sum over i is over the protons, and the factor $1/(2J+1)$ averages over the initial states.

To obtain the inelastic electron scattering form factor, one must insert the ground and excited state wave functions of the desired nucleus. These wave functions must be evaluated for all possible values of M $(-J, -J+1, \ldots, J)$, and M' $(-J', -J'+1, \ldots, J'-1, J')$. The corresponding contributions $F_{\text{in}}^{MM'}(q)$ must then be inserted into (34). Assuming that there is no scattering from the core protons, integrating over the core nucleons and using the orthonormality of the spinors and isospinors gives

$$F_{\text{in}}^{MM'}(q) = \frac{1}{Z} \left[\sum_{\mu'+M_S'=M} \langle 2\mu' 1 M_S' | 3 M' \rangle \delta^{(M'-\mu'),M} \int\int \langle n_k l_k n_j l_j ; 2\mu' | e^{i\vec{q}\cdot\vec{r}_k} | n_k l_k n_j l_j ; 00 \rangle d\tau_k d\tau_j \right]. \qquad (36)$$

We now express the radial eigenkets $(|n_k l_k n_j l_j ; 00\rangle)$ in terms of the relative and center-of-mass representation. Using the Talmi-Moshinsky transformations, and rewriting the exponential $e^{i\vec{q}\cdot\vec{r}_k}$ in terms of relative ($\vec{r}$) and center-of-mass ($\vec{R}$) coordinates, after much algebra, the inelastic form factor contribution $F_{\text{in}}^{MM'}(q)$ becomes

$$F_{\text{in}}^{MM'}(q) = \frac{1}{Z} \sum_{\mu'+M_S'=M} \langle 2\mu' 1 M_S' | 3 M' \rangle \delta^{(M'-\mu'),M} \times \int_0^\infty \int_0^\infty \tfrac{1}{2} \left[\frac{u_{00}^*(r)}{r} \frac{u_{02}^*(R)}{R} Y_{00}^*(\hat{r}) Y_{2\mu'}^*(\hat{R}) - \frac{u_{02}^*(r)}{r} \frac{u_{00}^*(R)}{R} Y_{2\mu'}(\hat{r}) Y_{00}^*(\hat{R}) \right] \times e^{i\vec{q}\cdot(\vec{R}-\vec{r}/2)} \left[\frac{u_{00}(r)}{r} \frac{u_{10}(R)}{R} - \frac{u_{10}(r)}{r} \frac{u_{00}(R)}{R} \right] Y_{00}(\hat{r}) Y_{00}(\hat{R}) d\tau_r d\tau_R , \qquad (37)$$

where $u_{nl}(r)$ and $u_{NL}(R)$ are the relative and center-of-mass radial functions.

Case 1. We insert the relative hard-core wave functions $u_{nl}^{\text{HC}}(r)$ and harmonic-oscillator wave functions for the center-of-mass motion. The individual form factor contributions $[F_{\text{in}}^{MM'}(q)]$ must be calculated and substituted into Eq. (34). Integrating over R we obtain

$$|F_{\text{in}}(q)|^2 = \frac{7}{12Z^2}(-W_1^{\text{HC}} + W_2^{\text{HC}} + W_3^{\text{HC}} - W_4^{\text{HC}})^2, \qquad (38)$$

where

$$W_1^{\text{HC}} = \left(\frac{-4\gamma^2}{3\sqrt{10}\nu_1} + \frac{\gamma^4}{3\sqrt{10}\nu_1^2} \right) e^{-\gamma^2/2\nu_1} \times \int_{r_c}^\infty \left(\frac{u_{00}^{\text{HC}}(r)}{r} \right)^2 j_0(\gamma r) r^2 dr, \qquad (39)$$

$$W_2^{\text{HC}} = \frac{\gamma^2}{\sqrt{15}\nu_1} e^{-\gamma^2/2\nu_1} \int_{r_c}^\infty \frac{u_{00}^{*\text{HC}}(r)}{r} \frac{u_{10}^{\text{HC}}(r)}{r} j_0(\gamma r) r^2 dr, \qquad (40)$$

$$W_3^{\mathrm{HC}} = \frac{\gamma^2}{\sqrt{6}\nu_1} e^{-\gamma^2/2\nu_1} \int_{r_c}^{\infty} \frac{u_{02}^{*\mathrm{HC}}(r)}{r} \frac{u_{00}^{\mathrm{HC}}(r)}{r} j_2(\gamma r) r^2 dr, \tag{41}$$

$$W_4^{\mathrm{HC}} = e^{-\gamma^2/2\nu_1} \int_{r_c}^{\infty} \frac{u_{02}^{*\mathrm{HC}}(r)}{r} \frac{u_{10}^{\mathrm{HC}}(r)}{r} j_2(\gamma r) r^2 dr, \tag{42}$$

where $\nu_1=\nu_{1s}=\nu_{1p}$ and ν_1 is the individual nucleon harmonic-oscillator frequency.

