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Application of the isobar-doorway model to pion charge exchange reactions

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The pion-nucleus optical potential in the isobar-doorway model is extended to isospin nonzero nuclei. Many-body effects in isobar propagation are parametrized in terms of an energy shift and width and an isobar nonlocality. Isospin dependence of these parameters is found to play an important role in determining pion charge exchange cross sections.

[NUCLEAR REACTIONS Pion charge exchange, isobar-doorway optical potential model, isospin-dependent modification of isobar propagator.]

The isobar-doorway model provides a convenient framework in which to parametrize the pion-nucleus optical potential. This was done in Ref. 1 where elastic scattering from isospin zero nuclei was discussed. In this paper we extend the isobar-doorway optical potential to nonzero isospin and calculate cross sections for single and double pion charge exchange in the resonance region. In an earlier paper[2] we showed that the (π^+, π^0) total cross section depends very sensitively on the choice of energy for the elementary pion-nucleon t-matrix in a factorized first-order optical potential description. Here we discuss this effect from the point of view of isobar propagation.

Using isospin invariance the single and double charge exchange amplitudes to isobaric analog states can be written as

$$\mathcal{T}^{0+} = \frac{1}{(2T+1)(T+1)\sqrt{T}}\,[2T\mathcal{T}_{T+1} + (T-1)(2T+1)\mathcal{T}_T - (2T-1)(T+1)\mathcal{T}_{T-1}] \quad (1)$$

and

$$\mathcal{T}^{-+} = \frac{\sqrt{2T-1}}{(2T+1)(T+1)\sqrt{T}}\,[T\mathcal{T}_{T+1} - 2T\mathcal{T}_T + (T+1)\mathcal{T}_{T-1}], \quad (2)$$

where $T = |N-Z|/2$ is the isospin of the target and $\mathcal{T}_I$ is the elastic scattering amplitude in the pion-nucleus isospin channel I. For isospin $\frac{1}{2}$ targets (e.g., ^{7}Li, ^{13}C, and ^{15}N) the charge exchange amplitude is

$$\mathcal{T}^{0+} = \tfrac{1}{3}\sqrt{2}\,(\mathcal{T}_{3/2} - \mathcal{T}_{1/2}), \quad (3)$$

while the π^+ and π^- elastic scattering amplitudes are

$$\mathcal{T}^{++} = \tfrac{1}{3}(\mathcal{T}_{3/2} + 2\mathcal{T}_{1/2}) \quad (4a)$$

and

$$\mathcal{T}^{--} = \mathcal{T}_{3/2}. \quad (4b)$$

For isospin 1 targets (e.g., ^{14}C and ^{18}O) the charge exchange amplitudes are

$$\mathcal{T}^{0+} = \tfrac{1}{3}(\mathcal{T}_2 - \mathcal{T}_0) \quad (5a)$$

and

$$\mathcal{T}^{-+} = \tfrac{1}{3}[(\mathcal{T}_2 - \mathcal{T}_1) - 2(\mathcal{T}_1 - \mathcal{T}_0)], \quad (5b)$$

while the noncharge exchange amplitudes are

$$\mathcal{T}^{++} = \tfrac{1}{6}(\mathcal{T}_2 + 3\mathcal{T}_1 + 2\mathcal{T}_0) \quad (6a)$$

and

$$\mathcal{T}^{--} = \mathcal{T}_2. \quad (6b)$$

The amplitudes $\mathcal{T}_I$ are calculated by solving the Lipmann-Schwinger equation using an optical potential given by the isobar-doorway model,

$$\langle \vec{k}'|V_I|\vec{k}\rangle = \langle \vec{k}'|V_I^{\rm NR}|\vec{k}\rangle + \frac{E - M_\Delta + i\Gamma_\Delta/2}{E - M_\Delta - \Delta E_I + \beta_I\Gamma_\Delta/2}\, t_{\pi N}(\vec{k}', \vec{k}) F_I(\vec{k}, \vec{k}'), \quad (7)$$