Correcting for the motion of the center of mass of the ${}^6_3$Li nucleus and for the finite proton sizes gives

$$|F_{1n}(q)|^2 = \frac{7}{12Z^2}(-W_1^{\mathrm{HC}} + W_2^{\mathrm{HC}} + W_3^{\mathrm{HC}} - W_4^{\mathrm{HC}}) \times e^{\gamma^2/A\nu_1} e^{-2\gamma^2 a_p^2/3}. \tag{43}$$

The integrals contained in Eqs. (39)–(42) were evaluated numerically on the ASU Univac 1110 computer. These results are shown in Fig. 3, $|F_{1n}(q)|^2$ vs q, for a hard-core radius $r_c = 0.4$ fm and for a harmonic-oscillator frequency $\nu_1 = 2\nu = 0.54$ fm^{-2}. When compared with the inelastic form factor calculated assuming just the harmonic oscillator wave functions, the inelastic form factor assuming a hard-core interaction between the valence nucleons shows a slightly better fit for larger q. However, for small q, the peak is still short of the experimental points.

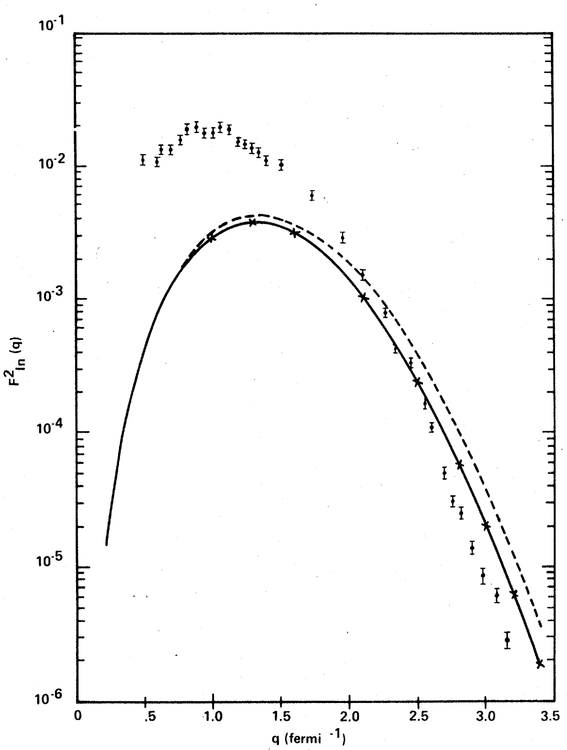

FIG. 3. Inelastic form factor squared versus momentum transfer for harmonic-oscillator wave functions ($\nu_1 = 0.54$ fm^{-2}) and unperturbed ($\nu_1 = 0.54$ fm^{-2} and $R_c = 0.4$ fm) hard-core wave functions: ------ harmonic oscillator $\nu_1 = 0.54$; —×— unperturbed hard core $\nu_1 = 0.54$, $R_c = 0.4$.

Case 2. The valence nucleons move in a hard-core harmonic oscillator well of frequency ν_1 plus a residual two-body interaction assumed to be a Gaussian.

Since the correlation between nucleons does not affect the center-of-mass motion, we may insert the harmonic-oscillator wave functions for the center-of-mass motion, and the perturbed hard-core wave functions $u'_{nl}(r)$ for the relative motion into Eq. (37). The corrected inelastic form factor may be written as

$$[F_{\mathrm{in}}(q)]^2 = \frac{7}{12Z^2}[-W_1' + W_2' + W_3' - W_4']^2 \times e^{\gamma^2/A\nu_1} e^{-2\gamma^2 a_p^2/3}, \tag{44}$$

where the W_i are obtained from Eq. (23) by replacing u_{Nl}^{HC} by u'_{Nl}.

We have evaluated Eq. (44) as a function of q using the ASU Univac 1110 computer, and graphed the results in Fig. 4 for the $V_0 = -150$ MeV, $R_0 = 1.36$ fm, $r_c = 0.4$ fm, and $\nu_1 = 2\nu = 0.54$ fm^{-2}.

The curve for the perturbed hard-core wave functions has a shape which is consistent with the experimental data.[17] For q between 2.0 and 2.5 fm^{-1} the graph, assuming the perturbed hard-core wave functions, fits the experimental data very well. For even larger values of q (2.5 to 3.4 fm^{-1}) the graph has a shape consistent with the experimental results but is slightly above them. Comparing the curve obtained using the perturbed hard-core wave functions and those using just harmonic-oscillator wave functions, we conclude that for small q, the perturbed wave functions have not changed either the location of the maximum or the actual magnitude of this maximum. For larger q, the perturbed hard-core wave functions seem to lower the form factor curve slightly making them nearer the experimental data. Comparison of Fig. 3 and Fig. 4 reveals that there is little change when the hard-core harmonic oscillator wave functions are perturbed by the Gaussian residual potential. The effects of correlation are primarily due to the hard core.

For small values of the momentum transfer q, the corrections due to the short-range correlations are of little, if any, help. As q becomes larger, these corrections become increasingly important and in fact have lowered the theoretical curve to be in fairly good agreement with the experimental data. These results were expected since electron scattering is the result of the long-range electromagnetic interaction. Perhaps to improve the results for low momentum exchange,

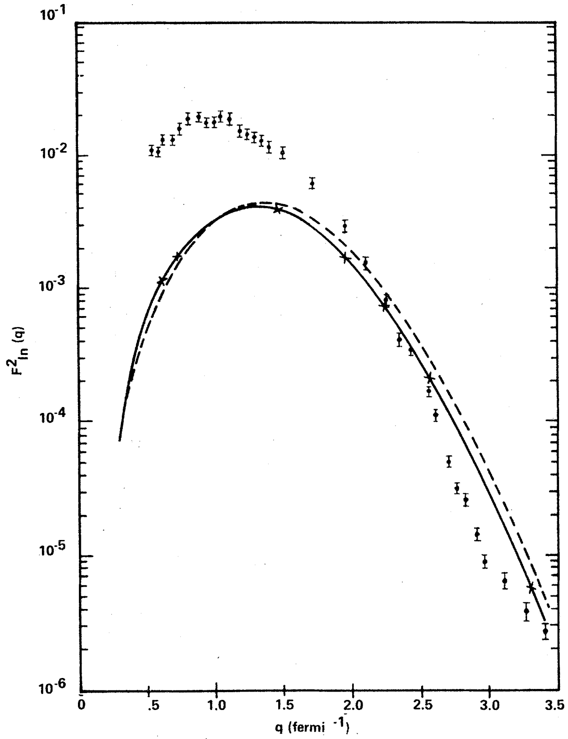

FIG. 4. Inelastic form factor squared versus momentum transfer for harmonic-oscillator wave functions and perturbed ($V_0 = -150$ MeV, $R_0 = 1.36$ fm, $R_c = 0.4$ fm, and $\nu_1 = 2\nu = 0.54$ fm^{-2}) hard-core wave functions: ------- harmonic oscillator $\nu_1 = 0.54$; —×— perturbed $V_0 = -150$, $R_0 = 1.36$, $R_c = 0.4$, $\nu_1 = 2\nu = 0.54$.

the residual potential should contain some type of long-range interaction.

VI. CONCLUSION

We have used the Born approximation to predict the elastic and inelastic electron scattering form factors for ^{6}Li. The result (Case 2, Fig. 2) for the elastic scattering form factors is in good agreement for both low and high momentum transfer. The diffraction minimum is predicted near $q \simeq 2.8$ fm^{-1} and the maximum has the proper magnitude.

The graph of our predicted inelastic form factor (Case 2, Fig. 4) shows an improvement over that calculated using the IPSM and Case 1, for high momentum transfer. There was, however, no improvement for small momentum exchange. This was expected because of the long-range characteristic of the electromagnetic interaction which has not been included here.

Although the elastic and inelastic form factor curves were improved using the perturbed hard-core wave functions there still remains some discrepancies with the experimental data. Our results, however, indicate that it is necessary to include some type of long-range correlation between the nucleons. It appears that the hard-core potential provides the dominant effect in the correlations.

It may be emphasized that we have not chosen the correlation in the relative wave function arbitrarily and adjusted the parameters in it as was done in the work of other authors.[6,3] We have solved for the correlated relative wave function once the two-body potential has been fixed and then predicted the nuclear form factors. In this way, an arbitrariness which existed in the earlier calculations of the form factors has been avoided.

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