where

$$F_I(\vec{k}, \vec{k}') = \sum_N \int \psi_N(\vec{r}_1)\rho_\Delta(\vec{r}_1, \vec{r}_2; \lambda_I)\psi_N(\vec{r}_2) \times e^{i\vec{k}'\cdot\vec{r}_1} e^{-i\vec{k}\cdot\vec{r}_2} d^3r_1 d^3r_2, \quad (8a)$$

and

$$\rho_\Delta(\vec{r}_1, \vec{r}_2; \lambda_I) = \frac{e^{-(\vec{r}_1 - \vec{r}_2)^2/\lambda_I^2}}{(\pi\lambda_I^2)^{3/2}} \quad (8b)$$

and $\langle \vec{k}'|V_I^{\rm NR}|\vec{k}\rangle$ is the optical potential due to nonresonant pion-nucleon interaction. The parameters ΔE_I and β_I are the energy shift and relative width of the isobar and differ from the free values due to binding and Pauli effects and coupling to inelastic channels (Q space). The quantity λ_I is a nonlocality parameter associated with propagation of the isobar in the nucleus.

Since the microscopic structure of the doorway states is different for different isospin channels there is no reason to expect ΔE_I, β_I, and λ_I to be the same for all I's. This can be clarified by considering the isobar-doorway states in the 1Δ-$1h$ picture. Taking the example of a pion plus ^{13}C possible 1Δ-$1h$ configurations are shown in Fig. 1. Assuming that the nucleons of the ^{12}C core carry isospin zero (filled $1S_{1/2}$ and $1P_{3/2}$ shells), Fig. 1(b) does not contribute to isospin $\frac{1}{2}$ doorway states whereas the isospin $\frac{3}{2}$ doorway states will contain Figs. 1(b)–1(d). It is obvious that binding effects are different for different isospins. We also expect that other many-body effects, such as Pauli blocking and coupling to Q space, lead to isospin-dependent effects. It should be pointed out that in our case we are parametrizing the cumulative effect of the Δ-$(A-1)$ interactions and coupling to Q space whereas in some microscopic calculations in the isobar-doorway model, the latter are included as a phenomenological complex energy shift to the Δ propagator.[3,4] In the spirit of this work we suggest that these parameters be taken as isospin dependent.

In the present calculation ΔE_I, β_I, and λ_I are phenomenological quantities. For isospin zero targets where there is only one isospin channel

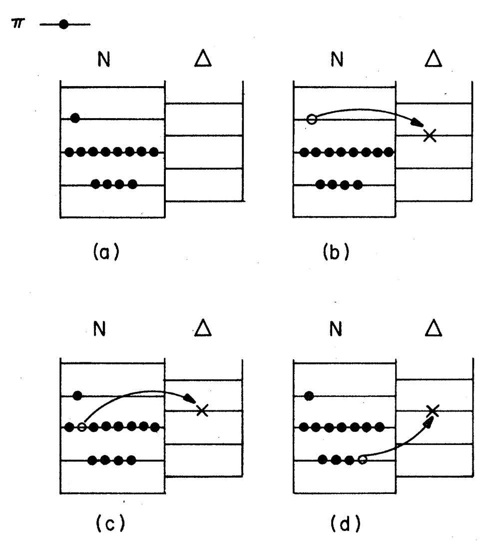

FIG. 1. The $\pi^+-{}^{13}C$ system in the isobar-doorway model: (a) pion outside the nucleus; (b) $1\Delta-1h$ states with ^{12}C core remaining in its ground state ($I=0$). These would contribute only to the doorway states with isospin $\frac{3}{2}$; (c) and (d) $1\Delta-1h$ states with ^{12}C core in excited state ($I=0,1,\ldots$). These would contribute to both $I=\frac{1}{2}$ and $\frac{3}{2}$ doorway states.

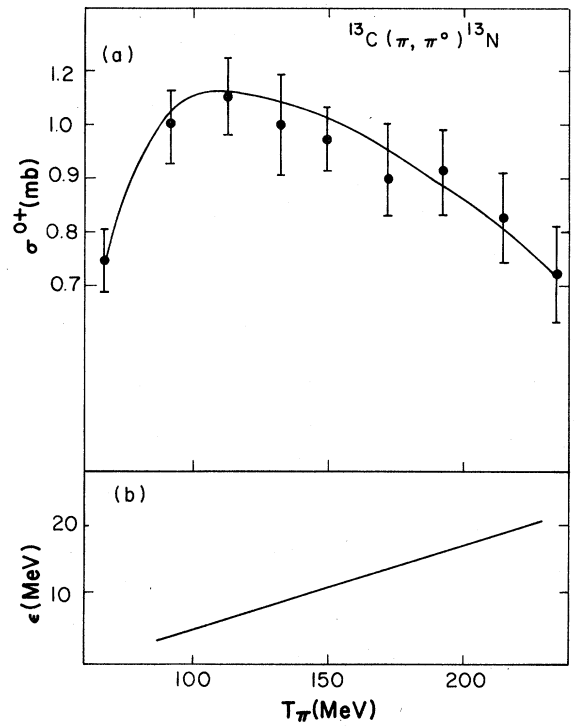

FIG. 2. The value of ϵ needed to fit π $^{13}C(\pi^+,\pi^0)^{13}N$(g.s.) total cross section. The experimental data are taken from Ref. 5.

the parameters were determined by fitting elastic scattering data.[1] For isospin nonzero a high precision fit to a large amount of elastic and total cross section data would be necessary to extract the isospin dependence of the doorway parameters. Since the data necessary to do this are not yet at hand we adopt a more heuristic approach. The parameters λ_I and β_I are taken to be isospin independent and equal to the values λ and β determined in Ref. 1 for isospin zero targets. For isospin $\frac{1}{2}$ targets the energy shifts are taken to be $\Delta E_{1/2} = \Delta E + \epsilon/2$ and $\Delta E_{3/2} = \Delta E - \epsilon/2$ where ΔE is taken from Ref. 1 and ϵ is a new parameter obtained by fitting the $^{13}C(\pi^+, \pi^0)^{13}N$ (g.s.) total cross section.[5]

Two points need clarification here. First, the above parametrization does not correspond to changing channel energies in a distorted-wave impulse approximation (DWIA) calculation of charge exchange as we do not change the energy at which Green's functions are evaluated. The result of changing channel energies to mock up Coulomb effects thus mimicking a difference of energy shifts was found to be small.[6] In fact, in actual DWIA calculations the effect of Coulomb interaction and $\pi^+ - \pi^0$ mass difference has been found to be less than 10%.[7] Second, the procedure used here is *ad hoc* in the sense that we

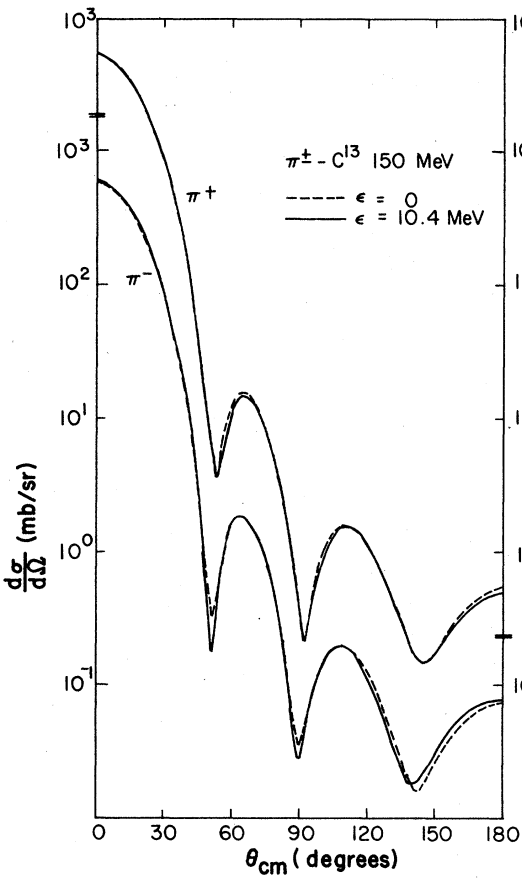

FIG. 3. π^+ and π^- elastic scattering on ^{13}C; (i) solid curve—with $\epsilon = 10.4$ MeV as determined by fitting charge exchange cross section; (ii) dash curve—$\epsilon = 0$.

are mocking up isospin differences in widths and nonlocality through the energy shifts. Indeed if the nonlocalities are taken to be slightly different in different isospin channels the value of ϵ needed to fit the ^{13}C data is reduced significantly. On the other hand the effect of changing the parameters in such a way that $\mathcal{T}_{3/2}$ and $\mathcal{T}_{1/2}$ change in the same direction will not affect the charge exchange cross section significantly. Thus $\beta = \beta_{3/2} = \beta_{1/2}$ could be changed by a small amount without changing σ^{0+} significantly. This implies that the usual explanation of the disagreement between DWIA calculations and the measured (π^+, π^0) total cross sections as due to too much absorption of the pion waves[8] may not be correct. Instead we suggest that this disagreement comes about because isospin-dependent many-body effects associated with isobar propagation have been neglected both in the distorting potential and the transition operator.

In Fig. 2 we show our fit to the $^{13}C(\pi^+, \pi^0)^{13}N$ total cross section along with the value of ϵ that

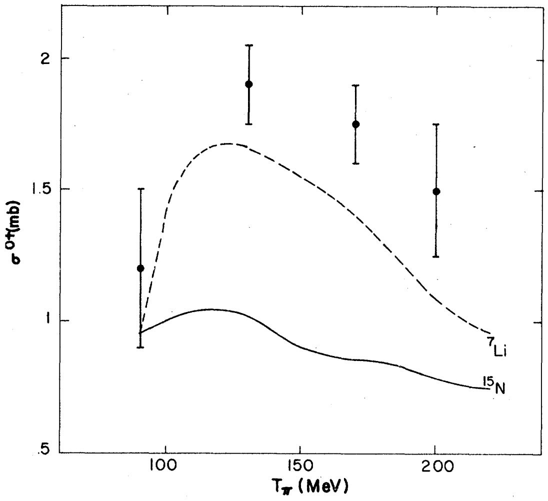

FIG. 4. Total charge exchange cross section to isobaric analog state for ^7Li and ^{15}N. The parameters of the doorway model are taken same as those for ^{13}C. The experimental data for ^7Li are taken from Ref. 5.

was used; the other parameters $\Delta E, \beta, \lambda$ were taken from a fit to ^{12}C elastic scattering. Figure 3 shows π^+ and π^- elastic scattering cross sections for ^{13}C at 150 MeV with these parameters (solid curve) and the corresponding ones for $\epsilon = 0$. As we can see the difference between the two is very small. This is to be expected, as for elastic scattering only coherent sums of the isospin amplitude occur [Eqs. (4a), (4b), (6a), and (6b)].

Integrated single charge exchange cross sections for other isospin $\frac{1}{2}$ targets (^7Li and ^{15}N) are

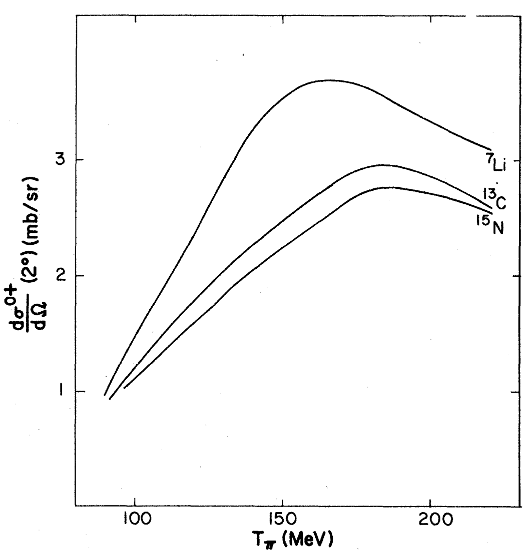

FIG. 5. Forward angle charge exchange cross section for isospin $\frac{1}{2}$ targets.

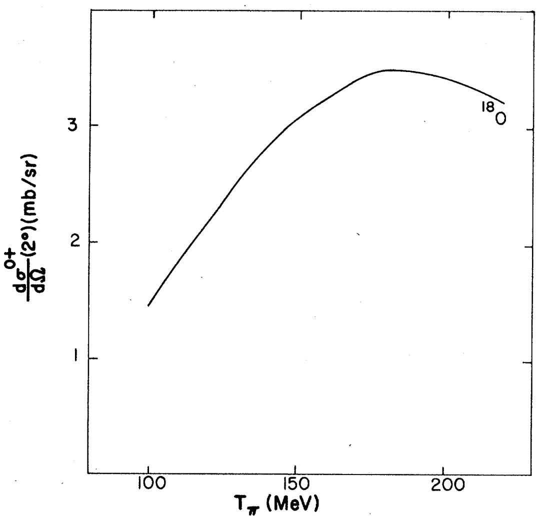

FIG. 6. Forward angle charge exchange cross section on ^{18}O. The parameters are again taken same as for $^{13}C(\pi^+,\pi^0)^{13}N$.

given in Fig. 4. The same isobar parameters (including ϵ) were used as in the ^{13}C calculation. The prediction for $^7Li(\pi^+,\pi^0)^7Be$ is in reasonable agreement with data. Figure 5 gives the energy dependence of the forward angle cross section. Although the magnitude of the calculated forward angle cross sections seems to be larger than the recently reported experimental ones,[9] the energy dependence is reproduced quite well. As pointed out by Bowman,[9] the differential cross section integrated up to 45° accounts only for about 50% total cross section of Ref. 8. Unless there is a very large contribution to the total cross section from large angles, which does not seem to be the case from our calculations, it will be very difficult to fit both the data simultaneously.

For an isospin 1 target there are three isospin invariant amplitudes and in principle we could have two independent energy differences. Since we have no independent information on the appropriate isobar-doorway energies we consider the following choices

$$\Delta E_2+\epsilon/2=\Delta E_1=\Delta E_0-\epsilon/2=\Delta E \tag{9a}$$

and

$$\Delta E_2+\epsilon/2=\Delta E_1+\epsilon/2=\Delta E_0-\epsilon/2=\Delta E\,, \tag{9b}$$

with ΔE taken from π-^{16}O scattering and ϵ taken from the fit to $^{13}C(\pi^+,\pi^0)^{13}N$. Note that the relative shift of channels where valence nucleon excitation contributes to those where it does not is the same as for the isospin $\frac{1}{2}$ case. The double charge exchange amplitude (5b) being a difference of differences is extremely sensitive to small variations of ΔE_I. At 164 MeV the difference between the $\epsilon=0$ cross section and choice (9b) for the energy shifts is about two orders of magnitude. On the other hand the shape of the calculated angular distribution changes very little with a minimum at an angle of 35° to 40° rather than 20° as observed by Seth *et al.*[10] Note that both (9a) and (9b) lead to identical single charge exchange as $\mathcal{T}^{0+}$ does not include $\mathcal{T}_1$. Our prediction for the $^{18}O(\pi^+,\pi^0)^{18}F$(g.s.) excitation function is shown in Fig. 6. This reaction provides a crucial test for the extension of our model to isospin 1 targets.

To summarize, we extend the phenomenological isobar-doorway optical potential to pion scattering by isospin nonzero nuclei. Charge exchange cross sections are very sensitive to isospin dependence of isobar-doorway parameters. By making a small difference in the isobar energy shift for different isospin channels the total cross section for $^{13}C(\pi^+,\pi^0)^{13}N$ can be easily fit. Using the same energy shifts the prediction for $^7Li(\pi^+,\pi^0)^9Be$ total cross section is in good agreement with data. The trend of our calculated excitation functions is also in agreement with recent measurements. We also calculate the excitation function for $^{18}O(\pi^+,\pi^0)^{18}F$ which would be a useful test of our model. Unfortunately, double charge exchange is much too sensitive to isospin differences in the parameters to be able to say anything conclusive. However, the shape of our calculated angular distribution is not in agreement with experiment. This may be due to the simple form of the nonlocality chosen in Ref. 1. It may turn out that the form chosen there is appropriate only for the processes in which the inelastic propagation of pions is not very significant. This point is being investigated further and will be reported in the future. In conclusion, we propose that the isospin dependence of the many-body effects that modify isobar propagation in nuclei plays an important role in determining pion charge exchange cross sections.

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